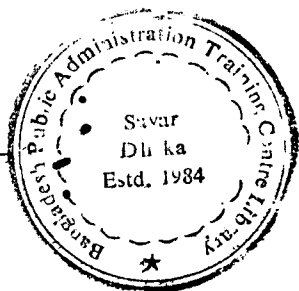


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A

MANUAL OF SURVEYING.

CHAPTER I.

INTRODUCTION.

1. "Surveying" is the art of ascertaining by measurements and of representing the form of the earth's surface and the relative positions of objects upon it, as projected on a horizontal surface. By the surface of the earth is meant not only its visible portion, but also the bottoms of lakes, rivers, the interior of mines, etc.

2. "Levelling" denotes the process of finding out by means of a levelling instrument, consisting of a telescope carrying a parallel and sensitive spirit-level, adjustable by means of screws, the differences in level between different points on the surface of the earth. In a comprehensive sense the term surveying also includes levelling.

3. **Plan and Section.**—The results of surveying plotted and drawn upon paper constitute a plan or a ground plan; those of levelling are drawn in the form of a Vertical Section, called more briefly a Section.

A plan is a miniature representation of the ground and the objects upon it, as projected on a horizontal surface; that surface being represented by the surface of the paper on which the plan is drawn. A plan differs from a map chiefly in the scale on which it is drawn, the scale of a plan being large enough to serve for the designing of engineering works, while that of a map is so small as to make it serviceable for the purposes of travelling and geography only.

4. **The portions of the earth's surface** represented by plans for engineering or other purposes are often so small compared with the whole earth that a horizontal surface, though its figure is very nearly that of a spheroid, may in most cases, be treated as if it were plane, without any error of practical importance.

5. **Measures of Length.**—The standard length, established by law in the United Kingdom of Great Britain and Ireland is the yard, being the distance, at the temperature of 62° Fahrenheit, and under the mean atmospheric pressure, between two marks on a certain bar which is kept in the office of the Exchequer, at Westminster. In India, the standard length adopted by the Public Works Department is the foot = $\frac{1}{3}$ yard.

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In addition to the yard, memorise the following units :

The inch = $\frac{1}{12}$ foot = $\frac{1}{36}$ yard = 25·399978 millimètres.
 „ foot = $\frac{1}{3}$ „ = 12 inches = 30·479 centimètres.
 „ fathom = 2 yards = 6 feet = 72 inches.
 „ chain (Gunter's) = 22 „ = 66 „ = 4 poles of $5\frac{1}{2}$ yds. each.
 = 20·116 mètres.
 „ statute mile = 1760 „ = 5280 „ = 80 G. chains
 = 8 furlongs = 1·6093426 kilomètres.

5 miles = 8 kilomètres.

The nautical or sea mile is the length of one minute of a degree of latitude at the mean level of the sea. The length of this mile varies in different latitudes, from about 6,107' at the poles to 6,045' at the equator, its mean value is nearly 6,075·6' = 1 knot = 1·1508 statute mile = 10 cables = 1000 fathoms, each of which is about $\frac{1}{80}$ th longer than the common fathom.

The French “mètre”, which is nearly $\frac{1}{10,000,000}$ th part of the distance from one of the earth's poles to the equator, is equal to 3·2808428' = 39·370113'' or $3'3\frac{3}{8}''$ nearly = 1·09361426 yds.

1 Décimètre = $\frac{1}{10}$ mètre = 10 centimètres = 100 millimètres.

1 Centimètre = $\frac{1}{100}$ mètre = 0·39370113 inch. [= 3·937 ins.]

1 Millimètre = $\frac{1}{1000}$ mètre = 0·039370113 inch.

1 Dekamètre = 10 mètres.

1 Hectomètre = 100 mètres = 0·0621 mile.

1 Kilomètre = 1000 mètres = 10 hectomètres = 3·28084278'.

1 Myriamètre = 10,000 mètres, = 6·213717 miles (English).

6. The Measures of Area are :—

English.

The square inch or sq. in. = 645·1589 sq. millimètres.

„ „ foot of 144 sq. ins. = 9·29 sq. decimètres.

The square yard of 9 sq. ft. = 1296 sq. ins. = 0·836126 sq. metres.

„ „ pole of $30\frac{1}{4}$ sq. yds. = $\frac{1}{16}$ rood of 1210 sq. yds.

„ acre of 10 sq. chains or 100,000 sq. links = 4840 sq. yds.

= 43560 sq. ft. = 4 roods = 160 sq. poles of $30\frac{1}{4}$ sq. yds..

= 0·405 hectare = about 40 ares:

The sq. mile = 640 acres = 3,097,600 sq. yds. = 2·599 sq. kilomètres.

Metric Square Measure.

1 Are = 100 sq. metres. } 1 Deciare = $\frac{1}{10}$ are.

1 Decare = 10 ares. }

1 Centiare = $\frac{1}{100}$ are = 1·196 sq. yd.

1 Hectare = 100 ares = 2·471 acres.

7. Bengal Linear Measure :—

2 spans or bighats = 18 inches = 1 hath or cubit;

2 cubits = 1 yard.

4 cubits = 1 Nat;
 120 ft. or 80 cubits = 1 bigha (linear) or rasi.
 5280' = 44 rasis = 1 English mile.

8. Bengal Square Measure :--

1 sq. cubit = 1 Ganda.
 20 gandas = 1 Chhatak.
 16 chhataks = 1 Cottah.
 20 cottahs = 1 Bigha or Rasi = 1600 sq. yds. = 14,400 sq. ft. = 330 $\frac{5}{8}$ acre.
 1 Acre = $\frac{121}{40}$ Bigha = 3.025 bighas = 3 bighas 8 chhataks.

9. United Provinces of Agra and Oudh Land Measure :—

20 Kanchawansis = 1 Biswansi.
 20 Biswansis = 1 Biswa.
 20 Biswas = 1 Bigha = 3025 sq. yds. = $\frac{5}{8}$ acre.

10. Bombay Land Measure.

39 $\frac{1}{4}$ sq. cubits = 1 Kathi.
 20 Kathis = 1 Pand.
 20 Pands = 1 Bigha = 3927 sq. yds.
 6 Bighas = 1 Rukeh.
 20 Rukehs = 1 Chahur.

11. The Punjab Land Measure.

9 Sarsai = 1 Marla.
 20 Marlas = 1 Kanal.
 4 Kanals = 1 Bigha = 1620 sq. yds.

12. Madras Land Measure.

144 sq. in. = 1 sq. foot.
 2400 sq. ft. = 1 Ground or manai.

13. Assam Land Measure.

12 Feet = 1 Loocher.
 20 Loochers = 1 Cottah.
 5 Cottahs = 1 Hillisah.
 4 Hillisahs = 1 Poorah.

24 Grounds = 1 Cawn = 6400 sq. yds.

484 Cawnies = 1 sq. Mile.

1 Madras } = 4 Bengal
 Cawnies } Bighas.

14. Orissa Land Measure.

4 Kanis = 1 Bishwa = 1 Padika square.
 16 Bishwas = 1 Goont = 4 Padikas square.
 25 Goonts = 1 Man.
 20 Mans = 1 Batti.

1 Jab = $\frac{1}{4}$ ''; 1 moosti = 3''; 1 Kani = 1'5 $\frac{1}{2}$ ''

The length of 24 dasti padikas = 10'5 $\frac{1}{2}$ ''.

1 Bishwa = 1 padika $\times$ 1 padika = (10'5 $\frac{1}{2}$ '')² = 108 sq. ft. 126 sq. in.

1 Goont = 16 $\times$ 108 sq. ft. 126 sq. in. = 1742 $\frac{2}{3}$ sq. ft.

1 Man = 25 $\times$ 1742 $\frac{2}{3}$ = 43560 sq. ft. = 4840 sq. yds. = 1 acre.

NOTE.—The word *kani* is the length from the elbow to the tip of the little finger. The word *Dasti* is derived from the Persian word *Dast* meaning the palm of a hand which is equal to $\frac{1}{4}$ cubit.

15. Burma Measures of Length.

8 Thits = 1 Maik or Palm.	20 Tas = 1 Uthab.
12 „ = 1 Htwa or Hand.	20 Uthabas = 1 Kawtha.
2 Htwas = 1 Daung (cubit).	4 Kawthas = 1 Gawut,
3 Maiks = 1 „ = 19.05 ins.	4 Gawats = 1 Yozana = $13\frac{1}{2}$
4 Daungs = 1 Lan (Fathom).	Eng. Mile.
7 „ = 1 Ta (Measuring Rod).	1000 Tas = 1 Taing.
= 11 1125 Ft.	6 Taings + 400 Tas =
1000 Tas = 1 Daing = 11, 1125 Ft.	1 Yozana.

16. Chittagong Land Measure used in Arakan.

8 Cubits = 1 Ta.	4 Kauris = 1 Ganda.
32 sq. cubits = 1 Kranta.	5 Gandas = 1 Kuni.
2 Krantas = 1 sq. Ta.	4 Kunis = 1 Kani.
3 „ = 1 Kauri.	16 Kanis = 1 Dong.

17. Arakan Land Measure.

15 sq. Tas = 1 Ywe.
8 Ywes = 1 Bè.
2 Bès = 1 Mu.
2 Mus = 1 Mat.
4 Mats = 1 Kyat or Dong.

18. Burma Square Measure.

1 Min Pè = 25 ² Tas = 1.772 acres.
1 Pègadi = 17.678 ² Tas = 0.8859 acres.
1 Acre = 1.1288 Pègadis = 18.78 ² Tas.
1 Gan = 10 ² Tas = 2835 acres.

19. The Practice of Surveying is divided into two branches, viz., (1) *Plane Surveying*, and *Geodetic Surveying* or *Geodesy*. In plane surveying the surface of the earth is treated as a plane. This is practically true up to areas of about 100 square miles. Geodesy, on the other hand, takes into consideration the curvature of the earth's surface. The methods of plane surveying are applicable, therefore, to areas of 100 square miles or less. They are sometimes used in connection with the surveys of larger areas, but only when a high degree of accuracy is not required. This book will treat of plane surveying only.

The Subject of Surveying naturally divides itself into—

- (a) The Adjustment, Use, and Care of Instruments.
- (b) The Methods of Field-work.
- (c) The Records or Field-notes, Computations and Final Products.

20. Practical geometry is the art of accurately delineating on a plane surface any plane figure. It is the most simple species of geometrical drawing, and the most useful. It has been defined as the art which directs the mechanical processes for finding the position of points, lines, surfaces and planes, with the description of such figures

on diagrams as can be intelligibly understood by definition, according to given dimensions and position of lines, points, &c.

It is founded upon the properties and relations of certain magnitudes, which are demonstrated in Euclid's Elements of Geometry, and other works to which the surveyor is referred, if in need; for the demonstration of the problems.

21. General Rules on Practical Geometry.—(1) Facility in the use of the drawing instruments can only be acquired by practice. The student will do well in beginning to ink in, to draw a series of lines, first fine, then gradually thicker, bearing in mind that a good line must be of the same thickness throughout.

(2) Avoid drawing unnecessary lines, to obviate subsequent erasure.

(3) Draw all the lines from left to right, and avoid pressing the pen against the edge of the set square or other instrument, as it produces an uneven line.

(4) Draw all the lines in pencil before beginning to ink in. For common use, a H H pencil is the best; for finer work use a H H H. The pencil should be cut evenly to a fine point, if not, bad lines and inaccuracy will be the result.

(5) In geometrical drawings, dotted lines are employed as construction lines, and to show those parts of an object which are not seen in the view presented. An attention to this matter adds to the neatness and general appearance of the drawing.

(6) Before drawing a line, try the pen upon a piece of waste paper. Practice will show the necessity for this.

(7) In using dividers, bowpen or bowpencil, press as lightly as possible to avoid puncturing the paper.

(8) When an ordinary drawing is to be made, it will be sufficient to secure the paper to the drawing board by pins; but when a finished drawing is to be executed, the paper should be secured by gluing its edges to the board, first damping by means of a sponge, the whole surface of the paper.

(9) All lines perpendicular or parallel to another should be drawn long enough at once to avoid the necessity of producing them.

(10) Arcs of circles or straight lines, by which an important point is to be found, should never intersect each other very obliquely, or at an angle of less than 15° or 20° . If this cannot be avoided, some other mode should be adopted to define the point precisely.

(11) When one arc of a circle is described, and a point in it is to be determined by the intersection of another arc, this latter need not be drawn at all, but only the point marked off on the first, as it is always desirable to avoid the drawing of unnecessary lines. The same observation applies to a point to be determined on one straight line by the intersection of another.

(12) Whenever the dividers can be used in any part of a construction, or to construct the whole problem, they are to be preferred to the rule, unless the process is much more circuitous, or unless the rule 10 above forbids.

(13) A right line should never be obtained by the prolongation of a very

short one, unless some point on that prolongation is first found by some other means, especially in any essential part of a problem.

(14) The larger the scale on which any problem or any part of one is constructed, the less liable is the result to error : hence all angles should be set off, and points determined by means of the largest circles which circumstances will admit of being described.

(15) Whenever it is practicable, lines should be drawn from a given point and not to it ; and if the line is to pass through two points, before drawing it, the pencil should be moved along the rule, so as to ascertain whether the line will, when drawn, pass through them both.

22. The Rules Applicable to Scales :—(1) A simple or plain scale is constructed by setting off a straight line about 6", not too long, yet of sufficient length to obtain conveniently any distance on the plan, and dividing it into major and minor divisions numbered in a useful manner. For instance, if a scale of 13 yds. to an inch is to be constructed to read yard, it will be convenient to take a line representing 70 yds., or a line $\frac{1}{13} \times 70 = 5.39''$ and to divide it into 7 equal major divisions, each representing 10 yds, and the first major division into 10 minor divisions, each representing a yard. It will not be convenient to take 1" for a major division of 13 yards and to divide it into 13 equal minor divisions, for (though such a scale can be constructed) it cannot be easily read, as some calculations will be necessary, if an odd distance is to be measured off the scale ; while in the former, the major divisions being 10 yds. each, and the minor ones 1 yard each, distance in yards can be quickly taken off.

(2) All scales must be numbered, with units of measure distinctly written on the right of 0 immediately after the last number, as without this the scales will have no meaning. The units of the minor divisions should also be written on the extreme left of 0 after the last number.

(3) The length of the scale should be taken such that it can be divided into such number of major divisions that the unit denoting each major division should be of an even number. This will get rid of the necessity of calculation before reading a scale when any odd number of any kind of units is to be taken off the scale.

(4) The decimal system of graduations is generally used, except when two different units are to be used on the same scale, i. e., yards and feet, feet and inches, miles and furlongs, when the major division representing yards, feet and miles, as the case may be, has to be divided into 3, 12 and 8 equal parts to show feet, inches and furlongs respectively.

(5) There is always a limit to which a scale can be drawn. This depends on the size of the scale and also upon the greatest number of sub-divisions that can be easily and clearly made in a major division with a pencil point or a drawing pen. The lines making the sub-divisions should be so placed that they can be distinguished by the naked eye. An inch in length divided into 80 equal parts is known to be the closest graduation that can be conveniently

read, beyond that the lines will almost overlap each other. A good scale should be convenient to use, and should read to the greatest exactitude required; in short, scales to be of use, should fulfil the following conditions:—(1) Divided with great accuracy and carefully numbered. (2) Long enough to measure the principal lines of the plan. (3) The zero (0) must always be between the units and its sub-divisions. (4) The name of the scale should be written on it and the R. F. shown. (5) Numbered, decimally in the major divisions except when two different units are to be used.

(6) The scale should be uniform and complete; for example 2" to 1 mile, and not 2½" to 1 mile.

(7) The scales should always be drawn in pencil first, then inked in on the plan, because the paper is liable to alter its dimensions with the changes of the atmosphere, but such changes will affect both the scale and the drawing. Paper expands when gets moist, and contracts when heated.

(8) For vernier scales, ($n-1$) parts of the primary scale are divided into n parts for movable verniers; for, the vernier and primary scale graduations can then be numbered in the same direction. The observer can easily find out, in this case, the reading on the primary scale, and follow up in the same direction the vernier graduations to find out the point of coincidence of the primary and vernier scales.

(9) In the case of the attached verniers ($n+1$) parts of the primary scale should be taken and divided into n parts, for the vernier and the primary scale graduations would then run together.

23. Scales in General Use in this Country.—The following are the scales usually adopted in this country for special purposes referred to:—

1" = 25 miles—The Great Trigonometrical Survey Maps are drawn on this scale

1" = 16 miles—Maps of Provinces are drawn on this scale.

1" = 8 miles } —For Geographical Maps of districts on small
1" = 4 " } scales.

1" = 1 mile or 80 chains to 1" }
2" = 1 " " 40 " " " } For extensive surveys of country
showing general topographical
features. Maps of Districts
are drawn on these scales
showing village boundaries.

4" = 1 " " 20 " " " }
8" = 1 " " 10 " " " } For surveys of towns, canton-
ments, large estates, and for
village surveys.

16" = 1 " " 5 " " " }
32" = 1 " " 2½ " " " } For Cadastral or Field-by-field
Survey of cultivation in vil-
lages, &c., where pathways,
tanks, roads, and houses are
distinctly shown.

64" = 1 " " 1¼ " " " } For Cadastral survey of Towns,

$50' = 1''$ }
 $30' = 1''$ } — For Site Plans of Buildings to be constructed.

$100' = 1''$ }
 $200' = 1''$ }
 $300' = 1''$ } — For preparing Plans for Engineering Projects.
 $400' = 1''$ }

$330' = 1''$ — For „ „ „ Land Acquisition purposes,
 and also for Engineering Projects.

24. Scales in Use in the United Kingdom.—The following are the scales of the Ordnance Survey of the United Kingdom, so called as the Surveys were carried out at their commencement under the Master General of the Ordnance :— $25'344''$ to 1 mile or $\frac{1}{25000}$; $6''$ to 1 mile or $\frac{1}{100000}$; $1''$ to 1 mile or $\frac{1}{63360}$; and 4 miles = $1''$. Ordnance sheets of large towns are also available on a scale of $10'$ to 1 mile or $\frac{1}{360}$, and $5'$ to 1 mile or $\frac{1}{720}$. For *working* plans of railways, &c., $25''$ Ordnance scale is most used, while for the *Parliamentary* plans $6''$ scale is used.

25. In the Continent of Europe, the scale is expressed by the ratio of the linear dimensions of the plotted survey to the actual linear dimensions of the ground surveyed, as $\frac{1}{5000}$, $\frac{1}{10000}$, &c.

CHAPTER II.

SURVEYING INSTRUMENTS AND THEIR USES.

26. The surveying instruments are intended for measuring horizontal, vertical or angular distances. It is intended here to describe those which are in use in modern practice for chain, compass, plane-table and theodolite surveys.

27. Names of Surveying Instruments used are :—

Linear Measuring Instruments.—(1) Pegs; (2) Flags; (3) Chains and arrows. (4) Tapes; (5) Offset rods.

Instruments for Determining Directions.—(6) Cross staffs; (7) Optical squares; (8) Prismatic compasses; (9) Surveying compasses; (10) Compasses attached to the levels; (11) Plane tables, alidades and rectangular compasses, etc. (12) Theodolites; (13) Sextants; (14) Circumferentors; (15) Line Rangers; (16) Mining Dials, &c.

28. The peg or stake is simply a pointed rod stuck into the ground to mark a station point.

Before commencing the pegging out in survey operations, it will be well to lay in a store of pegs and mallets. These latter are large wooden hammers for driving pegs into the ground. Each pegging

party should be provided with a small handy crowbar, a *da* (bill-hook), and a small hand-saw. Wooden pegs are used when permanent demarcation is not required.

The wood pegs should be 2" square, and made of *sāl* or *teak*; they should be made in various lengths of from 9" to 2' so as to have ready at hand pegs suitable for any kind of hard or soft soil. These pegs should be driven into within 3" of the surface, and their tops out of the ground should be painted red so that they can be found out easily.

In surveying through public thoroughfares, pegs should be driven flush with the ground; or if more permanence is desired stone blocks are imbedded below surface level, and two direction marks are erected on the road side and recorded, by which the station can at any time be recovered. When greater permanence is desired for a particular station mark, a block of stone may be used, having a cross marked on its top, the intersection showing the station mark. When a peg driven in at a station is likely to be disturbed, (1) the distance of the station from any two or more well-defined permanent objects such as corners of buildings, notches made at the foot of trees, may be measured and noted down; (2) or 3 more pegs may be driven, and the distances from them of the station measured and noted down; or (3) 4 pegs may be driven so that the intersection of the lines joining them diagonally may point out the station.

During the Revenue and Dearah surveys in Bengal, masonry pillars were built at all important points of tri-junction. Usually cylindrical coneheaded burnt mud-pillars as well as stone prisms are fixed in village boundaries during settlement operations. During the last Calcutta Survey, marks were cut on the kerbstones at the corners or junctions of streets.

29. Pickets, Ranging Poles or Rods, Flags, Signals or Jhandis.—These are generally made of thin bamboos or poles, say of 1" to 1½" in diameter, 10' to 15' long, with Δ or pieces of red or white cloth, called *Flags* attached to their tops. They are pointed at their lower ends (Fig. 1. *a*) for driving them into the ground, and used for marking station points. Ordinary iron-shod wooden poles are 10' long, painted in alternate lengths of black and white, or red and white every alternate foot or, better, half red and half white every alternate foot. This latter arrangement presents lines running down the pole by means of which its verticality may be better estimated by the observer. For the same reason poles having octagonal cross section are used. Iron range poles are smaller than wooden ones both in cross section and length. Flags are attached to these poles. For flags, white is the color that is seen farthest, but red is more generally used as it is easily distinguished from surrounding objects. At the ends of long chain lines, large flags, with poles 20' or 30' long, are used; these generally require rope stays to keep them upright. Picket or ranging rod, for use on hard ground or paved streets, may be fixed to a tripod stand with hinged legs to fold up. Sometimes "*whites*," which are bits

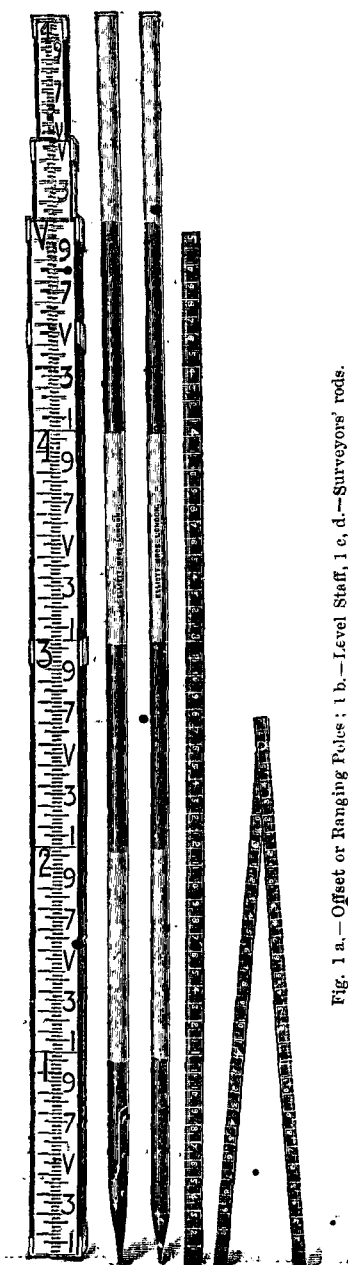


Fig. 1 a. — Offset or Ranging Pole; 1 b. — Level Staff, 1 c, d. — Surveyors' rods.

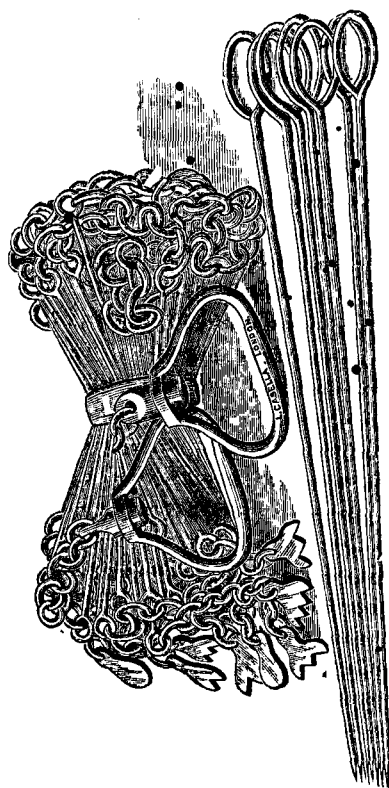


Fig. 2. — Chain and Pins.

of paper held in cleft sticks, are used for temporarily marking points. Flags are sometimes tied to branches of trees to show distant stations. Flags are called *bannerols* in England. Flagpoles are used to mark station points to which lines are measured and stations on which an instrument is turned. They are used for roughly prolonging lines and 3 of them are required to range out a straight line, but 6 to 12 are required to mark stations. In wet weather woollen cloth* should be used for flags as cotton clings to poles when wet.

30. The Chain and Pins.—The measuring chain (Fig. 2.) is a linear measure used for measuring distances required in a survey. Engineers have almost abandoned the Gunter's chain of 66', divided into 100 links of 7.92" in length each, and use one of 100' with links 1' long; and calculate areas in sq. ft. which divided by 43560 reduces them into acres and decimal parts, instead of roods and perches. For surveys in the hills where lot of "stepping" has to be done, and for measuring offsets off a chain line, a chain 50' long may be used with advantage, for a 100' chain becomes unwieldy to use. The 100' chain is used in all Engineering surveys and levels, because the measurements are required in feet by the P. W. D.

In this country besides the two above described, a 30' or 5-cottah chain, divided into 40 links of 9" each, is used. As 40 links equal 5 cottahs, so 8 links equal 1 cottah. It is now almost superseded by the Gunter's chain which is called *eleven-cottah chain*. One advantage in using a Gunter's chain is that acreage can be found out very easily, for (1 Gunter's chain)² = (66')² = 4356 sq. ft. = $\frac{1}{10}$ of an acre, and 80 Gunter's chains = 1 mile = 5280'. It is used in the Revenue Survey Department where extent of land and acreage are required. When a "chain" is spoken of without qualification, Gunter's chain is meant. In the continent of Europe steel tapes are preferred to chains, and are used for measuring horizontal distances. In railways, where miles and furlongs have to be shown, it is used also, for 10 chains = 1 furlong.

31. The 100'-chain possesses many advantages over the 66' feet one especially in the convenience it offers in the calculation of estimates of quantities of work to be done and gradients to be given to roads. It being the longest of the two is more conducive to accuracy.

There is another kind of chain, called the *decameter chain* in use in the European continent, divided into 100 links, each = 0.32809'. Therefore a decametre chain equals 32.809' = 10 metres. A 20 metre chain = 65.618 feet is also used.

32. Construction of Chains.—Land chains (100', Gunter's, and decametre) are made of iron, or the best cast steel wire, No 12 W. G. about $\frac{7}{16}$ " in diameter, hardened, tempered and japanned with three oval rings, properly welded between links, brass swivel handles at the ends, A, by which they can be dragged along the ground and tallies. S is the swivel. Oval shaped rings are better than round ones, and there should be three rings between two links. The joints of the rings and those of the eyes of the links should be welded; the chain is thus rendered much less liable to stretch than if those joints are open. Each chain is tallied at every 10 links with brass tallies, pointed, single notched, double notched, or treble-notched according as the brass tally lies at the 10th or 90th, 20th or 80th, 30th or 70th, and 40th or 60th links (Fig. 3) to enable surveyors to readily count the number of links, and in order that the chain may be used and

read beginning from either end. At the centre of the chain, *i. e.*, at the 50th link, the tally is a perfectly plain round piece of brass.

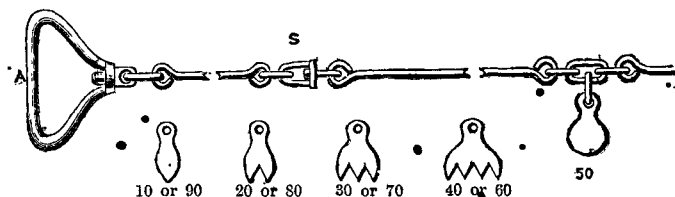


Fig. 3.—Chain Handle and Tallies.

33. Marking Pins.—To every chain, 100' or 66', are attached 10 pins, also called **skewers** or **arrows**, made of the best iron or steel wire, 15" long, pointed at one end, and having an eye or ring at the other made by bending the blunt end.

They are used for marking chain lengths and should be kept strung on a wire ring. They should be made of No. 6 wire.

34. Spécial Arrows for Hard Ground.—To bear hammering into hard ground, the pins may be 11" or 12" long, $\frac{1}{8}$ " thick, $\frac{5}{8}$ " wide, head $2\frac{1}{4}$ " wide, with a hole of $1\frac{1}{8}$ " diameter as in (Fig. 4).

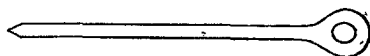


Fig. 4.—Arrow for Hard Ground.

Each arrow should have a bright-red flannel or cloth tied to its top to make it visible from a distance, also that it may be found out among

grass by the follower. Each 30' chain is provided with 4 arrows.

35. Chesterman's Patent Steel-Band Measuring Chain. with adjustable handle being practically unalterable, is recommended for use instead of the ordinary land chains. As compared with a chain it has greater correctness, is easier to clean and to coil or uncoil, and is much lighter and more compact. It is more reliable than the ordinary woven tapes, and will stand the wear of a number of tapes. It is stronger and more durable than the steel tape. It is divided into feet and numbered at every 10', available in sizes of 100', 50', 66', and 33' in length.

36. Steel-Band Chains.—In modern practice a continuous steel-band of nearly oval section is substituted for the above built-up chain. The steel-band should also be coiled on to a steel-cross (Fig. 5) or wound up in a metal case with handle (Fig. 6) when not in use. The

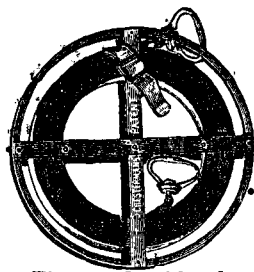


Fig. 5.—Steel-band on Cross.



Fig. 6.—Steel-band in Case.

band is made in four different widths— $3\frac{3}{8}$ ", $2\frac{1}{2}$ ", $\frac{5}{8}$ ", $3\frac{1}{4}$ " by about $\frac{1}{8}$ "

thick. They are made in links, feet, or metres, and of any length corresponding to land chains.

37. The Compound Steel-Band Chains are usually employed in two to five separate lengths, made attachable by swivels and hooks. The first length of each set is divided in the usual manner into links or feet and numbered at every ten, the others not being subdivided. Each set is wound upon a steel-cross, or a metal reel with handle can be had in place of the cross. The band is made in three different widths— $\frac{1}{4}$ " $\frac{1}{8}$ " $\frac{1}{16}$ ". This form of band chain will be found very useful in chaining large tracts of country when portability and lightness are of paramount importance.

38. Metric and English Steel-Band Chains, $\frac{5}{8}$ " wide, etched feet on one side and decimetres on the other, in lengths, of 33' and 10 metres, 50' and 15 metres, 66' and 20 metres, or 100' and 30 metres, are wound upon metal cased reels (Fig. 5). The metre chains are divided into fifths of a metre and tallied at every two metres. They are also etched decimetres or millimeters on both sides. Similar steel, metallic, or linen tapes are also used.

Steel-band chains are much lighter than built-up chains of equal strength, and are made of standard length. They are also lighter to use, being smooth and without any projection. On the other hand the reading is less distinct than with the chain, and they need more careful usage in chaining. They also require oiling before being put by. From the thinness of the metal they are altogether more delicate and less durable than the chains for hard wear, but they are of true length while they last in repair.

The steel-band does not accommodate itself to the unevenness of the ground, when laid down like the ordinary heavy built-up land chains, does not offer much resistance to traction, and is of small weight; but the brass swivel handles used for dragging it are liable to extension, and the points of measurement should be marked on the band itself and should not include the handles. To obtain the most accurate measurements the steel-band is suspended horizontally with a given tension, one end being made to coincide with a given mark, and the mark at the other end transferred to a nail driven into a peg or to some other record by means of a plumb-line. When used in this manner, the band must be 66 or 100 feet in length between the marks; and as this is the case at one particular temperature only (generally $52^{\circ} F$), temperature observations should be made simultaneously with each measurement, so that the true length passed over may be corrected by calculation. The rate of expansion of steel is 1 in 927 from 32° to $212^{\circ} F$, and is nearly constant within these limits. The steel-band is practically unalterable in length for ordinary small surveys while the built-up chain is liable to be stretched, or to have its links bent, and therefore to be tested and adjusted occasionally. It is to be used when long lines are to be measured as in chaining out the centre line of a railway, road, or canal.

39. Mining Chains used in mineral districts are made generally 10 fathoms or 60 feet, in 6" links counted off by tellers, in fathoms. Where used with the dial they are made either entirely of brass, or with one fathom at one end in brass and the other portion in steel. In collieries Gunter's chains are generally used.

40. Laying out Chain on Ground and Folding up.—

Before describing the method of chaining it will not be out of place here to describe the *art of doing up and throwing out the chain*. To lay out the chain on the ground, the handles should be held in the left hand with a few links loose, whilst the rest of the folded chain is held in the right; and then the chain should be thrown forward smartly, keeping hold of the two handles in a direction opposite to that in which the line is to be measured, so that the follower may watch it as it is drawn past and straighten out any kinks that may occur.

Then the leader walks away with one handle, dragging the chain forward. When doing or fastening up the chain after use, it should be held at the centre by the round tablet, and by and by, each pair of links towards the end should be folded up until the handles are reached. Fig. 2 shows the chain folded up and then fastened up with a leather strap with the arrows lying by. Arrows should be strung on a ring which has a spring catch to restrain them for carrying in the hand; or they should be carried in a quiver with a strap over the shoulder, which leaves the hands of the leader free to remove obstructions when they occur.

41. The Chain is Liable to Errors.—First, in itself; 2nd, in the method of using it; and, 3rd, in the uncertainty of pitching the arrows. Every possible precaution must therefore be taken in using it. If the chain be pulled too tight, the rings will open between the links, the arrows will incline, and thus will make the chain longer than it really is, i. e., the measured line will be shorter than the true line. Again, if it is not pulled sufficiently tight, the measure obtained will be longer than it really is.

A new chain should be tested daily until it has stretched to its utmost; and an old one, every 3 or 4 days. They have been known to stretch as much as 3" in a day's work. This, though trifling in one chain, is of great consequence in a day's work of say 250 chains, amounting to an error of $62\frac{1}{2}'$.

When a line has been measured with an incorrect chain (and such is usually the case) the true length of the line will be found out from the following proportion:—*Length of correct chain : Length of chain used : The measured distance : True distance.*

For instance, suppose the chain (100') to have stretched 3', and the measured distance is 1060 feet, Then $100 : 100\frac{3}{4} = 1060 : 1062\cdot65'$ (true distance).

Take another example.—Suppose the chain (100') is 1' shorter than it really is, and the measured distance is 2 miles, then the true distance will be $= \frac{10560 \times 1199}{100 \times 12} = 10551'$, i. e., 9' less than two miles.

The maximum error allowed in lineal measurement is 10 links in 100 chains, i. e., equal to 5.28' per mile.

42. Testing Chains.—The length of a chain changes from temperature, wear and various kinds of distortion. A change of temperature of 70° F. in a 100' chain will change its length by 0.05', or a change of 1 in 2000.

When the links are joined by 3 rings, then there are 8 wearing surfaces for each link, or 800 wearing surfaces for a 66- or 100-foot chain. If each surface wears 0.01 inch, the chain is lengthened by 8". It is common to survey about 300 miles of a railway with a single chain, which if of right length at the beginning of the survey, might be lengthened 6" at the end of it.

The chain may be lengthened from flattening out of the connecting rings, or from stretching it beyond its elastic limit, thus giving it a permanent set. It may be shortened from bending the links or by dirt getting into the rings. Both wear and distortion are less for a steel chain than for an iron one.

All unknown changes in the length of the chain produce *cumulative errors* in the measured lines, and therefore it is important that the true length of the chain should always be known, or better that the standard length (50' 100' 66' etc.) should be properly measured from one end of the chain and marked at the other. This chain test is performed readily with a standard steel tape at least as long as the chain. By the aid of such a tape a standard base may be marked out in the locality in which a survey is being done. Plinths of walls and platforms of railway stations, or the floor of a large room are convenient places upon which to mark the standard length. If these are not available at hand, a couple of stout pegs or two stone prisms may be driven into the ground at the proper distance apart, and sawn off flush with it. Upon their tops the standard length may be accurately marked. As a check it will be well to drive a third peg in the middle, and to mark on it the corresponding middle point of the standard length.

In laying out the standard base the temperature at which the unit of measure is the standard should be known (this temperature is marked on the best steel tapes) and if the base is not laid out at this temperature, a correction should be made before the marks are set. The co-efficient of expansion of iron or steel is very nearly 0.000065 for 1° F. If T_0 be the temperature at which the tape is standard, T the temperature at which the base is measured, and L the length of the base, then $0.000065 (T_0 - T) L$ is the correction to be applied to the measured length to give the true length. When the chain is tested by this standard base the temperature should be again noted, and if this is about the mean temperature for the field measurements no correction need be made to the field-work. If it is known, at the time the chain is tested, that the temperature is very different from the probable mean of the field-work, then the standard mark can be so placed on the chain as to make it standard when in use.

The chain may be tested also every day, and the standard steel rods may be used solely for this purpose. To do this the chain is pulled quite taut on a level piece of ground between two stout arrows. The two steel standard rods are placed alongside the chain from one end of it. Then keeping the second rod in its position, remove the first and place it beyond the second. The second rod is next removed and placed beyond the other, and so on until the other end of the chain is reached; when the portion deficient or in excess of the nearest foot may be measured by a foot rule or with a scale of equal parts.

In using or testing a chain the surveyor should see that the rings are free from dirt, and the links perfectly straight so that the chain may play freely along its whole length. To make it equal to the standard length, a few rings may be taken off or added to it. The bent links may be straightened, and rings cleansed of dirt. *The correct length of a chain is from outside of one handle to the outside of the other.*

The ordinary offset rods or levelling staffs (Figs. 1 a, b,) are sufficiently accurate for practical purposes, and may be used to test the chains.

43. Allowance for Elongation.—Even when the chain is tested daily, it is found to extend to some extent during a day's work when new. As the elongation occurs gradually, half the total elongation is taken as the mean elongation for the day, and the distance corrected according to proportion given above. A chain may be tested every day before going to use it as above described or by a standard steel chain used solely for this purpose.

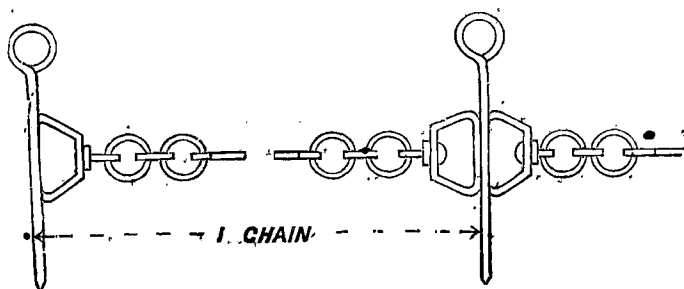
44. Use of the Chain and Arrows on Level Ground.—In using a chain two chain-men are employed, one of whom is called the "leader" who drags the chain forward, and the other "follower." Before a line can be measured, its ends are to be marked by pegs and flags; and if the line is very long, intermediate flags must be posted so that the chained line may not deviate from the line required. This posting of end and intermediate flags in a straight line with the end flags, as far as the eye can judge, is called "ranging the line."

In measuring the line, the follower in a *crouching* attitude places his end of the chain in contact with the starting flag, while the leader, who has with him the 10 arrows, goes ahead with the other end of the chain in the required direction until it is almost stretched. The follower then directs the leader, who crouches down at one side of the line holding near the ground an arrow exactly upright in the same hand which grasps the handle of the chain, in a line with the forward flag, and lifting the chain gives a jerk so as to stretch the chain in a straight line as far as practicable. When the follower sees that the chain is in the right direction after repeated jerks, he gives the word *down* or *stick* to the leader who, putting a firm pressure on the chain answers *stuck*, when he drives an arrow just outside his chain handle. They then, with their respective ends of the chain, proceed to the next chain length. The follower now coming up to the first arrow holds his end of the chain by it. (Fig. 7), and again directs the leader after he has stretched it to its full

length; and the latter then drives the second arrow just outside his chain handle. Then the follower takes up the first arrow, and walks towards the second. The process is repeated until the 10th chain-length is reached. On fixing the 10th arrow, the leader should cry in a loud voice "10" or "change." Then the surveyor records in the field-book that a length of 10 chains has been measured; the leader waits until the follower gives him the 9 previously picked-up arrows, and holds his end of the chain at the mark made by the 10th arrow which the leader then picks up, and proceeds with all the 10 arrows in his hand to commence the measurement of the next 10 chains.

The surveyor should keep constant watch over his chainmen by placing himself just in rear of the chain. He should see that the chain is perfectly in the straight line between the end flags, that it is straight without entanglements, that the arrows are vertically driven into the ground, and that the distances measured are horizontal lengths; and if there be sloping lengths, they should be measured separately, reduced to horizontal lengths, and then recorded. He must also see that the follower brings his end of the chain against the arrow in the ground, and that the leader after putting himself in line and stretching the chain puts *his arrow outside and against the end of the handle*. For ordinary chaining the total length of a chain should be equal to the true length of the chain minus the thickness of a pin. Then if both pins be outside of the handles a true chain length will be obtained from outside of one pin to the other. In hard ground to point the position of an arrow, a cross should be scratched on the ground.

The number of chains measured can be easily ascertained by counting the number of arrows in the follower's hands and the number of changes, each change being 10 chains; also the odd links are read off directly on the chain, the tallies assisting the ready reading of the chain.



Note :—True length of chain only is obtained.

Fig. 7.—Use of Chain and Arrows.

45. Second Method of Counting the Chainage on Uneven Ground.—In using the chain on uneven ground two flags are first

to be set up at the extremities of the line to be measured. The leader drags the chain forward with a ranging rod and 10 pins in hand. The follower carries a clinometer and the back end of the chain; he has also a tape tied to his button hole. A piece of cloth or paper is to be attached to the ranging rod at the height of the observer's eye. The leader drags forward the chain to the follower's direction towards the forward flag and pulls it tightly. The follower shakes it up to remove kinks and curvature in the chain. The leader then holds up the ranging rod, and waits for final direction, which when given, he puts the iron point of the rod through the handle of the chain to give the last stretch, and then puts in a pin and goes ahead. When he has put in his last pin, he runs out the next chain without a pin, aligns it by the rod, and, leaving the iron point through the handle of the chain, signals to the follower to come forward with the 10 pins. The follower first ties a knot in his tape to mark the 10 chains, and then comes on, leaving the chain to be dragged forward by the leader when he has received his pins. The leader then puts in one of the 10 pins at the rod, and proceeds as before. To assist in keeping the line straight as well as to serve for a check upon the number of chains measured, it is a better plan to set up a rod at each 10 chains, when the pins are exchanged.

The French and the Americans use an eleventh arrow which is never counted in chaining. When the eleventh pin has been stuck in the ground the follower delivers his 10 pins to the leader, both make a note of a "tally" and the work goes on. Note that by this method every pin stuck in the ground counts for 100', and therefore the number of pins picked up by the follower will give the length of the line in hundreds of feet.

When the slope exceeds a degree (or if the greatest accuracy is required, at every chain) the follower records the slope, and at the end of the line sums up the corrections and deducts the result from the chainage. He stands erect and directs the clinometer, to the mark on the ranging rod.

46. It would be better, if 9 iron or steel arrows and 10 brass ones are used. Then the leader, having fixed all the 9 arrows in fixing the first 9 chains, marks the end of the 10th chain with a brass arrow; and when the follower comes up to him, takes only the 9 steel arrows, leaving the brass arrows to be picked up by the follower when the next length has been measured. In this case the follower can tell, at any time, the number of entire tens of chains that have been measured by counting the brass arrows with him, and the number of chains over and above the entire tens of chains by counting the steel arrows; thus a very good check is kept on the number of entire tens of chains noted in the field-book. At the end of each 100 chains the leader gets back all the 19 arrows.

The leader should always advance by keeping his eye on the forward flag; and if he does so, he will be able to drag the chain in the true line with very little direction from the follower. The follower should, while proceeding, allow the chain to slacken, and should take care that it may not

entangle in either the arrow or other objects. As the chaining is proceeding, the surveyor should note when the chain line crosses all fences; boundaries, banks of streams, sides of roads and other objects to be drawn upon the plan, and also other station lines, and when suitable points for intermediate stations occur in the survey. The leader also should check the direction of the follower by keeping the latter and the back station flag in a straight line with himself. The follower does this as he directs the leader with the forward station, and thus by a mutual check greater accuracy is obtained.

47. The Surveyor is Liable to the following Errors in Using a Chain, and he should adopt measures to avoid them :—

(1) *In getting out of the chain line.* To avoid this, the surveyor should keep constant watch over his chainmen by placing himself just in rear of the chain, and checking his chainmen's directions.

(2) *Errors due to the follower giving, and the leader chaining too tight; or the leader chaining too loose, and fixing his arrows in a slanting direction.* This may be avoided by keeping watch on the follower, and seeing that he holds the handle of the chain and the pin firmly and vertically when the chain is stretched, and that the leader does not pull taut or slant his pins but fixes them vertically also.

(3) *A knot in the chain or a link bent or kinked.* New chains are most liable to form knots. This can be avoided by examination of the chain when stretched. After a day's work the chain should be properly put up, and unless so done, the links are liable to get strained and bent, to say nothing of the smaller space it occupies and the handy manner in which it can be carried when nicely packed. Any link bent should be straightened.

(4) *An arrow lost by the follower or carelessly omitted in counting up the number in hand.* As the sum of the arrows with the follower and the leader is 10, the surveyor can avoid this error by checking the number of arrows with each from time to time. Where offsets are to be taken an offset at the end of each chain will be a check on chainage.

(5) *Reading the odd links at the end of a chain line wrongly, either by mistaking the tally or being careless in counting from a tally.* Great mistakes are often made in reading the tallies of a chain, when at the 40th and 60th links, as the tallies for these are the same in shape, and the difference in chain-lengths on either side of them can only be determined by careful observation alone. At the other marks 10 and 90, 20 and 80, 30 and 70, the difference in chain-lengths on either side of these marks can be easily determined, for it is not likely that the surveyor will read 90 instead of 10, 80 instead of 20, and 70 instead of 30. The surveyor has only to see on which side of the centre mark, i.e., of the 50th link, he is reading, whether in front or back of it. This should be carefully born in mind. It would be better to have a round tag, instead of a notched tally, at each tenth link with the number of links, 10, 20, 30, 40, 50, 60, 70, 80, and so on, stamped on it. When the chain is dragged through mud, the tallies getting

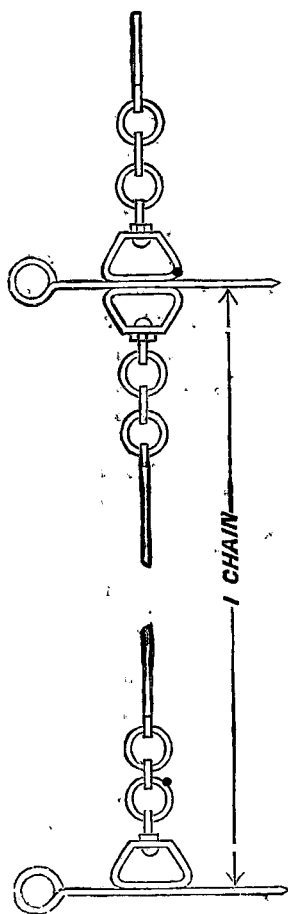
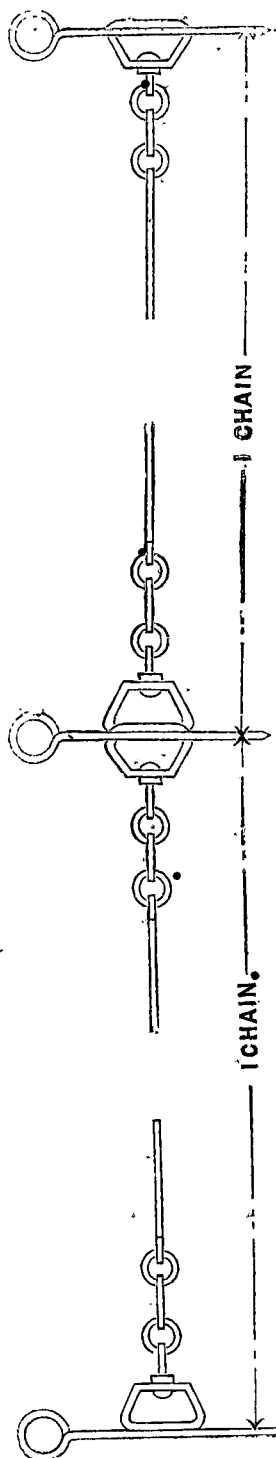


Fig. 8.—Use of Chain and Arrows.

besmeared with it, the surveyor should read the tallies after they are cleaned thoroughly; otherwise he may make mistake in distinguishing the tallies, and is liable to read 10 for 20, 20 for 30, &c.

(6) *In chaining on slopes and level ground indiscriminately when only horizontal distances are required.* Chaining on slopes is to be done by the process, called "stepping," as described under measurement of hills. True horizontal distances of sloping lengths are to be found and added to lengths measured on horizontal surfaces to find out the total horizontal chainage done.



Thickness of handle is lost.

Fig. 9.—Use of Chain and Arrows.

(7) *In pitching the arrows by which the thickness of an arrow is either added to or deducted from the true length of the chain.* True chain length can only be obtained in one of the manners following:—It will be seen from Fig. 7 that if the follower holds the arrow by the side of the chain handle, and the leader puts his arrow at the outside and against the handle, the true length of the chain is obtained if the chain were of full length. In Fig. 8; it is seen that at the end of each chainage the thickness of an arrow is added to the true length of the chain. Therefore to obtain a true chain-length the total length of a chain should be equal to true length minus the thickness of a pin in ordinary chaining if the pins are set in this manner, and in Fig. 9 the thickness of handle is lost. The true length of the chain should therefore be estimated from the outside of one handle to the inside of the other. The outside of the handle should be placed against the front edge of the back pin, and the fore pin set just inside the handle. The surveyor should therefore guide his men and instruct them properly.

(8) *In using an incorrect chain.* To avoid this, the chain should be tested every day with standard rods, chains or steel tapes, and corrected by removing or adding rings.

(9) *In forgetting to count a change of arrows showing 10 chains.* To avoid this, the surveyor must make it a point to record every change of 10 arrows after 10 chain-lengths are measured, or by using a brass arrow at every 10th chain.

(10) *In recording.* To record correctly, the surveyor must be very attentive to what is read aloud by the chainmen or offsetman.

48. The Result of Chain getting out of Line.—It has already been

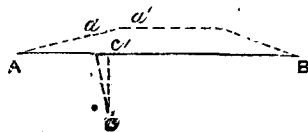


Fig. 10.—Result of Chain Getting out of Line.

pointed out, that the surveyor should keep constant watch over his chainmen, by placing himself just in rear of the chain, in order to avoid getting out of the chain line. If by careless chaining, the chainmen get out of the chain line, the length of the whole line between the flags is increased, and the lengths of the offsets and the points at which they are raised are also thrown out. If AB be the straight line joining A and B (Fig. 10), it is the shortest line between those points, and any other line $Aa'B'$ not straight will be longer than AB . If c be a point to be offsetted to, it will be found that Ac' is not $= Aa$, nor is the length of $cc' = ac$. When the field-work is plotted, unless the measured chain lines are correct, and offsets carefully and correctly taken, the plan will be a misrepresentation of the ground.

49. **Metallic Tapes.**—Measuring tapes (Figs. 11, 12) are made either of linen, or of linen interwoven with fine brass or copper wires, and called *metallic tapes*. They come in lengths of 50' and 100' with folding or flush handle. When really good, either of them may be trusted to half an inch. In using a tape in damp weather, or at any time when it gets wet, it should not be rolled up until it is quite dry,

It will spoil the tape, if it is wound up, when wet or dirty, in the

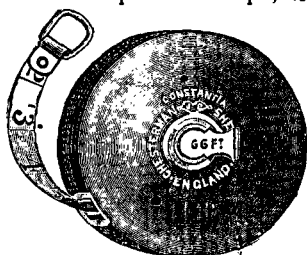


Fig. 11.—Linen Tape with Folding Handle.

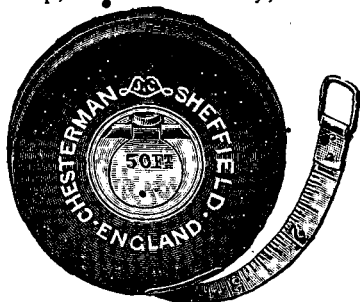


Fig. 12.—Metallic Tape with Flush Handle.

box. The tape after being washed should be coiled loosely up; and after carrying it for a short time in the open air, it will be dry enough to be wound up. In winding up a tape, it should be made to pass closely between the middle and forefingers. If this be not done, it is liable to get twisted inside the case. If in drawing it out it gets twisted and becomes liable to damage, the proper course, then, will be to unscrew the winding knob, to take out the tape through the central hole in the case, and to refit it again. New tapes may be bought separately and are easily inserted in the old leather case when the old tape is worn out. The metallic tapes of Chesterman's patent in dark brown leather cases are recommended as the best for wear.

50. Steel Tapes (Fig. 13) are also made $\frac{3}{8}$, $\frac{1}{2}$ and $\frac{5}{8}$ inch wide.

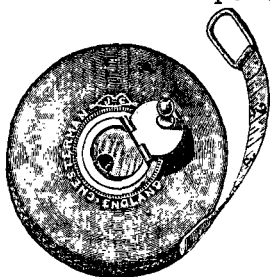


Fig. 13.—Steel Tape.

They are more correct and more costly than linen tapes, but less flexible and less durable. They should always be used in short important measurements. 100-foot tapes are usually divided into feet, with the foot at each end divided into tenths. The division marks are indicated by small rivets. They should be carefully handled as they are liable to break. Ribbon steel tapes in cases, of 50', 66', or 100' in lengths, are used. The ribbon can be detached from the case for use as chain and can be as readily

wound up as an ordinary tape. These will also prove valuable tests for other tapes as they can never stretch or otherwise vary.

51. Pocket Steel Spring Tape (Fig. 14) is useful in taking

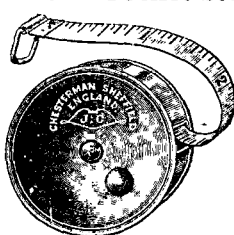


Fig. 14.—Pocket Steel Tape.

height of the axis of the level telescope occasionally required for town surveys. The case is $1\frac{3}{8}$ " in diameter of German silver. It is of ribbon steel $6\frac{1}{4}$ " long. On the one side on top edge are 62" divided in $\frac{1}{16}$ ths and under this 2 metres fully divided thus showing English and French measures. On the other side are the following fully divided scales, each 12": $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, 1, $\frac{3}{8}$, $\frac{3}{4}$, $1\frac{1}{2}$ and 3"; also 10, 20, 30, and 40 divisions to the inch.

52. The metallic tapes are useful in measuring offsets, dimensions of buildings, earth-work pits and all engineering works generally, but, on account of their liability to stretch cannot be depended upon for any degree of accuracy. In moist weather it shrinks and when dry it expands. The best tapes for strength and permanency are made of entirely handmade unbleached flax from Holland.

53. To measure a length, let an assistant hold the brass handle of the tape at the point from where the measurement is to commence, and take the case to the other end of the length, and read the graduation on the tape at the point after stretching it tight. Always keep the reading end yourself. When lengths of more than 100' are to be measured, marks are to be left at every 100', and the tape to be used like a chain.

54. **Offset-Staff or Rod.**—An offset-rod is made of a straight piece of bamboo, or a round piece of yellow pine, sal, ash, or teak wood, $1\frac{1}{8}$ " to $1\frac{1}{2}$ " diameter, diminishing to $\frac{7}{8}$ " at the top, 100 or ten links long, graduated to single feet or links. The 10'-rod is used with the 100'-chain, and the 10-links one is convenient when surveying with the 66'-chain. At one end of the rod there should be an iron



Fig. 15.—Jointed Offset-rod with Hook, H, Joint J.

ferrule with a steel point for driving it into the ground; at the other, there should be a flat-topped ferrule. A strong hook, H, large enough to hold the chain handle, would be useful in dragging the chain through fences if attached to its side near the top (Fig. 15).

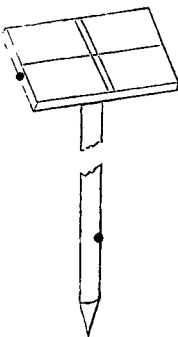


Fig. 16.—Wooden Cross.

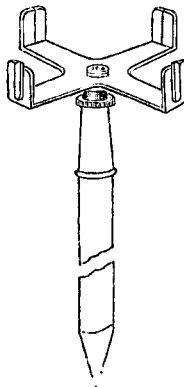


Fig. 17.—Open Cross.

55. **Use of the Offset Rod.**—When the offset to be taken is lined out,

the offset rod is turned over end for end as many times consecutively as may be required to bring it to the point to be offsetted to; but this method of offsetting is not preferable as errors accumulate rapidly even if it is most dexterously handled. Offsets of 50' or 60' lengths and above should be measured with a 100 feet tape.

56. Cross-Staff or Cross (Fig. 16) is a very ancient instrument for laying of $\perp$ s to a given chain line. This is a round or square piece of wood, $1\frac{1}{2}$ " thick and about 6" in diameter or sides, having two fine grooves, $\frac{1}{4}$ " deep, in it, which are at rt. $\angle$ s to each other. A staff with a pointed end is stuck upright into the ground, and the cross is fixed to the top of the staff about 5' high so as to form a small table.

Another kind of cross-staff is simply a staff with a spike at the lower end and two pair of sights at rt. $\angle$ s to each other at the upper end (Fig. 17).

French Cross-Staff.—Fig. 18 shows an octagonal brass tube with alternate sight slit and opposite window with vertical hair on each of the 4 rectangular sides of the octagon. On the other 4 sides are plain slits subtending 45° to those first mentioned. By means of these lines, an angle of 45° as well as 90° with the chain may be set off. The octagon is mounted on a socket-piece which fits on a pointed staff.

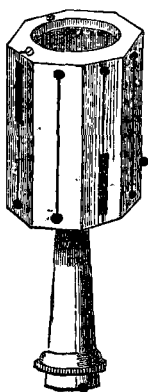


Figure 18.—
French Cross.

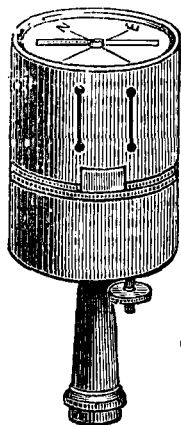


Fig. 19.—Adjust-
able Cylindrical
Cross.

A 4th kind is a brass cylindrical box, $3\frac{3}{4}'' \times 2\frac{3}{4}''$; divided into 360° in the centre, with sights along the sides in upper and lower half, rackwork adjustment and clamping screws, bar needle-compass on top, and with a ball-and-socket jointed tripod stand (Fig. 19). This may be used for setting off or measuring angles roughly. The meeting planes of the two cylinders are

graduated on the lower into degrees, and the upper with a vernier. The top part slides over the lower, and may thus be set to any required angle by means of the graduations. As in general long offsets are to be avoided the cross staff should only be sparingly used.

57. To test a cross, direct one of the grooves, $a'b'$, along the station line, ab (Fig. 20), and send an assistant with a flag at c'' in the

direction of the other groove, dc . Now move the cross horizontally round 90° as shown by the direction of the arrows so that the groove, cd , may be directed along ab . If the instrument is correct, the other groove, $a'b'$, will take up the former position of dc and point to the flag c'' . If the grooves be not at rt. $\angle$ s, then $a'b'$ will point to some other direction, $a''b'c'$ (Fig. 21).

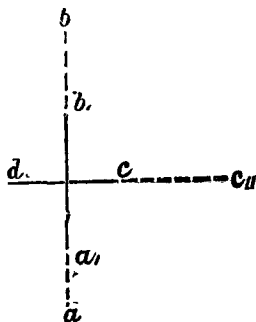


Fig. 20.—To Test a Cross.

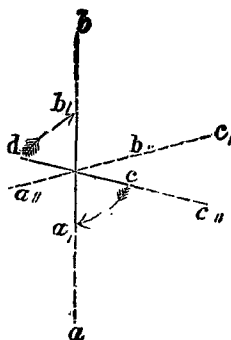


Fig. 21.—To Test a Cross.

58. To use a cross, it is placed on the chain line at the point from where an offset is to be taken, and one of the grooves, or a pair of sights is ranged along the chain line; then the other groove or the other pair of sights is as rt. $\angle$ s to chain line. When it is to determine the point on the chain line from where the offset is to be taken, the cross is set up as near the required point as possible, and is then moved to and fro on the chain line until the object of which the offset is to be taken is cut by the other groove or the 2nd pair of sights.

59. Optical Squares.—The cross has been superseded by the

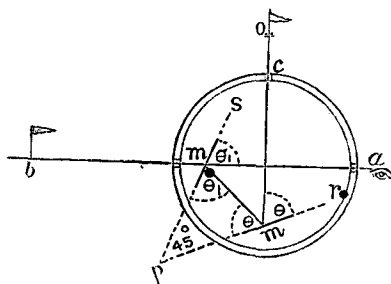


Fig. 22.—Round Optical Square.

optical square which (Fig. 22) is a round brass box within which two mirrors $\theta'\theta'$ and $\theta\theta$ are set at an angle of 45° to each other. Half the mirror $\theta'\theta'$ is unsilvered, so that an observer standing at a , and looking through the eye-hole at the side of the box can directly see, through the unsilvered portion of the mirror $\theta'\theta'$ the flag planted at b on the chain line. When a point is reached on the chain line

where an offset should fall from o , the flag posted at o is reflected through the opening c on the side of the box from the mirror $\theta\theta$

on to the other mirror $\theta\theta'$ and is by reflection seen through the eye-hole at a to be coincident with the flag, δ .

60. *Theory of the Optical Square.*—It can be easily proved that $\angle mps$ should be $= 45^\circ$, for $\angle omr$, the $\angle$ of incidence $= \angle pm m = \theta = \angle$ of reflection. Similarly $\angle pmm = \theta' = \angle am s$, and in order that the instrument should do its work, it is necessary that the original direction of the ray of light om should be at rt. $\angle$ s to its reflection bma . The $\angle s amm$ and mno , therefore, should be a rt. $\angle$, i.e., $(180^\circ - 2\theta + 180^\circ - 2\theta') = 90^\circ$, or $\theta + \theta' = 135^\circ$.

$\therefore$ the $\angle mps = 180^\circ - 135^\circ = 45^\circ$.

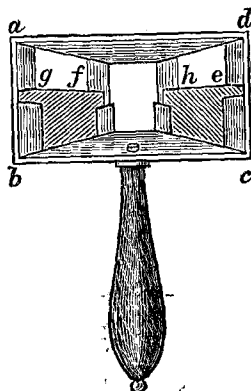


Fig. 23.—Indian Optical Square.

61. *Indian Optical Square* (Fig. 23). It is a brass wedge-shaped hollow box of 2" sides and $1\frac{1}{2}$ " depth, having a handle fixed below, about 3" long. It contains two rectangular mirrors bf and ch fixed at an inclination of 45° to the inclined sides of the box. Above these mirrors there are two rectangular openings. $abcd$ is the open face of the square, which is to be turned towards the object to be offsetted to.

62. *Use.*—Suppose AB to be a chain line proceeding in the direction of B at which a flag is planted (Fig. 24), and D an object to the left where a pole is set up, and to which an offset is to be taken. Stand on the chain line looking towards the flag B , and hold the optical square in the right hand, having its open face $abcd$ towards the pole; put your right eye to the opening over the mirror at the point e , and look through the opening over the other mirror along the point f to the flag. Now the image of the pole, after reflection from the mirrors, will be seen by you either to the right to the left of the flag. If to the right, you must advance slowly along the line, keeping the instrument carefully in position all the time until the pole coincides with the flag; if to the left, you must recede in the same way until the coincidence occurs; your position when the coincidence occurs is the point C at which the direction of the pole

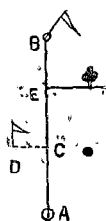


Fig. 24. or object at D is $\perp$ to the chain line, whence the required offset will be measured. Again, suppose, on your advancing further on the chain line, you see a tree to the right to be offsetted to. Hold the optical square as before with the open face towards the tree; then put your right eye to the opening over the mirror at the point g , and look through the opening over the other mirror along the point h to the flag. Now the image of the tree after reflection from the mirrors will be seen by you, either to the right, or to the left of the flag; if to the right, you must recede slowly along the line, keeping the instrument in position all the time

until the tree coincides with the flag; if to the *left*, you must *advance* in the same way until the coincidence occurs; your position now is the point *E* at which the tree is $\perp$ to the chain line. To take an offset with this instrument, the eye is always to be applied at the lower corner of either opening which is nearer the open face *abcd*, i. e., at *e* and *g* when the object is to the *left* or right of the chain line, and always look diagonally to the forward flag, i. e., from *e* to *f* and *g* to *h*. In using the instrument hold it quite erect.

The purpose of an optical square may be answered by a box-sextant, the index being set to 90° . This instrument will be described afterwards.

63. Properties of a Magnet.—Magnets are substances which have the property of attracting iron ore, iron or steel, and the term magnetism is applied to the cause of this attraction, and to the resulting phenomena.

When a magnetised needle is placed on a pivot on which it can move freely, it ultimately sets in a position which is more or less north and south. If removed from this position, it always returns to it after a certain number of oscillations.

The magnetic needle is utilized in carrying out survey operations. It is placed under a graduated card or ring, and fixed to it, the needle always pointing towards the north, and moving freely on a pivot attached to the centre of a brass box as in the prismatic compass to be described hereafter. The motion of the sights gives the reading which the direction of the object makes with the magnetic meridian. This reading can be easily read off the graduated card. In the surveyor's compass it is made to move freely, while the card is fixed to the box and sights. As the magnetic needle always points north and south the $\angle$ at the point of observation, through which the card and sights have moved to align the object, gives the direction of that object with regard to the meridian passing through the point of observation which is indicated by the needle. As will be explained hereafter, a plane table may also set up by the aid of the magnetic needle.

64. Prismatic Compass.—It is an instrument by which horizontal $\angle$ s can be measured to within about one quarter of a degree. It is very useful in filling in the interior details of a large trigonometrical survey, for sketching along a road, or river, or any continuous line of military works. It is also useful in surveying small towns, where a chain survey is not suitable as the country can not be divided into Δ s conveniently. Considering the continual changes going on in earth's magnetism, and the effects of local attraction, it is thought doubtful by the best authorities whether magnetic bearings can be relied on even to half a degree. Hence it is useful in filling up small details and making rough traverse surveys. It is not to be used where great accuracy is required.

The compass is often held in the hand, or when greater accuracy is required, it is screwed on to a tripod-stand, about $4\frac{1}{2}$ high; as the needle then comes to rest quicker and the work is altogether more satisfactory. The top of the tripod is provided with a ball-and-

socket joint, for the purpose of taking the bearing of an object after adjusting it horizontally.

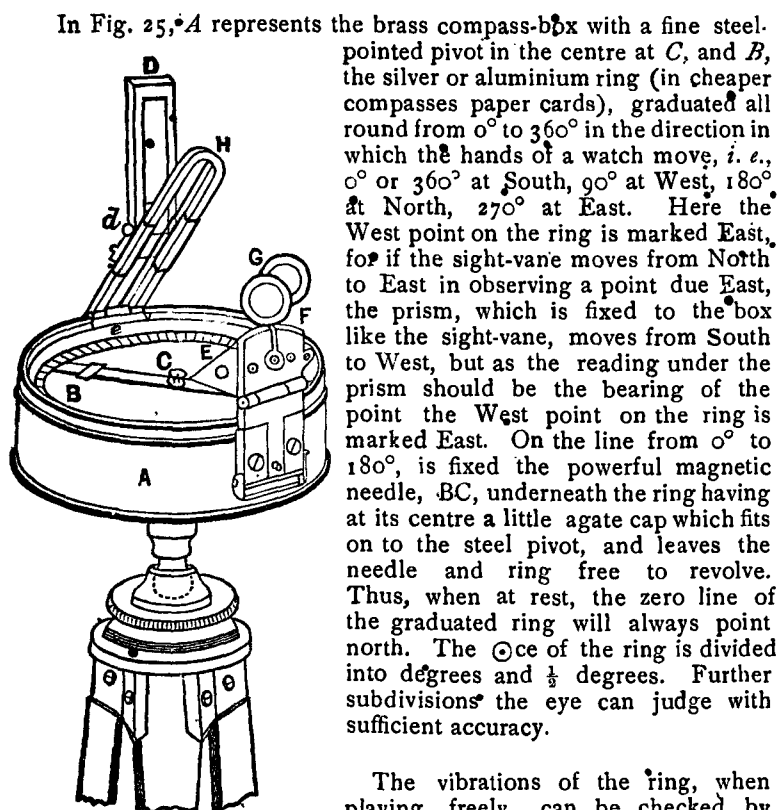


Fig. 25.—Prismatic Compass. The vibrations of the ring, when playing freely, can be checked by touching a small spring, *e*, in the side of the box below the sight-vane, *H*. The sight-vane, *H*, has a fine thread or horse-hair stretched along its opening in the direction of its length, which should bisect the object under observation by turning the box round horizontally. The sight-vane is mounted on a hinge-joint, which enables it to be turned down flat on the box, when it is not in use, for the convenience of carriage. Below the dark glasses, *G*, is a totally reflecting magnifying prism *E*, (so placed as to show to the eye of the surveyor a reflected and magnified image of that part of the edge of the graduated ring which is directly below the line of sight), through which the observer looks in taking bearings of objects; also mounted on a hinge-joint by which it can be turned upon the box, or turned over the side of the box for putting it into the case. The prism is fixed to a plate sliding in a socket, so that it can be raised or lowered as desired. The portion of plate *F*, which projects above the prism, has a narrow slit, through which an observation is made

by raising or lowering the prism in its socket, when the divisions on the compass ring immediately under the prism are seen, and these divisions, seen through the prism, appear, as each is successively brought into coincidence with the horse-hair by rotating the instrument, as continuations of the hair, which is seen directly through the slit above the prism. In order to show bearings in degrees to the east of north, and owing to the prism being placed at the reverse side of the graduated ring and exactly opposite the sight-vane, the zero of the ring is placed at the south. One face of the prism is convex, and as the image is seen through this face, it is magnified, and as it is seen by reflection, it is inverted, so that to read the figures correctly, graduations are written upside down: by this means the number seen through the prism is the reading required.

A sketch of the flat graduated ring of the prismatic compass is shown in Fig. 26. The divisions are not all shown, but some of the figures are shown, and written upside down.

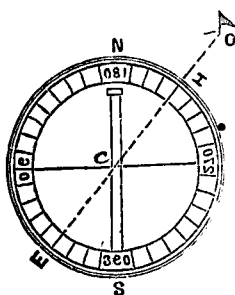


Fig. 26.—Dial of Prismatic Compass.

SN represents the direction of the magnetic north which is stationary, while the prism, *E*, and the sight-vane, *H*, are moved by turning the box to take the bearing of the object *O*. Here the angle under the prism, *E*, on the ring is 40° , i.e., $\angle SCE = 40^\circ = \angle NCH =$ magnetic bearing.

A mirror is shown, *Dd*, in Fig. 25 which slides up and down the sight-vane, *H*, or can be removed at pleasure. Its use is to reflect the sun or other object when it is much below or above the level of the observer. This operation is done by inclining it by its joint, *d*, at any $\angle$ required. The mirror will remain stationary on any part of the sight-vane by the friction of its slides. There are two dark glasses, *G*, which may be made to turn over the sloping side of the prism, and are useful when observing under a strong glare of the sun. There is also a stop near the hinge-joint of the sight-vane, and it is so arranged that the mere act of laying the sight-vane flat on the glass face throws the needle off the pivot. As the accuracy of the instrument depends on the fineness of the agate centre upon which the needle swings, it should not be allowed to be worn out by the constant playing of the needle, which should be thrown off its centre, when not in use, by laying the sight-vane flat on the glass face. The silver or aluminium rings are generally made from $3''$ to $4\frac{1}{2}''$ in diameter according to the size of the compass-box. A cover fits on to the box, and the whole is packed in a leather case which is slung over the shoulder by a strap for convenience in travelling.

65. The Method of Using the Prismatic Compass is as follows:—The sight-vane, *H*, and the prism below *F*, being turned up vertical on their hinge-joints, the instrument is held in the hand as nearly as possible horizontal; or if it be used on a tripod stand, set it as nearly horizontal as can be done by moving the legs of the stand, so that the card or ring may play freely. The

prism is then raised in its socket until the divisions on the ring are seen distinctly through the prism, and the instrument turned round until the object to be observed is seen through the portion of the slit projecting above the prism in exact coincidence with the hair of the sight-vane. The graduated ring is then brought to rest by touching the spring, and the reading at the division upon the ring, which appears in the line of the hair, gives the magnetic azimuth or "bearing" of the object observed, or the $\angle$ which a straight line drawn from the eye to the object makes with the magnetic meridian. The magnetic azimuth of another object being taken in the same manner, the difference between the two azimuths or bearings is the $\angle$ subtended by the objects at the eye. Suppose, the former bearing is $50^{\circ} 40'$, and the latter $25^{\circ} 20'$, both read from the north point on its right side, then the $\angle$ will be $50^{\circ} 40' - 25^{\circ} 20' = 25^{\circ} 20'$ which is the "relative azimuth" or angular distance from the first to the 2nd object. The bearing of a forward station is called the "forward bearing," and that of a back station "back bearing."

66. **Prismatic Compass, Hutchinson's Form** (Fig. 27) is similar in construction to that shown by Fig. 25 except that there

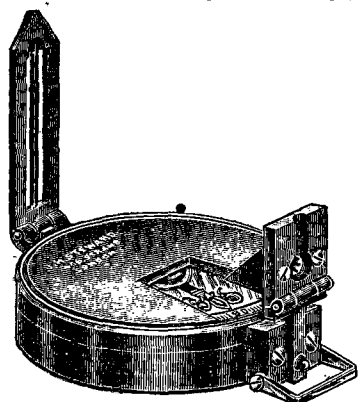


Fig. 27.—Hutchinson's Prismatic Compass.

prism folds down where the handle of a watch is, and the objectsight is made of a round piece of glass in the cover, having a vertical line, D, scratched on it. This compass may be used as a clinometer.

is an aperture in the fixed metal cover below the prism. The aperture has a stout glass let in. This arrangement allows of the instrument being brought into use directly it is taken out of the case. It has no mirror nor dark glasses.

67. **Pocket Prismatic** (Fig 28) is in shape and bulk like a watch. The

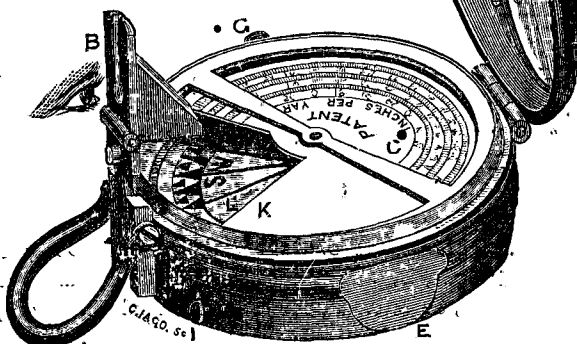


Fig. 28.—Pocket Clinometer Prismatic Compass.

68. **The Surveyor's Compass** (Fig. 29) consists of a compass box, magnetic needle, and two plain sights $\perp$ to the meridian

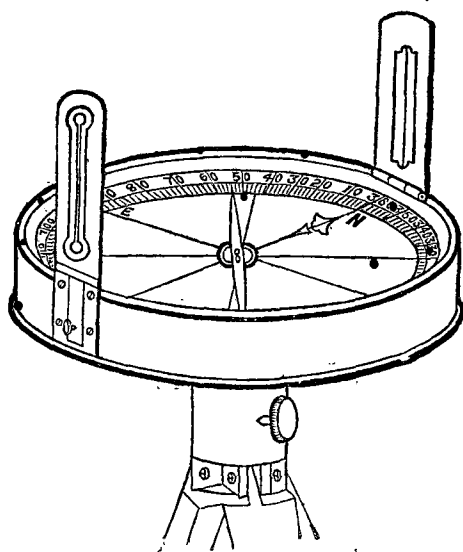


Fig. 29.—Surveyor's Compass.

sights being fixed, move together. In the card the East is marked W, and the West marked E. The angle has to be observed with the naked eye as pointed to by the needle after the object has been bisected. The graduated card is divided into degrees only, and numbered 10° , 20° , 30° up to 360° . The zero begins at the north point of the card, and the graduations run in the contrary direction to the motion of the hands of a watch. The north, south, east, and west points are also marked. The magnetic north being stationary, and the north point on the card being directed towards the object of which bearing is required, the bearing of the object is then the same angle as that through which the card has been moved. For instance, an object bears 60° east, this can be read off easily by the prismatic compass, but in the surveyor's compass the 60° east point on the card is now under the north point of the needle. The reason for changing east for west, and *vice versa* is clear; for were they not changed, exactly opposite reading would have been taken; again, for instance, if a point bears due east, the north and south points of the card and the sights move through 90° from left to right when the point is observed, and the W point is marked E. because that is the point which comes under the needle, and is read as the bearing of the point.

The surveyor's compass, which can not be used without a stand, is also used like the prismatic compass for filling in the interior details of a survey by means of bearings.

line in the box, one of which has a longitudinal slit through which the observer looks and puts the horse-hair in the other sight in a line with the object of which the bearing is required. The sights are attached in various ways for portability, but are now generally made to turn down on a hinge in order to lessen the bulk of the instrument, and render it more convenient for carriage. In the prismatic compass the magnetic needle is attached to the graduated ring or card; while in the surveyor's compass the graduated card is fixed, and the needle swings freely. The card and

69. Method of using the Surveyor's Compass.—Set up the compass over the station by means of the plumb-line, adjust it horizontally, and bring the needle to zero or the north point of the card. Then looking through the eye-sight, turn the box round from left to right till the object is bisected by the horse-hair of the other sight. Read the graduation pointed to by the north point of the needle, which is the bearing required.

70. The Points of Difference between the Prismatic and the Surveyor's compasses.—From the above descriptions of the prismatic and the surveyor's compasses, it will be seen that in the latter the graduated card being fixed moves with the sights, while the needle, not being attached to the card, always points to the magnetic north; while in the former the card and the needle are attached, and are constant in direction when the prism-box and the sight-vane move.

(2) In the prismatic compass, the sighting of an object and reading of the bearing are done simultaneously; whereas in the surveyor's compass, the surveyor has to see first, and then go round and read the bearing with the naked eye, i.e., the graduation that the needle points to on the card.

(3) The prismatic compass can be used with the hand without a stand, while the surveyor's compass can not be so used.

(4) The graduations on the card of the surveyor's compass are made in a different way. In the prismatic compass the zero point is at the south point of the needle under the prism when the needle points north, and the graduations are made from 0 to 360° in the direction in which the hands of a watch move. In the surveyor's compass the zero or 360° point is at the north point of the card in a line with the horse-hair, and the graduations are made in the direction opposite to the motion of the hands of a watch; the true east is marked W (west), and the west marked E (east), but the graduations are written ordinarily and not upside down.

71. The Prismatic and Surveyor's Compasses are Liable to the Following Errors:—Errors due to carelessness in reading the bearings and recording them. (2) Errors due to magnetic changes in the atmosphere. (3) Local attraction, due to the presence of iron or steel near by, chain pins carried by the surveyor or steel spectacles worn by him. (4) Variation of the magnetic needle daily and annual and different at different places. (5) Error due to the needle getting rusty, losing its magnetic power, and not moving freely on the pivot. (6) Error due to the dip of the needle, which in the northern hemisphere is due to the north point of the needle being slightly depressed owing to attraction from the north pole of the earth, similarly in the Southern hemisphere the needle dips to the South, and therefore the needle does not move perfectly horizontally, or play freely on the pivot. (7) Errors due to the wrong centering of the compass on the point where it is set up. (8) Errors due to the compass-box not being horizontally set up. (9) Errors due to imperfect intersections of objects, to the sights being defective, or to the vertical horse-hair

too thick or loose. (10) Errors due to the bad construction of the instrument in having the pivot not exactly in the centre of the box or the graduated ring, to the line of sights not passing through this centre, or to the bad construction of the needle whose axis not passing through its north and south points.

72. Testing the Compass.—To ascertain whether the compass to be used is a trustworthy instrument or not, sight to any distinct object and read the compass, then move the sights away from the object and resight it, taking care to bisect the same object; the instrument should read the same now as it did before. Another way is, having bisected and read the instrument, to oscillate the needle by bringing a key or a pen-knife near it; if the needle is sensitive, and there is no friction on the pivot, it will return to the same reading. The horse-hair should be fine. If the needle be not straight, or if the graduations be not correct, the two ends of the needle will not point to readings differing by 180° . To find, if the pivot is exactly in the centre of the card, and if the needle is straight, set the needle to point north-south and move the box through 180° ; if the needle now points south-north exactly, it may be assumed that the needle is straight, and the pivot is in the centre. To see if the line of sights passes over the pivot, stretch a horse-hair through the middle of the slits in the sights: it should also pass over the pivot. If the pivot is not vertical and not in the centre, and the needle not straight, the instrument should be sent to the instrument maker for adjustment. The sight-vanes should be vertical and fixed diametrically opposite to each other. To test this, a plumb-line may be suspended before the compass at a little distance from it, and when the instrument is in level, the eyesight, the horse-hair, and the plumb-line should be in the same line. If the needle dips it is adjusted to a horizontal position by weighting the end which is above the horizontal position. This is done by the maker. Needles sometimes lose their magnetic properties. They must then be remagnetised after removing it from the pivot. An apparently sluggish needle may be due to a blunt pivot. If so, this should be removed, and ground down on an oilstone.

73. The Compass attached to the Level (Fig. 30).—In this compass the magnetic needle is attached to a graduated aluminium or silver ring which swings on a steel-pointed pivot at the centre of a brass compass-box as in the prismatic compass. The compass-box is generally in one piece with the bar which carries the Ys upon which rests the telescope, A B, of the levelling instrument. The ring is graduated into degrees and half-degrees all round from 0 to 360° . The zero begins at the West point, 90° at the North point, 180° at the East, and 270° at the South point. Thus on the line from 270° to 90° is fixed the magnetic needle underneath the ring. The number of graduations written not upside down but as ordinarily written. On the left of the telescope is a magnifying glass, G, attached to the side of the glass-covered box for the observer to read the bearings. In recent instruments prisms are used to read the bearings,

the graduations being written upside down. Below this glass is a needle-point or index, P, fixed to the side of the box, in a line $\perp$ to the axis of the telescope, to indicate the bearing of an object. The

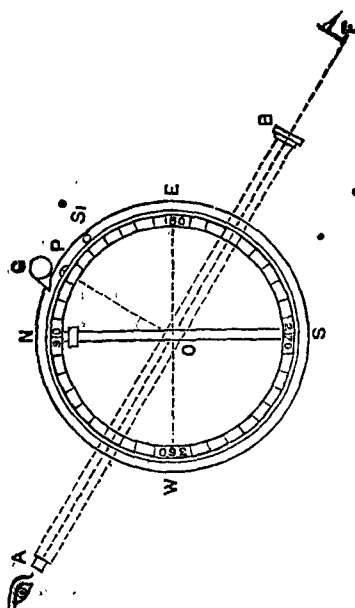


Fig. 30.—Compass attached to the Level.

To get the direction of the object on paper, it will have to be plotted on the east side of the north line. For $\angle WOA = \angle EOF$, and $\angle AOP$ and $\angle NOE$ are equal to one another being rt. $\angle$ s. $\therefore \angle WOA + \angle AOP = 120^\circ = \angle NOE + \angle EOF =$ ordinary bearing.

Full description of the level, and its temporary and permanent adjustments will be found in the latter part of this work.

75. Setting up the Compass.—It is of great importance to set up the compass in its proper place from where the bearing of an object is to be taken. The place is to be marked by a peg, and the tripod stand placed just over it. This is done by letting fall a piece of earth, or a broken brick, or stone, from below the centre of the disc at the top of the stand on to the top of the peg. The position of the compass has to be moved until the piece of broken brick falls on the peg. This can also be effected by suspending a plumb-bob from the centre of the disc, and by the pointed end of the bob just touching the centre of the peg. When this is done, the box should be adjusted horizontally, and all precautions

the screw S', attached to a spring and placed in a hole in the rim of the box, is tightened to throw the needle off its centre when not in use. The telescope carries the spirit-level. The instrument is adjusted on a tripod stand for use.

- 74. To use the instrument, the level is adjusted horizontally by the legs and the foot-screws, supposing the instrument to be in permanent adjustment, then it is directed towards the object which is to be bisected by moving it horizontally. To bisect any object, first obtain a clear image of it by moving out the eye-piece until the cross-wires can be seen distinctly and then turning the focussing screw at the side of the telescope. In Fig. 30, suppose, the flag, F, is bisected by one of the vertical hairs of the diaphragm and the index, P, reads 120° on the graduated ring. This is the bearing of the object. It is read from 0 to 120° .

taken against the liability of the compass to error, before securing the greatest possible accuracy in taking bearings which, if only a compass is used, consists in taking both forward and back bearings of lines.

76 The true bearing or bearing of a line is its inclination to the direction of the true meridian, and the **magnetic bearing** of a line is its inclination to the magnetic meridian, *i. e.*, to the line shown by the direction of the magnetic needle. The true bearing is measured from the true meridian, and the magnetic bearing is measured from the magnetic meridian which does not coincide with the direction of the true meridian. The definition of an "angle" is the inclination of two lines to one another. The prismatic compass does not show this inclination, but only shows how the line from an object to the observer is inclined to the magnetic meridian. As has been stated before to find out an $\angle$ subtended at the observer's eye, it is necessary to take the difference of the bearings of two objects.

77. Declination of the Needle.—The geographical or **true meridian** of a place is the imaginary plane passing through this place and through the two terrestrial poles, and the meridian is the outline of this plane on the surface of the earth. Likewise the **magnetic meridian** of a place is the vertical plane passing at this place through the two poles of a movable magnetic needle in equilibrium about its vertical axis. The magnetic meridian does not coincide with the true meridian, and the horizontal $\angle$ which the magnetic makes with the true meridian is called the **declination of the magnetic needle**. The declination is said to be east or west according as the north pole of the needle is to the east or west of the true meridian. It is different at different places, and at a given place undergoes a gradual change from year to year besides smaller changes which are continually taking place. The earth is a huge magnet whose north pole is situated at about 68° north latitude and longitude 95° west of Greenwich. But the needle points only approximately to this pole. The point for the surveyor, to remember, however, is that he possesses the means of determining the declination for any particular time, place and instrument.

78. The Variations of the Declination.—At no place on the earth is this declination a *constant*. This *change* in this angle is called the *variation of the declination*. The position of the needle varies from the following causes:—

- (1) *The annual variation.*—This is very small being only about half a minute of arc from the mean position for the year;
- (2) *The lunar inequalities* are still smaller, being only about 15 seconds of arc from the mean position.
- (3) *Magnetic Disturbances*, which may amount to 1° or $1\frac{1}{2}^\circ$, due to *magnetic storms*, may occur at any time and cannot be predicted, and may last a few hours or even several

days. These appear to coincide with the appearance of the aurora borealis, earthquakes, and volcanic eruptions. Irregular changes sometimes quite large are noted during magnetic storms.

- (4) The *geometric axis* of the needle may not coincide with its *magnetic axis*, and hence the readings of two instruments at the same station may differ slightly when both are in adjustment. In this case the declination should be found for each instrument independently. The changes due to the *annual variation* and *lunar inequalities* are so slight that in practical work they may be neglected. The only two kinds of regular variations with which the practical surveyor is concerned are:—
- (5) The *Secular Variation*, and (6) the *diurnal variation*, the secular variation being much the more important.

79. Secular Variation of the magnetic declination, takes probably 2 or 3 centuries to complete a cycle. Observations have shown that, year in and year out, the north end of the needle is moving either towards the east or towards the west. Observations bearing on this subject have been made at Greenwich where records of the magnetic declination have been kept for about $3\frac{1}{4}$ centuries.

Year.	Variation.
1580	$11^{\circ}36'E$
1663	0
1818	$25^{\circ}49' W$ -max
1880	$18^{\circ}38' W$
1900	$16^{\circ}30' W$

The marginal record shows the variations. The needle is losing west declination about $7'$ annually. At present the declination is west in Europe and east in Asia. This phenomenon of a long continued movement of the needle in one direction is observed all over the world. From the fact that it continues through so many years, this change in the direction of the needle is known as the *Secular variation*. The cause of these remarkable changes is unknown.

Supposing that the traverse shown in Fig. 31 was run out

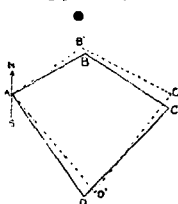


Fig. 31.—Retracing Lines with same Bearing.

50 years ago, and that during that time and at the place of survey the secular variation has been $3^{\circ}20'$ west. If the bearings in the original field-book were magnetic, and we run out the survey with the same bearings, the result will be the dotted lines as in Fig. 31. This shows the necessity of recording in any compass field-book and on any plot of a compass survey, whether the bearings given are true or magnetic, and, if the latter, what the amount and direction of the declination were at the time and place of survey. Knowing the declination at the time of original survey and also the declination at the present time magnetic bearings of any field-book can be corrected and using

the corrected bearings, retrace the lines run by the original surveyor. If the amount of the declination is not stated in the original notes, or if the declination at the present time is unknown, the change in the declination may be computed approximately as shown in art. 86, p. 39, by means of map showing isogonic lines.

80. The Diurnal or daily Variation in the Declination consists in a swinging of the needle through an arc of 5 to 15 minutes daily, the north end having its extreme east variation at 8 A.M., and its extreme west position about 1.30 P.M. It has its *mean or true* declination about 10.30 A.M., and 8 P.M. For determining the declination or important bearings the compass should be read at about 10.30 A.M. In ordinary compass work, it is customary to neglect the daily variation. It varies with the latitude and with the season and the cause has been stated to be the influence of sunlight.

81. The Variation of the Compass Needle must be taken into account; for if any survey were made with reference to the true meridian, then the variation of the needle must be added to or deducted from the observed bearing according as the variation is east or west to obtain the true bearing. This can be explained by Fig. 32. Let N S be the true north, $n s$, and $n_1 s_1$ the magnetic north directions. Suppose, the variation is west and P is an object of which the bearing is being observed. The magnetic bearing of P is $\angle nOP$, and the true bearing of P is $\angle NOP$. But $\angle NOP = \angle nOP - \angle nON = \angle nOP - \text{magnetic variation}$. Again supposing the variation to be east as shown by the dotted needle $n_1 s_1$, the magnetic bearing of P is $\angle n_1OP$, and the true bearing is $\angle NOP$. But $\angle NOP = \angle n_1OP + \angle NON_1 = \angle n_1OP + \text{magnetic variation}$.

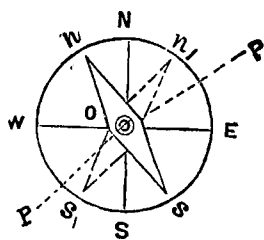


Fig. 32.—Variation of the Needle.

If the magnetic bearing of a line O P' be 225° , and the variation of the compass be $3\frac{1}{2}^\circ$ east, then the bearing of the line with reference to true north is $228\frac{1}{2}^\circ$. If at the point O, an $\angle$ of $131\frac{1}{2}^\circ$ to the right of the line is plotted, that line produced both ways is the true north and south line.

In India the magnetic variation is about $2\frac{1}{2}^\circ$ East.

82. The True North and South Line of any Place can only be determined with accuracy by astronomical observations, but the following methods are correct for all practical purposes :—

83. Method I. By the Sun's Shadow.—

Drive a rod, having a pointed top, into an even piece of ground vertically at O (Fig. 33) by means of a plumb. Mark the point where the shadow of the top of the rod is thrown, and with O, the rod, as

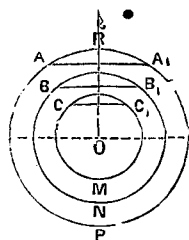
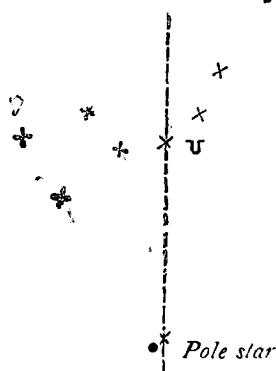


Fig. 33.—Finding True Meridian by Sun's Shadow.

centre and that length of shadow as radius, describe two or three concentric $\odot$ s of varying radii as AA_1P , BB_1N , and CC_1M . Suppose, the shadow from the point on the rod cuts the $\odot$ s before noon at equal intervals of time at A, B , and C , and again in the afternoon at the same equal intervals of time at A_1, B_1 , and C_1 . Now join AA_1 , BB_1 , CC_1 , and bisect them. These should have a common bisector, and if this be not the case a mean may be taken. This bisector is the true North and South line OR .

Now, if a compass be set up at O on the line OR , and the bearing taken, that bearing will be the variation of the compass at the point O ; only if its bearing be east the variation is west, and *vice versa*.

84. Method 2. By the Polestar.—The polestar is on the



meridian, either above or below the pole, when a vertical plane passes through it and the first bright star of the tail of the Great Bear, called *Ursæ Majoris* α (Fig. 34). Therefore twice during 24 hours the polestar comes on the meridian. Now set up a fine plumb-line at a place where it is not likely to be disturbed by the currents of air. The observer, placing himself behind the plumb-line, waits for the moment when the polestar and the *Ursæ Majoris* are in a line with the thread of the plumb-line. If the bearing of the polestar is then taken it will determine the variation of the compass. But as it is not easy to observe the

Fig. 34.—Finding True Meridian by Pole-star.

bearing at night, it is convenient to place a stick with a candle on it in the line of the polestar at some distance from the position of the observer. The line joining the observer's position and the candle being on the meridian, its bearing which may be taken the next morning is the variation of the compass.

At Madhupur on the E. I. Ry. the magnetic variation was $2^{\circ}37'$ east in 1888. At Calcutta it was observed to be $1^{\circ}47'15''$ east in 1887.

85. Method 3. By Observing Delta Cassiopeiæ, etc.—One of the simplest methods of determining the true meridian, and which calls for no mathematical instruments, or knowledge of the celestial sphere, is by observing Delta Cassiopeiæ over the vertical of Polaris. The method is this: The dotted line from the peep-sight, attached to the block of wood lying on the coal-tar barrel, to the pole star, Polaris, is continued in the same plane to Delta Cassiopeiæ (Fig. 35). When this ray is in perfect alignment, as seen at the peep-sight, with the plumb line, Polaris, and Delta Cassiopeiæ, Polaris is then within 3 m. 42 s. from the meridian. Watch in hand, the peep-sight is then moved westerly, keeping the star hid by the plumb-line. At the expiration of 3 m. 42 s. the star is on the meridian, and the observer is looking directly north, as shown

by the dot and dash line. Zeta Ursa Majoris is also in the meridian, approximately at the same time as Delta Cassiopeiæ, but is too high up to observe through a peep-sight. One eye, however, placed just before the plumb-line, as shown in the picture, can range Zeta with Polaris very well. The dotted line shows the visual ray from the eye, going almost vertically through Zeta in the constellation of Ursa Major, and thence curving onward below the Pole to Polaris. Zeta is however, in alignment with Polaris 42 seconds after Delta. The heavy block will lie unmoved on the cask till morning, when the meridian may be laid out by sighting to a stake some 100 feet to the north, as seen in the figure.

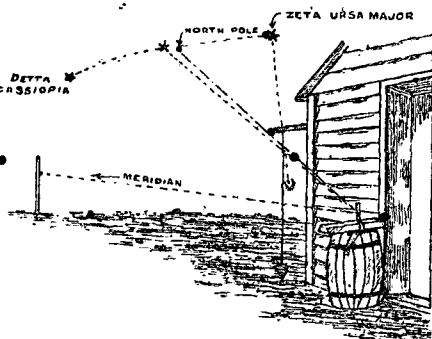


Fig. 35.—Finding Meridian by Delta Cassiopeiæ.

The north-west corner of a building is the best position, as shown in the drawing. The plumb-line may be 10 or 12 feet long, and 4 feet from the corner of the building. The cask may be set about 5 feet south of the plumb-line. An ordinary compass sight is the best to screw on the scantling, though a piece of tin, with a slit will answer (*Scientific American*, January 14, 1905).

86. Method 4. By Means of Map showing Isogonic Lines.—

If the declination at the present time is unknown, the change in declination may be computed approximately by means of a map showing *Isogonic lines* which are imaginary lines on the earth's surface joining points whose declinations are equal at any given time. The isogonic line joining points having no declination is called the *agonic line*. At the ends of the lines is shown the amount of declination at the points which they join. The map states whether the declination is west or east. At all points in Asia and Europe (See Hints to Travellers) east of the agonic line, the declination is east and *vice versa*. The declination for the year of the map, for any point not on an isogonic may be obtained by interpolation. Thus if a point is between the 2° and 3° east declination lines and one-half the perpendicular distance between them from the 3° line, its declination is $2\frac{1}{2}^{\circ}$ east, etc. To obtain the declination at any point for a year other than that of the publication of the Map, obtain first the declination at the point for that year. At numerous points on the map will be found the yearly change in the declination given at those points with + or - sign, showing annual increase or decrease. If the point in question is not one of those for which the yearly change is stated, it may be obtained approximately by approximation by the method stated above.

87. Method 5. By a Watch.—Point the hour-hand towards the sun. Half-way between the place of the hour-hand and XII on the dial will be the

south point in north latitude, and the opposite point of the dial will be the north point. In south latitude the reverse of this would be the case, while in the tropics the position of the north and south points would depend on whether the sun, when on the meridian, is to the north or south of the observer.* When the sun is near the zenith this method is useless.

88. All Indian Revenue Survey maps are plotted with reference to true meridian, and the field-books give true bearings. Before using a compass, its magnetic variation should always be determined. If a line is to be *relaid* according to field-books giving true bearings the magnetic variation of the compass in use has to be deducted from the recorded bearings when that variation is east. Similarly when the variation is west, it has to be added. But it has been stated before that when the variation is east, it is to be added to the magnetic bearing, and to be deducted when it is west to obtain true bearing. This should be explained fully. In the latter case magnetic bearing is first observed, and then the true bearing is obtained; while in the former case the true bearing is given, and the direction of the line is to be ascertained. Referring to Fig. 32, P is an object of which the direction is to be ascertained. The true bearing, suppose, is given in the field-book and is $\angle NOP$. The magnetic variation is first determined, supposing it to be east, and is $\angle NO_n$. The magnetic bearing of P is, therefore, equal to $\angle n_1OP$ which is equal to $(\angle NOP - \angle NO_n) = (\text{true bearing} - \text{magnetic variation})$. Similarly when the variation is west, magnetic bearing of P is $\angle nOP$ which is equal to $(\angle nON + \angle NOP) = \text{true bearing} + \text{magnetic variation}$.

89. Sources of Error in Magnetic Needle Readings: Local Attractions.—The magnifier for reading the verniers and needle-bearings, having black hard rubber or vulcanite frames, is liable to become electrified when the frame is cleaned by rubbing and the lens by wiping, and attracts the needle. Even magnifiers with brass, German silver and horn frames are found to be afflicted with magnetic attraction arising from impure metal in their cases, rivets, etc. In connection with this subject the following remarks, while not new, may not be out of place: "Be sure that nothing to attract the needle is carried by the surveyor in the shape of pen knives, watch-chains, buttons, or iron rivets in the magnifier used to read the graduations. Of late years new causes of error from this have arisen. The new watches in which the movements are made of nickel are almost as powerful in attracting the needle, as if those movements were made of iron. The felt-hat makers' modern fashion of stiffening the rims of their hats by inserting around the edge a small iron wire should make the surveyor examine both his hat and his watch." Aluminium has also been used instead of brass for instrument. But on account of the metal having "magnetic attraction" it is not suitable for needle instruments; this attraction is especially noticeable after the hammering process that is necessary to condense the aluminium after casting; this hammering, 'polarising it' as it were.

Prismatic compass should not at all be used near iron or by a surveyor using steel spectacles. The instrument should not be set up near telegraph wires, rails, &c. Umbrellas having steel or iron frames should not also be used by the surveyor. The reason is that iron or steel attracts magnetic needle which then does not indicate true results. The numerous disturbing influences, to which the compass is subject, render its indications of bearings very unreliable. Local attraction from iron or gaspipes in the ground, iron lamp posts, fences, building fronts in cities and towns, from iron ore in the soil, ferruginous gravel, trap rocks, &c., is another source of inaccuracy; as are also the annual and secular variations. Electricity, either atmospheric, or excited by rubbing the glass cover, sometimes gives trouble. It may be removed by touching the glass with the moist tongue or finger or by breathing upon it. It is plain that none of these causes except the last will affect the measurement of angles by the compass. By local attraction is, however, meant the attraction due to fixtures on the tract only.

90. General Remarks on the Tests, Adjustments and Use of the Compass.—Unless the instrument has fallen down or met with some like accident, it will not be necessary to make any test or adjustment. The greatest trouble is apt to be a sluggish needle, and no work should be undertaken with the instrument in this condition. Handle the compass carefully. Whenever the instrument is being moved, be sure that the needle is held against the glass, and when put away, that the needle has been allowed to come to rest in the magnetic meridian and then lifted against the glass. The instrument has no equal for general work when the highest accuracy is not required, especially in a wooded region. For country work unless the ground is entirely clear it is a waste of time to use more accurate instruments.

91. Combined Surveying and Plotting Instruments.—Plane tables are of this class and are described below.

The Plane Table is a drawing board, A (Fig. 36), mounted on a portable tripod stand, and available in sizes of 23" x 16", 32" x 23" and 41" x 28", and 1" thick. The standard planetable of the Indian survey measures 30" x 24" x 1" with transverse edge bars below, to prevent warping and buckling. It is set up on a stand, usually a braced tripod, to which it is clamped by a powerful hand-screw passing through the head of the stand into a brass socket fixed centrally under the table; the screw, when relaxed, serves as a pivot round which the table may be turned horizontally and set in any required position. The table is firmly clamped so as to maintain a constant position during the subsequent laying off of bearings.

Plane tables are made of well-seasoned, best dried pine, oak, teak and toon; deal and other soft woods are not fit for it, as they imbibe moisture quickly and expand across the grain. The surface of the table is level and well-planed. The back of the table has often two battens about 2" x 1" screwed on to it to prevent warping. Very good tables should be battened underneath with mahogany

and brass slots, one edge inlaid with ebony, with teakwood or mahogany tripod stand and clamping screw. For use, a sheet

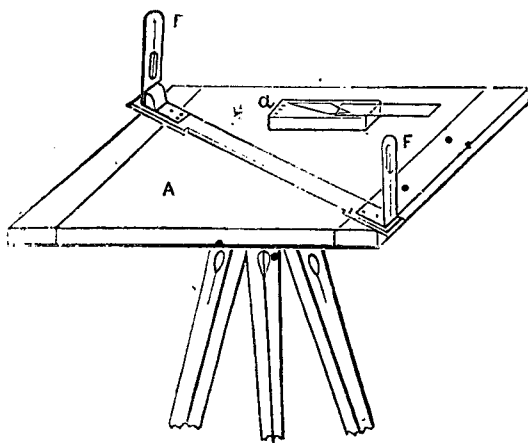


Fig. 36.—The Plane Table with Alidade and Trough-Compass.

of paper is strained on it with drawing pins. When this is done, prominent station points of the survey, the positions of which have already been fixed either by a theodolite or a compass, are marked on the paper; the points are joined by red lines; also the scale of the map is drawn in a spare corner, and the north line of the survey plotted. The table will then be ready for use:

92. Trough-Compass.—A rectangular wooden glass-covered box *a*, $6'' \times 1''$, having graduations on silver on either side of zero at the inside ends, and containing a powerful and sensitive magnetic needle, which should play freely, is put on the table to fix it with reference to the magnetic meridian. The construction of the box is such that when the needle points zero, its axis is parallel to the sides of the box along which north lines are drawn. With some tables a round compass-box is attached to the left side of the table. The magnetic needle should play freely, be strongly magnetised, be very sensitive, and yet settle quickly.

93. Alidade—Each plane table is also provided with an Index-rule, also called Sight-vane or Alidade, made of ebony, teakwood, or brass about the length of the diagonal of the table, available in sizes of from $12''$ to $30''$ long, $1\frac{1}{2}''$ wide, $\frac{1}{8}''$ thick, having at the ends bronzed sights $5''$ high, either fixed or folding, like that of the surveyor's compass, F F. (Fig 36). The edges of the Index-rule are bevelled. Sometimes it has one bevelled-edge, called the *fiducial edge*; the vertical hair and this edge are in the same plane.

Testing Sight Vanes.—To test whether the sights are $\perp$ to the index-rule, point the rule on some well-defined object at a distance of say $500'$.

Bisect the object with the upper or lower portion of the sights, and moving the eye up and down, see if the slit, the wire, and the object still remain in the same line; if they do not, the wire or even the sight must be altered until they do so.

94. A divider, a plotting scale, a pencil, an eraser and some pins are also required when the plane table is used.

95. **How Planetabling Operations are commenced on a New Sheet of Paper.**—Planetabling operations are commenced on a new sheet of paper at a fixed point by aligning with the alidade on another fixed point which brings the meridian line of the table on that of the station. The magnetic needle may now be placed on the table, and a position assigned to it for future reference; for example, A and D are two fixed points previously fixed by triangulation or otherwise, and *a* and *d* are two points projected on the table representing them. In fixing the position of the table at D, place it over this point, and move it horizontally until the alidade touching the line *da* on paper bisect the flag at the station A. Then clamp the table, and draw a line along the compass-box, when the magnetic needle points north, for future reference.

Position of the Table.—It should be observed that in setting up the table over any station, the point on the table representing such station,

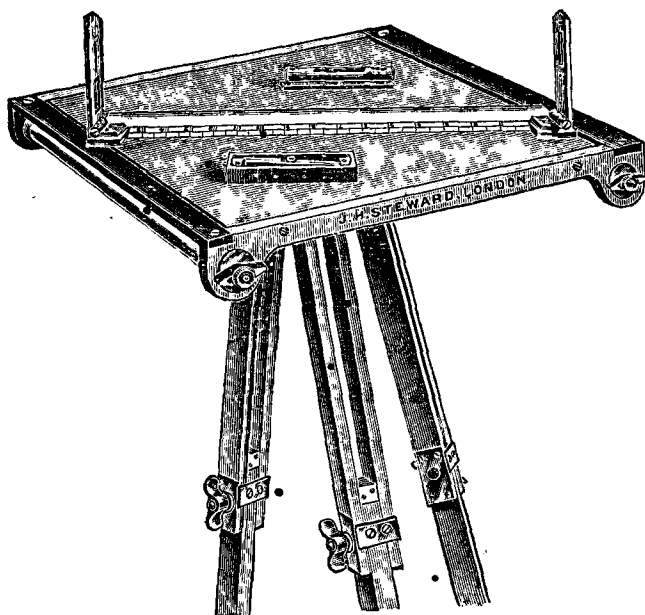


Fig. 37.—Topographer's Plane Table.

and not the centre of the table should be over the station on the ground;

it may be so placed by dropping a pebble, or a piece of broken brick or, a piece of earth from the corresponding point beneath the table.

96. Stewards' Topographer's Plane Table.—It consists of a best quality framed board, with rollers to carry a long sheet of paper (Fig. 37). The rollers (18" long) are fitted below the surface of the board, so that there is no interference with the movements of the alidade, and the paper passes from one roller to the other through slits near the edge of the board so that either edge can be worked from with a T-square. By this form of board a stretch of country, or course of a river, can be mapped on one strip of paper. The tripod stand has sliding legs for quickly levelling the board, height extended to 4' 6", and when closed 2' 9", with a large and firm head and revolving socket and clamp for board. Other accessories of this plane table are a metal 18" alidade with folding sights and divided bevelled edge, a metal trough-compass with arcs of degrees and a metal mounted spirit-level. The sizes available are 24" x 18" and 18" x 14" with waterproof canvas case, leather bound, with pockets, and knapsack shoulder straps.

The advantage of a plane table is, that it enables a survey to be made without the aid of, and in less time than with other instruments. All its other parts are mere additions to render this operation more convenient and accurate. Though the principle on which all plane tables are constructed is the same, they vary considerably in

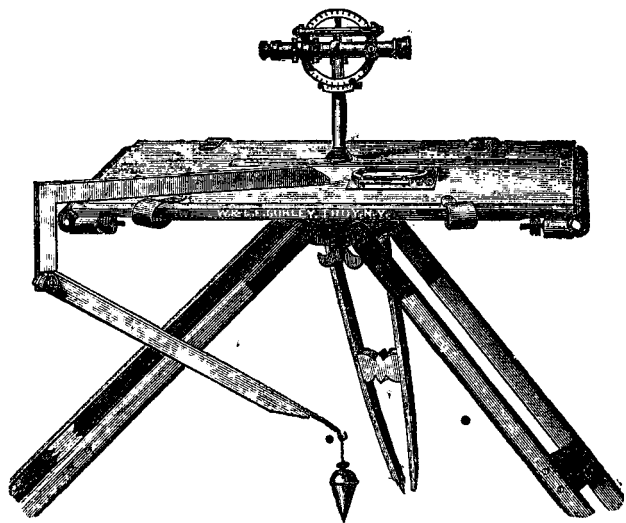


Fig. 38.—Gurley's Plane Table.

details. Those, for instance, constructed by Messrs. W. F. Stanley & Co. (Figs 39—41) and those used by the U. S. Geological Survey, and several of the European Governments are very elaborate instruments

with parallel plates and levelling screws, having also a telescope in lieu of plain sights rendering the plane table an instrument of precision. The levelling screws enable the surveyor to plant his instrument much more quickly and accurately than he possibly could without them, and with the telescope distant objects could be easily seen.

97. Gurley's Plane Table.—Fig. 38 shows the instrument which consists mainly of a drawing-board, 30" × 24", upon a tripod, having upon its upper surface a movable alidade arranged either with sight-vanes or telescope 9" long, power 20 diameters, with stadia. The vertical $\odot$ reads to one minute.

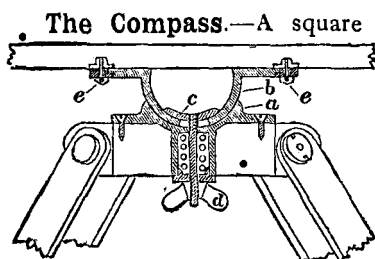


Fig. 39.—Socket of Gurley's Plane Table.

The Compass.—A square brass plate, to which is attached a compass with two spirit-levels, is also shown, and serves both to level the table and, when applied by the edges parallel to the zero-lines of the compass-circle, to determine the magnetic bearing of the lines drawn on the paper, or the direction of the table itself.

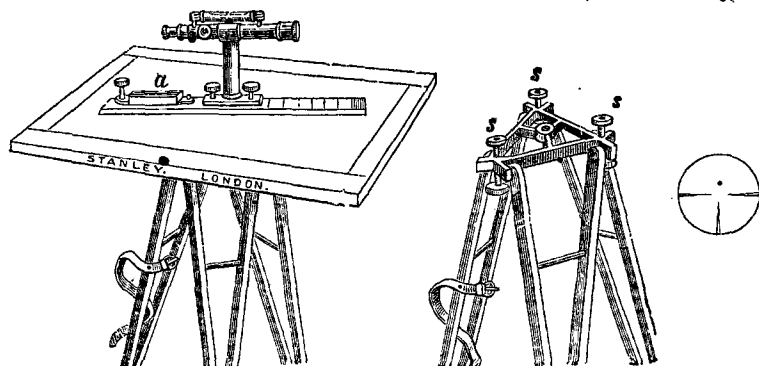
Drawing-Board.—The table is made of wood arranged in sections so as to prevent warping, and has an adjustable wooden roller at each end, by which the paper is brought down snugly to the board or upon which a long sheet can be rolled or unrolled at will. Sometimes in place of the rollers and often in combination with them, a number clamps, as shown, are used in holding the paper firmly.

The Plumbing-Arm shown has its end brought to a point, that it may be set at any given point on the paper, the plummet hanging from the under-arm determining the corresponding point on the ground.

The Socket and Tripod.—The construction of the socket and tripod head is shown by Fig. 39, *a* representing the hemispherical concave metal cup fastened by six screws to the wooden top of the tripod, *b* the upper or convex part fitting nicely into the cup and clamped to it at will by the clamping piece, *c*, and nut, *d*. A strong spiral spring in the hollow cylinder between *c* and *d* serves to hold the two spherical surfaces of the socket together and allows the easy movement of one within the other in the levelling of the table. The flange of the socket, *b*, supports the table and is connected with it by 3 segments of brass, two of which are shown at *e*, *e*. The table can be oriented at will and clamped by a milled-head screw.

98. Stanley's Plane Table with Telescope.—Where greater refinement of observation is required than is possible with plain sights, a telescope is mounted on the alidade, which moves in the vertical plane upon an axis, so that it may be directed in a linear direction with the fiducial edge of the rule to any point in azimuth.

The telescope sometimes carries a level, so that the table may be set level by means of the alidade. In Fig. 40 is shown a class of plane



Figs. 40-41-42.—Stanley's Plane Table with Telescopic Alidade, Stand and Subtense Points of Telescope.

table which meets all necessary refinement for ordinary filling in of field-work. The drawing surface of this table consists of a loose panel which stretches the sheet of paper by pressing it into its frame where it is afterwards held by a pair of ledges which fit at their ends into long slots. The panel of the board is mounted upon a firmly-braced tripod-stand. The head of the tripod-stand (Fig. 41) is secured to the board with a central screw (not shown) which permits the board to be set in any direction, it being the rule that the edge *W* should always take a north to south direction. Three screws, *s, s, s*, at the corner of the Δ head can be raised or lowered by milled heads from the under side. These screws permit about 15° of adjustment to the table without any unsteadiness, as the centre screw clamps the table finally hard down upon them when all adjustments are made. A small trough-compass, *a*, is placed upon the rule to strike the magnetic north line, to which all angles are referred in transposing the work of the plane table. The diaphragm of the telescope is provided with a platino-iridium point fixed vertically at the mutual focus of the object-glass and the eye-piece. A pair of points to subtend an angle to measure a staff for distance (Fig. 42) is a convenient addition.

Stanley has another design for a plane table which has rollers at each end of table upon which a long slip of paper is rolled. The rollers for small tables are made of brass tubes about $\frac{5}{8}$ inch diameter.

99. Kern's Planè Table.—A more elaborate instrument is made by Messrs Kern & Co., of Aarau, Switzerland, who make a speciality of instruments for *precise work*.

Fig. 43 shows the alidade of latest construction, (smallest pattern) specially for topographical surveys in mountainous parts. The telescope, 23—190 mm., magnifying 22 times, is reversible at the eye-piece end. The vertical circle of 14 cm. diameter is fixed on the telescope axis, whilst the vernier turns around same with the collimation level

Position of the vernier corresponds with 45° inclination between horizon and zenith to eye-piece. The divisions on the periphery of the circle gives direct reading from the eye-piece. Heavy division $360^\circ \frac{1}{2} 19-20-1'$ or $400^\circ \frac{1}{2} 19-20-1'$. The rule is 43 cm. long with parallel rule. There is a loose level on telescope and a collimation level on the vernier-arm. There is also another level at the foot of pillar.

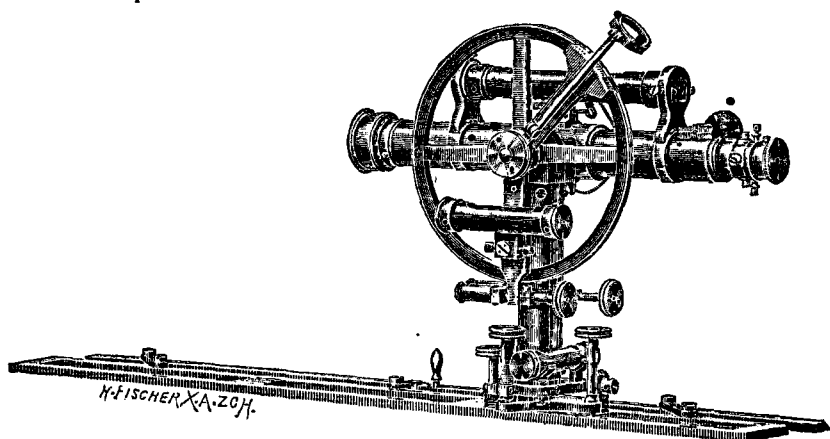


Fig. 43.—Kern's Plane Table Alidade.

There are also medium and large patterns of the latest type with telescopes 25—245 mm., 27—325 mm., magnifying 24 and 26 times respectively, vertical circle 14 cm. diameter. Rule 50 and 60 cm. with parallel rule. In other respects same as the smallest pattern. The above heads with stadia lines 1 : 100 on glass, or fixed hair distance-measurer, 1 : 100, are also available.

100° Kern's Plane Table Stand.—Corresponding to the 3 patterns of the alidade as above described there are 3 patterns of the plane table stand (Fig. 44). The stand-head consists of a metal hexagon the three levelling screws of which serve for the horl. adjustment of the triangle, on which the board is screwed with handscrews. The pivot is strong for the horl. adjustment with clamp and micrometer screw. The most suitable sizes of the board are 42×48 cm., 54×60 cm., and 60×66 cm., with leather caps for protecting the metallic parts.

Fig. 44 shows such a plane table stand head with the large pattern alidade of the latest type. The instrument is provided with Prof. Wild's calculating slide and long type orienting compass the magnetic needle of which is suspended in a sapphire centre and projects on both sides somewhat over the circle divisions. To avoid parallax in the needle reading, two sided mirrors are fitted under the divided circle sectors, the correct reading of the needle position takes place when the mirror image of the needle is covered. The instrument is provided with a plumbing fork and paper clamps for stretching the paper on the board. There is also a level with best ground tube on the rule. The rule and mount are all made of brass.

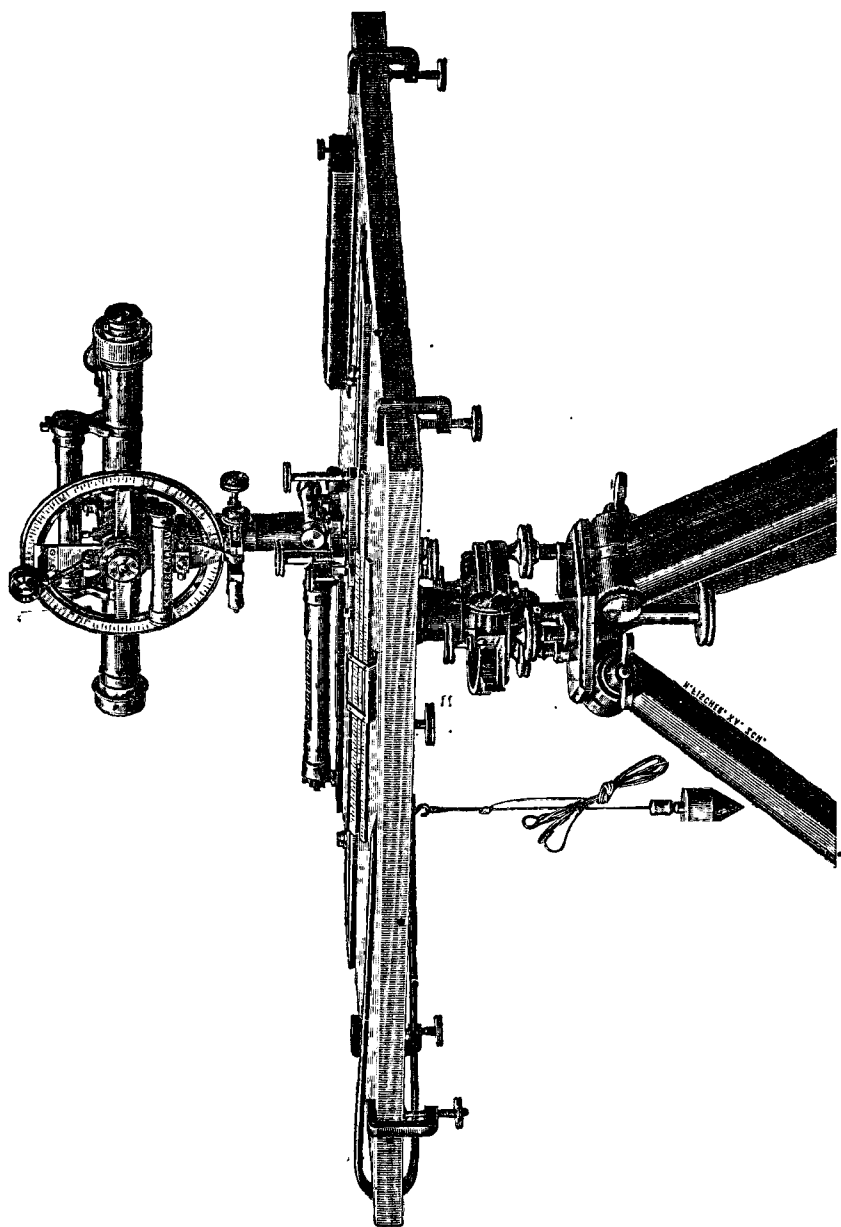


Fig. 44.—Kern's Plane-Table with Stand and Alidade.

The students, who will care to follow the subject further, are referred to the following papers :—

Paper by Mr. J. Pierce, Jun., read before the Institution of Civil Engineers, February 1888, *Proc. Inst. C. E.*, vol. xciii., part iii., paper No. 2308. See also *Military Surveying in the Field*, by Major The Hon. M. G. Talbot. *Prof.* ; *Papers Royal Engineers*, vol. xiv., p. 25.

101. Military Sketching Board.—This makes a very convenient exploring sketching board, buckled on the wrist, permitting sketches to be made on horse back *en route*. It consists of a board (Captain W. Verner's) $9\frac{1}{4}'' \times 7\frac{1}{2}''$, at two sides of which there are small rollers to hold paper 7" wide, and from 3' to 6' long, according to its thickness. Two indiarubber bands passing over the board hold a straight-edge to scale in any position on the paper with sufficient firmness to be able to draw a line against it. The small compass attached to it shows direction. After one sketch is made, a fresh part of the paper is rolled forward.

Fig. 45 shows the board as made of well-seasoned Spanish mahogany. The rollers are solid with a thin slit only to allow the

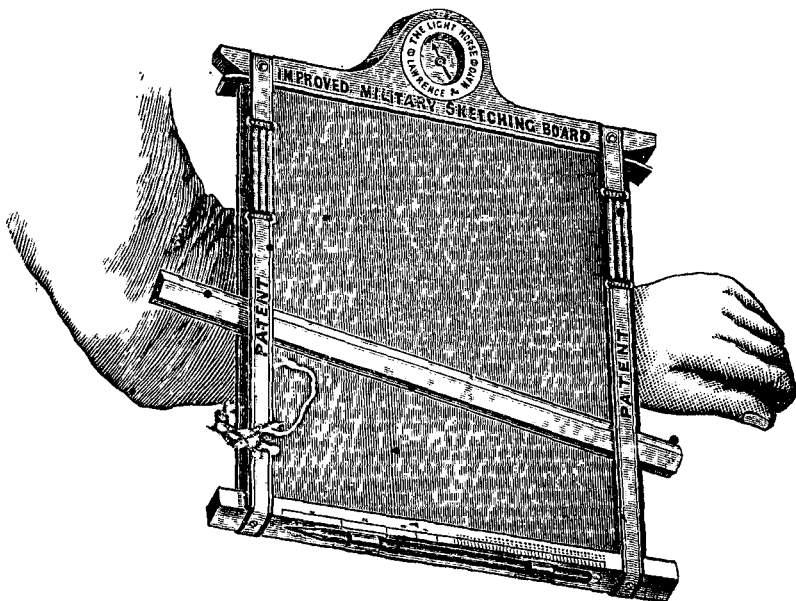


Fig. 45.—Military Sketching Board.

threading of the paper. A further improvement has been added to the sketch-board, by substituting leather straps with nickel-plated spiral springs, for the rubber bands usually supplied, which soon become useless in tropical climes.

A pair of pencil clips are provided for the reception of a pencil.

For a detailed description on sketching with the Military Sketchboard, see p. 41, Gribble's "Preliminary Survey."

102. Waterproof Case.—The surveyor should provide himself with a waterproof case into which he can slip his plane table in the event of heavy rain.

103. Tin Case.—A tin case should always be used for keeping papers mounted on very thin cloth, and cut to the size of the plane table. These papers will be found serviceable, as it will not easily tear, and can be rolled up and kept in the tin case until wanted.

104. A Box Sextant is a valuable adjunct for plane-tableing, as in certain cases the objects may be so crowded in one direction as to confuse the rays if they are all drawn on the table. Angles observed and booked in a note-book can be plotted afterwards when working up the plan in the tent.

105. The Theodolite is an alt-azimuth instrument which is chiefly used to measure and set out horizontal and vertical angles and for ranging out lines.

Theodolites are of various modes of construction. No idea can be formed of the theodolite from figures given, as it is a somewhat complicated instrument; and the beginners are often confused by the number of its parts. But if the student will compare the description with the instrument before him, he will find it quite simple. He should first of all learn the names of the several parts, and then their use by handling the instrument. The theodolite should be used very carefully and gently. Force or pressure should never be applied to any of its parts, specially when taking it out of, or packing it in the box. Each maker's instrument varies in small details of construction, but the main features in all are the same.

106. Description of the Transit Theodolite.—This instrument (Fig. 46) is usually made of gun-metal or aluminium except the limb or graduated ring of each of the $\odot$ s, which is made of silver or platinum. Its vertical and horizontal $\odot$ s are of the same diameter. The various parts of the instrument, as made by Messrs Elliott Bros., are:—

i. The Telescope, *ab*, consists of two tubes, one sliding inside the other. The inner tube has, at its end, *a*, the *object-glass*, which forms at its focus an inverted image of the object looked at. The outer tube has, at its end, *b*, the "*eye-piece*", which magnifies the inverted image. In these instruments, "*erecting eye-piece*" may be used, which makes the object appear erect: but this gives less light and secures no particular advantage; and is formed by the use of an additional tube, and certain additional glasses. By shifting the inner tube in and out by a rack and pinion, turned by the milled-

head screw on the other side of the telescope, the foci of the object and eye-glasses are adjusted till they coincide, when the image becomes steady and distinct.

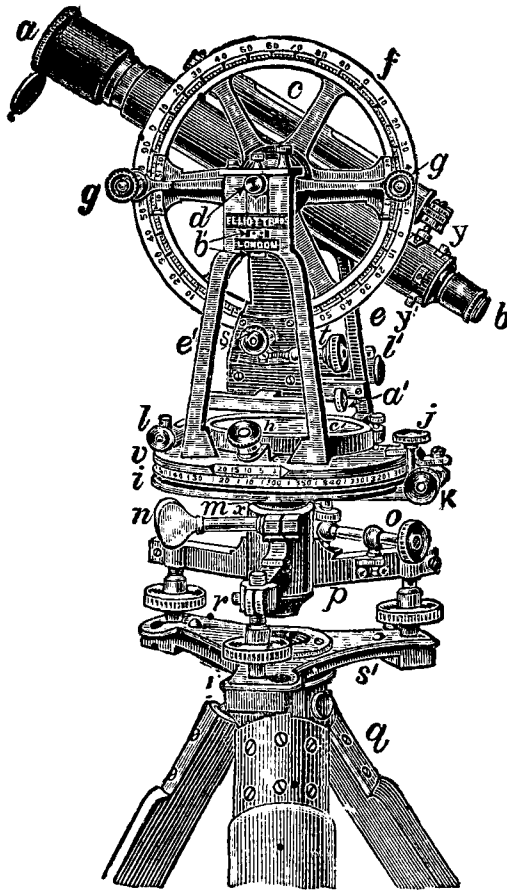


Fig. 46.—Elliott's Transit Theodolite.

The Diaphragm (Fig. 47) is either a glass with fine lines etched on it, or a brass ring with a round hole in the middle crossed by 3 spider's lines or equally fine platinum wires, one, *ab*, horizontal, and the other two, *cd*, *eg*, inclined, and is placed at the common focus of the object and eye-glasses where the inverted object is seen. The point *f* of intersection of the wires should be in the line of collimation of the telescope, and marks in the field of view a fixed point which is slowly made to coincide with a point of the image, thus making

the observation practically exact. The four screws, y, y', y, y' , are for adjusting it to that position in Figs. 46 and 47. Stadia-hairs should be added as shown at, s, s . (Fig. 47). For 'Line of Collimation,' see p. 67.

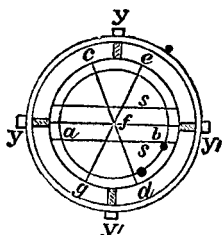


Fig. 47.—Diaphragm.

ii. The Bubble-Tube, c , (Fig. 46), is fixed to the outer telescope-tube by the 'capstan-screws, by means of which it can be set exactly parallel to the line of collimation; so that when the bubble retains the centre, the collimation line of the telescope is horizontal.

iii. The Horizontal or Transit Axis, d , (Fig. 46) of the telescope is exactly at right angles to the line of Collimation and exactly level, when the instrument is in adjustment; so that the telescope may revolve round it in a truly vertical plane. To effect this, the transit axis is raised or depressed by means of adjusting-screws, b', b' . The transit axis is conical in shape, and its larger ends are fixed to the outer telescope-tube; its smaller ends are shaped into equal cylinders, called *pivots*, which rest on the Y 's formed at the top of the vertical frames, and are held down by cleats.

iv. The Supports, e, e' , (Fig. 46) of the horizontal axis, are high enough to admit of the telescope being revolved round completely in a vertical plane, a motion which is useful in making certain observations.

v. The Vertical or Altitude Circle, f , called the *Vertical Limb* or *Face*, (Fig. 46), and fixed upon the transit axis, is divided on one side into 4 quadrants each way from 0° to 90° . The two zeros are at the ends of the diameter parallel to the line of Collimation of the telescope; and the two 90° s at the ends of the diameter $\perp$ to the former. The vertical limb is sub-divided by the verniers at the opposite ends of a horizontal bar, and read by the microscopes g, g . The bar is centred on the transit axis, and is slightly movable only by the clip-screws, d, d , when the vertical clamp is released. When the line of Collimation is horizontal, each of these verniers reads 0° . The axis, d , must rest, in a position truly parallel to the horizontal limb, upon the supports, e, e' , so as to be horizontal when the horizontal limb is set truly level; and the plane of the limb, f , should be accurately $\perp$ to its axis. The *face* being attached to the transit axis rotates with the telescope. On the vertical limb the verniers are fixed, and the arc movable, i.e., the arm ending in verniers remains fixed, while the telescope revolves with its vertical limb, for the vernier-arm is fixed $\perp$ to another adjustable vertical arm, the lower end of which is clamped by clip-screws, d, d , against a stud in the vertical frame. Above the clip-screws this arm carries also the clamp and slow-motion screws of the vertical $\odot$ which are used as under:—

In directing a telescope to any object, it is turned at first by the hand approximately in the required direction, then the vertical $\odot$ with its horizontal axis is *clamped* by the *clamping screw*, s ; and then

- | | |
|--|---|
| $\alpha'\alpha'$ —Clip screws for vertical verniers. | l —Bubble tube. |
| ab —Telescope. | l' —Lower plate. |
| c' —Trough compass. | m —Vertical axis. |
| d —Horizontal axis. | o —Tangent screw for lower plate. |
| ee —Supports of the horizontal axis. | q —Tribrach. |
| f —Vertical circle. | r —Locking plate. |
| gg —Vernier reading microscopes for vertical circle. | t —Tangent screw for vertical circle. |
| hh —do. for horizontal circle. | v —Vernier plate. |
| j —Clamp for vernier plate. | x —Collar. |

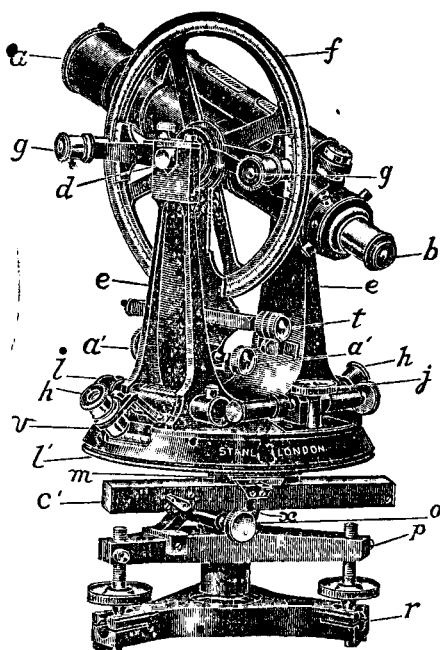


FIG. 46A.—Stanley's Transit Theodolite.

by the *slow-motion* or *tangent-screw*, *t*, a slow-motion is given to the $\odot$ and the telescope, until the line of collimation points towards the object.

vi. The Vernier or Upper Plate, *v*, (Fig. 46) is a round plate, fixed on the top of, and $\perp$ to the *inner vertical axis* (concealed in the figure). The supports, *e*, *e'*, of the transit axis are fixed on it at its sides; in its centre is the glass-topped compass-box, *c*, and near its edge is a bubble-tube, *l*, set at rt. $\angle$ to another bubble-tube, *l'*, fixed on the frame supporting the vertical limb.

vii. The Horizontal $\odot$ or Lower Plate, *i*, (Fig. 46) with the plate, *v*, is called the *horizontal limb*. The plates, *v*, and *i*, are concentric, and fit accurately one upon the other. The horizontal $\odot$ projects a little beyond the vernier-plate, and its projecting edge is sloped off or chamfered, and graduated at every $\frac{1}{2}^\circ$. At two points on its edge, diametrically opposite to each other, the vernier-plate has portions of its edge chamfered off, so as to form with the chamfered edge of the lower plate continued portions of the same conical surface. These chamfered portions of the upper plate are graduated to form verniers, by which the limb is subdivided to single minutes, and by estimation with the eye to half and quarter minutes. In recently made instruments the verniers read to $30''$. The degrees in the lower plate are numbered continuously round it towards the right up to 360° (in the direction of the motion of the hands of a watch). The verniers are read by microscopes, *h*, *h*. An arm attached to the vernier-plate carries the *clamp*, *j*, for clamping the upper plate when the telescope has been turned approximately by hand towards an object, and the *tangent-screw*, *k*, for giving the vernier-plate a slow-motion, until the line of collimation points exactly towards the object. The illustration given is that of a theodolite of 5" horizontal $\odot$ s, which has two such verniers 180° apart. The instruments are styled according to the size of their horizontal limbs. Thus a 5" theodolite means one, the diameter of whose horizontal circle is 5".

viii. The Outer Vertical Axis, *m*, (Fig. 46) fixed to the lower plate, is a conical tube containing within it, and accurately fitting the inner vertical axis. It turns round in a socket in the centre of a tribrach, *p*, and is clamped in any required position by means of a collar, *x*, with a tightening-screw, *n*. The *slow-motion* or *tangent-screw*, *o*, gives a slow-motion in azimuth to the lower plate when the outer vertical axis is clamped. The fixed nut of this screw is fixed to the tribrach. To the internal axis the upper or vernier-plate of the horizontal limb is fixed, and thus, while the whole limb can be moved through any horizontal angle required, the upper plate only can also be rotated through any desired angle, when the lower plate is fixed by the clamping screw, *n*, which tightens the collar, *x*; or in other words—both the plates have clamping and slow-motion screws, by means of which the lower plate may be clamped leaving the upper free to rotate on it, or the upper may be clamped on the lower which can move freely carrying the upper in its motion.

ix. The Tribrach or Upper parallel Plate, *p*, (Fig. 46) carrying at each end of its arms a levelling-screw which serves to place the vertical axis truly vertical, *i.e.*, to set the horizontal limb truly horizontal, supports the instrument on the tripod stand, *q*. The vertical axis passes through the tribrach and is clamped to it. The lower end of each screw has a conical knob which fits into a pear-shaped hollow slot, by which and by sliding the locking-plate, *r*, by means of a spring it is held down to the Δr lower parallel plate, *s*, which forms the top of the stand-head.

In the middle of the under side of the stand-head or directly under the vertical axis, and passing through a hole in the Δr plate, is screwed a hook from which a plummet is hung in order to ascertain whether the centre of the theodolite is exactly over the station on the ground.

In the older instruments, there are two parallel plates and 4 adjusting foot-screws which are liable to jam when screwed up far. They are to be adjusted with both hands, two screws at a time. A tribrach stage permits the instrument being used on a wall without the legs and gives no strain on the axis.

x. The Stands of smaller theodolites are generally movable and about 5' in length. The ground on which the theodolite-stand is fixed, should be firm, and when loose, the way to remedy the evil is to drive 3 tent pegs nearly flush with the ground, and to rest the stand on them; this will secure the isolation required. Stands are either 3 ∇ -shaped or framed tripods, which latter are the best to use.

xi. Detached Parts of a Theodolite.—A plummet is supplied in the box of the theodolite and is made to hang from the hook by means of a soft silk cord looped to hold in the hook. Fig. 48 shows a plummet which has a contrivance for shortening the cord. This is effected by concealing a reel, *R*, around which the cord is wound by turning the milled-head, *K*, on top. The friction upon the reel will hold the plummet at any desired point of the line. In ordinary plummets a hollow milled-head, with one end of the cord knotted at the screw end of it on entering at the top, is screwed on them. The plummet cord is adjusted to the station mark by forming a loop in the cord by passing it through two fine holes in a small ivory bar and knotting at the other end of the cord. The friction of the cord by thus passing through the bar permits extension and contraction of the loop for the adjustment of the plummet.

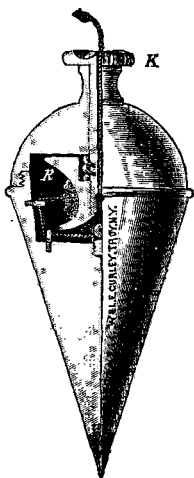


Fig. 48.—Gurley's Plummet. xii. Two Screw-Drivers, a tommy pin and a camel-hair brush are also supplied with the case of the theodolite, the pin to turn the capstan-heads. A piece of

wash-leather, a bottle of vaseline, and a bottle of watch oil will be found useful.

107. Stanley's New 6" Telemetrical Theodolite.—This is (Fig. 49) unquestionably a great improvement upon the general

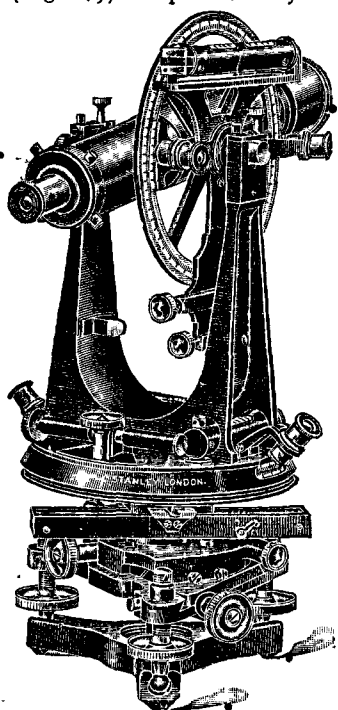


Fig. 49.—6" Telemetrical Theodolite reading to 20'.

construction of transit theodolites as the work is not built up so much in separate castings and pieces, as is usual, but every possible casting is shaped out of the solid to the finished form: It has the following improvements:—A Tribrach stage instead of 4 parallel screws, which permits the instrument being used on a wall without the legs, facilitates setting up, and prevents all possibility of straining the centre. By recent arrangement the Tribrach is made quite solid without spring plate, and so can not get out of order. The instrument has a mechanical stage with two adjusting-screws at right angles to the foot-screws for shifting the centre with exactness over the desired spot. The size of the bearing on the centre has been increased, and the telescope supports and the centre are in one casting. The diaphragm is fitted at the option of the purchaser

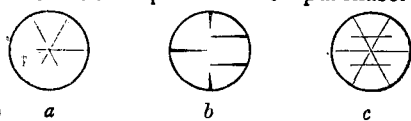


Fig. 50.—Diaphragms of Telemetrical Theodolite.

with (a) Webs, (b) Platinum-iridium points, or (c) lines on glass (Fig. 50). The two latter are set for stadia reading, 1 : 100, so that the distance of the staff is read in the telescope without further measurement. The points do not cover up the divisions of the staff and are permanent in all climates. The head of the tripod is of an improved form, giving greater rigidity.

108. In Cooke's and Stanley's instruments, the level is not fixed to the outer telescope-tube, but attached near the vertical vernier-arm. The graduations are also covered. There are two levels at right angles to each other attached to the upper plate.

109. For ordinary surveying purposes a 5" theodolite, divided into degrees, and $\frac{1}{2}^{\circ}$ or $\frac{1}{3}^{\circ}$, and reading by two verniers to single minutes and $\frac{1}{2}$ minutes, is the most useful size. For the survey of railways, and for ranging curves a 6" instrument divided into degrees and 20 minutes, and read by the verniers to 20 seconds, is

useful. 5" and 6" Transits without vertical circles, with long telescope and centering apparatus to the stand, are used also for railway surveys.

110. Centering Plates.—Several improvements have been

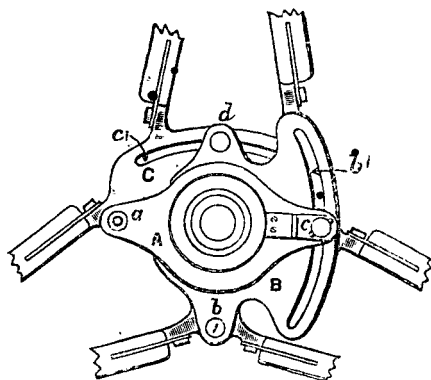


Fig. 51.—Plan of Centering Plate to Stand.

made in accurately adjusting the vertical axis of the instrument over a station by means of centering stands, whereby the plummet may be adjusted nicely. Figs. 51 and 52 show Messrs Troughton & Simms' arrangement for adjusting the instrument bodily over any required station. The plate, A, to which the theodolite is attached, works upon a pivot at *a*, and is clamped to the plate, B, by the hand-screw, *c*, which moves in the segmental slot, *b'*. The plates, A and B, thus clamped together may be rotated again round the pivot, *b*, and are clamped by another hand-screw at *d* underneath the plate, C, which moves in another segmental slot, *c'*, in the lowermost plate. C, to which the legs are attached. Fig. 51 is a plan of the stand-head, applicable to framed stand. This arrangement admits of two motions at right angles to each other, and permits a movement of about 3" each way, thus admitting of the plumb-bob being adjusted to a hair's breadth.

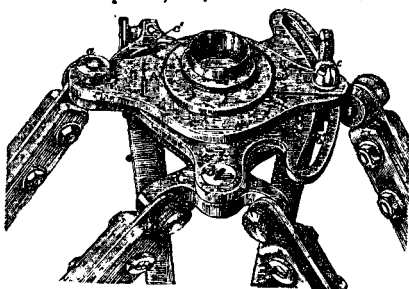


Fig. 52.—Perspective View of Centering Plate.

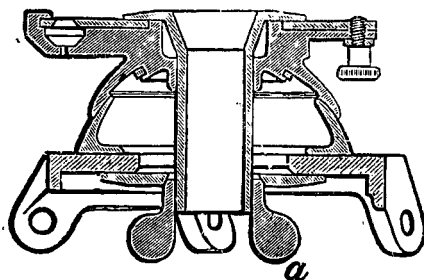


Fig. 53.—Section of Cooke's Centering Arrangement.

The same eminent firm has also an arrangement adapted to the ordinary tripod stand by which the stand-head can be moved laterally by about an inch.

Figs. 53 and 54 show Littlejohn's combined centering and levelling arrangement to the stand of 5-in. or 6-in. instruments. By the aid of the thumb wing-screw,

α , the stand-head can be moved in any position without shifting the legs of the stand.

111. Sliding Adjustable Telescopic Tripod.—These tripods can be adapted for levels and theodolites. Each leg is provided

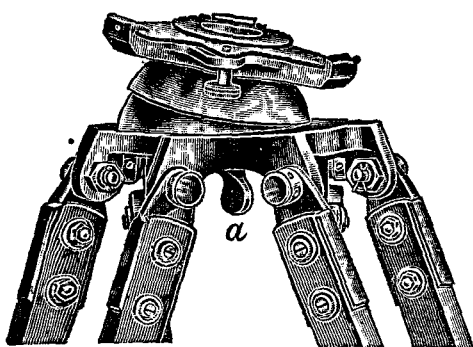


Fig. 54.—Cooke's Centering Arrangement.

with the sliding adjustable arrangement between two side pieces, and can be fixed between them securely in any position or height desired by means of large thumb wing-screws. Particularly in hilly or uneven ground the manifest advantages of this form of tripod will be apparent (Fig. 55). It is strongly shod.

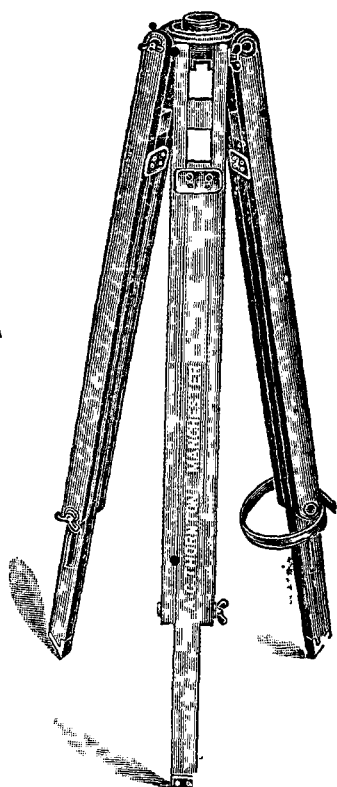


Fig. 55.—Sliding Adjustable Telescopic Tripod.

112. Description of Cooke's Pattern Everest's Double Arc Theodolite (Fig. 56).—The parts are of the same description as that described for the Transit Theodolite, with the exception of the following:—

In the Everest's theodolite the vertical limb is mounted in a way similar to that of the transit theodolite. Over the vertical axis a bar is laid, whose ends are bent to U-shape to form the supports for carrying the pivots of the horizontal axis of the telescope; the vertical axis is connected with the vernier-plate carrying the verniers, and the supports of the horizontal axis of the telescope are made low for the sake of compactness; the telescope can not be revolved completely on its horizontal axis as it can be in the transit theodolite; but it may be turned completely over, when required, by lifting the horizontal axis out of its bearings, and replacing it after turning end for end. Instead of a complete vertical circle, there are two

opposite sectors of about 105° each so as to enable a surveyor to measure *altitudes* and *depressions* as far as 45° and 60° respectively.

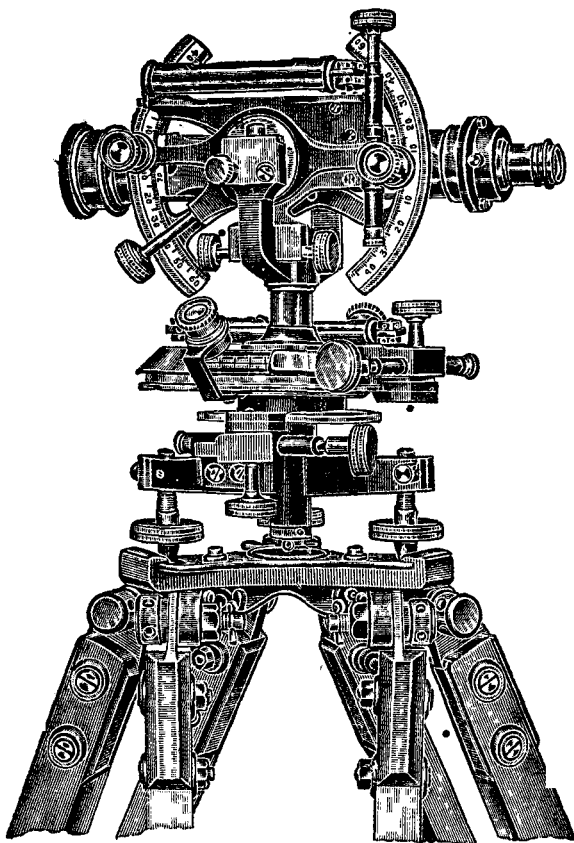


Fig. 56.—Cooke's Pattern Everest's Double Arched Theodolite.

The bubble-tube is not fixed on the top of the telescope, but on a plate fixed to the vernier-arm. Over the telescope a trough-compass may be attached when necessary; the arrangement of the vertical verniers and clip-screws is like that of the transit theodolite. The horizontal limb of this instrument is similar to that of the Transit but instead of two it has three equidistant verniers at the edge of the vernier-plate marked A, B, C, for reading the horizontal angles. There are similar *clamping and slow-motion screws* for fixing the upper and lower plates. There are two spirit-levels at right angles to each other fixed on the vernier-plate by capstan-screws by which they are adjusted so as to be parallel to it. These theodolites are much more light and portable than the transit instrument.

113. The Old Pattern Everest Theodolite (Fig. 57) has a flat horizontal limb which consists of the lower plate graduated at its $\odot$ ce. The index is formed with 4

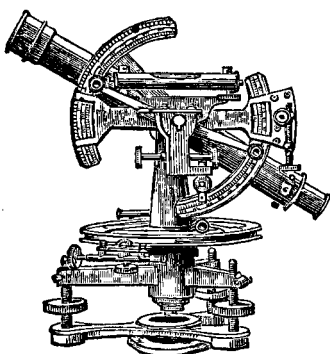


Fig. 57.—Negretti Zambra's Everest Theodolite with Flat Horizontal Circle.

radiating bars having verniers at the ends of 3 of them marked A, B, C; the fourth carries a clamp to fasten the index to the horizontal limb, and a tangent-screw for slow-motion. The radiating vernier-arms are diverging from the top of the inner vertical axis at equal angles of 120° . Instead of 2 bubble-tubes on the vernier-plate, there is one fixed parallel to the horizontal axis on the flat bar which carries the vertical supports of the horizontal axis of the telescope. The theodolites generally made are the 4, 5, 6, 7, 8, and 10 inch ones depending entirely on the degree of accuracy required.

114 Troughton & Simms' Pattern Everest's Double Arc Theodolite is similar to the old flat pattern with the exception that

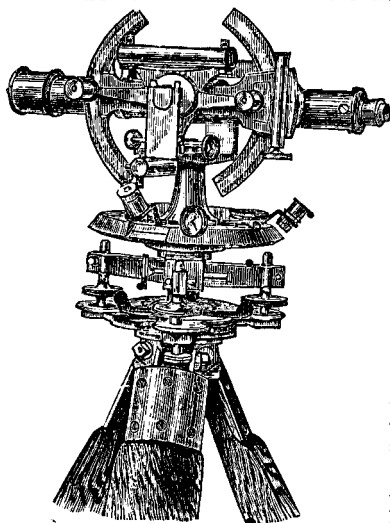


Fig. 58.—Troughton & Simm's Everest Theodolite with Sloping Horizontal Circle.

the horizontal circles are bevelled as in Cooke's pattern, and the graduations are covered (Fig. 58), but it differs from the Cooke's pattern in that it has one horizontal level parallel to the horizontal axis on the bar which carries the vertical supports of the horizontal axis of the telescope.

For executing traverse lines in Revenue Surveys of nearly flat districts 5" instruments without vertical arcs are used in this country.

115. Description of the Y — Theodolite. — In this plain

theodolite (Fig. 59) the telescope is not fixed in the middle of the horizontal axis as in a Transit or Everest theodolite, but is supported in two Y's, at the ends of a bar which is fixed at right angles to the horizontal axis, and on the top of the telescope can be turned end for end by lifting it out of the Y's. When not necessary to be lifted out, the telescope is secured firmly in its Y's by two semicircular arcs, called *clips*, which are hinged to the Y's at one side, and fastened with pins at the other. Instead of a vertical $\odot$, there is a semi- $\odot$ only, having but one index and vernier fixed

below the horizontal bar. The horizontal axis can be fixed by a vertical clamping screw, and the vertical limb can then be moved

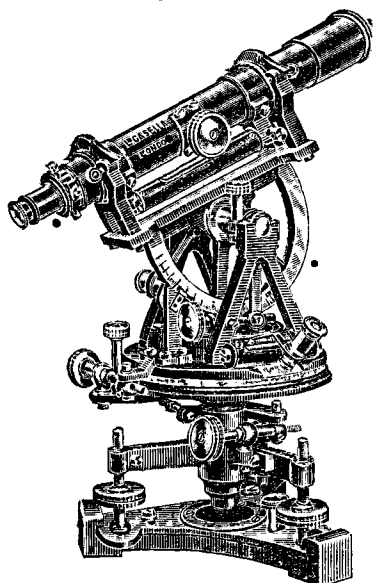


Fig. 59.—Casella's Y-Theodolite.

through a small space by the slow-motion screw. The vertical limb is graduated on one side at every 30 minutes, each way from 0° to 90° , and sub-divided into single minutes by the vernier, which is fixed to the compass-box. Upon the other side are engraved the number of links to be deducted from each chain, for various angles of inclination, in order to reduce distances, measured on ground rising or falling at these angles, to the corresponding horizontal distances. The axis of this limb must rest in a position truly $\parallel$ to the horizontal limb, upon the supports, so as to be horizontal when the horizontal limb is set truly level, and the plane of the vertical limb must now be $\perp$ to its axis. Underneath the telescope is a bubble-tube, attached to it at one end by a joint, and at the other by a capstan-headed screw. In other respects it is similar to the Transit theodolite. It has either a tribrach and locking plate with 3 foot-screws or two parallel plates with four foot-screws for adjustment on the stand-head which is supported by 3 wooden legs.

In the Everest and the Y theodolites the great disadvantage is that the telescope can not revolve in a truly vertical plane; for when producing a line from a back point, the telescope has to be taken out of its bearings, and replaced with the ends reversed; and then the instrument is liable to be disturbed. This may be avoided by moving the vernier-plate through 180° .

116. Clamp and Tangent Adjustment of Surveying Instruments*.—Before the invention of the *clamp and tangent-screw motion* by Helvetius, the famous Danzig astronomer (about 1650), the vernier-reading to the circle could not be taken at its real value, when it was adjusted by the hand.* By this mechanical adjustment, the circle is left free to rotate about its axis, until the clamp is screwed down, which then fixes it firmly. The fixing device of the clamping-screw is attached to a substantial part of the instrument, but is so constructed that when it is clamped it may yet be moved without releasing, in relation to the fixed part of the instrument, by

* Taken from Mr. Stanley's Surveying Instruments.

the tangent-screw which, as its name indicates, is placed in a direction tangential to the circle or arc.

The form, common to vertical circles and arcs generally, of a theodolite, arc of sextant, circles upon some mining-dials, and pro-

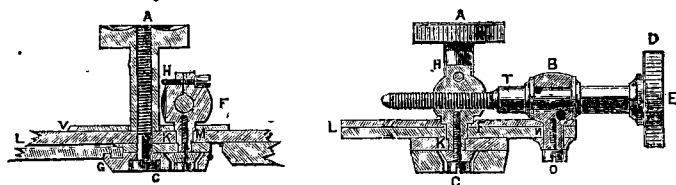


Fig. 60-61.—Sections of Clamp and Tangent in two Directions.

tractors, is illustrated by Figs. 60, 61 which show a clamp and tangent motion in two sections at right angles to each other. Fig. 62 is partly a front elevation of the same, but with part of the clamp screw, A, broken. The stem of the tangent-screw is shown in section at E. In all the figures, L is the limb of the circle or arc. This has a groove at its under side at G, which has been omitted from the cut, into which a fillet of the clamping-piece, C, is inserted to make the clamp slide freely about the periphery of the circle when the clamping-screw, A, is loose. A spring is sometimes inserted to open the clamp between the sliding piece, K, and the clamp, C, and part Section of clamp FF, (Figs. 60, 61) is the tangent-nut to E. and Tangent.

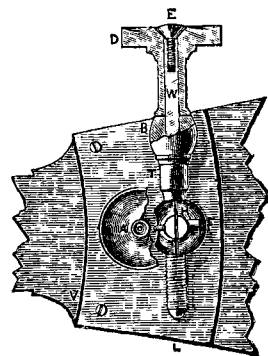


Fig. 62.—Elevation between the sliding piece, K, and the clamp, C, and part Section of clamp FF, (Figs. 60, 61) is the tangent-nut to E. and Tangent.

This nut is sawn down and has a cross-screw to keep sufficient tightness to prevent *loss of time*, and yet to allow the tangent-screw to work smoothly at the same time that it holds the circle and vernier quite dead to the position to which it is adjusted by the screw. The tangent-nut, F, has to move to the direction horizontal to the plane of the tangent-screw; therefore it has an axis vertical to the plane of the clamp. This is shown at K. The axis is held down firmly by a nut and a washer fitted with a square hole, to prevent the nut unscrewing. The tangent-screw has a collar fitting at the tangent boss B, which is turned down from the full-sized metal of the screw. The fellow collar on the outer side of the boss is formed by the shank of the milled-head of the tangent-screw, D. The hole through the milled-head is made square, so that it can be adjusted up to the boss without risk of after unscrewing by the friction of the nut, E. This is generally tightened up by means of a forked screw-driver applied at E. The boss B has a vertical axis, N, similar to the tangent-nut, which is attached to a solid part of the instrument by the washer and nut shown at O.

117. Axis Clamp and Tangent.—Fig. 63 shows the arrangement which is generally used to bring the horizontal axis of an instrument to bearing, and is made independent of the circle and vernier.

This form is adopted for clamping the vertical axis of a theodolite, mining-dial, Y-level, and some other instruments. The clamp C forms a collar round the axis, from which two lugs in the same casting, are projected at *a*. These are tightened upon the outer axis socket, B, by means of the clamping-screw, W.

—The tangent-screw adjustment is illustrated at T, moved by the milled-head, M, the boss, E, being fixed to the instrument. This part is exactly similar to that described above for a vernier tangent. This form of clamp tends to become weak after a time from the constant clamping and releasing, which causes loss of elasticity in the metal. This weakness occurs no doubt from the metal of the clamp not being good gun-metal, or if brass, not thoroughly pressed or hammered before the piece is made up.

118. German Plan of Clamp and Tangent to Vertical Axis.—A common German plan of avoiding the above supposed

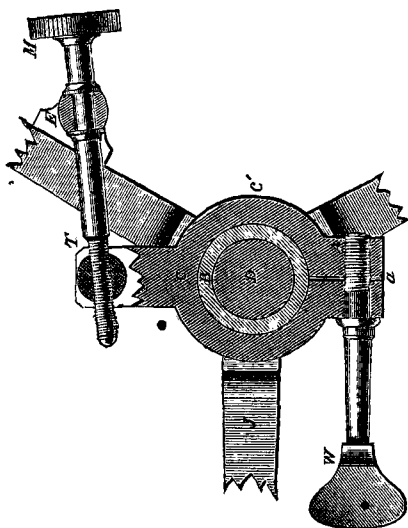


Fig. 66. — Clamp and Tangent to a Vertical Axis.

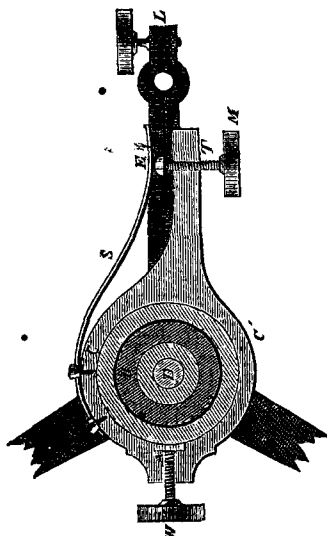


Fig. 64. — Clamp and Tangent (German Plan.)

source of weakness is to bring up a *tumbling piece* direct on the axis by a screw. This is shown in Fig. 64 by screw, W; tumbling piece, A, which produces a direct clamp upon the axis socket B. The clamp-ring, CC', is made loose on its socket. Mr. Stanley found the tumbling piece clamp, Fig. 64, although it holds firmly, disturbs the centre much more than the clasping clamp, Fig. 63. Therefore when the former is used the clamp should be upon a strong flange.

The arrangement (Fig. 64) shows also a spring S falling upon a stud at E, fixed upon a part of the instrument upon which it acts as a fulcrum. The spring should be of hard rolled German silver. In this case the tangent-screw needs no split or other adjustment to

make it tight, as all *loss of time* is taken up by the spring. This plan is found practically to answer fairly; but unless this is very carefully made there is a want of solidity in the movement which a well-fitted, direct-acting tangent-screw possesses. •

119. French Axis Clamp and Tangent.—The French generally in all their superior instruments clamp upon a flange carried out from the lower rim of the socket, with the screw placed longitudinally to the axis. When this plan is very carefully carried out, so that the clamping has neither tendency to raise or lower the socket-piece, it is no doubt very good. In large instruments, where weight is no object and the flange may be made large, it is certainly

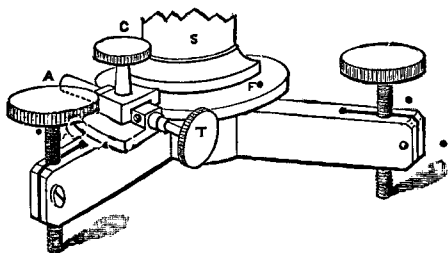


Fig. 65.—French Axis Clamp and Tangent.

the best plan. In such cases the clamp may be released as a free fitting to prevent the possibility of strain. Fig. 65 shows the French plan attached to a tribrach; *S*, socket, *F*, flange, *C*, clamping-screw, *T*, tangent-screw. The tangent in this arrangement acts against a spiral spring contained in a tube *A*, which gives a very steady motion to the instrument.

120. Testing and Adjusting the Fitting of the Tangent-Screw.—The clamp should be tightened down, and the ball, *B*, (Fig. 61) held tightly between the thumb and forefinger; then, by using a gentle reciprocating motion in the direction of the tangent just sufficient to move the circle, if there is any looseness in the screw or ball fitting, *B*, it will be felt as a jar, or technically, a slight *loss of time*. If this is in the ball, *B*, it can be taken up by the screw, *E*, at its end. If it is in the screw, it can be taken up by the cross clamp-screw. If it is in neither of these, it may be in one or both of the axes, *N* and *K*. In this last case it will need refitting.

121. Protected Tangent-Screw with Helical Spring.—The wear of a tangent-screw is due principally to the fact that this

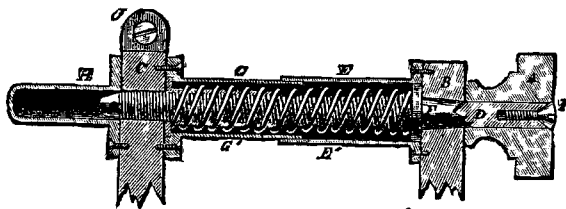


Fig. 66.—Protected Tangent-Screw with Helical Spring.

screw is necessarily oiled to make it work freely, and that the oiled part being exposed to dust, this dust attaches itself and works into the thread with the oil so as to cut both the screw and the nut. *A*

good plan, common in American instruments, to prevent wear from dust, is to insert the end part of the screw beyond the nut in a closed tube. This plan, combined with a spring arrangement by Stanley, renders it safe from loss of time and much wear. This arrangement, being expensive to make, is applied only to high-class instruments. In Fig. 66, *C* is the nut, through which tangent-screw passes; *B*, tangent-boss, *A*, milled-head, *H*, covering tube to the point of the screw, *GG'*, *EE'* pair of telescopic tubes which cover the screw during adjustment. A German silver or platinum spring works inside these tubes, keeping a constant separating pressure between *C* and *B* to take up any loss of time in the screw.

122. Free Tangent-Screw.—There is always a risk of a tangent-screw of any fixed kind producing a certain amount of strain

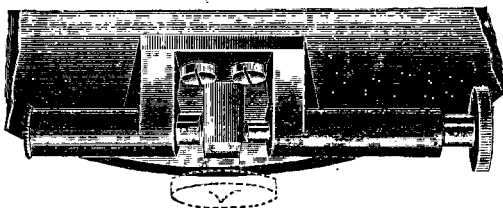


Fig. 67.—Free Tangent Adjustment.

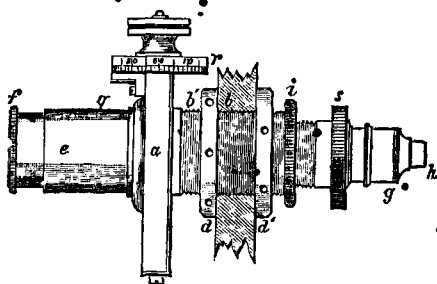
upon the instrument, therefore, where practicable, it should be made free. Fig. 67 shows the form of the free tangent Mr. Stanley applies to many instruments. The centre stud is clamped to the lower part of the instrument by the screw shown in dotted lines.

To the left hand a piston containing a spiral spring carries a pressing-rod against which the screw to the right-hand works.

123. Micrometer Microscopes.—A micrometrical microscope is used to obtain more exact readings than that can be measured by means of the vernier, as in the case of the reading of the circles of 12" or over to seconds. This instrument gives the means of reading the distance from line to line of the division upon the limb by the movement of a web or line set in motion by a fine screw with a divided head. Generally 2, 3, or 5 micrometers are applied to a circle. The structure of the framework of the instrument which carries the micrometers should be made extremely rigid, as it becomes impracticable to read to seconds of arc when there are slight vibrations. The vernier is generally dispensed with when micrometers are used to read the graduations. The circle is graduated to read in 5'.

124. The Micrometer is a compound microscope consisting of 3 lenses, with measuring apparatus at the mutual focus of the field-glass and of the pair of lenses of the eye-piece. The field-glass is generally an achromatic microscopic lens of an inch or more in focus and is placed nearest the graduated circle. The side elevation of Troughton's micrometer is shown by Fig. 68, the longitudinal section by Fig. 69, the micrometrical slide by Fig. 70; *a* is the micrometer, *g* microscope body tube which has a male screw outside at *b'b* upon which there are two capstan-headed collars *d'd*. By means of these collars the microscope is set at any required distance from the limb, on the reading frame *b*, to secure proper focal

adjustment. The objective tube, *g*, screws into the body tube and permits adjustment of the limb and the micrometer index web by the milled head, *s*. The jamming nut, *i*, keeps this tube from being shaken after it is once properly adjusted. *h* is an achromatic object-glass of half-an-inch or over in focus. *e* is the tube into which the eye-piece, *f*, is slid by friction to obtain distinct vision of the spider lines in the micrometer.



**Fig. 68.—Side Elevation of Troughton
Micrometer.**

125. The Micrometer

Fig. 68.—Side Elevation of Troughton Micrometer.

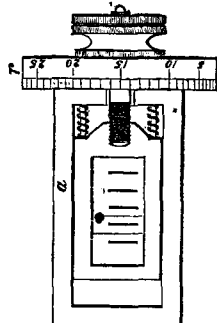
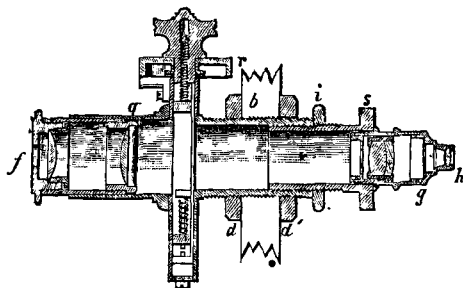


Fig. 69.—Section of Troughton Micrometer. Fig. 70.—Micrometer Slide.

eye-piece. The index web frame is worked by a screw of about a hundred threads to the inch, projected through the centre of the drum from the milled-head. There are two springs, between the index frame and the outer frame, which keep the drum up to its collar. The screw is moved by the milled-head above the drum so that it is not necessary to touch the divided surface of the drum.

The drum *r* is graduated on its edge into sixty equal parts to read seconds generally to a single line of index. The limb of the theodolite is divided to 5 minutes. Five turns of the drum or micrometer screw serve to bring the index web through a division of 5' of the graduated limb; therefore we have

5 turns of drum = 5' of graduated limb.
 $\therefore$ 1 turn " " = 1' " " "
 $\therefore$ 1 division " " = 1" " " "

The fixed scale or *comb* is commonly placed in the focus of the eye-piece with fixed webs upon it, fixed to agree with five turns of

the screw on a rack with points at the side. These racks divide the $5'$ of arc in minutes, and indicate the number of revolutions of the screw, as shown by the displacement of its index line. In modern instruments a pair of webs are placed $1'$ apart to assure certainty of reading by the means of two observations.

126. Adjustment of the Micrometer Microscope.—

(1) Make the diaphragm and spider lines distinctly visible, by putting the eye-piece, f , the proper depth into its cell. Then make the graduated limb also distinctly visible without parallax by turning the collars, d, d' , unscrewing one, screwing up the other, till the desired object is attained.

Now bring the spider line on the micrometer frame upon a stroke of the limb, and turn the micrometer head, r , to zero; then turning the screw through 5 revolutions, if the spider line has not moved over the whole of one of the divided spaces on the limb, the object-lens must be screwed up to diminish the power by rotating the objective tube, g , by the milled-head, s ; if it has moved over more than one of the divided spaces, it must be unscrewed to increase the power, and then altering the position of the microscope, by turning the collars, d, d' , till distinct vision of the limb is again obtained, the measure of the space, moved over by 5 revolutions of the screw, must be repeated as before. When after repeated trials, the result is satisfactory, the collars, d, d' and the jamming nut, i , must be screwed tight-home, to render the adjustment permanent.

(2) When the microscope has thus been adjusted for distance, the zero of the division of the limb must be brought to the spider line of the index frame, and the drum, r , rotated, till zero is pointed to by its index, and then, if the zero on the comb be not covered exactly by the index, the comb must be moved by turning the screw, which enters the bottom of the micrometer box, till its zero is covered by the index. The adjustment of the reading microscope will now be perfect; and the graduated limb to be read by it, being divided at every $5'$, the degree and nearest $5'$ of an observed angle will be shown by the pointer or index to this graduated limb; while the number of complete revolutions, and the parts of a revolution, of the screw, in the order of the numbers upon the micrometer head, r , required to bring the index line upon a division of the graduated limb, will be the number of minutes and seconds, respectively to be added to the degrees and minutes shown by the index of the circle. The complete revolutions, or minutes, to be added, are shown by the number of teeth the index has passed over from zero, and the parts of a revolution, or seconds and tenths to be added, are pointed out upon the micrometer head, r , by its index.

127. Comparison of the Theodolite and Compass.—The compasses are to be used for unimportant or preliminary surveys. It is an invaluable instrument for filling in details.

When, however, it is used for large and important surveys, there

is a likelihood of great inaccuracies occurring. The daily variation of the declination of the needle is nearly 10 minutes. Errors of 10 minutes or more may arise between 8 a.m. and 1 p. m., when the declination of the needle passes from its minimum to its maximum. With a radius of 1, the chord of 10' is '0029'. Therefore, in a length of 1000' there will be a lateral displacement of 2'9".

The compass readings are never exact. With compasses of the ordinary size, the needle cannot be read more accurately than to $\frac{1}{5}$ of a degree. Thus, errors of $\frac{1}{10}$ of a degree or 6 minutes, are unavoidable. In a length of 1000', this error in the bearing gives a lateral error of 1'74".

Magnetic storms and the influence of ferruginous rocks in the neighbourhood of the survey may give rise to considerable error, often difficult to detect.

With the theodolite the results are different. A 5" instrument reads to 30" even without repetition. The chord of this angle being '000145, in a length of 1000' the lateral error will not exceed 145". The accuracy of the theodolite is therefore 31 times as great as that of the compass, while with repetition the theodolite gives more accurate results. With the compass if the bearing is read repeatedly the same error occurs each time.

The compass must remain unmoved when a bearing is taken. The surveyor is therefore obliged to put his eye, his head and sometimes his whole body into a particular position with reference to the instrument. The theodolite gives the reading correctly in any position of the surveyor as the horizontal circle may be rotated to any required position, without changing the angle shown by the vernier.

With the vernier-compass or circumferentor, readings to 3 minutes are only obtained. The chord of this angle being '000872, in a length of 1000' the lateral error will be 872". The theodolite is therefore 6 times more accurate than the circumferentor which is practically a theodolite of a simple and compact form specially adapted for underground surveying. The theodolites are much heavier than the compass and are unsuitable for underground work.

128. **Line of Collimation and other Terms.**—Before describing the methods of adjustment the student should acquaint himself with the terms used in connection with the theodolite and its manipulations:—

(1) The **Optical Axis of a Lens** is the line which joins the centres of the spherical surfaces by which the lens is bounded.

(2) The **Optical Axis of the Telescope** is the straight line which joins the centres of the object-glass and the eye-glass.

(3) The **Line of Collimation or the Principal Line of Sight** is the imaginary straight line which joins the optical centre of the object-glass, with the point of intersection of the cross-wires of the diaphragm, and is perpendicular to the transit axis.

(4) **The Virtual Line of Sight** is the straight line joining the points bisected by the cross-wires after proper focussing and removal of parallax.

(5) **How to Change Face.**—An observer standing at the eye-end of the telescope of a theodolite, Transit or Everest, has its vertical limb on his left-hand over vernier A. The instrument is then said to be *face left*.

When it is necessary to *change face*, it is only necessary to bring the vertical limb on his right-hand over the vernier A.

In a Transit Instrument of which the vertical limb is a complete circle, move the left-hand vernier, i.e., A, 180° in azimuth so as to bring the object-glass of the telescope before the observer; then revolve the telescope round vertically through 180° on its horizontal axis; its eye-glass will now be in front of him and vertical limb on his right-hand.

In an Everest's Theodolite, the instrument being at *face left*, open the cleats which hold the pivots down in the supports, unclamp the two clip-screws, lift the telescope with the vertical limb out of its bearings in the supports, and hold its object-end in the left-hand. Release the vertical clamp, and turn the vertical verniers round 180° , so that they change places and clamp again. Then reverse the telescope end for end in the hand by bringing the level upward, replace it in the supports and the pivots resting in their original places; this will bring the face over the vernier A, and make the released clip-screws pinch. The object-glass end being now before observer, turn the telescope 180° in azimuth; the eye-piece will be before him, and the vertical limb on his *right-hand*. The reading of the verniers will differ by 180° from what they had with *face left*. The compass-box must be removed before a change of face is attempted; otherwise the box will come into contact with the nut of the vertical axis.

(6) 'Changing Pivots,' 'Reversing the Telescope in its Supports,' and 'Inverting the Telescope' mean the same manipulation of the instrument. To do this, go on as in *changing face*, but instead of reversing the telescope, end for end in the hand, replace the opposite pivots in the supports. The object-glass end will remain in its former place, and there will be no change in the vernier readings of the horizontal circle.

(7) **The Zero of Vernier** signifies that division of the lower plate which is shown by the arrow of the vernier A, when the telescope bisects the left hand of the two objects, the angle between which is to be measured. It may not be Zero or 360° , but any graduation, such as $50^\circ 20'$, which is the origin of the readings, and is expressed by saying Zero $15^\circ 20'$; the left hand station is called the Zero station.

129. The Adjustments of the Theodolite may be distinguished into **Temporary adjustments** which are made by the surveyor every time the instrument is set up, and **Permanent Adjustments** which are made by the maker, and only tested and corrected occasionally by the observer when he has reason to think.

that the instrument has been shaken by some means, and the adjustments are out.

130. The Temporary Adjustments will now be described on the supposition that the permanent adjustments are correct. They are described as follows :—

(a) **To Set up the Instrument over a Station.**—Place the theodolite on its stand over the station peg by the aid of the plumb-line suspended from a small hook which is attached exactly under the vertical axis of the instrument by first setting it approximately over the peg by the legs of the stand, and then exactly by the adjusting-screws or sliding-plate. When there are no adjusting-screws or sliding-plate, the plumb-bob is centered over the station point by means of the legs only, shifting and pressing into the ground one leg or other until the bob is exactly over the point. If the levelling up of the instrument disturbs the position of the plumb-bob, re-adjust and level up again.

(b) **To "Level up the Instrument"**—i.e., to place the vertical axis truly vertical, and to make the horizontal $\odot$ s truly horizontal. This is done by making the vernier-plate truly horizontal by means of the bubble-tubes, l, l' . The instrument is to be rotated to such a position, for that purpose, that the two bubble-tubes shall be parallel respectively to the two diagonals of the square formed by the foot-screws (when there are four such screws). Then the bubble is to be brought to the centre of each level by turning the pair of foot-screws to whose plane the level lies parallel. Or if there be 3 foot-screws, rotate the instrument horizontally until one of the levels is parallel to two of the foot-screws. By means of these screws, bring the bubble of this level to the centre of its run, and by the 3rd screw bring the bubble of the other level to the centre of its run. If the bubbles of the levels still retain their centres, on rotating the instrument horizontally through 180° , the adjustment is effected.

A More Exact Adjustment is effected by means of the level, c , fixed to the telescope. To do this, turn the vernier-plate till the telescope is over one pair of foot-screws: by the aid of the tangent-screw, t , adjust the vertical limb to 0° ; turn the pair of foot-screws under the telescope until the bubble retains the centre, turning them both inwards, or both outwards, according as the bubble is required to go to the right or left; turn the vernier-plate round through 180° ; if the bubble now deviates from the centre of the bubble-tube, correct one-half of the deviation by the slow-motion screw, t , and the other half by the foot-screws. Repeat the operation, until the bubble retains the centre when the instrument is rotated through 180° . Now turn the vernier-plate through 90° , so as to bring the telescope over the other pair of foot-screws (when there are four), or over the third foot-screw (when there are three), and by means of this bring the bubble to the centre again: the vertical axis is now truly vertical. If the bubbles of the vernier-plate levels are not

at their centres now, those levels are not truly $\perp$ to the vertical axis; but the correction of this error belongs to the permanent adjustments (Fig. 46).

In Everest's theodolite, the vertical axis can also be adjusted with the level fixed to the bar over the support, or when there is only one fixed on to the vernier-plate, by first placing that level, over two of the 3 foot-screws, and bringing the bubble to the centre by turning them both inwards; or both outwards, according as the bubble is required to go to the right or left; turning the telescope through 90° , and bringing the bubble to the centre of the level in its new position by means of the third foot-screw, turning it to the left or right until the bubble retains the centre. In effecting the last operation, the level will be perhaps thrown out on moving the instrument back to its first position; in which case it must again be levelled by means of the first two foot-screws as before described. When the bubble remains stationary in the centre during a whole revolution, the instrument is said to be level.

In Everest's theodolite, the vertical level, i. e., the level attached to the plate fixed to the vernier-arm is adjusted by means of the two *clip-screws*; and by their means the bubble is brought to the centre of its run, turning them outwards or inwards as it is desired to make the bubble come to the left or right. The clip-screws should always press tightly the shoulder against which they work.

(c) To Adjust the Telescope for the Prevention of "Parallax," i. e., to bring the Foci of the Glasses to the Cross-wires.

Focus and Parallax.—When the image of the object viewed, formed by the object-glass, falls either short of, or beyond the place of the cross-wires, the error arising from this cause is called *parallax*. The existence of parallax is determined by moving the eye about, when looking through the telescope, and observing whether the cross-wires change their position, and are flitting and undefined. To correct this error, first shift the eye-piece in and out by means of the movable eye-piece tube, till the cross-wires are clearly defined, and sharply marked against any white object. To do this, hold the socket of the eye-piece firmly with one hand, and move the eye-piece with a gentle screwing motion with the other. Do not pull the whole instrument by the eye-piece by using one hand only, as it is often stiff. Then by moving the milled-head screw at the side of the telescope, the internal tube is thrust outwards or drawn inwards, until the proper focus is obtained according to the distance of the object, and the object can be clearly seen, and the intersection of the wires, clearly and sharply defined, before it. This operation is called *focussing* the object-glass. The existence of parallax is very inconvenient, and, where disregarded, has frequently been productive of serious error. It will not always be found sufficient to set the eye-glass first and the object-glass afterwards. The setting of the object-glass, by introducing more distant rays of light, will affect the focus of the eye-glass, and produce parallax or indistinctness of the wires, when there was none before:

the eye-piece must, in this case, be adjusted again. Generally, when once set for the day, there is no occasion for altering the eye-glass, but the object-glass will of course have to be altered at every change of distance of the object. The nearer the object, the further must the inner tube be drawn out. In adjusting the instrument, the parallax should be first corrected, and then the error of Collimation.

The temporary adjustments should be made with care before testing the permanent adjustments which are described below.

131. The Permanent Adjustments of the Transit Theodolite are :—

1. Adjustment of the Levels on the Vernier Plate $\perp$ to the Vertical Axis of the Instrument.—(a) This is effected exactly in the same way as the first adjustment of the Everest theodolite described below.

1. (b) The Same Adjustment can also be made by reference to the level which is mounted on the telescope or the vernier-arms of the vertical $\odot$, sometimes called the *azimuth level*. This bubble is of more delicate construction than those on the vernier-plate, so that by its use greater accuracy may be attained. The operation is as under ;—

Adjustment of the Horizontal and Vertical Limbs.—In the case of transit theodolite, set the instrument up as accurately level as possible by the eye by moving the legs of the stand. Tighten the collar, x , by the clamping-screw, n , and, unclamping the vernier-plate, turn it round till the telescope is over two of the foot-screws. Bring the vertical circle carefully to zero by turning the tangent-screw, t . Turn the vernier-plate half round, bringing the telescope again over the same pair of foot-screws, and, if the bubble of the level be not in the centre of its run, bring it to the centre, half-way by turning the foot-screws over which it is placed, and half-way by turning the tangent-screw, t . Repeat this operation till the bubble remains exactly in the centre of its run in both positions of the telescope ; and then turning the vernier-plate round till the telescope is over the 3rd foot-screw, or the other pair of foot-screws (where there are four such screws) bring the bubble again to the centre of its run by turning the 3rd screw, or the other pair of screws.

The bubble will now retain its position, while the vernier-plate is turned completely round, showing that the internal azimuthal axis about which it turns is truly vertical. The bubbles of the levels on the vernier-plate being now, therefore, brought to the centre of their tubes, will be adjusted to show that the internal azimuthal axis is vertical. If the bubble-tubes on the vernier-plate do not retain their centres, they must be brought so wholly by means of the adjusting-screws at either end of the level-tubes. Now, having clamped the vernier-plate, loosen the collar, x , by loosening the clamping-screw, n , and move the whole instrument slowly round upon the external azimuthal axis ; and if the bubble of the level above the telescope maintains its position during a complete revolution, the external azimuthal axis is truly

parallel to the internal, and both are vertical at the same time; but if the bubble does not maintain its position, it shows that the two parts of the axis have been inaccurately ground, and the fault can only be remedied by the instrument-maker. When the horizontal limb is in adjustment, i. e., when the bubbles of the two levels on the vernier-plate remain in the centre of their runs, during all positions of the revolution of either the vernier-plate or the whole instrument around the vertical axis, the bubble above the telescope should be in the centre of its run, when the vertical circle is set to Zero. Should this not be the case, there is an index-error, which must be allowed for in all observations made with the vertical limb.

2. The Adjustment of the Horizontal Axis exactly $\perp$ to the Vertical Axis.—(a) This adjustment is effected by means of the antagonistic screws, b', b' , situated just below one of the Y bearings thus: set up the instrument at about 40' or 50' distance from a high wall or other object of considerable altitude, level it carefully, and direct the telescope to some point situated as high up as possible: and bring the cross-wires to bear exactly upon it. All parallax must now be carefully eliminated by means of the focussing-screw, and by gently moving the eye-piece in or out of its socket, until the cross-wires and the object are brought exactly into the same focus. Then tilt the telescope, and make a small mark on the wall near to the foot, at the point of intersection of the cross-wires. Now turn the telescope 180° in azimuth, transit it, and again direct it to the upper mark: after which tilt it as before, and see if the point of intersection of the cross-wires exactly corresponds to the lower mark. If it does so, the transit axis is truly horizontal, but if not, half the error must be corrected by means of the antagonistic-screws, b', b' , and the operation repeated in order to ensure accuracy.

2.(b) This same Adjustment may be carried out with the aid of a *striding level* (Fig. 71), which is a spirit-level with two upright legs exactly equal in

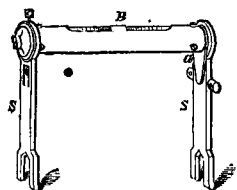


Fig. 71.—Striding Level.

length, fixed to the ends of its bubble-tube: the bottom of the legs being forked, the level can be placed over a round axis like the transit axis, as well as, on a flat surface to test their horizontality. When the theodolite is provided with a striding level the adjustment may be effected thus:—Having levelled the instrument carefully with the aid of the bubbles on the vernier-plate, and noticed that this adjustment is perfect, turn back the caps which cover the ends of the transit axis in its bearings. Now erect the striding level on the top of the axis just over the bearings, and observe it carefully. Reverse it in order to examine its own adjustment, and, if any error be discovered, this must be corrected before proceeding further. If the bubble does not remain in the middle of its run, it must be made to do so by slightly opening or closing the slit at the bottom of the adjustable

Y by means of the antagonistic-screws, b', b'' , which have the effect of lowering or raising that end of the transit axis. When the adjustment is completed for one position, the bubble in the striding level ought to remain steady, while the instrument is rotated in azimuth through an entire revolution.

(c) When the instrument is not provided with a striding level the adjustment can be tested either as described above in para. 2 (a), or by seeing if, when the instrument is clamped in azimuth, the cross-wires traverse an object and its image as reflected from a level surface of fluid; or by setting up the instrument accurately level, directing the intersection of the cross-wires to some well-defined vertical line, such as the quoin of a building, and moving the telescope up and down. If the intersection of the cross-wires coincide during the whole motion of the telescope with the edge of the quoin, the construction is good, if not bad; and if the instrument be not provided with the adjusting-screws, the fault can only be remedied by the instrument-maker.

3 (a). The Adjustment of the Line of Collimation in a transit theodolite as well as in Everest's consists in setting that line exactly $\perp$ to the horizontal axis about which the telescope rotates. To effect this, set the cross-wires on some very distinct distant object by means of the tangent-screw, o , of the lower plate after clamping the upper and lower plates by the screws, j and n . Now unclamp the vertical circle, and lift the horizontal axis, carrying the telescope with it, out of its bearings in the supports, e, e' ; and replace it in its bearings with the ends reversed, so that the telescope is upside down. If the cross-wires now coincide exactly with the same point as when the telescope was in its original position, the line of Collimation is $\perp$ to the horizontal axis; if not, one-half of the deviation must be corrected by moving the cross-wires by means of the horizontal adjusting-screws of the diaphragm y, y' , and the other half by the tangent-screw of the lower plate, o . Reverse the telescope again, and repeat the operation, until the adjustment is accomplished.

In the transit instrument, the telescope may be reversed for the purpose of effecting this adjustment in another way, which consists in turning the telescope over on its horizontal axis, and revolving the horizontal limb through an angle of 180° .

To adjust the line of collimation in a vertical direction, if necessary, same process has to be gone through except that the vertical adjusting-screws of the diaphragm are to be used instead of the horizontal.

3. (b) Accurate Method of Adjustment of the Collimation Line of Transit Theodolite.—An objection to the foregoing method is that in lifting the telescope out of its bearings and replacing it, the instrument may be disturbed, unless too much care is taken. Also when with a transit theodolite to avoid this, the telescope is revolved vertically, and the instrument turned through 180° horizontally, any error due to the graduations of the horizontal circle, or reading the vernier is introduced. To collimate a transit theodolite properly, set up the instrument at a , (Fig. 72), and bisect a well-defined mark on the top of a peg stuck into the ground at b . Revolve the telescope

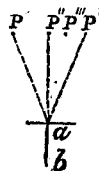


Fig. 72.—Accurate Adjustment of Collimation Line.

vertically, and mark where the cross-wires cut a distinct point p . Again set the cross-wires on to b , with the ends of the transit axis reversed. This is done, with the telescope still directed to p , by turning the whole instrument through 180° horizontally, and then bisecting b again by the tangent-screw, o . Next revolve the telescope vertically, and mark if the cross-wires still bisect p . If they do not, but bisect the object say in p' , then the point p'' half-way between p and p' is the right point, i.e., p'' is in a line with a and b , and bp'' is a straight line. To collimate, correct $\frac{1}{2}$ the total deviation, i.e., $p''p'$ by the horizontal diaphragm screws, and $\frac{1}{2}$ by the tangent-screw, o , of the horizontal circle, i.e., $p''p'$. Now, when the telescope is revolved vertically, the cross-wires should bisect both the points b and p'' in both positions of the transit axis. Repeat the process till the adjustment is perfect. By using both the faces of the instrument in this way, and taking p'' half-way between p and p' each time the telescope is revolved vertically, a straight line may be ranged out correctly even if the collimation line is not in adjustment.

NOTE.—The points b and p should be on a level piece of ground, and ap should be much greater than ab . It will be seen that, although the instrument may have been rotated horizontally through 180° , the cross-wires are set exactly on to the point b before the telescope is directed on to the point p for the second time. The instrument is not liable to be disturbed, therefore, as the telescope has simply to be revolved vertically. By changing pivots of the telescope, there can be introduced no error due to the supports, (adjustment 2) as the position of the supports is not changed. It is, however, difficult to lift out the telescope and replace it without shaking the instrument. In this case, however, it is better to change the pivots, as the cross-wires are always set on to the point, b , again by means of the tangent-screw, o ; before the telescope is turned over on to the point, p . We may then be sure that none of the errors is due to adjustment 2 of the supports being out.

4 (a). The Adjustment of the Level on the Telescope.—This level should be parallel to the line of Collimation of the telescope. In the Transit theodolite it can be adjusted exactly like the Dumpy level.

4 (b). The Adjustment of the Level attached to the Vernier-Arms of the Vertical $\odot$, so that when the Collimation (central line of vision) line of the telescope is horizontal, and the zero lines of the vertical verniers coincide with the zero diameter of the vertical $\odot$, the bubble may be in the middle of its run.

This adjustment is effected by means of the capstan-headed screws that attach the level-bubble to the vernier-arms. Having levelled the instrument by means of the bubbles on the vernier-plate, bring the bubble in the azimuth level to the centre of its run by means of the clip-screws. Now set the zero diameter of the vertical $\odot$ to coincide exactly with the zero lines on the vertical verniers, and clamp it there. Direct the telescope on a levelling staff

held at as great a distance as it can be distinctly seen, and take the reading by the horizontal wire. Now release the clamp and transit the telescope and again adjust the zero diameter of the vertical $\odot$ to the zero lines on the verniers. Revolve the head in azimuth one-half turn, bringing the telescope to its former position, and once more take the reading of the staff. If it is not the same as previously observed, correct half the error by the clip-screws, and then repeat the operation, until all error is by this means eliminated. When the adjustment is complete, correct the azimuth level by means of the capstan-headed screws, so that the bubble retains the centre.

5. To ascertain the Index-error of the Vertical Circle, set the vertical axis truly vertical with great care as described under temporary adjustments; set the spirit-level on the telescope or the vernier-arm exactly level; observe the reading on the vertical circle; if it is 0° , there is no error; if it differs from 0° , the difference is an error in the position of the index of the vertical circle, to be allowed for in each vertical angle measured.

132. The Permanent Adjustments of the Everest's Double Arc Theodolite are:—

1. Of the Lower Levels.—To set the Levels parallel to the Upper Plate.—Set up the instrument nearly level by its legs. Turn the telescope, so that one of the lower levels may be parallel to two of the foot-screws, and by their motion bring the bubble exactly to the centre of its run; and by the third screw bring the bubble of the other level to the centre of its run. If they remain so, on turning the telescope half round, the bubble-tubes are parallel to the upper plate; if not, correct the error by bringing the bubbles to the middle of their runs, half by the foot-screws, and half by the capstan-headed adjusting-screws at the ends of the levels. Repeat the process till the bubbles retain their centres. The upper and lower plates should be clamped together during this adjustment. If there is only one horizontal level, go on as before with two foot-screws and adjusting-screws; then the levelling of the instrument is perfected by turning the telescope through 90° , so that it may be over the 3rd foot-screw; and by it bring the bubble to the centre of its run.

2. Of the Line of Collimation (in azimuth).—To set the Line of Collimation at right angles to the Horizontal Axis.

This is done precisely in the same way as the adjustment 3 (a) of the transit theodolite, after having levelled the instrument as above.

3. Of the Upper Level and Virtual Line of Sight.—To set the Upper Level at the same Inclination to the Zero Line of the Verniers, as is included between the "Virtual Line of Sight" and Telescope-axis.

Step 1. On a tolerably level piece of ground, drive two pegs at A and B, say 150' apart, nearly flush with the ground. Set up the instrument exactly midway between the pegs, and adjust the lower bubbles to the middle of their runs by means of the foot-screws; tighten the stud-screws and clamp the vertical arc—say at zero. Read the staff placed upright on

both pegs; and find the true difference of level between the top of the pegs at A and B by subtracting the readings on the staff placed successively on the pegs after duly focussing without parallax in each case. Next gently tap the higher of the two pegs (this corresponds to the *lesser* reading) into the ground. Place the staff on it, and again observe the reading. Continue tapping whichever peg is the *higher* into the ground, until the reading on the staff is the same on both pegs. After a few trials this can be effected, and then the tops of the pegs will be on the same level.

Step II. Next remove the instrument on to the line A B produced say 50' from B, and level the horizontal plates by means of the lower levels and foot-screws. Clamp the vertical arcs so that their mean reading shall be zero. Tighten the stud-screws (clip-screws) so as to bring the bubble in the upper level near the centre of its run, so that the "*virtual line of sight*" may be *nearly horizontal*; direct the telescope on the staff placed successively on either peg after duly focussing without parallax in each case, and note the readings down. If these readings be the same, then the "*virtual line of sight*" is level; if not, it must be tilted by gently tilting the whole upper works by means of the stud-screw—recording the reading on the staff after each operation. The object-glass is to be *depressed*, if the reading on the further staff exceed that on the nearer, and *vice versa*. The process is to be repeated until the readings become *exactly equal*. The "*virtual line of sight*" will then be horizontal. The above procedure will probably have disturbed the telescope level; its bubble is now to be brought back to its centre by means of the upper level-screws. The upper level axis and virtual line of sight will now be both horizontal, whilst the vertical arcs are at their zero, so that in future a zero mean reading on the vertical arcs with the telescope level bubble at the centre will indicate the *horizontality of the 'virtual line of sight.'*

Another way of this adjustment is this:—Level the lower and upper levels, direct the telescope to some well-defined object, bisect it, and read the vertical angle, which is, say $E\ 0^{\circ}24'0''$; next change face, level the upper level, and bisect again the same object which now reads, say, $E\ 0^{\circ}32'0''$. Take the mean of these readings which here would be $E\ 0^{\circ}28'0''$, and set the verniers to read that mean reading; then bisect the object by using the clip-screws (stud-screws), which operation must have displaced the bubble. Now by means of the two adjusting-screws at either end of the level, bring the bubble to centre, and the upper level will be in adjustment. The process to be repeated till the result is satisfactory.

Note.—The 2nd and 3rd adjustments are not absolutely necessary. The simple means of observing every point twice or an even number of times with the face of the vertical arcs in an opposite position on each occasion, eliminate all errors due to non-adjustment of the line of collimation and mean line of the (vertical) verniers.

133. The Permanent Adjustments of the Ordinary Y-Theodolite are:—1. The Adjustment of the Line of Collimation.—

First direct the telescope to some well-defined object at a distance off of not less than 300', then make the temporary adjustments, and see that the intersection of the cross-wires cuts it exactly; then loosen the clips that confine the telescope in the Y's; turn the telescope half round in its Y's about its own axis, observing if the center of the wires continues to cut the same object. If it does, it is in adjustment; if not, the line of collimation, not coinciding with the axis of the "telescope-bands" (on which the telescope turns), half the deviation is to be corrected by the diaphragm-screws, and the other half by the tangent-screw of the lower plate. Repeat the operation till the adjustment is perfect. This adjustment makes the line of collimation coincide with the axis of the Y's. The adjustment of the latter line 1 to the horizontal axis is done by the instrument-maker.

2. The Adjustment of the Level attached to the Telescope or of the Vertical Limb for setting the Level beneath the Telescope to indicate the Horizontality of the Line of Collimation.

Having levelled the level attached to the telescope by the slow-motion screw of the vertical limb, lift the telescope out of the Y's, and reverse it end for end again in the Y's. If the bubble does not retain the centre correct half the error by the capstan-headed screw at the end of the level, and the other half by the tangent-screw of the vertical circle. Repeat the operation till the result is satisfactory. Next turn the telescope round a little both to the right and to the left, and if the bubble does not retain the centre, the level must be adjusted laterally by the screw at the other end.

3. Adjustment of the Horizontal Limb for setting the Levels on the Horizontal Limb to indicate the Verticality of its Axis.

This is similar to the adjustment 1 (b) of the Transit Theodolite.

134. Centering of the Object-Glass and Eye-Piece.—
The centres of the object and eye-glasses should exactly coincide with the collimation line of the telescope. These adjustments are generally left to the instrument-maker as no means are provided for making them. The centering of the object-glass is performed by focussing it on a very near point, and then turning the telescope half round on its axis. If the cross-wires continue to cut the point, the object-glass is correctly centered; but if not, half the deviation is to be corrected by moving the object-glass. Repeat the process until the cross-wires continue to intersect the same object. Repeat the same operation for a very distant point and again for a near point, if necessary, until the adjustment is perfect. Before centering the object-glass the cross-wires should be put exactly in the axis of the telescope.

When a level or a theodolite is purchased, it is a good plan to first screw the object-glass firmly home to its place; and then make a continuous scratch upon the ring of the glass and its slide, so as to be able to see at any time that the glass is always in the same

position relatively to its slide. Otherwise, if the glass be not accurately ground and a little eccentric, the adjustments may be put out if it is taken out to be cleaned, and not screwed up again to exactly the same place. The glass may be restored to its previous position by means of the continuous scratch. The glasses of the eye-piece should also be similarly centered to the axis of the telescope so as to see the intersection of the cross-wires exactly in the centre of the field of view.

135. To test whether the Cross-Wires are truly Horizontal and Vertical in the Diaphragm.—In some instruments two cross-wires are fixed on the diaphragm-ring, one truly horizontal and the other vertical by the maker, instead of 2 inclined and 1 horizontal as in Fig. 47, but it often happens that in collimating, the ring may be turned a little. To test if one of the cross-wires is vertical (in which case the other being at right angles to it will be horizontal), after performing the temporary adjustments, take sight with the telescope at a plumb-line, or other vertical straight-edge. If the vertical wire coincides with this plumb-line, the cross-wires are in adjustment; but if not, then *slightly* loosen two *adjusting-screws* of the four diaphragm screws, y, y', y, y' (Fig. 47); and with a knife, key, or other small instrument tap very gently against the screw-heads, so as to turn the ring a little in the telescope; persevering until the vertical wire coincides with the plumb-line. When this is done, tighten the screws up again. In the absence of a plumb-line or vertical straight-edge, fix the intersection of the cross-wires on some sharply defined point, such as the lightning conductor of a building, or failing any similar point, drive a peg 300' off with a nail put on it, and see if the wire still cuts that point, when the telescope is raised or lowered by revolving it on its axis. *This adjustment* should always be done before the *adjustment of the collimation line*. When there is no vertical wire, adjust the horizontal wire on to a levelled straight-edge.

136. To Replace a Spirit-Level or Bubble-Glass.—Detach the level from the instrument; draw off its sliding ends; push out the broken glass vial, and the cement which held it; insert the new one with the proper side up (the upper side is always graduated by the maker), wrapping some paper around its ends if it fits loosely. Finally, put a little putty, or melted bees-wax over the ends of the vial, to secure it against moving in its tube.

137. To Replace the Cross-Wires in the Diaphragm.—The diaphragm is nicked to receive one horizontal and two diagonal wires. When the cross-wires break, it will be necessary to replace them. To do so, remove the four adjusting-screws which hold the eye-end of the telescope to the sliding tube. Draw out this end together with the eye-piece, and take out the diaphragm keeping the screws by carefully, so that they may not afterwards change places. The wires should be of fine silk fibres, or spider's lines; the former being stronger should be selected; fine platina

CHAP. II.] PRINCIPLE OF THE VERNIER.

wires are much better. Human hair is entirely too coarse. Before wiring, the surface of the diaphragm and the nicks on it should be cleaned with spirits of wine. The wires can be fixed to the diaphragm with Canada balsam, or common lac dissolved in spirits, or any gum moistened in diluted spirits, or with a little beeswax. The gum should be put lightly on the diaphragm, and the wires taken between the points of a pair of compasses gummed also to make it stick, and opened wider than the diaphragm. The wires can then be carefully laid on the diaphragm. When dry, cut off the ends of the wires, carefully replace the diaphragm in the telescope, and fix on the screws in their proper places. This operation should be done on a sunny day. Glass diaphragm or diaphragm fitted with Platinum-iridium points will save this trouble of replacing the cross-wires.

138. Terms used in Connection with Angles and their Measurements are defined as under :—

1. *A plumb or vertical line* is perpendicular to the spherical surface of the earth and passes through its centre. A plane perpendicular to it is the *horizontal plane* or *horizon*.
2. *The Zenith* is the point in the heavens directly above the head of the observer. *The Nadir* is the point directly below the feet of the observer, hence opposite to the Zenith.
3. *A Vertical circle* is a great circle passing through the Zenith and Nadir of a place, and is at right angles to the *horizontal circle* which is a circle whose plane is a tangent to the Earth at the place of the observer.
4. *A horizontal angle* is the angle formed by the inclination of two straight lines in the plane of the horizontal circle or in one parallel thereto, which meet in its centre.
5. *A vertical angle* is the angle formed by the inclination of two straight lines in the plane of the vertical circle which meet in its centre.
6. *An angle of elevation* is an angle which an object makes with the horizon above it at the place of the observer.
7. *An angle of depression* is an angle which an object makes with the horizon below it at the place of the observer.

139. The Principle of the Vernier.—The vernier is a contrivance by means of which angles may be read or set off to minutes and seconds, although the graduated lower limb of the theodolite may be divided into degrees and $\frac{1}{2}$ degrees or third parts of a degree only.

(a) In the case of a theodolite the horizontal circle of which is divided into degrees and $\frac{1}{2}$ ° or 30', any number of minutes less than 30' is measured by the verniers which move along its edge. The length of a vernier is taken equal to 29 divisions or 14°30' of the graduated limb, and is divided into 30 equal parts which therefore enable us to read off to an accuracy of 1' (Fig. 73); for,

1 " = 29 ÷ 30 " = 29',
i.e., there is a difference of 1' between the scales of the limb and the vernier.

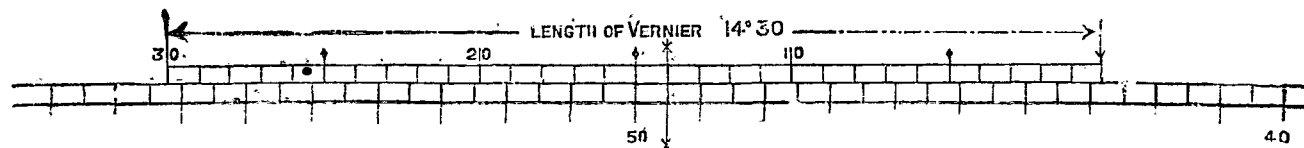


Fig. 73.—Vernier Reading to one minute.

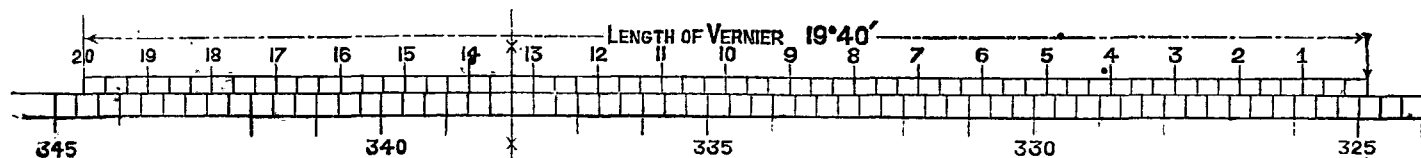


Fig. 74.—Vernier Reading to $20''$.

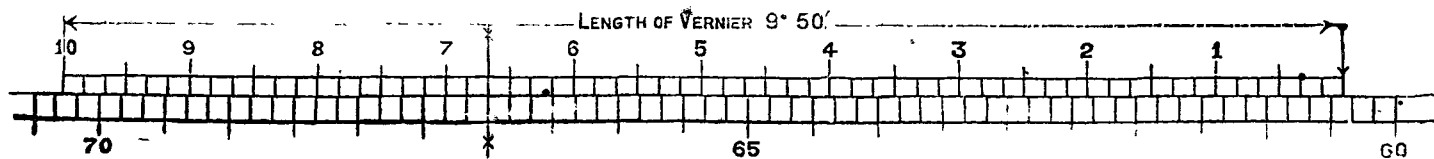


Fig. 75.—Vernier Reading to $10''$.

(*b and c.*) In large instruments more minute divisions are obtained. For instance, where the limb is divided to $10'$, and it is required to read an angle to $10''$ by means of the vernier (Fig. 75), or when the limb is divided to $20'$, and it is required to read to $20''$ by the vernier (Fig. 74)—in both cases 59 divisions of the limb are made equal to 60 divisions of the vernier. For, a and b being lengths of the division on the limb and vernier respectively.

$a - b = \frac{a}{n} = 10'' = \frac{a}{60}$; $\therefore a = 10' \times 60 = 600''$. Similarly $a - b = \frac{a}{n} = 20'' = \frac{a}{60}$; $\therefore a = 20' \times 60 = 1200''$; $\therefore 60a - 60b = a$ or $59a - 60b = 9^\circ 50'$ and $19^\circ 40'$ respectively = Lengths of Verniers.

(*d*) Again, if the primary scale of the limb shows $15'$ or $30'$, and the vernier reads to $15''$ or $30''$, then also 59 divisions of the limb are to be taken equal to 60 divisions for the vernier, for $a - b = \frac{a}{n} = 15''$, or $30'' = \frac{a}{60}$; $\therefore a = 15' \times 60 = 900''$, or $a = 30' \times 60 = 1800''$. $\therefore 59a = 60b$. Length of verniers = $14^\circ 45'$ and $29^\circ 30'$ respectively.

(*e*) If the limb is divided to $10'$, and the vernier reads to $15''$ then 39 divisions of the limb are taken and divided into 40 for the vernier; for $a - b = \frac{a}{n} = 15'' = \frac{a}{40}$; $\therefore a = 10' \times 60 = 600''$ and $\frac{a}{40} = 15''$. $\therefore 39a = 40b = 6^\circ 30' =$ Length of Vernier.

(*f*) If the limb is divided to $20'$, and the vernier reads to $30''$ as in Cooke's $5''$ theodolites, then 39 divisions of the limb will be taken equal to 40 divisions of the vernier; for $a - b = \frac{a}{n} = 30'' = \frac{a}{40}$; $\therefore a = 20' \times 60 = 1200''$ and $\frac{a}{40} = 30''$. $\therefore 39a = 40b = 13^\circ =$ length of Vernier or $b = 19^\circ \frac{1}{2}'$. $\therefore a - b = 20' - 19^\circ \frac{1}{2}' = 30''$.

140. Horizontal Limb Graduations.—The horizontal limb of a theodolite is a complete $\odot$ graduated from 0 to 360° , and numbered from left to right. In a theodolite reading to $1'$ the graduations are at every $\frac{1}{2}^\circ$. Every 10th degree is marked by figures, every 5th degree by a dot, every degree by a long line and every $\frac{1}{2}^\circ$ by a short line. The 0° is termed the **Zero of the limb**. There are 30 divisions on the vernier beginning at the arrow. Every 10th minute is marked by figures, every 5th minute by a dot, and every minute by a short line. The **arrow** is termed the **Zero of the vernier**. It is also called the *Index*, and points to the reading of the angle on the horizontal limb.

141. To Obtain the Reading on the Horizontal Limb.—If the arrow of the vernier does not coincide with any of the divisions, representing degrees and subdivisions, of the horizontal limb, then note down the number of degrees it has passed, and run your eye from the arrow along the vernier to see which of its lines

coincides with one of the lines on the limb; and the number of divisions on the vernier scale, from Zero to where such coincidence occurs, are so many minutes and seconds to be added to the degrees and subdivisions already noted down for the required reading. If none of the lines of the vernier and limb coincide, the number of seconds must be estimated by the eye on that portion of the vernier where two contiguous lines on it very nearly coincide with two corresponding lines on the limb. This might better be explained by some examples:—

(1) *Required the reading of the vernier where the limb is divided to 30' and the vernier to read to 1' (Fig. 73).* Here the index is beyond the 30' division of the limb after 42°, and the fourteenth line of the vernier coincides with a line on the limb, the reading therefore is 42° 44' 0."

(2) *Required the reading of the vernier when the limb is divided to 20' and the vernier to read to 20" (Fig. 74).* Here the index is beyond the 2nd 20' division after 324°, and the first division after 13' on the vernier coincides with a division on the limb; the reading therefore is 324° 53' 20".

(3) *Required the reading of the vernier when the limb is divided to 10' and the vernier to read to 10" (Fig. 75).* Here the index is beyond the second 10' division after 60°, and the fourth line after 6' division on the vernier coincides with a division on the limb; the reading therefore is 60° 26' 40".

142. To Set off any Angle on the Horizontal Limb.—

The process of setting off angles is simply the converse of the process of reading off given angles. In the case of a theodolite with the horizontal circle divided into degrees, and $\frac{1}{2}^\circ$ degrees, if the index of the vernier be made to coincide with the zero of the horizontal circle, then, as the space between the index and the first division of the vernier is 29', while the space between the zero and the first division of the horizontal circle is 30', the first division of the vernier falls behind the first division of the horizontal circle by an amount equal to the difference of 30' and 29' or 1'. Likewise the second vernier division falls 2' behind the 2nd horizontal circle division, the 3rd division 3' behind, and so on up to 30th division which is 30' behind. *To set off any angle containing degrees and minutes, set the index of the vernier to the given number of degrees on the horizontal circle, and then bring the vernier division corresponding to the given number of minutes to coincide with the next division in advance of it on the horizontal circle. For instance:—*

(1) *If the instrument is required to be set to 105° 43', set the index of the vernier to 105° 30' on the horizontal circle, and clamp the vernier-plate; then by the tangent-screw of the vernier-plate bring the 13th division of the vernier to coincide with the next division in advance of it on the horizontal circle. The zero of the vernier will then be at 105° 43'.*

In the case of instruments having the horizontal circle divided to 30', and the vernier to read to 30", the length of the vernier is taken equal to 29° 30', and the space of each division of the vernier,

$29^{\circ}30' \div 60 = 29\frac{1}{2}'$. Therefore the vernier reads to $30' - 29\frac{1}{2}' = \frac{1}{2}'$ or $30''$ instead of $1'$ as above. For instance :—

(2) If an angle of $51^{\circ} 48' 30''$ is required to be set off, the index of the vernier is first made to coincide with $51^{\circ}30'$ on the horizontal circle. The division of the vernier corresponding to $18'30''$ is then made to coincide with the next division in advance of it on the horizontal circle.

(3) Similarly when the horizontal circle is divided to $20'$ or $10'$, and the vernier reads to $20''$ or $10''$, the angles are to be set off exactly as described for the vernier reading to $30''$.

143 Vertical Limb Graduations in Everest Theodolite and Reading Vertical Verniers.—The graduations on the vertical circles of theodolites are made in different ways to their horizontal limbs. Ordinary $5''$ Everest theodolites are divided to $\frac{1}{2}^{\circ}$ s. The method of numbering the vertical arcs differs from that of the horizontal; each of the arcs has its Zero in the middle from which both upwards and downwards the degrees are numbered up to 45° and 60° respectively. The arcs are attached to the telescope and move with it. Every 10th degree on the vertical arc is figured every 5th degree marked by a dot, every degree by a long line and every $\frac{1}{2}^{\circ}$ by a short line. The horizontal arm, which carries a vernier at each end for reading the vertical angles on the two arcs, is fixed and does not move with the telescope like the arcs. The arrows of the vertical verniers, A and B, indicate the direction of the horizon when the instrument is level. *When the Zero of the vertical arc near the object-end of the telescope is above the index of its vernier, the angle between the horizon and the direction of the telescope is an angle of elevation; and when the Zero is below the index, the angle is an angle of depression.* The vertical verniers are constructed like the horizontal verniers, but their numbering is different, and they are 180° apart. They are figured at every 10th minute with two sets of numbers, so that their sum at any given graduation always equals 30. The internal numbers are intended for reading depressions on face left and elevations on face right; and the outside numbers are intended for reading elevations on face left, and depressions on face right. In short—in any given position of the vertical arcs, use that set of numbers on the vernier, which increase in the direction from the Zero of the limb towards the Zero on the vernier. The arrows of the vertical verniers point out the number of degrees of the angle on the arc, and are reckoned from the Zero on the arc, otherwise they are read like horizontal verniers. Suppose the reading on the arc, as shown by the Zero of vernier, B, is 6° , and that on the vernier 15', the angle, therefore, is $6^{\circ} 15'$; and so vernier A is $6^{\circ} + 14'$ or $6^{\circ}14'$. In recording the readings of both the verniers, the degrees of A are only to be noted down, and the minutes and seconds of both A and B; the mean of which is taken for the correct angle.

144. Measuring Horizontal Angles with the Theodolite.—To measure the horizontal angle subtended at a given station, a, by the direction of two objects, b and c, set up the theodolite on

its stand exactly over the station by means of the plummet, and perform the temporary adjustments as described before. In the Y or transit theodolite the lower plate being clamped, and the vernier-plate and vertical circle unclamped, direct the telescope towards the left-hand object, *b*, as correctly as possible by the hand; clamp the vernier-plate, and by its tangent-screw make the intersection of the cross-wires nicely bisect the object. Read the degrees, minutes, and seconds indicated by one vernier, which call A, and the minutes and seconds only indicated by the other, which call B, and note them down and take the mean of the two readings thus:—

$$\begin{array}{r} A = 162^\circ 30' 30'' \\ B = \quad \quad 37' \quad 0'' \\ \hline \text{Mean} = 162^\circ 33' 45'' \end{array}$$

Next unclamp the vernier-plate, direct the telescope towards the other object, *c*, and go on as before, taking care to read the entire degrees on the same vernier. The difference between the mean of the last readings, and the mean of the first readings will be the horizontal angle, *bac*, required.

145. Using both Faces of the Instrument.—If very accurate measurement of the angle is required, *change face* of the instrument: if it was *face left* at first, make it *face right*. There will be now a difference of 180° in the Zero readings of the verniers. Go on as before and get a mean reading. The mean of the two means will give the more accurate measurement of the angle. By *changing face*, the error of horizontal collimation as well as errors arising from the transit axis not being perfectly level, either owing to the lower bubble-tubes not being parallel to the vernier-plate, or owing to the bearings of the transit axis in the supports not being exactly level, are removed; and as readings are taken over a different part of the graduations, the errors of graduation are also partly removed.

146. Elimination of Error of Eccentricity.—The object of reading the minutes and seconds on both verniers, and taking the mean, is to correct the effect of any errors that might arise from the vertical axis not being exactly concentric with the upper and lower plates. Let *ab* and *cd* be two straight lines cutting each other in *o'*, a point not in the centre of the circle *bca*d. The eccentricity of that point produces equal and opposite deviations in the arcs *ac* and *db*

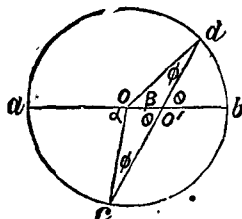


Fig. 76.—Elimination of Eccentricity.

from the arc which would subtend an angle equal to $\angle bo'd$ at the centre of the circle; so that the mean of those arcs is exactly equal to the arc which correctly measures the angle $\angle bo'd$ however great the eccentricity may be. This is proved as follows:—Let *o* be the centre of the circle (Fig. 76), and *o'* the point round which the circle revolves. Now let the circle revolve round *o'* through an angle, θ , so that the line *ab* occupies the position *cd*; then the angles α and β subtended at the centre by the arcs *ac* and *bd* will correspond to the two readings at opposite sides. Then by Euclid (1.32) $\beta + \phi = \theta$;

$$\begin{aligned}\therefore \beta &= \theta - \phi, \\ a &= \theta + \phi; \\ \therefore a + \beta &= 2\theta; \therefore \theta = \frac{1}{2}(a + \beta)\end{aligned}$$

In Everest's theodolite the same object is gained by taking the mean of the three equidistant vernier readings thus :—

$$\begin{array}{r} \text{Vernier A} = 121^{\circ} 29' 30'' \\ \text{,, B} = \quad 29 \quad 0 \\ \text{C} = \quad 30 \quad 30 \\ \hline \text{Mean reading} = 121^{\circ} 29' 40'' \end{array}$$

147. Another Method of Observing an angle with the theodolite generally applied in practice is the following : Clamp the zero of vernier marked A to 360° on the lower plate by means of the upper plate clamping and tangent-screws. Then turn the lower limb round, and with the lower clamp and tangent-screw fix the cross-wires in the telescope on any object. Next unclamp the vernier and turn the upper limb round, fixing the cross-wires by the same clamp and tangent-screw on any other object ; the angle subtended can then be read off the graduations. If one of the verniers is set to 360° , the others should also be read before measuring the angle ; as in many instruments the 2nd vernier does not read exactly 180° when the first reads 360° ; this is due to errors of graduation and position of the verniers, &c. Similarly with 3 verniers the 2nd and 3rd do not generally read exactly 120° and 240° when the first is set to zero.

148. To Ascertain Angles by Observing Bearings.—Clamp the zero of vernier to 360° of the limb, and turn the whole instrument round, until the magnetic needle lies over the north and south line NS ; and then clamp it firm in that position. Release the vernier-plate and bring the telescope to bear consecutively on *b* and *c*, clamping the vernier-plate each time, so that the readings on the objects *b* and *c* are their bearings. The difference between the two bearings is the required angle *bac*.

149. The Magnetic Bearing of an Object may also be obtained by rotating the upper plate (keeping the lower plate clamped) till the needle reads Zero, at the same time reading off the horizontal limb ; then turn the upper plate round, bisect the object, and read again ; the difference between this reading and the former will be the bearing required.

• **150. To Repeat an Angle.**—When very great accuracy is required in measuring horizontal angles, the effect of errors of graduation may be diminished to any required extent by the process called *Repetition* which is as under :—With the vernier or upper plate and the theodolite clamped to zero, direct the telescope to the first (left) object to be sighted, clamp the lower plate, and bisect the object by means of the lower tangent-screw ; read all the verniers, unclamp the upper plate, and bisect the right-hand flag. The reading of the verniers will give the angle required. Next unclamp the lower plate and bisect the first object, release the vernier-plate, and direct

the telescope to the right-hand object, bisect the signal and again read the verniers which should record twice the first angle. The operation should be repeated at least once round the circle. Then the difference between the first and last readings divided by the number of repetitions, or readings to the right flag, will give the required angle. The following is an example of repetition with a 5" Everest theodolite reading to 30".

OBJECTS	VERNIER A	B	C	MEANS
BACK	0° 0' 0"	120° 1' 0"	240° 0' 0"	0° 0' 20"
FORE	91° 36' 30"	211° 37' 30"	331° 36' 30"	
FORE	183° 13' 30"	303° 14' 30"	63° 13' 0"	
FORE	274° 49' 30"	34° 51' 0"	154° 51' 0"	274° 50' 30"
				3)274° 50' 10"
				Mean Angle = 91° 36' 43"

Then the telescope must be transited, and the process repeated to take the mean of the two results obtained with the telescope in its two positions.

By the process of repetition, the errors of graduation are diminished, but the errors of observation are accumulated, as an observer tends to make the same error of observation each time the telescope is directed upon the same object. Errors due to instability of clamping and tangent-screws, are also introduced. In theory, the method of *Repetition* is apparently perfect; but from the defects of materials and workmanship, and from want of stability of the clamping and tangent-screws, it is found in practice to *repeat the error* along with the angle measured. Errors of graduation are eliminated by measuring the angle several times on different parts of the instrument.

151. Time for observation.—The best time for observing horizontal angles is during the cool hours of the morning and evening from about sunrise to 11 A. M. and from 4 P. M. to sunset; for vertical angles between 1 and 3 P. M., when there is least refraction, *i.e.*, during the time when the object appears in its real position, and does not rise from it as at other times of the day. For ordinary work any time convenient may be utilised.

152. Care necessary in Setting up and Observation.—Care should be taken that, before going to level the instrument up with the levelling screws, it should be firmly planted in the ground by pressing in the legs over the station peg, and set up as nearly level as possible by means of them. In observing a point where a flag or a ranging rod is posted, bisect the lower part of the rod just above the ground, or if that is not visible, take care to plumb the

rod and bisect the lowest visible part of it. In the case of a church steeple the apex should be observed. Never use the theodolite or any other instrument on a windy, foggy or rainy day. The reasons are obvious. Set up instrument exactly over the station point, unhook the plummet, for if left in a wind, it will set up a vibration. Level up the theodolite, lightly brush the verniers and give only just enough pressure to make the clamp-screws grip. A clamp should be just sufficiently tightened so that a little pressure in the contrary direction will release it easily. Always turn the instrument gently by handling the upper and lower plates and *not the telescope*. Be careful to obtain a correct focus after eliminating parallax. If the instrument goes out of level slightly during one set, do not mind it, but go on till the set is completed.

153. Method of Observing Vertical Angles by the Theodolite.—Plant the theodolite over the station by means of the plummet, and perform the temporary adjustments, having both the horizontal and vertical levels truly level. Then direct the telescope to the object whose angle of elevation or depression is required, bisecting it by the intersection of the cross-wires, by clamping the vertical clamp and using the vertical tangent-screw; next read off the vertical verniers A and B, and take their mean for the vertical angle on face left. Again, if greater accuracy is required, release the vertical clamp and change face; relevel the upper level, and bisect the object as before; and read off the verniers A and B, and take their mean for the vertical angle on face right. The true vertical angle will be the mean of both face left and face right thus:—

<i>Face left.</i>		<i>Face right.</i>	
Vernier	A = E 3° 17' 30"	Vernier	A = E 3° 15' 0"
"	B = E 19' 0"	"	B = E 14' 30"
Mean of verniers = E 3° 18' 15"		Mean of verniers = E 3° 14' 45"	
∴ Mean of both faces = E 3° 16' 30".			

In entering vertical angles, E and D are made to represent angles of elevation and depression. In measuring vertical angles, the upper horizontal clamp should be free. The mean on both faces will neutralise the effect of error that may exist in the line of collimation. In the Y theodolite after observing the telescope in its proper position, it should be repeated with the telescope turned half round in its Y's, *i. e.*, with level uppermost.

In observing with the theodolite the altitudes of terrestrial objects, the refraction of the lower strata of the atmosphere introduces a good deal of error which cannot be removed.

154. Measuring Distances by the Stadia Diaphragms.—The diaphragms in Figs. 47, 50b, 50c are arranged for subtense measurement of distances, with the form in Fig. 47 this is done by means of the top and bottom horizontal webs, *s, s*; with points-diaphragm (Fig. 50b) the two points on the right-hand side of the illustration; with glass diaphragm (Fig. 50c) two short horizontal lines are employed. The subtense webs, points or lines respectively

are so adjusted that on observing a distant levelling staff, a space amounting to exactly one hundredth of the distance it is away appears between them. So every hundredth of a foot or metre on the staff between these webs, points or lines, as the case may be indicates 1 foot or metre of distance. For instance, a space of 1.34 feet would indicate a distance of 134 feet. Owing to the optical arrangement of the telescope these distances are taken from what is known as the front focus of the instrument, but as the distances required are from the centre of the theodolite it is necessary to add a *plus constant* to all subtense readings. This simple addition, varying from about 12 to 24 inches with different instruments, is marked in the lid of the case, and is the constant for all distances.

155. **Remarks on the Care of Theodolites.—Packing.**—When unpacking a new instrument, note the position of the instrument, and if not already done, write on the edge of the box the position of the telescope or face, &c., so that there shall be no difficulty in replacing the instrument. Never use force. In placing a Theodolite back into its box after use, care should be taken that the foot-screws are screwed up sufficiently high, so as not to rest on the bottom of the box during transit. For want of this precaution the lid is sometimes jammed too hard on the telescope, bending the screws and injuring the pivots. Care should be taken that the reading Microscopes, the object-glass, and the clamping-screws are screwed home. In transit, these things are liable to shake out, causing injury to the glasses and leaving marks on the circle. Always see that the Plummet is securely fastened to its place, and cannot possibly get loose and damage the instrument. Do not leave screws, nails, and pins in the boxes of theodolites. Clear the box of everything but the instrument and its appurtenances. It cannot be too strongly recommended that all Theodolites sent anywhere on a long journey should be packed carefully with no screws jamming against the bottom or sides, with all extras in their places, with screws and clamps tightened up, and with the additional precaution of padded paper all round the instrument.

Ill-use of Instrument.—The experience is that Theodolites do not show signs so much of ill-use as of want of attention; for instance, when the instrument has turned stiffly, it has not been overhauled at once, but worked in its bad condition. Nothing else can account for the guttered axes, the worn screws, and burred circles. In the old-pattern theodolites, with the flat horizontal circle, the divisions, immediately under the edges of the verniers, are generally worn out, and the vernier edges turned up on account of too great pressure, while the same result is still more marked in those whose limbs are divided vertically.

To prevent the injury here spoken of, only ordinary and simple precautions need be taken, *viz.*, to keep the edge of the circle free from dirt, and the verniers from pressing too heavily on the circle. It is easy to ascertain by the sound and feel of the instrument whether it is in proper working order.

The horizontal limb of a small 5" or 6" Theodolite should be free enough to be twirled round with the finger and thumb through two whole revolutions with vernier clamped or unclamped, and without any grating sound. After cleaning and oiling an instrument, it should be seen, when putting back the female axis, carrying the verniers and clamping arm, that one vernier does not rest on the circle harder than another, and that the clamping arm is free. They are usually thin and easily bent; a tap with the finger-nail will at once show whether a vernier is pressing unduly. If the sound be dull and dead, the vernier should be lifted up a bit; when properly adjusted, a tap with the finger-nail will produce a slightly ringing sound.

Oil should on no account be run between the vernier and the circle or any part of the instrument except the axis, and here only sparingly (pure olive oil is very suitable). Tallow is the best lubricant for tangent-screws, pivots, and edges of verniers, and even tallow should only be put on to rub off again leaving the merest shade of grease behind. As a rule it is inadvisable to lubricate any part that is exposed to the air and consequent dust.

156. Instruments for measuring Angles by Reflection.—Of these the Box or Pocket Sextant, the Nautical or Astronomical Sextant, the Double Sextant, the Line Ranger, the Optical Cross, the Apomecometer, the Prismatic Compasses and the Optical Squares are the most useful. The last two are already described.

157. The Pocket or Box Sextant is illustrated in Fig. 77, and

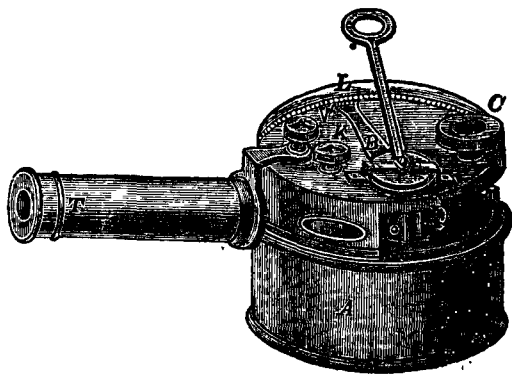


Fig. 77.—Box Sextant.

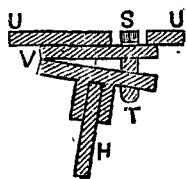


Fig. 78 —Section of Horizon-Glass of Box Sextant.

forms, when shut up, a brass box of 3" diameter and $1\frac{1}{2}$ " depth. It requires no support; and is held in the hand when observations are made. In using the sextant, the cover *A* is first unscrewed from the top, and then fitted on to the bottom of the box, forming a handle for holding the instrument. Many sextants are provided with telescope, *T*, which is then drawn out; and the dark glasses having been depressed, through the opening at the side of the instrument (concealed in the

figure being on the other side of the box) by raising the two small levers, it is ready for observation. The telescope is about 3" long and may be dispensed with. *B* is an **index-arm** having at its end a vernier *v* of which thirty divisions coincide with twenty-nine of the divisions upon the graduated limb, *L*, which is 70° in extent, but graduated to $\frac{1}{2}^\circ$, and numbered up to 140° only as the sextant will not measure an angle greater than about 125° . As the graduations on the silver arc denote 30 minutes each, the angles observed are read off by means of the vernier to a single minute. The index is moved by turning the milled-head, *C*, which acts upon a rack and pinion within the box. To the index-arm is fixed inside the box a mirror, called the **Index-glass**, which moves with the index-arm, having its plane exactly $\perp$ to the plane in which the index-arm moves, and which is called the **Plane of the Instrument** which again is identical with the plane of the face of the instrument or of the graduated Limb, *L*. In the line of sight of the telescope, and inside the box, is placed another glass, called the **horizon-glass** (because when measuring vertical angles of celestial bodies it is directed towards the horizon) having only upper half of its surface silvered, so as to allow the left one of the two objects, between which the angle is to be observed, to be seen directly through its lower half, and through an opening in the side of the box beside this mirror, while the image of the other object is seen in the upper half, reflected from the index-glass. The two dark glasses can be raised or lowered with two levers, so as to be interposed, when necessary, between the mirrors and any object, too bright to be otherwise conveniently observed, as the sun. The eye-end of the telescope has also a dark glass. That the instrument may be in adjustment, ready for use, these two mirrors must be at right angles to the *plane of the instrument*; i. e., to the underside of the top of the box, to which they are attached; and must be also parallel to each other, when the zeros of the vernier and of the limb coincide. The zero of the limb is at one end. The index-glass is already permanently fixed by the maker, and requires no other adjustment. But the horizon-glass has two adjustments which are made by a key, like that of a watch, having a milled-head, *K*. It is screwed into the top of the box, so as to be always at hand for use. When needed it is unscrewed. This key fits upon two small square-heads (like that for winding a watch), one of which is between the milled-head, *C*, and the limb on the top of the box; while the other is near it, but on the side of the box. These squares are the heads of two small screws. If the horizon-glass, *H*, (Fig. 78) should, as in this sketch (where it is shown endwise), not be at right angles to the top, *U U*, of the box it is brought right by turning the square-head, *S*, of the screw, *S T*; and if, after being so far rectified, it still is not parallel to the index-glass when the zeros coincide, it is moved a little backward or forward by the square-head at the side. When not required for use, the sextant is packed in a small box for carrying it in the pocket, or in a small leather case slung over the shoulder.

158. To Adjust a Box Sextant, bring the two zeros to coincide precisely; then look through the eye-hole and the lower or unsilvered part of the horizon-glass at some well-defined object, over half a mile distant. If the instrument is in adjustment, the object thus seen directly will coincide precisely with the reflected image, seen at the same time, at the same spot. But if it is not in adjustment, the two will appear separated either horizontally or vertically, or both, thus*,; in which case, apply the key, *K*, to the square-head *S*; and by turning it slightly in whichever direction it may be necessary, *still looking at the object and its image*, bring the two into a horizontal position, or on a level with each other, thus**. Then apply the key to the square-head in the side of the box; and by turning it slightly, bring the two to coincide perfectly. The instrument is then adjusted. In some instruments, the horizon-glass has a hinge at *v*, to allow it to play while being adjusted by the single screw, *S T*; but others dispense with this hinge, and use two screws like *S* on the top of the box in addition to the one in the side. This is the only adjustment that the box sextant requires, and the observer should be careful not to attempt to make even this until he is certain that they are required, and that the object observed is at the distance of fully half a mile.

159. To Measure Horizontal Angles.—To obtain the angle subtended by two objects nearly in the same horizontal plane, hold the sextant horizontally in the *left-hand*, look through the telescope, and by turning the milled-head, *C*, with the *right-hand*, bring the reflected image of the right-hand object into coincidence with the direct image of the left-hand object; and the reading of the vernier will give the horizontal angle between the two objects.

160. To Measure Vertical Angles.—To obtain the angle subtended by two objects situated nearly or quite in the same vertical plane, hold the instrument vertically in the *right-hand*, and bring down the reflected image of the upper object by turning the milled-head, *C*, till it exactly coincides with the direct image of the lower object; and the reading of the vernier will give the vertical angle between the two objects.

161. Observing with the Box Sextant.—It is needless to add that by sighting through the telescope, the left-hand or lower object is seen directly through the lower unsilvered half of the fixed horizon-glass; and by turning the movable index-glass by means of the milled-head, *C*, until the right-hand or upper object is made to reflect from the index-glass to the upper silvered half of the horizon-glass, the image of the right-hand or upper object is seen just over the first object as seen through the lower unsilvered half as shown in Fig. 79.

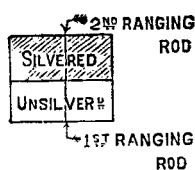


Fig. 79.—Observing with the Box Sextant. object is seen just over the first object as seen through the lower unsilvered half as shown in Fig. 79.

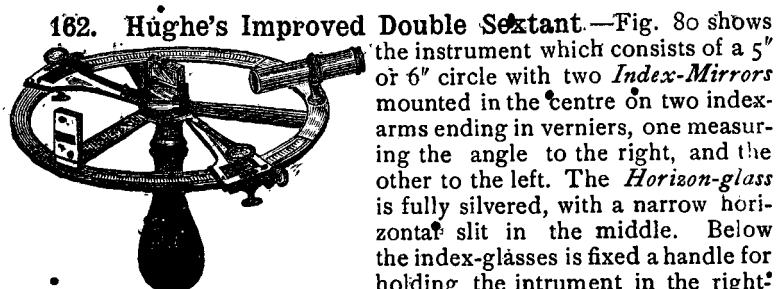


Fig 80.—Hughe's Double Sextant. the instrument which consists of a 5" or 6" circle with two *Index-Mirrors* mounted in the centre on two index-arms ending in verniers, one measuring the angle to the right, and the other to the left. The *Horizon-glass* is fully silvered, with a narrow horizontal slit in the middle. Below the index-glasses is fixed a handle for holding the instrument in the right hand, while the index-arms are rotated by means of the left. It may be made to fit on to a tripod-stand with a ball-and-socket movement, whereby greater steadiness and more accuracy may be attained. This instrument has the advantage of measuring nearly double the angle which can be observed with the ordinary sextant.

163. Method of using the Double Sextant.—With this instrument two angles between 3 objects can at once be obtained. Sight with the telescope to the middle object through the slit in the horizon-glass, and turn the index-arms, until the reflected images of the right and left-hand objects from the index-glasses are made to reflect into the silvered portions of the horizon-glass. When the 3 objects are in one vertical line, the angles are observed with the verniers.

The Adjustments of the Index and Horizon-glasses are made as usual, the upper index-glass being adjusted last.

164. Nautical Sextant.—Theodolite is the most suitable instrument for making astronomical observations except the lunar distance observations for longitude, which can only be observed with the sextant. This instrument being much more portable, is useful for making detached observations for latitude, longitude, time, meridian, in rough explorations and reconnaissance of routes. It is chiefly used in navigation and hydrographical surveying, &c. It is constructed on the same principle as the box sextant, and is seldom used for land surveying; for which purpose the box sextant is employed.

The nautical sextant, as shown in Fig. 81, is held by the wood handle at back. There is a hole in the middle of the handle for fixing it to a stand which is useful when an observer wants steadiness. *AB* is the graduated arc, *CD* the index-arm which moves about a pivot placed at the centre of the arc, *M* a microscope for reading the vernier divisions. At the back of the instrument is a clamping-screw for putting the index in any required position and a tangent-screw, *F*, for giving it a slow-motion to complete its adjustment. *I* is the *index-glass* fixed to the index-arm, and *HH*, the *horizon-glass*. Both the glasses have their planes perpendicular to the plane of the arc, and are parallel to each other when the index is at zero of the graduated

arc. The horizon glass is carried by the frame of the arc, and has its lower half only silvered. Both glasses should be made of strong plate glass with its surfaces exactly plane and parallel. The horizon at sea, or the reflection of the observed object in the *artificial horizon*, is seen directly through the upper unsilvered half of the horizon-glass *HH*, while the reflection of the object as reflected from the index-

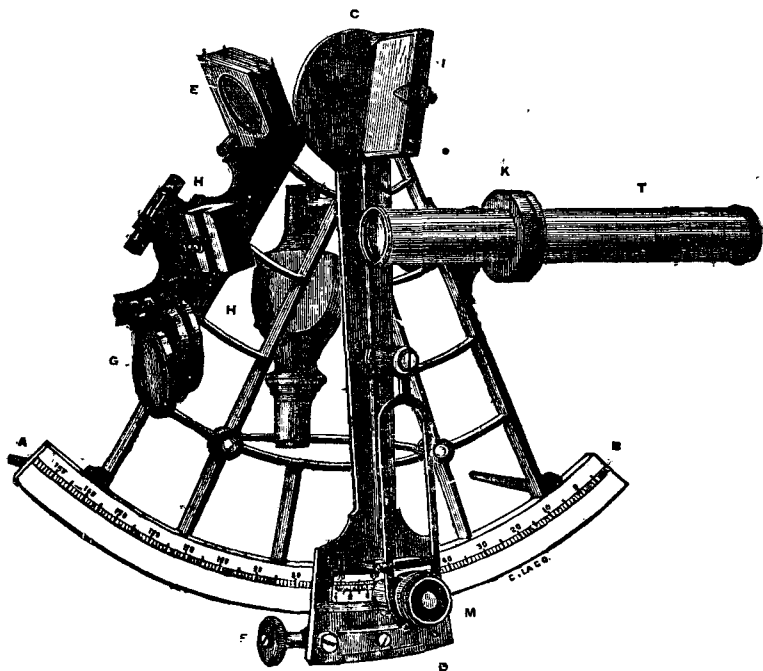


Fig 81.—Barker's Nautical or Astronomical Sextant.

glass *I*, is seen in the lower silvered half. *T* is a telescope directed towards the horizon-glass. It is carried by a ring *K*, and is capable of adjustment to a greater or less distance from the plane of the arc. *E, G* are sets of colored glasses which may be interposed between the sun or its reflection and the observer's eye. The same purpose is effected by fixing a dark-glass to the eye-end of the telescope which is sometimes dispensed with, and a hole or a plain tube provided instead. The arc, *AB*, is usually $\frac{1}{2}$ of a circle, and measures angles up to 150° , the divisions on the graduated arc being numbered with twice their actual value so that actual angle between two objects is read at once on the vernier. The arc is of 6" to 8" radius, and is divided into 10' commencing from *B*, and read by the vernier to 10". The divisions are also continued on the other side of zero towards *B* forming what is called the *Arc of Excess*, which is useful in finding the *Index-error* of the instrument as will be explained hereafter.

165. The Adjustments of the Nautical Sextant are:—

1. **To Place the Index-glass exactly at Right Angles to the Plane of the Arc.**—Move the index near to the centre of the arc; then holding the instrument horizontally with the graduated arc from the observer, and the index-glass to the eye, look obliquely down the glass so as to see the circular arc, by direct view to the right-hand, and by reflection on the left, in the glass at the same time; and if they appear as one continued arc, the index-glass is in adjustment. If not, the arc will appear broken where the reflected and direct parts of the arc meet. Then put the glass at right angles to the plane of the arc by the screws behind it.

2. **To Place the Horizon-Glass at Right Angles to the Plane of the Arc.**—Set the index near to 0° ; look at some distinct distant object, such as a star, and turn the tangent-screw of the index till the object as seen directly and its reflected image are made to seem to coincide, if possible. If the horizon-glass is in adjustment, it will be possible to make the coincidence exactly; if not, it must be adjusted by the screws behind it.

3. **To Examine if the Two Glasses are Parallel when the Index is at Zero.**—Set the zero on the index to the zero of the arc, and if you look at some object, a star for instance, you will see that the direct and reflected images appear as one, if the glasses are exactly parallel to each other; but if the two images do not coincide, the index will not be at zero, and the difference is the index-error. The effect of this error on an angle measured by the instrument is equal to the error itself; therefore it is considered preferable to determine its amount, and apply it with proper sign to each observation. The amount of index-error may be found as follows:—clamp the index at about 30 minutes to the left of zero, and looking towards the star, the two images will appear either nearly in contact or overlapping each other: then perfect the contact by moving the tangent-screw, and note the reading on the vernier. Next place the index about the same quantity to the right of zero or on the *arc of excess*, and make the contact of the two images perfect as before, and note the reading on the arc of excess; and half the difference is the index-error; additive when the reading on the arc of excess is greater than that on the arc, and subtractive when contrary is the case. When reading off the arc of excess, the vernier must be read backwards, or from its contrary end.

Make the direct and reflected images of a star coincide. If the index is at zero, no correction is required; if not, the difference is the *index-error*. The index-error may be adjusted by moving the horizon-glass by means of the screws provided, until the images do coincide when the index is at zero, and in that case the perpendicularity of the horizon-glass to the plane of the instrument must again be tested and corrected, if necessary, until both adjustments are correct.

4 **The Parallelism of the Line of Collimation of the Telescope to the plane of the instrument** is tested by placing the index so as to produce

the apparent coincidence of two distinct objects whose directions make an angle of 90° , or thereabouts, and observing whether a slight motion of the plane of the sextant about an axis traversing the object seen by reflection disturbs the apparent coincidence, which it should not do, if the adjustment is correct.

166. Care to be taken in observing with the Sextant.—The sextant is to be held *loosely*, so that its plane is in the plane of the two objects to be sighted. Observe the *left-hand* or *lower* object *directly* through the *unsilvered* half of the horizon-glass. Move the index-arm until the reflection of the other object is seen in the *silvered* half of the horizon-glass, and the two are made apparently to coincide. The vernier then indicates the angle between the two objects. Always look at the brighter of the two objects by reflection. If necessary to do this turn the instrument upside down.

167. The Advantages of the Sextant are.—(1) It can be used while travelling, or on a moving body as on board a ship, and does not require any stand; (2) it can take sights to two moving bodies, as the moon and star at the same time. Its disadvantage is that the angles measured by it being oblique angles are to be reduced to their horizontal projection.

168. Parallax of the Sextant.—The angle between the rays, bi , coming from the object b to the index-glass, thence to the horizon-glass, and from there to the eye, and the rays, he , coming from the other object directly to the eye, *i.e.*, the $\angle bea$, is measured by the sextant (Fig. 82). The eye may be at e^1 or e^2 on either side of e . The error in the observed angle due to the eye not being exactly at the point where be cuts ae is called the *Parallax of the Sextant*. A difference of 1 inch in the position of the eye makes a parallax of 3 seconds when the reflected object is 1 mile distant and 3 minutes when 100' distant. In order to eliminate the effect of parallax as much as possible, sight to the more distant object by reflection. When both objects are near, take some distant point in line with one of them, and observe this point by reflection and the other object directly. For distant objects, such as a star, the parallax is very small. The parallax is the angle subtended at the reflected object by the distance ee^1 or ee^2 .

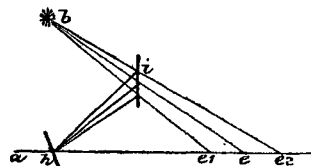


Fig. 82.—Parallax of the Sextant.

The error in the observed angle due to the eye not being exactly at the point where be cuts ae is called the *Parallax of the Sextant*. A difference of 1 inch in the position of the eye makes a parallax of 3 seconds when the reflected object is 1 mile distant and 3 minutes when 100' distant. In order to eliminate the effect of parallax as much as possible, sight to the more distant object by reflection. When both objects are near, take some distant point in line with one of them, and observe this point by reflection and the other object directly. For distant objects, such as a star, the parallax is very small. The parallax is the angle subtended at the reflected object by the distance ee^1 or ee^2 .

169. The Principle of the Construction of the Sextant.—The sextant is essentially a reflecting one, and in consequence its principle is founded upon the same data which govern all instruments of a similar character. *The principle upon which reflecting instruments act is this:—that if there are two plane mirrors whose reflecting surfaces make a given angle with each other, and a ray of light, in a plane $\perp$ to the planes of both mirrors, is reflected from both successively, its direction after second reflection makes with its original direction an angle which is double of the angle made by the mirrors with each*

other. This is explained by Fig. 83. Let a represent the index-glass which is moved by the index-arm as required, and b the horizon-glass which is fixed in a plate perpendicular to that of the instrument. The angle of inclination between the two glasses in any one given position will therefore equal in the figure the $\angle adb$. If we take r to represent a ray of light impinging on the mirror, a , it will be reflected from a to b , and the angle of incidence, rae = angle of reflection, bad . On arriving at b , the second mirror, the ray of light will again be reflected in the direction, bc , making $\angle abf = \angle cbd$. The total deviation of the ray of light is measured, by the $\angle abc$, and by the conditions of the problem this must be equal to twice the $\angle adb$ or that between the index and horizon glasses.

In the two $\triangle ahc$, bhd , the $\angle ahc = \angle bhd$. $\therefore$ the remaining $\angle shac$, hca of the one $\triangle$ equal the remaining $\angle shbd$, hdb of the other; but $\angle hbd = \angle abf = \angle hdb + \angle bad$. $\therefore \angle hac + \angle hca = \angle bad + 2 \angle hdb$, but $\angle bad = \angle rae = \angle hac$. $\therefore \angle hca = \angle acb = 2 \angle hdb$; or the angle of deviation of the ray equals twice the angle between the glasses. This angle is practically equal to that subtended by the object and its image at the eye of the observer. The difference between it and the actual theoretical angle is usually called the *parallax* of the instrument, which may be altogether eliminated by properly handling it. When the eye of the observer, the centre of the index-glass, and the object, form three points in the same straight line, this error becomes reduced to zero. In order to obviate the necessity for first registering an observation, and then doubling it to obtain the angle required, the divisions on the arc or limb are marked double of what they really represent, so that the correct angle is read off by the vernier at once. In reading the angle care should be taken to bring the microscope perpendicularly over the vernier; or the true reading will not be obtained.

170. Use of the Pocket Sextant.—The pocket sextant is a reflecting instrument for measuring the angle between two objects in either a horizontal or vertical plane, the reflected image of one object being brought to coincide with the other seen direct. It, being very portable, can be used on horse back, on tops of trees, on boats and places where other instruments can not be carried and in all weathers. It is used for filling in plans. For determining heights and distances the sextant is very useful to a military surveyor. The fact that it reads to single minutes, renders it at times very useful to the Engineer. It is useful for obtaining latitudes by the aid of an artificial horizon and in finding heights and distances.

Pocket Sextant very useful for taking Off-sets.—To use it, set the index to 90° , and walk along the chain line to see where an off-set will fall, then look through the sextant at the left-hand object, and move to and fro along the chain line until the two objects, *i. e.*, the off-set mark and that on the chain line are brought to coincide.

Or to lay off a line at right angles to another, as in the case of setting out curves, send an assistant with a staff in the required direction, and having set the index at 90° as before, make him move right or left until his staff and the other mark are made to coincide.

171. Objections to Sextant.—The chief and indeed only objection to the sextant as a surveying instrument, arises from the angles taken with it not being always, like those measured by the theodolite and compass, *horizontal ones*. To measure horizontal angles, the two objects and the observer's eye must be at the same level. When this is not the case, the following methods may be adopted:—(1) An approximate method is to select by the eye two objects in the same vertical planes with the objects whose relative azimuth is to be found on a level with the observer's eye, and to measure the angle between these. (2) Two vertical poles are to be ranged and adjusted by the plumb-line, or two plumb-lines may be suspended in the directions of the two objects, and the angle between them measured with the plane of the sextant horizontal. In using the box sextant for the details of a survey, this method is sufficiently accurate, if the ground is not very hilly.

172. To Reduce an Oblique Angle to its Horizontal Projection.—The most correct method is to measure the angle by holding the sextant in the plane of the two objects, and to take also the angle of altitude or depression of each. In Fig. 84, let $\angle aob$ be the observed oblique angle, and let $a'ob'$ be its horizontal projection. Let o be the observer's station, z his zenith; then za and zb are the zenith distances of a and $b = 90^\circ$ —respective altitudes of a and b .

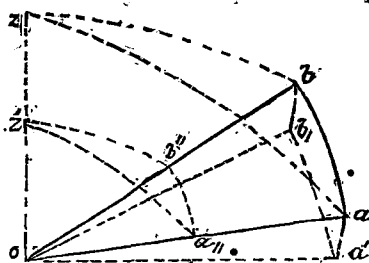


Fig. 84.—Reduction of Oblique Angles.

The spherical $\angle azb$ is equal to the required horizontal projection $\angle a'ob'$. Then in the spherical $\triangle azb$, the three sides are given, viz., $za = (90^\circ - \text{altitude of } a)$, $zb = (90^\circ - \text{altitude of } b)$, the zenith distances, and $ab = \text{observed oblique angle between the two objects}$. From these the spherical $\angle azb$ which is equal to the horizontal projection $\angle a'ob'$ can be calculated from either of the following formulæ:—

$$\sin \frac{1}{2} a'ob' = \sin \frac{1}{2} azb = \sqrt{\frac{\sin (s-za) \sin (s-zb)}{\sin za \times \sin zb}};$$

$$\text{or } \cos \frac{1}{2} a'ob' = \cos \frac{1}{2} azb = \sqrt{\frac{\sin s \times \sin (s-ab)}{\sin za \times \sin zb}};$$

$$\tan \frac{1}{2} a'ob' = \tan \frac{1}{2} azb = \sqrt{\frac{\sin (s-za) \sin (s-zb)}{\sin s \times \sin (s-ab)}}.$$

re $s = \frac{1}{2}$ the sum of the sides of the $\triangle azb$.

In Fig. 84, the objects a and b are assumed to be at equal distances from the observer, i.e., $ao=bo$. If the objects are at different distances, precisely the same reasoning and formulæ will apply.

Example :—The altitudes of a and b are $52^{\circ} 50' 0''$ and $46^{\circ} 22' 22''$, and the observed $\angle aob=68^{\circ} 46' 2''$. Find the spherical $\angle azb$ which is equal to the required horizontal projection.

Here $za=(90^{\circ}-52^{\circ} 50')=37^{\circ} 10' 0''$; $zb=(90^{\circ}-46^{\circ} 22' 22'')=43^{\circ} 37' 38''$; $ab=68^{\circ} 46' 2''$; $s=\frac{za+zb+ab}{2}=74^{\circ} 46' 50''$. $\therefore \log \sin \frac{1}{2} a'ob'=\log \sin \frac{1}{2} azb$

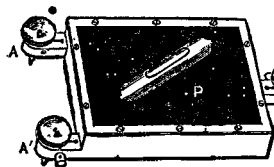


Fig. 85.—Artificial Horizon of Black glass.

$$\begin{aligned}
 &= \frac{1}{2} [\text{arith. comp. } \log \sin (s-za) + 10 + \text{arith.} \\
 &\quad \text{comp. } \log \sin (s-zb) + 10 - (\log \sin za + \log \sin zb)] \\
 &= \frac{1}{2} [\text{arith. comp. } \log \sin 37^{\circ} 36' 50'' + 10 + \text{arith.} \\
 &\quad \text{comp. } \log \sin 31^{\circ} 9' 12'' + 10 - (\log \sin 37^{\circ} 10' 0'' \\
 &\quad + \log \sin 43^{\circ} 37' 38'')] \\
 &= \frac{1}{2} [9.7855698 + 10 + 9.7137678 + 10 - (9.781134 \\
 &\quad + 9.8388261)]
 \end{aligned}$$

$$\therefore \log \sin \frac{1}{2} a'ob' = \log \sin \frac{1}{2} (60^{\circ} 29' 53'') = 9.9396885.$$

$$\therefore a'ob' = 120^{\circ} 59' 46''.$$

173. The Artificial Horizon is a reflector, the surface of which is perfectly horizontal, it is used in combination with the sextant for observing altitudes. Though the principle of all is the same, there are several forms of this instrument which are described as under :—

174. Artificial Horizon in Black Glass :—It consists of a plane of black plate glass in a metal frame (Fig. 85) of circular or square form and levelled by a bubble. There are 3 adjusting-screws A, A', A'', to level the instrument by a loose level-tube ground on its under surface P. The level-tube is placed parallel to two screws, A, A', and afterwards at right angles to its first position with one end of the tube towards A''. It is $4'' \times 3''$, and weighs 2 lbs. This form answers well during the day when the sun is the object observed, but at night owing to much loss of light with the black plate it is very difficult to use in star observations. In order to overcome this difficulty artificial horizons of this class have been constructed with a brass frame containing a black plate on one side for day observations, and a silvered mirror on the other for night. This though a portable instrument, can only be recommended in the absence of a mercurial horizon.

175. Mercury Artificial Horizon (Fig. 86) is the most common and best form. It consists of an oblong tray, $6'' \times 3'' \times \frac{3}{4}''$ of wrought iron with a plateglass roof. The tray contains the mercury which when not required for use is poured into a stoppered cast iron bottle. Two forms of this are shown below.

176. Steward's Artificial Horizon (Fig. 86) is so arranged that : (1) no separate mercury bottle is required. (2) The mercury cannot be disturbed by air or wind when instrument is in use. (3) The mercury cannot be lost. (4) The mercury is never exposed.

external air. (5) It can be carried about or overturned at all times without loss or exposure of mercury. The roof of the instru-

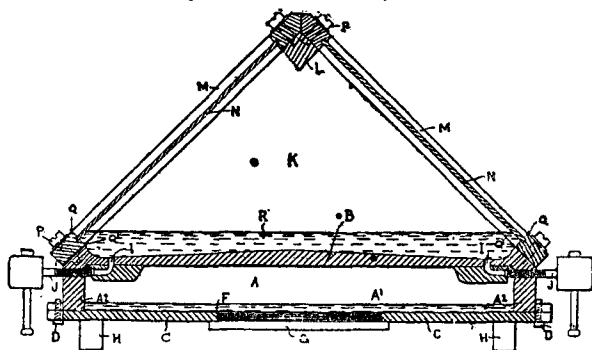
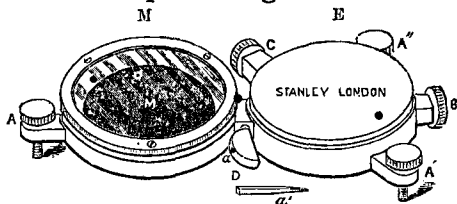


Fig. 86.—Steward's Artificial Horizon.

ment is shaped on the old lines, but the lower part forms a reservoir, A, sufficient to contain the whole of the mercury when not in use, and the covering plate of the reservoir, which also forms the bottom of the upper chamber, B, is sloped down each way, so that when the mercury is done with, the screws, J, J, can be released, and it naturally drains off through the small holes, I, I, into the reservoir between the plate B, and base A' and any scum or dirt is left behind on the plate, B. The scum can at any time be removed after unscrewing one of the glass sides, M, of the roof, which are so framed and constructed with wash-leather edging as to prevent any escape of mercury or the penetration of external air. The construction of this instrument permits the surface of the mercury to be raised to such a height in the instrument that the whole or nearly the whole of such surface is within view of the observer, so that very small angles of altitude can be measured. The mercury passes through the upper chamber when valves, J, J, are open, by simply tilting the instrument, and when the required amount reaches the level of Q, and impinges on the glass plates of the roof, the valves, J, J, are closed, and the instrument is ready for use.

177. Capt. George's Artificial Horizon (Fig. 87) is



made of iron and more portable. Two chambers, E, M, are connected together by a tube with a cock at *a*. The chamber, E, forms the bottle which contains about 1 lb. of mercury introduced by re-

Fig. 87.—Capt. George's Artificial Horizon.

B. The chamber, M, is an open tray with a plateglass cover, fixed under an iron rim which screws it down. A milled-head, C, forms the air plug. The cock moves very stiffly by the leverage given by a tommy-pin shown at *d* which is inserted in the

hole at *a*. The chamber, *E*, is slightly elevated to cause the mercury to flow from it to *M*, the cock being turned on and the air-screw, *C*, released a little at the same time. In the same way, *M* being raised the mercury flows back into *E*.

178. Use of George's Horizon.—When the mercury is poured into the tray, *M*, it is levelled by the 3 screws, *A*, *A'*, *A''*. A separate disc of plateglass which fits the tray, *M*, loosely floats on the surface of the mercury and keeps it quite still. This artificial horizon is carried in a leather case slung over the shoulder. It weighs $4\frac{1}{2}$ lbs, size $9\frac{1}{2}" \times 4" \times 1\frac{1}{2}"$. Surface of mercury is $3"$ in diameter.

179. Use of the Artificial Horizon.—When the altitude of a celestial body is taken at sea, it is brought down, as the term is, to the natural horizon; and the measure of the angle is read off on the graduated arc, of the sextant and the correction for *dip* of horizon is applied to the observed altitude; but on land, due to its unevenness, the natural horizon cannot be used; therefore an *artificial horizon* such as a vessel of water, oil, molasses, or mercury is used instead of the natural horizon. The observer then looks directly at the reflected image of the sun or other body in the fluid: he has only then to bring down the image, as reflected from the index-glass, until it reaches its reflection in the fluid; the altitude will then be half the number of degrees, indicated by the graduated arc, subject to certain corrections (for semi-diameter of the observed body; parallax and refraction, first two being additive and the last subtractive) not necessary to be explained here.

180. Theory of Artificial Horizon.—The reason of taking half

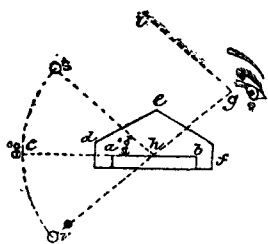


Fig. 88.—Diagram of Artificial Horizon.

the number of degrees will be clear from the following:—Let *ab* (Fig. 88) represent the surface of quicksilver in a vessel whose plane is continued to *c*; *de* and *ef* are two glass plates forming the roof of the vessel to protect the quicksilver from the influence of the wind. The surfaces of the glass plates are plane and parallel to each other. ☉ is the sun at *s* whose altitude is required. The ray *sh* from the sun's lower limb to the surface of the quicksilver, will be reflected thence to the eye at *g* in the direction *hg*, and the upper limb of the sun's image, reflected from the quicksilver, will appear in the line *gh*, continued to *r*; by optics the $\angle$ of incidence *sha* or *shc* = $\angle$ of reflection *ghb*; and the $\angle ahr$ or $\angle chr$ = $\angle ghb$ - $\angle shc$ the altitude of the sun's lower limb above the horizontal plane; so that, supposing the $\angle shr$ measured by a sextant to be 80° , the altitude of the sun's lower limb will then be 40° , subject to the corrections as above.

In observing the altitude of the sun it is difficult to tell whether the upper or lower limb has been observed in the mercury. The following will show this:—In the forenoon, or when the sun is

rising, if the *lower limb* is observed, images will be continually separating; if the *upper limb* is seen they will be continually overlapping; and *vice versa* in the afternoon or when the sun is setting, with an inverting telescope.

181. To Observe Very Small Angles with the Artificial

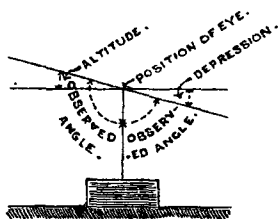


Fig. 89.

Horizon—Fix a string over the artificial horizon (Fig. 89). Put your eye so that the string appears to overlap its image in the mercury. Then the string and the eye are in the same vertical plane. Keeping the eye in the same position, observe with the sextant the angle between the string as seen directly, and the reflection of the object as reflected from the index-glass. This angle $= 90^\circ + \text{altitude of object}$, and the required altitude $= \text{observed angle} - 90^\circ$. In the case of a depression, the required depression $= 90^\circ - \text{observed angle}$.

182. Adie's Line-Ranger.—This instrument is made up of

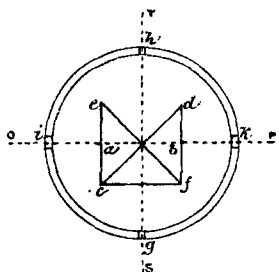


Fig. 90.—Plan of Line Ranger.

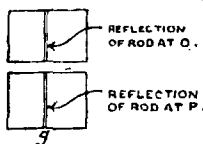


Fig. 91.—Use of Line Ranger.

two square prisms *a* and *b* (Figs. 90 and 91) having their hypotenusal sides *cd* and *ef* silvered and at rt. $\angle$ s to each other, Fig. 90 being a plan of the instrument. They are fixed one above the other in a circular box like the optical square, having openings at *i* and *k*, and slits at *g* and *h* in the direction, ST. On sighting through the slits, *g*, *h*, in the direction, ST, if the instrument is in the line OP, the reflections of the ranging rods at O and P will appear to be in the same vertical line at *g* in the mirrors as shown in Fig. 91.

183. Use.—To use the instrument, move it back or forward until the lining rods appear as shown in Fig. 91. A point vertically under the instrument will then be in the straight line OP. This instrument is very useful for finding an intermediate point in a long line without requiring the surveyor to go to one end of the line to range it out. This method being very accurate should be adopted in ranging out long lines in chain or compass surveys.

184. The Optical Cross or Double Optical Square.—Fig. 92 shows the optical parts of the instrument. The two index-mirrors, *c* and *d*, are fixed at equal angles to the direct line of sight, *eo*. The two horizon-mirrors, *a* and *b*, are placed one above the other with a space of $\frac{1}{16}$ " between them. The horizon-mirrors are each separately adjusted, so that their reflecting planes are respectively 45° to the index-mirror from which they get the reflections. The instrument is made, $2\frac{1}{4}$ " in diameter and $\frac{7}{8}$ " in depth, of either gun-metal or aluminium and is in fact made up of two optical squares, the one placed exactly over the other, the one reflecting to the right-hand, and

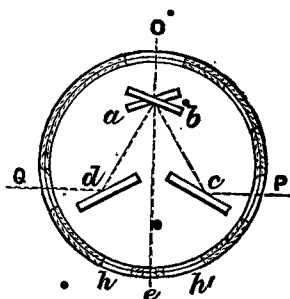
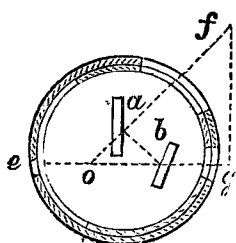


Fig. 92.—Optical Cross.

the other to the left. This arrangement makes it more useful than the single optical square. First as regards the 90° , this need not depend in any way on the position of the observer, as two objects may be observed, one to the right, and the other to the left, to appear to cut the direct forward line of sight, and therefore to cut the chain line at the exact position of the instrument perpendicular to it. Secondly, an intermediate station can be ascertained in direct line between any two points, as the $90^\circ + 90^\circ$ forms this line.

185. Apomecometer.—This instrument (Fig. 93) is like the



optical square (Fig. 22), except that the planes of the mirrors are fixed at an angle of $22^\circ 30'$, so as to give reflection of 45° . In Fig. 93, *a* is the index-mirror, *b* the horizon-mirror, *e* the eye-sight. There is one opening in front of the index-mirror, *f*, and another behind the horizon-mirror, *g*, each sufficiently wide to obtain a field of view at about 45° , and in the direct line, *e* to *g*. These openings close by rotation of the casing of the box as in the optical square. It is dust-tight when closed. The instrument is $1\frac{1}{4}$ " in diameter, $\frac{3}{8}$ " thick, and can be carried in the waistcoat pocket. It is intended to measure the height of buildings, trees, &c., by measuring the distance from the vertical upon the surface of the ground. The angle measured being 45° , the vertical will be known, this being equal to the horizontal which is measured. Horizontal distances may be measured in the same manner from a perpendicular to any line.

186. Method of Using the Apomecometer.—An open level ground in front of a building is selected to measure its height,

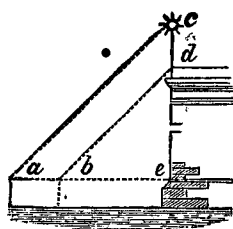


Fig. 94.—Use of Apomecometer (house).

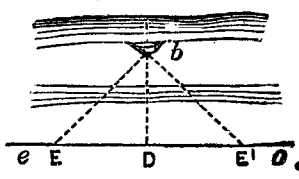


Fig. 95.—Use Apomecometer (boat).

and the observer stations himself at a distance thought to be approximate to the height. The instrument is held edgewise with the eyesight to the eye, and a point of the building almost on a level with the eye is observed by direct

vision through the instrument; at the same time a reflection of the top of the building will be seen on the horizon-mirror. By walking backwards or forwards, keeping sight of a level object, as for instance, in Fig. 94, the plinth of a building, at a certain distance the summit of

the building will appear by coincident reflection. The distance from the object is the same as the height of the point of sight to the summit, assuming that it is vertical to it. This distance may be measured on the ground, the principle of which is, shown by Fig. 93 in the line og , being equal to fg . If a part of an object is required to be measured, such part may be taken on the horizontal plane, as for instance, the height of the lightning conductor in Fig. 94, by ab being $=cd$, as the base ab can easily be measured.

The distance of an inaccessible object may be measured, as for instance, a boat in the river, by measuring in any straight line double the distance, and taking equal angles thereto by the apometer on any direct line. As for instance, b being a boat in the river (Fig. 95), walk from e , at which a ranging rod may be set up, towards an object, o . At E the boat, b , and the object, o , will appear to coincide. Then mark this position of the instrument. Walk on till beyond E , and turn towards e . Now in returning, the boat and the object, e , will appear to coincide at E' which is marked. The distance EE' is equal to $2D$ b .

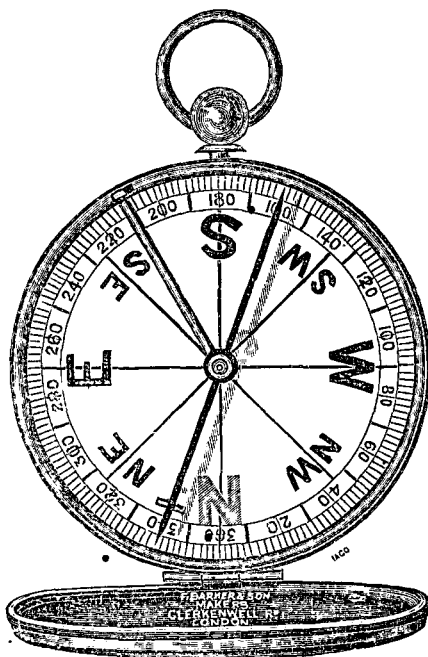


Fig. 96.—Pocket Compass.

verses. The instrument (Fig. 96) should be protected in a case like the hunting case of a watch.

188. Description of Theodolites for Special Purposes :—

Casella's Traveller's Transit Theodolite.—It is a 3" theodolite (Fig. 97) for explorers, and is a small portable instrument. In designing this instrument, the object has been to condense into the least possible bulk the smallest instrument with which useful work can be made. It packs in a mahogany case, $7\frac{1}{2} \times 5\frac{1}{2} \times 4\frac{1}{2}$ inches, the whole weighing $4\frac{1}{2}$ lbs with tripod stand complete. Its telescope, fixed at one end of transit axis gives it the advantage of optical power; and it has complete 3" circles, both horizontal and vertical with verniers reading to one minute; it can therefore be used not only as

187. Pocket compass.—

When going to survey in an undeveloped country, jungle or dense forest a surveyor should take with him a pocket compass. It will prevent him from losing his way in case of accident to his other instruments. Sometimes it is useful for making rough tra-

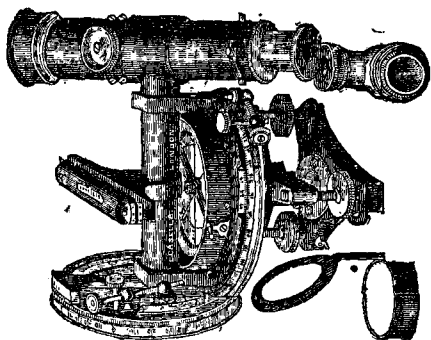


Fig. 97.—Traveller's Transit.

theodolite for terrestrial surveying, but also as an *altazimuth* for determining time, latitude, and azimuth, astronomically. It will be found convenient in use, its diagonal eye-piece admitting of zenith stars being observed with perfect facility. It is supplied with a reflector for illuminating the wires at night, a dark glass for solar observations, a finely-divided level, a compass, and the means of performing all the adjustments necessary in such instruments.

The same theodolite, but with a telescope in centre, in case complete, $10\frac{1}{2} \times 5 \times 4$ inches, having a weight of $5\frac{1}{2}$ lbs., is also available.

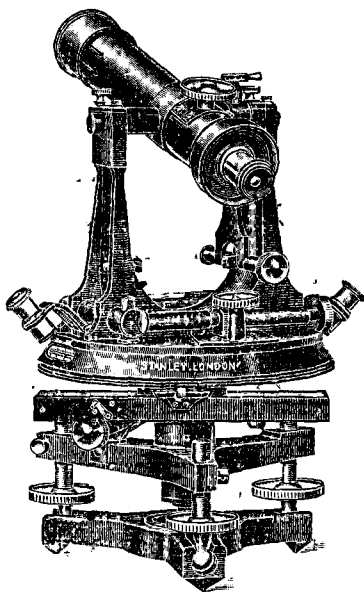


Fig. 98.—Railway Transit.

189. Railway Transit Theodolite.—Ordinary transit theodolites, being heavy and large, are not suitable for Railway work. Instrument with much shorter uprights making it lighter and very compact and rigid is shown by Fig. 98 which has been specially designed for taking horizontal angles. It has no vertical circle. The telescope transits over the eyepiece end only. The telescope has a stadia diaphragm for checking measurements and is fitted with a clamp and tangent for vertical adjustments. 4" and 5"—instruments read, by verniers to 1', and 6—inch to 20". 5" and 6"—instruments fitted with micrometers read to 10" and 5" respectively.

190. 7 and 8-inch and Larger Theodolites have the Following

Additional Parts.—(1) **The Illuminated Axis.**—The larger instruments, such as are chiefly used in constructive works as in setting out tunnels, are fitted with a small spirit lamp, resting on two pins or a bracket attached to one of the supports. The transit axis is bored on one side through to the interior of the telescope. Through the bore a small ray of light is sent by the lamp with a plano-convex lens front, to a lens placed at the end of the axis. This by a slight adjustment of the lamp on its stand, focusses the light upon a small mirror placed within the telescope, which

reflects its rays to the diaphragm. The lamp gives a faint light only sufficient to distinguish the cross-wires for night and underground observations. The mirror is about $\frac{1}{16}$ inch in diameter, and is generally mounted upon a milled-head screw placed below the telescope. The point of the screw is extended as a thin stem into the axis of the telescope where the mirror is held by it. This arrangement permits the mirror which is made of silver or platino-iridium to be removed for cleaning. The bracket may be removed when the lamp is not required and placed in the case.

If it is required to observe with 5" or 6" instruments in the dark, a lamp may be held against the instrument, which answers the same purpose as above; the lamp may also be used for reading the verniers. 4, 5 and 6-inch instruments are sometimes provided with lamps and bored axes.

(2) **The Diagonal Eye-piece.**—In large instruments the diagonal eye-piece is used. It permits the telescope to be used by the observer looking at right angles to its axis. Thus by the natural direction of the eye, stars or the sun may be observed to near the zenith, or the direction of a line cut by two lights at the bottom of a shaft may be observed from above by a theodolite on its ordinary stand, to check the magnetic bearing of the needle below ground, if this is assumed to be subject to local disturbance.

(3) **The Trough or Long Compass used in place of the Circular Compass.**—A long compass is often applied to a theodolite, either upon the top of the telescope, or more generally and conveniently for reading, under the limb. In this last case, the trough needle is a separate piece, which is only attached to the limb of the theodolite, when required to take a bearing, by means of *loop slides* or *bayonet fittings* under the limb.

The trough needle is generally made 5" or 6" long, and reads into a short scale of about 10° at each end. The divisions are best placed upon sliding fittings, so that they may be adjusted by four screws (at the sides near the ends) from the outside of the box. This brings the needle to its own axis, and also to the 0° reading of the horizontal limb of the theodolite. In larger instruments a reader is placed at each end of the needle.

(4) **The Striding Level.**—For the adjustment of the transit axis of a theodolite a very sensitive spirit level called the striding level is used (see p. 72). In the construction of the striding level (Fig. 71) the two striding standards, *s, s*, are carried down from the ends of the casing tube, *b*, of the spirit-level. These are adjustable, one by raising or lowering the end of the bubble-tube by the capstan-screws, *c, c'*, and the other by a lateral adjustment of the capstan-screws, *p, p'*, that act upon the stud, *s'*; which is fixed upon an arm centered upon the axis of the tube (Fig. 99.) This connection is shown by the dotted lines. By these two motions the standards

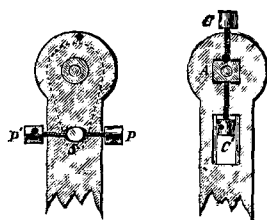


Fig. 99.—Construction of the Striding Level.

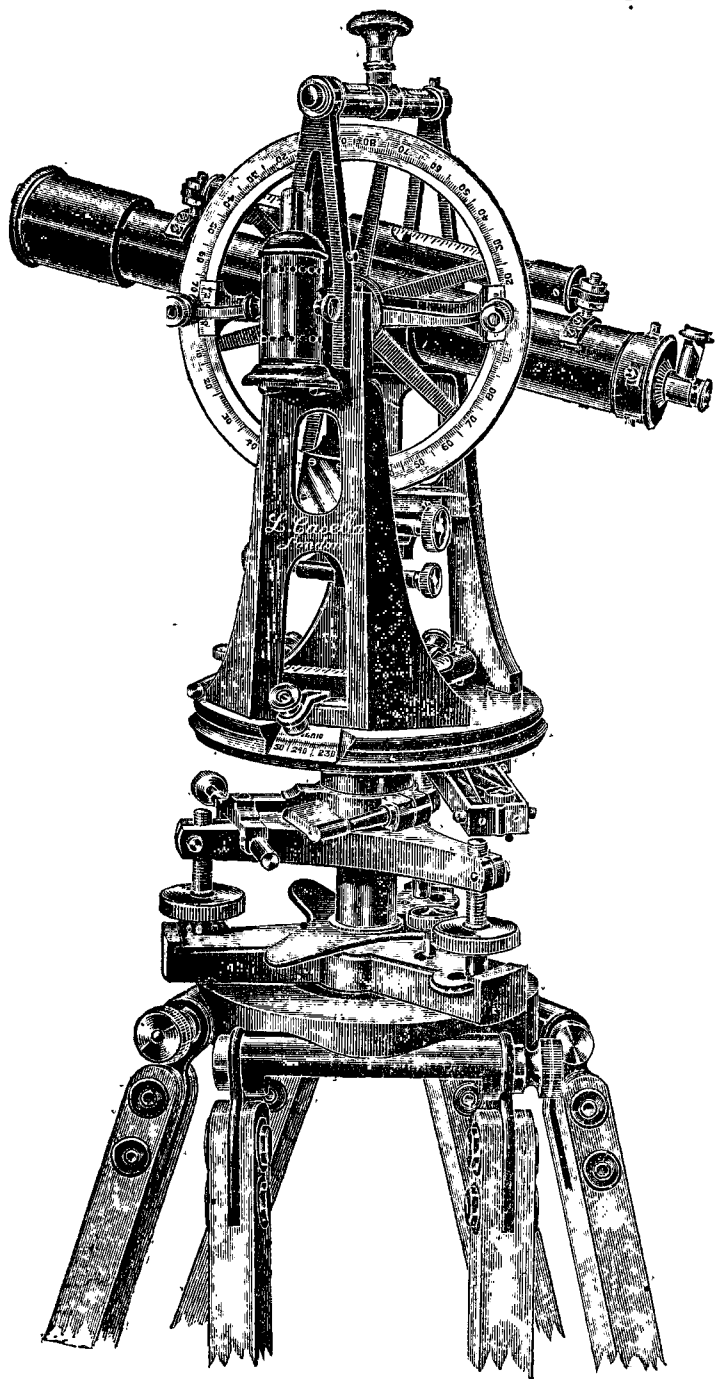


Fig. 100.—Casella's 8" Transit.

are brought to perfect parallelism with each other for their bearing surfaces and adjustment of the crown of the bubble-tube.

Fig. 100 shows Casella's pattern 8-inch Transit Theodolite with movable stand-head and with all the most modern improvements and the additional parts mentioned above. The limbs are divided on silver to 10 seconds.

The 8-inch or a large instrument is generally used for opening a survey in new countries for superior triangulation, particularly for

observations, at night of distant lights. The larger circle gives more exact reading of the limb which is generally graduated to 10 seconds, and by estimation to 5 seconds. When the instruments exceed 8", or even possibly of this size, exact readings can be obtained by means of micrometer microscopes. Theodolites of 8" and over are uniformly packed in two cases. The lower part goes in a case by itself, the upper parts connected with the vertical limb and all separate parts, eye-piece, plummet, etc., forming contents of another case, each part being sufficient for one man to carry without the tripod.

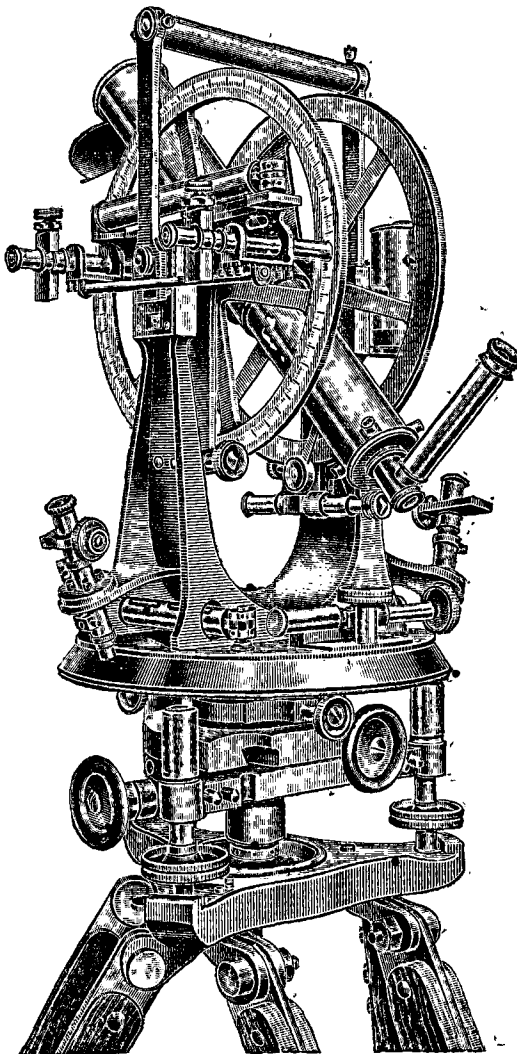


Fig. 101.—Stanley's Geodetic Theodolite.

191. 10 or 12-inch Geodetic Theodolite—This instrument approaches the limit in size of portability for movable stations in triangulation. In general construction of the instrument illustrated by Fig. 101 it has a mechanical sliding stage, and an extra powerful clamp and tangent arrangement

for the lower limb, the adjusting screws being all covered to exclude

dust. The circles are divided to 5 minutes, and are read by micrometers to to 1" of arc. Two vertical arcs are used, the second one carrying the clamp and tangent arrangements, which also serves to balance the trunion. The

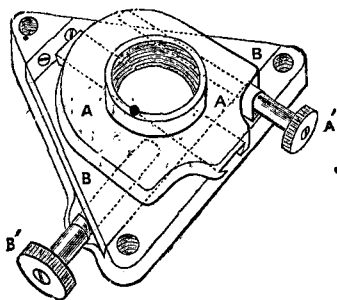


Fig. 102.—Tribrach Stage.

The 10-inch instrument carries a $14\frac{1}{2}$ -inch telescope with 2-inch object-glass, and the 12-inch instrument a 16-inch telescope with $2\frac{1}{8}$ -inch object-glass. The tangent-screws all act against springs to avoid loss of time. The foot-screws have caps to exclude dust.

175. Mechanical Tribrach Stage.—This addition to the theodolite permits exact adjustment over a station. The upper plate of the tribrach with the movable stage is shown in Fig. 102. A dovetail slide is fitted

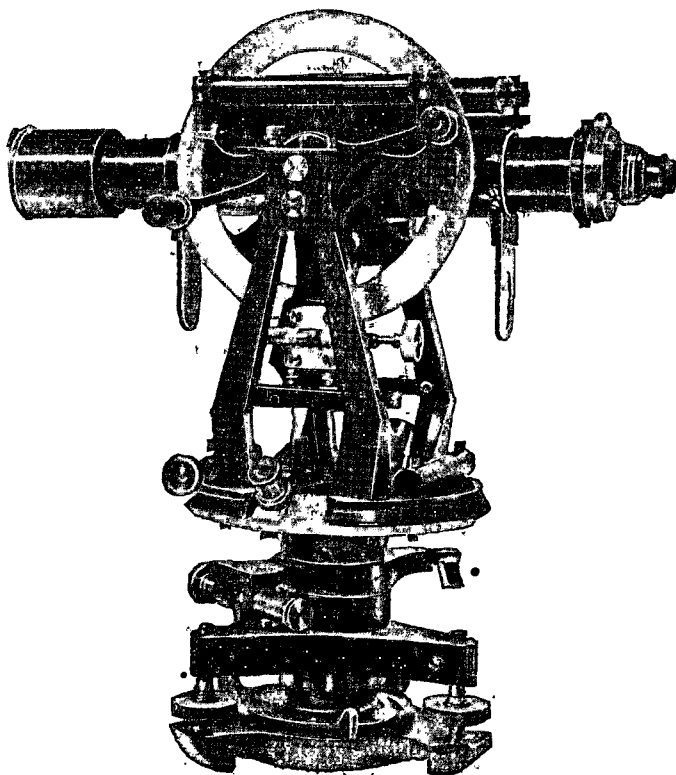


Fig. 103.—Hoskold's Mining Theodolite by Davis & Son.

upon the base of the stage adjustable for wear by a slip-piece with two

screws at the narrow part. The slide is adjusted to position in the direction of its dovetail fitting by the screw B' so as to move the whole instrument above it for centering in this direction. A slide

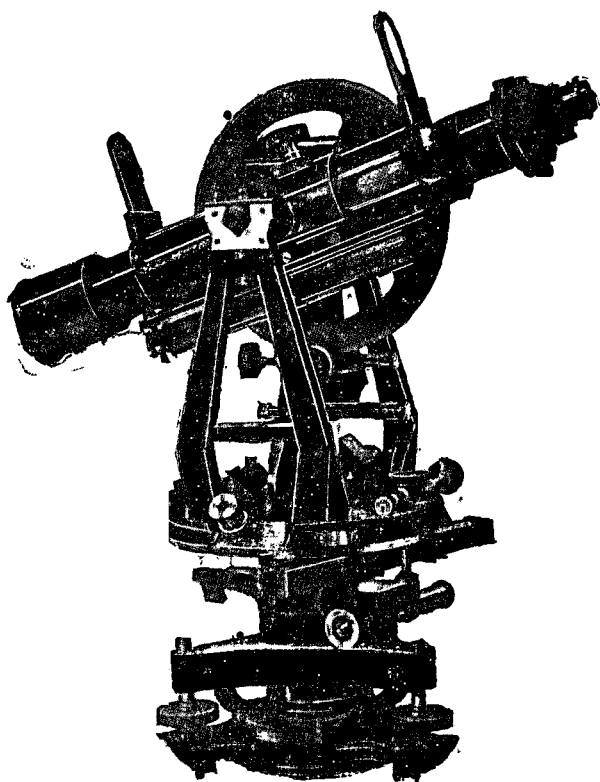


Fig. 104. —Hoskold's Mining Theodolite.

acting in the same manner, with dovetail fitting pieces at each end only moves the slide for an equal distance for centering transverse to the slide B' by the milled-head A'. The motion given the screws A' and B' permits the perfect adjustment of the theodolite over a point on the ground corresponding with the suspended plummet, after the instrument is set up to nearly its true position by movement of the tripod legs. The range of motion is from $\frac{3}{4}$ " to 1". (See Fig. 49, p. 55).

192. Hoskold's 6" Civil and Mining Engineers' Transit Theodolite as manufactured by Messrs Davis & Son is illustrated by Figs. 103, 104, 105. In Fig. 103, the instrument is shown with *face left* with the bubble-tubes over the vernier bar, and telescope in front, and two sights attached below the telescope for observ-

ing short lines. Fig. 104 shows the instrument with the prism reflectors to light the verniers, and the trough compass fixed under horizontal circle, with the *face right*. The bubble-tube is fixed below the telescope. In Fig. 105 are shown the telescope *E* raised

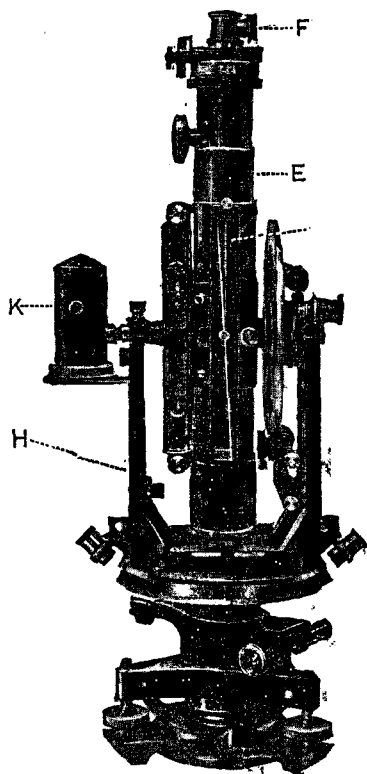


Fig. 105.—Hoskold's Mining Theodolite.

through sleeve or axis-collar to horizontal base enabling standards to be constructed shorter; the diagonal eye-piece, *F*, fixed for sighting down shaft through the hollow centre *H*; *K*, the lantern and *L*, the trough compass fixed above the telescope. The horizontal $\odot$ is 6" in diameter, graduated on solid silver to $\frac{1}{2}^\circ$ s with two rows of figures 0 to 360° with two double verniers reading to 15" verniers protected with glass covers and illuminated with prism reflectors. The vertical $\odot$ is also 6" in diameter, reading to $\frac{1}{2}$ degrees, with two verniers reading to 15". The telescope is 14 $\frac{1}{2}$ " long, having power 24 diameters, reading objects inverted. The diagonal eye-pieces are provided for sighting up or down a shaft inverting and erect. Special movement of telescope in sleeve is adjusted by rack and pinion. The instrument has rain-guards, very sensitive spirit-levels, hollow centre, three-screw parallel plates with centering movement, trough compass to fit either on the telescope or under the limb of horizontal $\odot$, and the following accessories:—sunshade with small aperture; 10 oz plumb-bob, mahogany case with wrench, screw-driver, adjusting pins and mahogany tripod.

193. Method of Using the Hoskold's Transit.—This instrument is acknowledged to be the best adapted for all kinds of surveying operations. One of its novel features is a perforated internal axis, like a cylinder, and, when the telescope is turned in a perpendicular direction, a sight can be taken through it, the centre of the horizontal circles and the vertical axis and may be continued down a vertical shaft. This is an exceedingly important and useful provision, because by its means the first line in any underground tunnel or heading joined to a shaft can be reproduced upon the surface

in the same direction with the greatest facility and accuracy. This operation is effected, first by placing two powerful electric lamps or other similar illuminated objects, at the bottom of a down cast pit, as far apart as may be possible, and in the direction of the centre line of the heading or main road of the pit; and secondly, by placing a properly constructed temporary platform over the centre of the shaft at the surface, and then a tripod stand having a traversing head with the theodolite, screwed upon it, is placed upon the platform, levelled, and the telescope placed perpendicularly; the traversing head is then slid right or left until the electric lamps appear in the field of the objective. The slow-motion screw is then turned until the vertical hair in the focus of the telescope coincides with or intersects the centre of the illuminated lamps. If the spirit-levels remain correct, the telescope is gently raised to the horizontal, and the underground line is sighted and pegged out in the same direction upon the surface. This line may be extended to any convenient distance, and becomes a base for all underground and surface surveys, with the certainty that all surface objects observed and plotted will occupy their true relative position to the underground workings.

When pits are sunk in the line of a railway tunnel, or other similar works to shorten the time of its execution, the transit theodolite is planted over the shaft in the centre line of the tunnel, telescope turned perpendicularly, and two illuminated marks are sighted out with the vertical spider line of the telescope at the bottom of each shaft, and the tunnel commenced in that direction and afterwards corrected by repeating the operation. The telescope of this instrument being extra powerful, a sight can be taken to the bottom of the deepest mining shafts.

Another important improvement consists in constructing the body of the telescope tube in a single length, and when mounted with its eye-piece and focussing tube, the central part of the telescope is made to slide in its horizontal axis, socket, or sleeve, and turning a special rack and pinion screw, either end of the telescope can be lengthened or shortened. When a perpendicular sight is to be taken, the object-end is thus shortened until it passes the vernier circle, and when it is required to take a high altitude sight, or up to the zenith, then the eye-end is shortened. It is evident that this part of the invention makes it possible to employ a longer and more powerful telescope, with much shorter standards or Y's, than is the case in other instruments. When the instrument is used for railway surveys, tunnel work, or land surveying operations, the telescope is slid half-way to its normal position.

In countries where many geographical positions have to be fixed in national and particular surveys, the observations of two stars near the zenith, for latitude determinations by the Talcotte method, is common, consequently this Transit Theodolite is admirably adapted for this object, as also for determining azimuths and for setting out the true meridian.

The vertical circle has a very sensitive spirit-bubble attached to the vernier bar, and on the opposite side a larger, longer, and still more sensitive spirit-bubble is fixed to the side of the telescope and level with the top of it, so that this addition renders the telescope a sensitive spirit-level, for all spirit-level operations, independent of the other parts of the theodolite and its uses.

A horizontal revolving glass prism is fixed behind each reading microscope, so as to illuminate the verniers, a most desirable convenience underground, in dull weather or at night. A finely-constructed micrometrical eye-piece is provided for measuring fine angles for seconds of arc, to be employed in determining distances without measuring with a chain. This mode consists in setting up a wooden rod in a horizontal position of any convenient length, say 5, 10, 15, or 20 feet or other measure, at any distance within the range of the telescope and measuring the subtended angle with the micrometer between a circular disc, with a white cross upon it, placed at each end of the rod. The following is an easy mode of finding the distance by this method :—

Say, measured angle = $3^{\circ} 10' 31''$. 4, and subtense bar = 30'.

Then Log. of base 30',	= 1.4771213
and Log. Coscant $3^{\circ} 10' 31''$. 4	= 11.2565632
	= 12.7336845
	- 10.0000000

And Log. of distance = $541.60725' = 2.7336845$

When the sub-tense base passes the practical limits of a wooden rod, two poles, with a white thin line drawn perpendicularly upon each, may be employed instead. The micrometrical eye-piece is useful for a variety of purposes, including an auxiliary for measuring horizontal angles. All parts of the instrument excepting the bearings, bushings, and screws have been made of composite aluminium metal; the instrument is, consequently, much lighter than those of the old type of the same size and quite as strong, and is not subject to flexure. The instrument has an axis level, lantern, and inside reflector for night or underground work. The adjustments are the same as for other instruments of the transit class.

A special traversing stand has been designed for use with this instrument, when the telescope of that instrument is employed for sighting down vertical shafts, the principal object being solidity with the least possible bulk and weight. The frame, supported by the legs, is circular and 12" in diameter. This supports a tube across the centre, a corresponding tube running parallel being attached firmly to the circumference. Along these tubes a carrier traverses, to which is screwed the usual centering apparatus common to all first-class theodolites. The theodolite is brought roughly over two points fixed at the pit bottom; it is then traversed on the stand, which allows of a movement of $6''$, and it can also be adjusted in any direction for about $1''$ by the centering motion. The

centering motion is then clamped, the long traverse is also clamped and the fine adjustment comes into operation, bringing the vertical spider line into coincidence with the two illuminated marks placed at the bottom of pit.

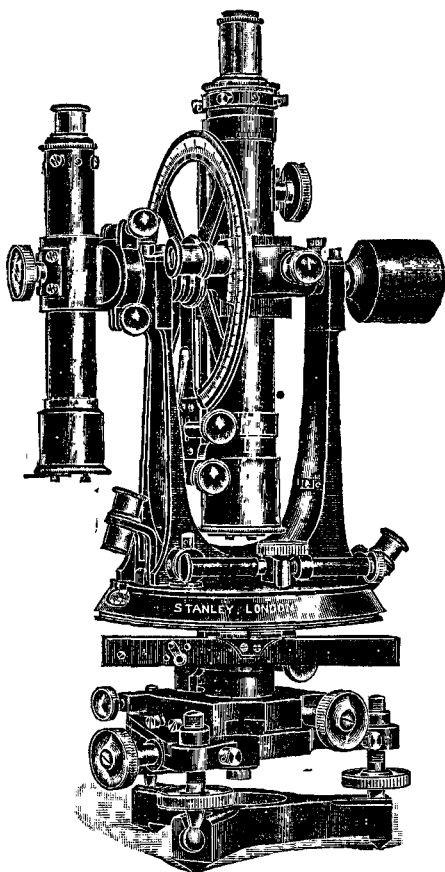


Fig. 106.—Stanley's Universal Transit.

194. Stanley's Universal Theodolite.—Figs. 106 and 107 illustrate an instrument recently brought out and may be deemed a universal theodolite, since, whilst occupying the same sized box only as an ordinary civil engineer's transit, it is provided with an attachment fitting it for shaft sinking work. This attachment consists of a small telescope, which can be screwed on to either end of the main telescope axis, as shown in Fig. 106, or on top of the main telescope, as shown in Fig. 107. In the former case the weight of the attachment is counterbalanced by a small weight screwed on to the opposite side of the main telescope, as shown in the illustration.

The objection to all such attachments hitherto, has lain in the impossibility of securing permanent parallelism between the two telescopes, as, even if the attachment was arranged by the maker to be accurate at the outset, this accuracy soon disappeared in use. In the instrument illustrated no attempt is made to secure a permanent parallelism between the optical axes of the two telescopes, but means are provided by which the user can secure this whenever the attachment is to be used. To this end the auxiliary telescope is not "made solid" with the socket by which it is screwed into place, but is secured to it by an adjustable ring, for which a slow-motion is provided. Its eye-piece is, moreover, provided with a single cross-wire only, which is arranged parallel to the axis about which this auxiliary telescope is adjustable. Hence, after screwing this telescope into place, the necessary parallelism can be secured by setting the

main telescope on some distant mark, and then sighting the same object through the auxiliary telescope, the fine adjustment of which is manipulated till the single cross-hair cuts the fiducial point. When the distant point sighted on is simultaneously cut by the cross-hairs of both telescopes, the optic axes of these are necessarily in the same plane.

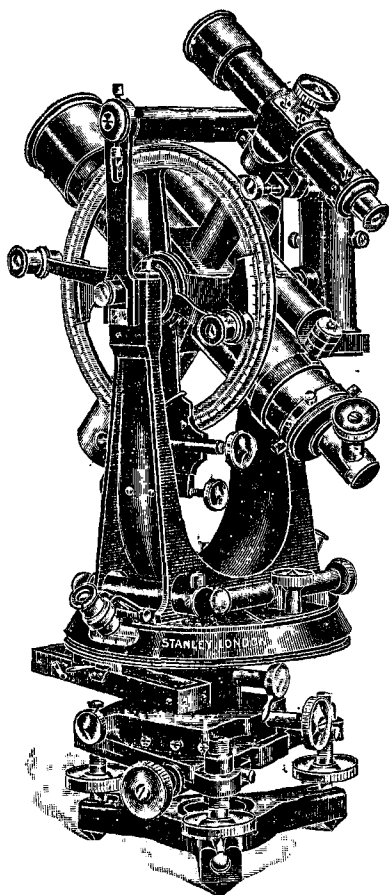


Fig. 107.—Stanley's Universal Theodolite.

In using the instrument for transferring a line to the bottom of the shaft, the instrument is set up over the shaft, so that the main telescope transits along the desired line on the surface. The auxiliary telescope is then mounted on top of the main telescope, and adjusted until its cross-hair also transits along the same line, which can then be transferred down the shaft by simply turning the telescope round its horizontal axis. The centre of the instrument can finally be transferred down the shaft by rotating the upper parallel plate through 90 deg., and the line of sight of the auxiliary telescope will then describe a truly vertical plane at right angles to that in which it previously moved. The intersection of these two planes is, of course, the vertical through the axis of the theodolite. This method of transferring a base line down a shaft is, it will be seen, a very simple one, and free from the difficulties inherent in the use of plumb-lines, with which great trouble is often experienced, owing to the action of air-currents preventing the lines from remaining in a truly vertical position.

By fixing the auxiliary telescope at one end of the axis on which the main telescope turns, vertical angles can be read when desired. In this case the horizontal cross-hair in the main telescope is first aligned on some distant mark, and the auxiliary telescope is then adjusted by a tangent-motion till its cross-hair also cuts the same mark. The optic axes of the two telescopes then lie in the same plane, and vertical angles, too steep for reading in the main telescope, owing to

the line of sight of the latter fouling the base of the instrument, can then be read with the auxiliary. (Engineering. Nov. 18, 1904).

195. Instruments generally Used in Mine Surveying but may be Used for Surface Work :—

196. Semi-circumferentor.—Fig. 108 shows the instrument which is supported on a ball-and-socket joint. The socket is made

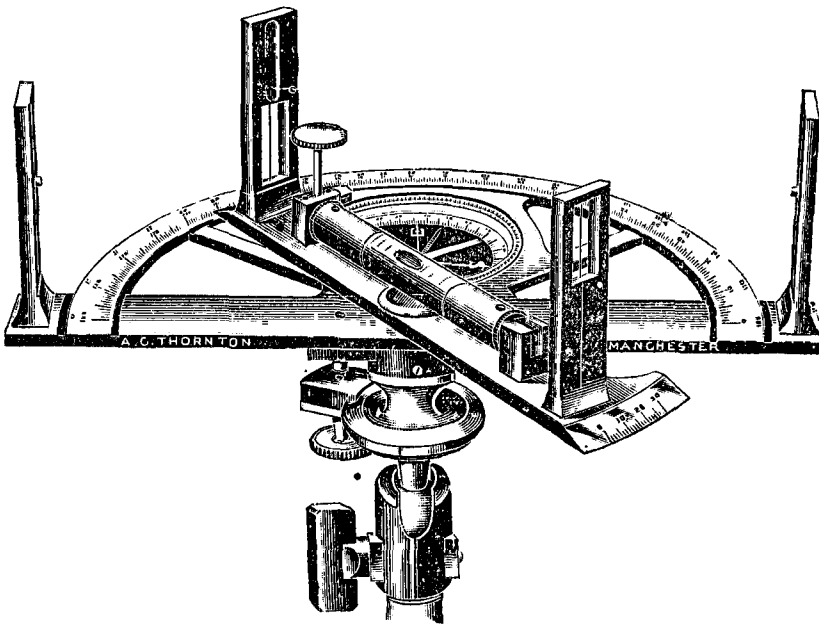


Fig. 108.—Semi-circumferentor.

up of two pieces which are clamped together to hold the ball by a wing-headed screw. The semi-circular arc which forms the limb is fixed upon the vertical axis, and is graduated to half degrees. The diameter has two sights mounted at its ends. A sight-vane is jointed on the vertical axis to move centrally between the first pair of sights. The length of the sight-vane with the verniers at its ends is made shorter to pass within the first pair of sights to any angle around the arc, except the small angles with which the sights themselves interfere when they are superimposed. The verniers are read on the arc to 2'. There is a small compass fixed to the limb. This cheap instrument is particularly useful in setting off angles from drawings plotted from a survey by a better instrument. The diameter of the circle is made from 6 to 8½ inches. The instrument is provided with a wooden tripod, and is useful both for surface and underground work.

197. Miners' Dial or Circumferentor.—The miners' dial is illustrated in Fig. 109. It consists of a compass 6" in diameter,

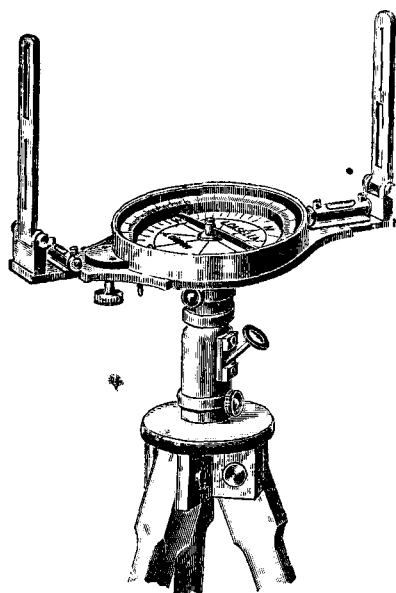


Fig. 109—Casella's Mining dial with Rack adjustment.

divided by a pointed edge-bar needle mounted on a jewelled cap. The divided compass is raised on a step so that, when the compass is horizontal, the upper surface of the needle is on a level with the plane of the graduations. The bottom plate of the compass box is extended (north to south) by strong arms at the ends of which sights are hinged to turn down to the top of the box for portability. The box is mounted upon a ball-and-socket joint fixed upon a tripod stand by which it may be placed over a station. The sights have in each two slits, one narrow and the other wide. The wider one, called the window, contains a vertical hair. In the pair of sights the window of one sight is placed at the lower position, and the narrow slit in the upper. In

the fellow sights the position of these parts is reversed, the observation being always taken from the slit through the window. The duplication of the parts in each sight permits it to be used in either direction. There are also two bubble-tubes fixed on the compass-plate between the sights and the box for the purpose of fixing the dial perfectly horizontal, which is effected by the ball-and-socket arrangement. By means of the ring-screw at the side of the joint, the instrument can be clamped or fixed. The arms with the sights and the rim of the circumferentor are turned round by the screw shown below the compass-plate on the left hand, by which the

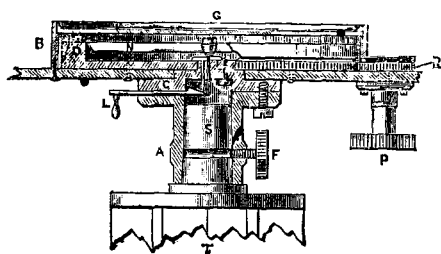


Fig. 110.—Section of Compass-box of Lean's Dial.

instrument may be accurately adjusted. This is explained as under :—The revolving compass forms part of this dial as well as of Lean's dial described below. The compass is shewn in section (Fig. 110) In this instrument the socket-piece, *A*, is fitted on to the inner vertical axis, *S*, and rests on the ball-head or lower parallel plate, *T*, so that it may rotate easily* to set the needle to magnetic north of the

compass, if desired. The socket-piece is attached to the compass-box by means of a collar, *C*. The compass has a step, *D*, graduated to degrees on its inner edge to read to the point of the needle, and similarly to degrees on its outer edge to read with a vernier, shown above *D* to 3'. The bottom-plate of the compass-box is graduated to 10°: in some difficult positions in the use of the instrument this last is the only reading that can be sighted. The arm-plate carries the compass-box to which the vernier, *B*, is fitted. The outside rim of the compass-plate is formed into a toothed wheel, as shown in section on the right-hand side, into which a small pinion, *R*, is fixed in a box upon the arm-plate that works by means of a large milled-head, *P*. By means of this milled-head the instrument may be rotated about the compass, so that the line of division on the compass-step reading into the vernier performs the functions of the horizontal limb of a theodolite. In this way angles are read by means of the vernier, quite irrespectively of the reading of the needle. When the compass is set to the zero of the vernier at north (360°), it may be fixed in this position by means of a pin fitting in opposite holes to the arm-plate and bottom-plate of the compass, not shown; when thus fixed the needle only is used as in the surveyor's compass (Fig. 29), or in plain dial without racked adjustment. Between the collar-piece, *C*, and the socket-piece, *A*, a wedge-shaped lift raises the needle off its centre by pressing in a slide shown at *L*.

198. To take an observation, set up the instrument by means of the plumb-line over the station point, fix the two plates by the pin underneath the compass-box, and bring the sights round, so that the needle and 360° coincide. Having levelled the instrument, clamp it by the ring-screw, then release the pin, and then by the screw turn the sights until they cut the points set up; the vernier on the rim will give the angle to 3 minutes taken from the meridian. This mining dial without rack adjustment and vernier is also available, and is a strong serviceable instrument very suitable for rapid surveying. It has the advantage of much larger compass and longer needle, and it is steadier and not so liable to get out of order. In point of lightness, speed, and portability it compares favorably with the prismatic compass. For route survey or trial location in jungle or dense forest this is the most suitable instrument.

199. Henderson's Dial.—Fig. 111 is a 6-in. instrument. Its special feature consists in the attachment of two pairs of sights revolving within each other. The fixed sights are always in a line with the back object when surveying with the verniers, whilst the inner or revolving sights adjusted to the forward object, gives the angle of deviation from the original direction. The sights are of the split kind, and are so constructed as to fold over to be out of the way in case the forward and backward draft should be in close proximity. The back and forward angles can always be checked without shifting the instrument when once fixed, and deviation is detected immedi-

ately. The instrument combines some of the parts of the Lean's dial, as just described for the plain Dial.

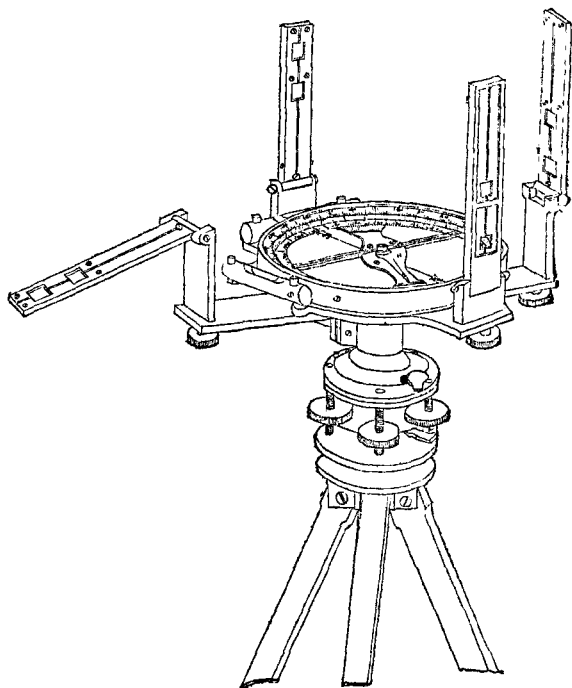


Fig. 111.—Letcher's Henderson's Dial.

200. Lean's Dial.—In general construction the sights and limb on which they are mounted are the same as in the dial (Fig. 109) first described. The legs are also the same. Fig. 112 shows the 5" instrument as made by Elliott—the additions being the telescope, level, and the arc mounted together on the limb. In the engraving the sights and the vertical arc with the telescope are shown separately to show that either of them may be used. The telescopic arrangement is used generally above ground, and the sight arrangement below. The tripod of the mining circumferentor has the legs fitted directly between the *book-pieces* which are fixed to the lower parallel plate, thus dispensing with separate tripod head common to theodolites and levels. The parallel plates have four foot-screws and used in the same manner. The upper parallel plate in this dial carries the vertical axis which fits into a socket attached below the centre of the limb in the way described for the Miners' dial (Fig. 109).

201. The Revolving compass forms also a part of Lean's dial as described before.

202. **The Vertical Arc** is fitted by the compass-box to leave room for a level to be fixed outside the seatings of the arc. To

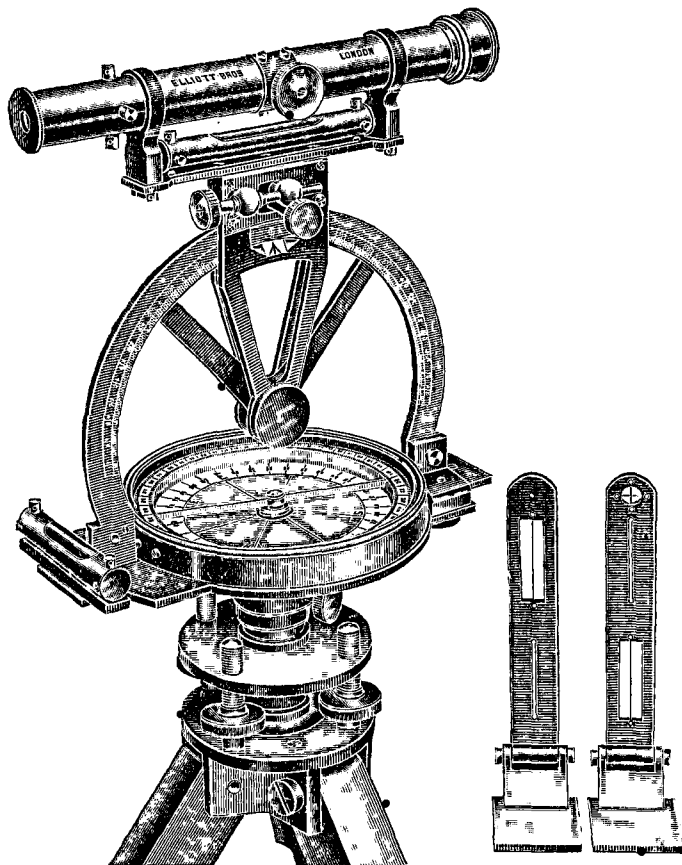


Fig. 112.—Elliott's Lean's Dial.

make the instrument as low as possible, the axis of this arc is placed near the surface of the cover which protects the glass of the compass-box. The telescope traverses the arc tangentially, and can be adjusted at any position by means of the clamp and tangent-screws for reading the arc by its verniers. The arc is graduated on one side into degrees, and reads by the vernier to $3'$ in the same manner as the horizontal circle; on the other side it is divided with a percentage scale of difference of hypotenuse and base which reads to an index line. A bubble-tube is placed over the axis below the telescope adjustable by means of capstan-headed screws. The telescope fixed is placed just sufficiently above the arc to permit it to be adjusted to a vertical position at 90° or a degree or two over this. By this means a bearing may be taken of any object upon the ground from the top of a shaft, and a line may be sighted to the bottom of the shaft in exact azimuth with this.

The vernier to the compass being placed below the vertical arc, can only be read obliquely when the telescope is mounted, and this is a defect in the instrument.

203. The Vernier, as fixed to instruments in Figs. 109-112 is

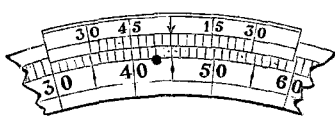


Fig. 113.—Vernier reading centrally to 3'.

set off on each side of zero line in 10 divisions, which are figured 30, 45, 0, 15, 30 (Fig. 113). The principle is this:—If the limb is divided to 60', and vernier reads to 3', then $a - b = a \div n = 3' = a \div 20$, $\therefore a = 60'$; $\therefore 20a - 20b = a$ or $19a = 20b$,

i.e., 19 divisions of the limb are taken and divided into 20 for the vernier.

204. Adjustment of Lean's Dial.—This is the same as that of the Y-theodolite, so far as this can be carried out.

205. To Set a Line in Azimuth with One taken Above Ground.—This is necessary where there is local attraction to the needle below, or there is suspicion of this, so that the needle can not be depended upon with certainty. The use of the instrument is then like that described for the Hoskold's theodolite.

206. Hedley's Dial.—The peculiar feature of this form of dial (Fig. 114) is that the sights move upon a frame-work centered upon

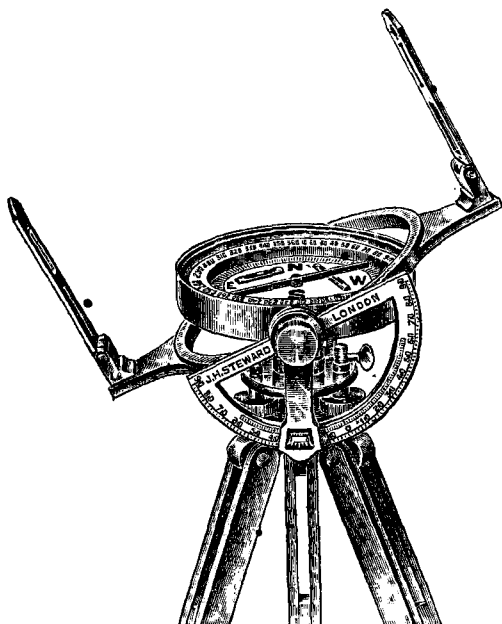


Fig. 114.—Hedley's Dial.

a horizontal axis, so that they may by a rocking motion take horizontal angles within wide azimuth without obstruction to sight of the compass. From opposite points of the side of the compass two pivots are projected. These are set at right angles to the vertical axis which is placed above ball-and-socket. The pivots and the centre of the vernier are in a line with the *E* and *W* points of the compass when this is set at zero (360°). The pivots form the axes of a *rocking ring* which surrounds the compass-box with a space sufficient to clear it when the ring is rocked

about its axis. The extended arms of the ring carry the sights 12" apart. One of the pivots is prolonged for about $\frac{3}{4}$ " beyond the

outer circumference of the ring. This prolongation is generally triangular in section, and forms a fitting to the vertical arc which is attached by a milled-head screw, when required, the arc being an encumbrance when this dial is used for making horizontal plans only. The vertical arc with its index-arm forms a separate piece. The arm is centered upon the arc with a ground fitting. The arm-piece forms the axis, through the centre of which a triangular hole is made to fit the triangular prolongation of the pivot, so that the index-arm remains fixed, and the arc moves with the rocking ring. The arc is graduated into degrees on the outer edge of its surface, and has a scale of difference of hypotenuse and base upon its inner edge. Hedley's dial can be locked by a pin which is attached to the under-side of the compass-box, so as to work by the compass only. The ring can also be locked level with the compass by a sling *latch-piece* so as to convert it into a plain dial. The raised horizontal $\odot$ is divided to 360° with solid fixed vernier reading to one minute. There is also a concealed rack adjustment for fast dialling. Compass dial is divided to 10° . The needle has agate centre and stop and sliding dip-adjuster. The instrument is mounted on a tripod stand with adjustable sliding legs, for rapid levelling and for use on uneven ground and in shallow seams; it is fitted with parallel plates and 4 foot-screws.

207. Improvements in Hedley's Dial.—Fig. 115 shows the

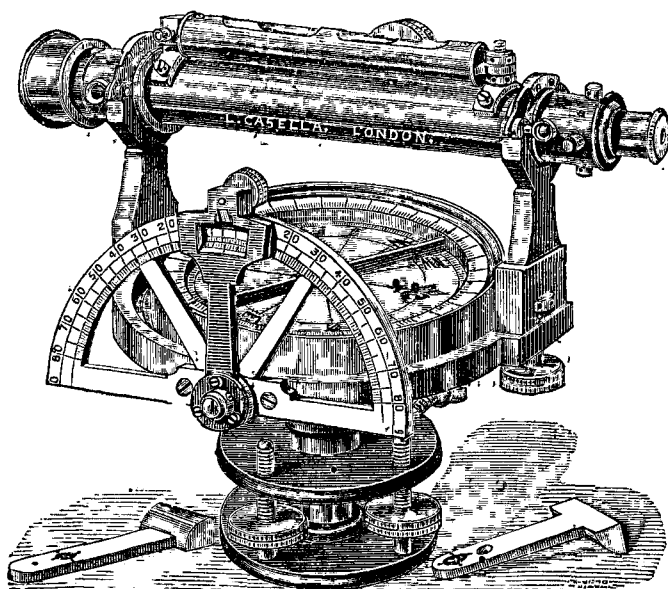


Fig. 115.—Telescopic Improved Hedley Dial.

instrument as improved by Casella by the addition of a telescope which detaches exactly like Lean's dial. The instrument shown has

parallel plates suitable for fairly level workings. The telescope is placed on Y's, and is of exactly the same form as that described for the plain theodolite. The Y's in this instrument offer a great convenience for reversing the telescope for back sights in range when the vertical axis is fixed. The level above the telescope enables it to be converted into a drainage level when the rocking ring is locked with the compass.

208. Davis's Improved Hedley Dial.—This instrument (Fig. 116) contains, in addition to the outside vernier of the theodolite,

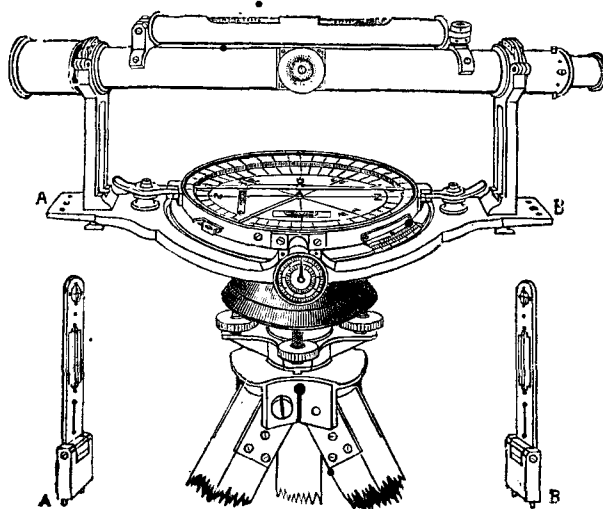


Fig. 116. — Davis' Hedley with Hoffman Head, Telescope and Sights.

which is more easily read than when the vernier is placed inside the compass-box, and which enables horizontal angles to be read with great accuracy, all the latest improvements of the original Hedley dial. This is the best instrument for colliery use. For surface surveys it may be used as a theodolite when a telescope is substituted for the sights. It is made up of a compass-box 5 or 6 inches in diameter divided into 360° on the compass-ring, and into four times 90° on the plate, 90° being at the east and west points and 0° at the north and south points. In the face of the instrument, protected by a glass cover of the compass-box are set at right angles to each other two spirit-levels. There is a circle or limb divided into 360° underneath the main-plate. By means of a pin the upper and lower limbs of the instrument may be fixed together at 360° . Instead of the side arc for taking vertical angles as in the original Hedley dial, the improved instrument has a fixed circular box, $1\frac{3}{4}$ inches in diameter, with a hand moving round dial plate. The advantage of this new form of arc is that it always remains in position, and does not interfere with the manipulation of the screws under the body of the dial. The main feature of the Davis's

instrument is that by its means bearings may be taken simultaneously with the magnetic needle and with the vernier, the latter automatically checking the former. Hence any error due to incorrect reading is at once found out. The dial is numbered from the North from left to right and the vernier ring is also numbered from left to right. The graduations of the vernier ring and of the needle ring being arranged accordingly, the readings correspond.

209. Cranked 6-inch Gimbal Dial.—Fig. 117 represents the highest modern refinements in Mining Dials; it is fitted with sliding

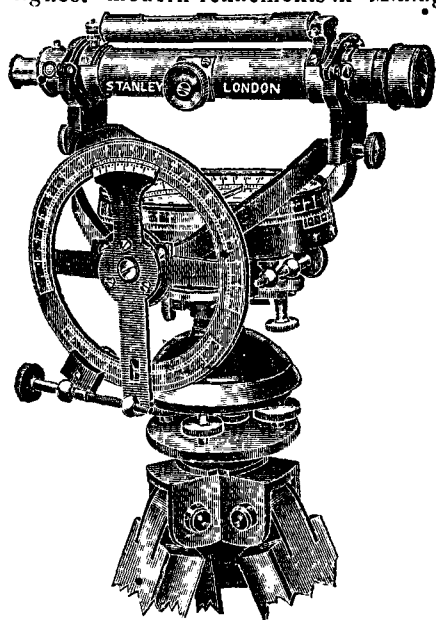


Fig. 117.—Combined Mining Dial, Theodolite and Level.

stand, with solid head, and an improved form of Hoffman head, Clamp and Tangent movement to the lower axis, also to the Limb, which is outside reading by two Verniers to single minutes. The Gimbal is cranked so that THE TELESCOPE WILL READ 90° BOTH WAYS, thus enabling a bearing to be taken of any object upon the surface from the top of the shaft, and a line being sighted at the bottom of the shaft in the same vertical plane with it, without changing the horizontal adjustment of the instrument. It is also useful for taking Vertical sights up a shaft. The Vertical Arc has a Clamp and Tangent, and is divided to read single minutes. The telescope is fitted with Stadia Points for reading distances *without the use of a chain*. It is also fitted with folding sights for use without the telescope and a needling pin to lock the compass. The entire instrument is constructed as low as possible for use in shallow workings.

210. Description of the Grubb-Sight.—A rectangular brass tube about $\frac{3}{4}$ inch square, 3 inches in length, and open at both ends, forms the sight, the object being viewed through the tube. There is a plate of glass coated with a semi-transparent and highly reflective film, and fixed at an angle of 45° about midway in the tube. Fig. 118 shows the original conception of the sight as it is necessary to describe it in order to render the new construction quite simple. Opposite to the reflector is a circular tube fixed at right angles to the square sight. There is a diaphragm at one of its ends, *d.* coated with opaque material, and through it cross-lines, a star, a scale or some other convenient device are cut or photographed. At

the base of the circular tube is an achromatic lens, *o*, of a focus suitable to the distance between the lens and diaphragm.

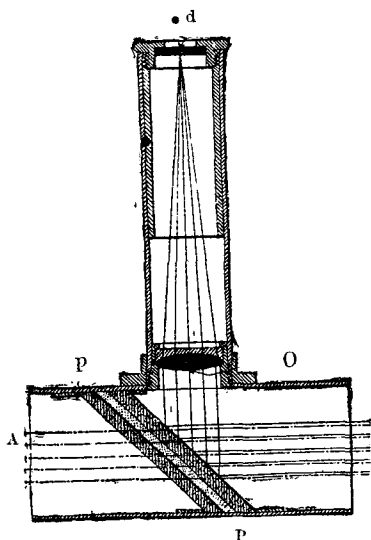


Fig. 118.—Grubb-Sight.

The luminous rays from the markings on the diaphragm run parallel after passing through the achromatic lens, and are bent at right angles and directed into the eye of the observer. The result is the same as if the rays from the diaphragm enter the eye from a distant object, and the observer sees the image or sight lines at the same distance as the object *i.e.* virtually superposed on it. The lines are sharply and clearly defined, there being no parallax. At whatever part of the sight-tube the eye is applied, the cross-lines will coincide with the object. This is due to the rays being parallel after passing the lens, *o*, under exactly the same conditions as if coming from the cross-lines at a greater distance instead of a small cross at a shorter distance.

As previously mentioned sufficient brilliancy is obtained as a result of coating the inclined glass plate with a semi-reflective film.

A modification of the above sight has been made in which the diaphragm is placed at one side of the sighting-tube (Fig. 121) as the original one having a tube at right angles to it becomes more or less cumbrous when applied to surveying instruments. Now the divergent rays from the diaphragm are reflected from a silvered part of the back window of the sighting tube made of plate-glass, upon the concave surface of the front window, and are by it rendered parallel and again reflected into the eye of the observer in an exactly

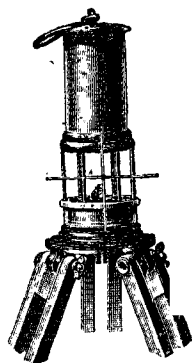


Fig. 119.—Grubb-Sight or Cross thrown on a Safety Lamp.

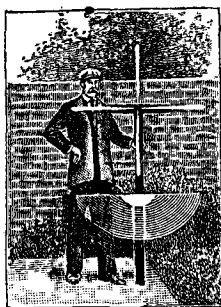


Fig. 120.—Grubb-Sight.

similar manner described before in Fig. 118. A brilliant image of the + or other device is visible, when using the sight, its effect being similar to observing an object through faintly smoke-tinted glass. The eye may be applied near the centre of the sight and at a greater or less distance from it, and the object can be easily seen.

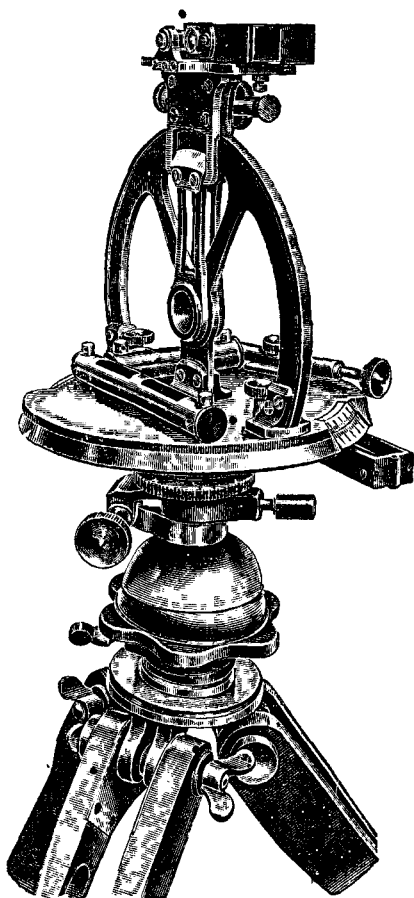


Fig. 121.—Grubb Miners' Dial with Trough Compass.

is also seen a level-bubble from a Grubb pocket level.

211. Grubb Miners' Dial.—As the Grubb-sight is only three inches long, the usual distance between the fore and the back-sights is done away with, in the construction of the dial, so that the greatest space required is regulated by the diameter of the compass-box which is usually 5 or 6 inches. Fig. 121 illustrates the miners' dial with the sight attached, and with this construction angles can be measured up or down to 90° , and may be read to 3 minutes of arc; and the sight is hinged, so that back-sights and fore-sights may be taken by simply reversing that portion. The instrument may be fitted with a circular compass, outside vernier reading to 3 minutes,

The new sight, does not claim to displace the telescope in standard surveying instruments, such as the theodolite and level; but these instruments are used for work for which the theodolite and level are not suitable on account of their delicacy, weight, and size which are regarded as disadvantageous. The new sight, when attached to the instrument for underground surveying, will be most useful to the mining engineer; for with a lamp, a brilliantly illuminated cross or any other device may be thrown on a staff or any other object used in the pit for measuring vertical or horizontal angle. Fig. 119 shows the Grubb-sight or cross thrown on to a safety lamp, it being taken from an actual photograph clearly indicates the ease of surveying by its means. Fig. 120 shows the sight on a brick wall where

and the Grubb sight, mounted on an arc reading to 90° both-

ways, with a vernier reading to 3 minutes. Fine adjusting-screws are fitted for reading both horizontal and vertical angles.

Instead of a circular compass-box, Fig. 121 has a trough-compass which is fixed under the horizontal plates, the ends projecting beyond the circumference. There is also an additional fine adjustment, so that the needle may be read with ease and brought to zero accurately. Two large and sensitive levels are also fixed at right angles to each other, there being ample room owing to the circular compass-box being displaced.

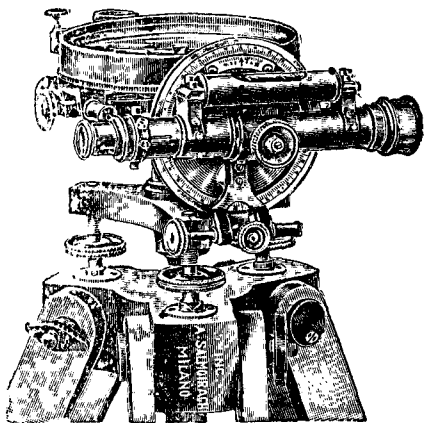
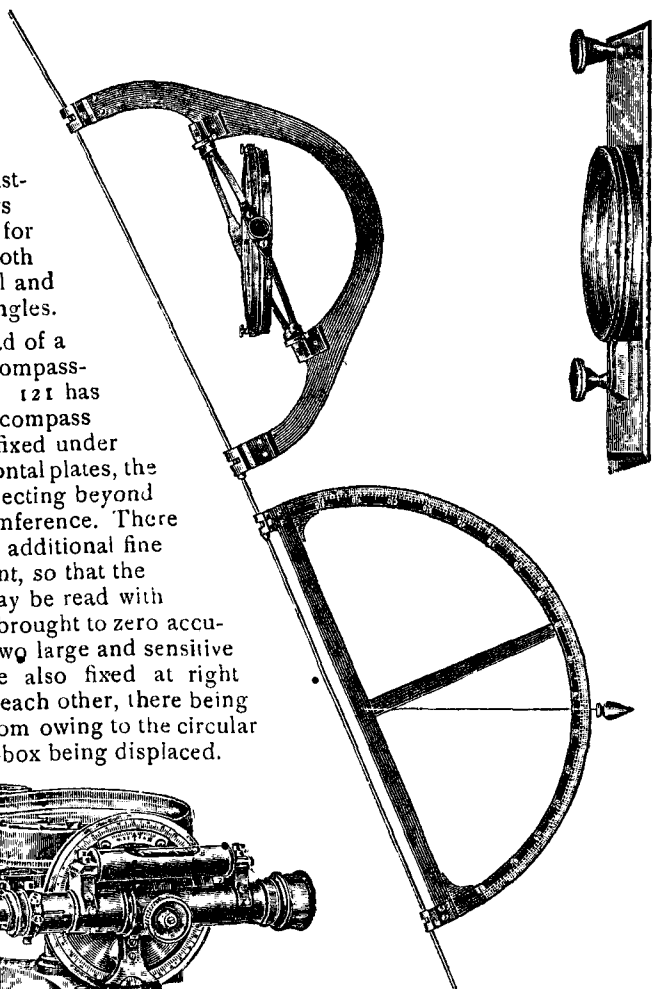


Fig. 122.—Mining Survey Transit.



Figs. 123, 124.—Hanging Compass and Clinometer.

212. Mining Survey Transit.—In this instrument (Fig 122) the compass is placed clearly in view. The vertical axis has a clamp and tangent-motion to bring the compass to exact bearing or to permit surveying with the compass only. The axis has also a clamp and

tangent-screw to the exterior divided circle which reads with verniers. The telescope is placed on the side of the instrument, and has clamp and tangent-motions to read the vertical circle which the vernier traverses in transit. The instrument is balanced by a counterpoise weight to keep its vertical axis in equilibrium.

213. Hanging Compass or Dial and Hanging Clinometer.—The instruments shown in Fig. 123-124 are used in rough underground surveying in mineral districts where, mineral veins being tortuous, sighting is difficult. The measuring line upon which it is used is either a hempen or copper cord or a chain. The compass is hung upon the cord which may be stretched to any point out of sight and the compass will then indicate the bearing of the line. In Germany two instruments are used simultaneously—the hanging compass for taking the bearing, and a clinometer composed of a light brass semicircle graduated to degrees with a small plummet for taking inclination.

214. Mining Survey Lamp (Fig. 125).—In lighting underground survey lines, two lamps are provided. The vertical line of the cross+ of either of which will take the position of the vertical axis of the mining theodolite or dial when the lamps are placed upon the tripods formerly occupied by it. Three tripods are provided for each instrument with head adjustments complete. The tripods are made in such a manner that the instrument can be placed on any one of them, in a level position. By placing the instrument in the middle, fore and back-bearings can be taken with the aid of the lamps and so the angular positions of lines can be extended with accuracy.



Fig. 125.—Mining Survey Lamp.

215. The Instruments for finding Approximate Measurement of Distances are:—

216. A 2-ft. Rule.—Approximate measurements of distances

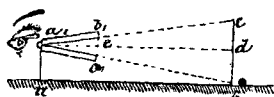


Fig. 126.—2-ft Rule for Finding Distances.

may be obtained with a 2' rule, by opening the rule so that, when held up to the eye, its two limbs include a known height at the end of the line whose length is required. For instance, in Fig. 126, let the distance ab be required, and let bc be any known height. bc may be a pole or a ranging rod of known length, or a man on foot or on horseback. If now the foot-rule is held up to the eye at a_1 , and opened until it includes the points b and c , we have $\frac{a_1d}{a_1e} = \frac{bc}{b_1c_1}$ or $a_1d = \frac{a_1e}{b_1c_1} bc$; and as a_1e is 1', $a_1d = ab = \frac{bc}{b_1c_1}$ = height bc divided by the

width b_1c_1 between the open ends of the foot-rule. b_1c_1 may be measured with an ordinary 10-scale of inches divided into tenths. Thus if bc is $30' = 360''$, and $b_1c_1 = 1'20''$, then $ab = 360 \div 1.20 = 300.'$

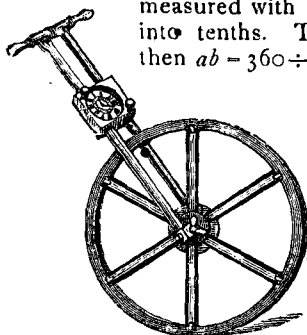


Fig. 127. — Casella's Perambulator.

217. Perambulator.—This instrument (Fig. 127) consists simply of a wheel with a handle which is rolled over the line to be measured, and the number of revolutions is shown on a small indicator connected by gearing to the axle. This indicator expresses the revolutions in feet, and so gives the total distance on inspection.

218. Pedometer.—This instrument records the distance walked over, and consists of a simple arrangement of weight, and pendulum, acting on plain toothed-wheels by which the distance walked over by the wearer is accurately measured. It should be suspended by the hook *A* in the waistcoat or side pocket, and must be maintained

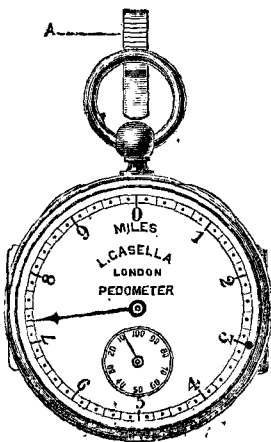


Fig. 128. — Pedometer.

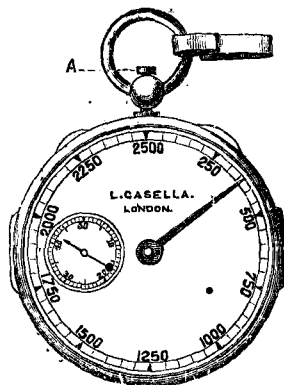


Fig. 129. — Passometer.

in an upright position. The figures on the dial (Fig. 128), represent miles, and the spaces between the figures are quarter miles. When the pedometer is to be used, the dial should be turned by means of the keyless button until the figure 10 is brought exactly under the hand. It can be readily adjusted to the step of the wearer. The case of the instrument is made of bronzed metal, $1\frac{3}{4}''$ diameter.

219. Passometer or Step Measurer (Fig. 129) is a similar-sized instrument to the Pedometer, arranged to record the number of paces or steps taken by the wearer. These are indicated on the face of the instrument up to 2500; a stop *A* passing through the pendant, puts the instrument in or out of action. The passometer should be

fixed to the centre of the person. The steps are numerically indicated by a separate hand which reads up to 50 steps upon a small dial. Each revolution of the small hand reads through gearing one division of the central hand which continues the enumeration up to 2,500 and higher.

220. Direction for use.—Shake the instrument until the small hand stands at 50, after which move the large hand backwards with the finger, until it stands accurately at 2,500; the instrument is then ready for use, and it should be suspended by a hook from a waistcoat

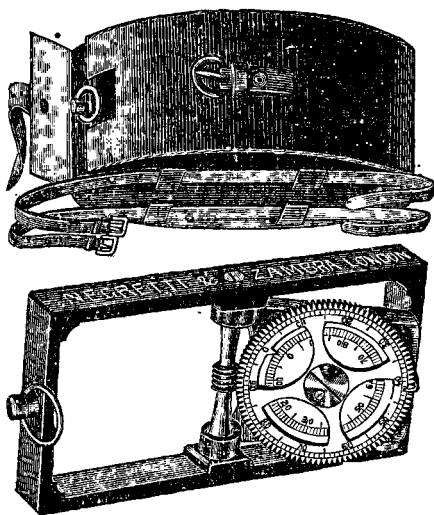


Fig. 130.—Trocheometer.

button-hole. In using a pas-sométer, the distance corresponding to a given number of paces should be ascertained by walking over a fixed distance, and a *scale of paces* constructed by which the plotting may be done directly from figures given by the instrument which registers the number of paces up to 25,000 or over 14 miles.

221. Trocheometer.—

This (Fig. 130) is a contrivance for registering the number of revolutions of carriage wheels, etc., in copper case with leather strap. By fixing it to the axle of a carriage, trolley, or other vehicle, the total distance passed may be found from the number of revolutions and the known circumference of the wheel.

222. Steward Pocket Surveying Telemeter.—This portable instrument (Fig. 131) which is only $4\frac{1}{2}'' \times 1\frac{1}{2}''$, will rapidly deter-

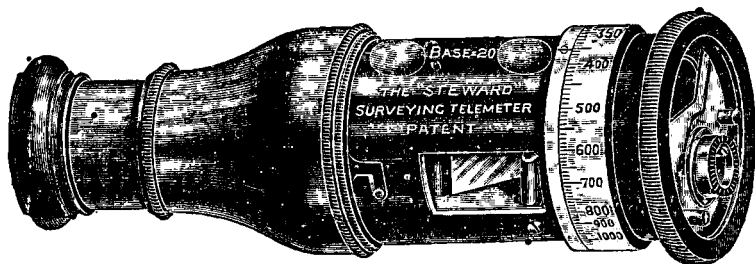


Fig. 131.—Pocket Surveying Telemeter.

mine with accuracy the distance of any fixed object, or the distance separating any two objects inaccessible or otherwise, and will solve

many other problems met with in surveying. It is especially useful for rapid preliminary survey when the transport of cumbersome instruments is inconvenient. The mean error working out at less than one per cent. The principle of the telemeter is to solve a triangle by measuring the two angles at the extremity of the base the length of which is known. The base may be of fixed definite length, or variable in the ratio of 1 to 50 of the distance of the object: or proportionate and subsidiary bases can be employed according to the nature of the country and the extent of the survey.

The telemeter consists of two plane mirrors mounted on pivots and adjusted so as to form an angle of 45 degree in their initial position, and serve as index and horizon-mirrors as in the ordinary sextant. The mirrors are mounted upon a base enclosed in a metal tube to one end of which is attached a small telescope. To the other end is adapted a movable collar, connected to the index-mirror by means of a metal arm. By rotating this ring the index-mirror is moved in azimuth, and the angular displacement is measured by reference to a graduated scale upon the exterior of the tube. A small toothed wheel attached to a finely threaded screw acting upon the horizon-mirror enables the angle of double reflection to be varied about 5 degrees, in order to facilitate the determination of the direction of the base. At the same time this screw furnishes the means of re-establishing the right angle should the zero of the instrument get displaced. A rectangular opening is provided at the side of the tube to enable distant objects to be reflected in the mirror, and another opening is provided in front of the tube so that the landscape can be observed over the horizon-mirror. This latter is covered by a sliding shutter to keep out the dust. The scale is divided on the German silver ring fixed to the rotating collar, and upon the fixed portion of the ring attached to the body of the instrument the index-zero or reading-point is engraved. Upon a stud in front of the telemeter is engraved an arrow head, and a corresponding arrow head is engraved upon the small toothed wheel. When the instrument is set to zero, so that the mirrors reflect an absolute right angle, both the zero of the scale and the index zero and the two arrow heads should be in coincidence. The scale of distances is calculated for a normal base of 20 units, and is figured at every tenth division from 200 to 1000, each division representing 10 units of distance. The proportion that the base bears to the distance decreases from $1/10$ to $1/50$ as the range increases. In certain operations it may be convenient to employ a base of constant ratio to the distance, and the telemeter is adapted for this system of measurement. The distances are, however, in this case calculated and not read from the graduations. For this purpose an angle has been selected that provides a convenient ratio for mental calculation, the distance being in the proportion of 50 to 1 of the base.

223. The following is one of the methods of using the **Telemeter** for measuring distances and known as the *fixed base method*. The length of the base employed is never less than $1/50$ th of the

distance measured, and the telemeter mechanically computes the distance which is indicated on the scale in terms of units of the base. In Fig. 132 the arrow head denotes the direction in which the observer is looking, O is the object whose distance is required, M the guide point or reflecting mark, A the first station and B the second position of the observer. AB is therefore the base, and AO the distance measured.

Indirect Method.—The object is seen by reflection and the

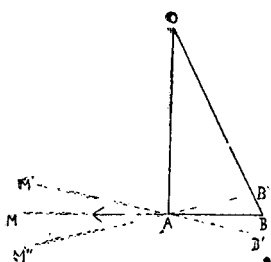
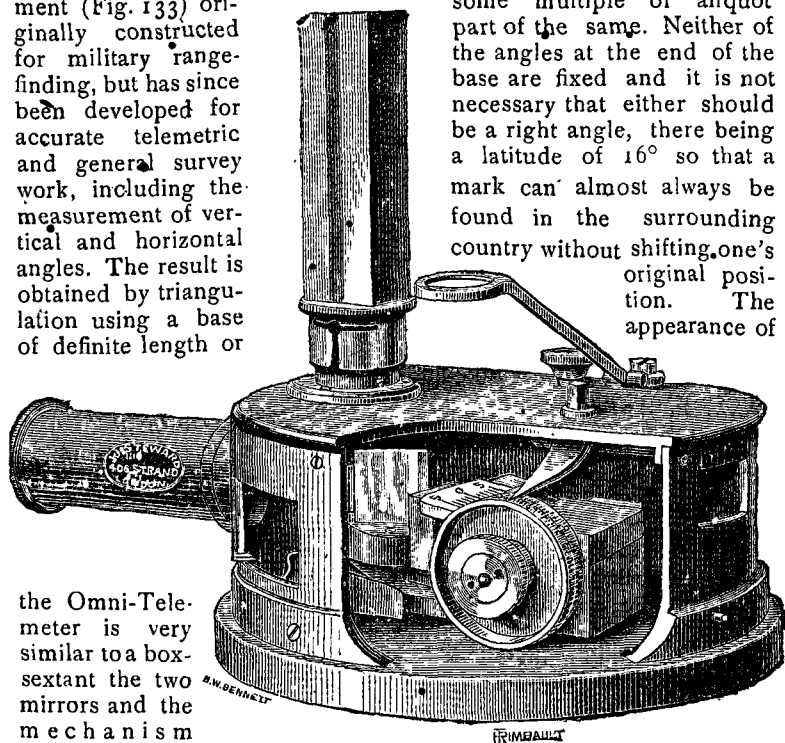


Fig. 132.

which is hereafter called the mark M, the observer proceeds to make coincidence with the reflected image of O. If M is to the right (M') or to the left (M'') of the reflection of O, the observer paces to the right or left until O nearly coincides, exact coincidence being effected by turning the toothed wheel at the end of the instrument. Care must be taken to make the coincidence of O and M very exact as it has an important bearing on the accuracy of the result. Having established the right angle BAO or its equivalent the exact position under the eye of the observer at A is marked by a picket, stone, or any article that is readily visible. The observer now turns right about face, and marches in a direct line away from M, M' , M'' , as the case may be. Arrived at B the end of the base observer turns right about, and again faces the mark M (M' , M''), and moves to the right or left until he is in a true line with A and M. As a deviation from the straight line introduces an error into the observation, it is, important to ensure the accuracy of alignment. Facing M and holding the telemeter as before in the left-hand, the observer turns the revolving part of the cylinder, until the reflection of O, which is seen to the left of the mark, is made exactly to coincide with M (M' , M''). The observer then refers to the graduation opposite the fixed zero point, and reads at once the range AO. The pamphlet, by Mr. J. H. Steward, London, contains instructions for solving various surveying problems.

224. The Dredge-Steward Omni-Telemeter is an instrument (Fig. 133) originally constructed for military range-finding, but has since been developed for accurate telemetric and general survey work, including the measurement of vertical and horizontal angles. The result is obtained by triangulation using a base of definite length or

some multiple or aliquot part of the same. Neither of the angles at the end of the base are fixed and it is not necessary that either should be a right angle, there being a latitude of 16° so that a mark can almost always be found in the surrounding country without shifting one's original position. The appearance of



the Omni-Telemeter is very similar to a box sextant the two mirrors and the mechanism being contained in a metal box 4" in diameter and 2" deep. Its compactness and accuracy recommend it as an instrument for preliminary and rapid surveys.

Fig. 133.—Dredge-Steward Omni-telemeter.

The construction of the Omni-Telemeter will be understood by reference to the figures which will also explain one of the methods employed for measuring the distance of an object from the place of observation. In Fig. 135, H is a micrometer-screw which actuates the index-mirror and rotates a graduated drum, G, divided into 100 parts with a central circumferential line. Before making an observation this circumferential line is set to the central line on the knife-edge scale which is divided to 10 parts. Then by means of the milled-head, E, (Fig. 134) the same circumferential line is set to the fiducial mark, e, Figs. 134 and 135. The telemeter is then set and ready for use. To make an observation, the observer uses the instrument in the hand, or for greater steadiness and accuracy attaches it to a light tripod, observations being made through an aperture-sight or through the telescope (Fig. 133). Taking Fig. 136

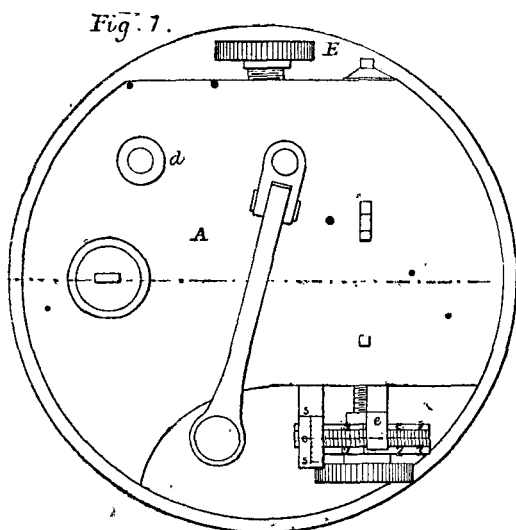


Fig. 134.—Top view of instrument showing index and adjustments.

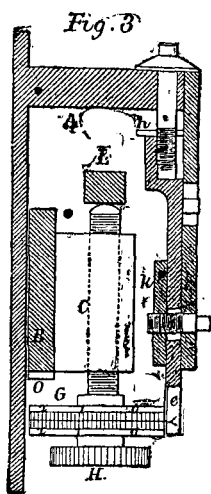


Fig. 135.—Section showing screw of Micrometer and adjustment for 90 degs.

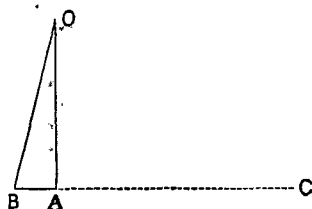


Fig. 136.

the observer at A wishes to ascertain the distance of O or in other words the length of the line AO. He looks through the telemeter either in the direction AO or AC approximately at right angles to AO. The objects O and C are seen simultaneously, one by direct vision and the other by reflection and are made to superpose by turning the milled-head, E, (Fig. 134)

which has the effect of altering the relative position of the two mirrors and projecting the image of one object on to the other. This operation determines the angle OAC which, as previously mentioned, need not be a right angle. The observer then moves from A to B for a distance of 50 units or an aliquot part of that distance taking care to make a perfectly straight continuation of the line CA, (the unit may be a yard, metre, foot or any other convenient measure). At B another observation is made the observer looking through the telemeter in the same direction as previously and causing the same two objects, O and C, to superpose again by turning the drum-head, H. This determines the angle ABO. The indications of the drum and knife edge are then read off by means of a lens mounted in an arm mounted on top of the instrument, the hundreds being read from the knife-edge and the units from the drum. The reading given is referred to a table attached to the instrument and if the base employed was 50 units the distance AO is ascertained without calculation in the same terms of units as employed for the base. Supposing however

a base of 100 units were employed, the distance indicated by the telemeter should then be doubled or for a base of 25 units the distance indicated should be halved. If any other proportionate base is used the same proportion of the result must be taken.

There are various other methods of using the Omni-Telemeter but informations on these can be obtained on application to Mr. J. H. Steward, 406 Strand, London.

225. Electric Cars and Surveying Instruments.—There is a new danger to the compass needles of surveying instruments when such instruments are carried in electric cars. The powerful electro-magnets beneath the floors of cars may either partially or entirely demagnetise the compass needle when the cases containing compasses or transits are set down upon them, the damage done bearing a strict relation to the distance of the needle from the magnets and the length of time the needle is exposed to their influence. The north end of the needle would stick to the glass, and under some circumstances the polarity of the needle may be entirely changed by demagnetising and re-charging, so that the north end pointed to the south. A compass needle is originally a simple bar of unmagnetised steel nicely balanced on a centre pin. When it is magnetised, the north end “dips,” and to counteract this dip, a few coils of wire are usually wrapped about the south end of the needle. It is evident then, that, if the needle is demagnetised, the effect of the magnetic dip is lost, and the needle is thrown out of balance, with the north end touching the glass. To detect whether an electric car has affected the needle of his compass, the surveyor has only to try the balance of the needle upon the centre pin. If it is balanced, no harm has been done. If the south end sinks, however, and the north end strikes the glass covering, the compass needle has been affected, and it should be sent to the maker for repair.

Surveyors are advised to enter and leave trolley cars by the rear door, and to keep their instruments as far as possible from the motors, say, on the rear platform. Under no circumstances should the instrument be set on the floor of the car. If it must be carried inside the car it is better to put the box on a seat, or on the owner's lap. Neither rubber nor glass will protect from the *magnetic* effect, though rubber covers do insulate against *electric* currents, as do the rubber “bumpers” placed under all boxes. Needle instruments that are kept in modern very high office buildings are also liable to be effected injuriously, owing to the presence of electric currents and powerful magnets. It is also well to remind the surveyor, that the needle may be made to stick to the glass by static electricity, produced by rubbing the glass against the clothes, or with a silk handkerchief. This effect is easily removed by discharging the electricity by the touch of a moistened finger to the glass. This experiment should be first tried in the case referred to, and if the trouble is not remedied by the needle at once resuming its normal position, then the needle has been demagnetised, as heretofore explained (Engineering News; New York, Mar. 6, 1896.)

226. General Care of Surveying Instruments.—All surveying instruments should be well-protected from rain, dust, and damp. A waterproof cover should be used to protect the instruments when mounted on the stand and not in use. Exterior parts should not be oiled; otherwise dust falling on will stick to them. The axis should be cleaned occasionally with chamois leather and a very little watch oil. Graphite may be used to clean the glasses and the verniers. Soft wash handkerchief freed from dust and moistened with spirits of wine or liquor ammonia, should be used for cleaning dirty glasses. If there be dust on the cross-hairs, it may be blown off after removing the eye-piece and the object-glass. This should be carefully done: otherwise the wires may break unless they are made of wires. Every part of an instrument should be kept clean and dry. Wipe off all dust carefully on commencing and leaving off work. Instruments should never be carried in carts but always by men by means of stout straps fixed on to the boxes. Instruments should always be handled by the substantial parts, and less their axes and screws are worked, the longer they are likely to last in an efficient condition.

CHAPTER III.

USEFUL PROBLEMS IN SURVEYING.

227. Prob. I.—To Draw upon the Ground a Straight Line through Two Given Points.—Plant a ranging rod at each of the given points, then fix others between them in such a manner that, when the eye is fixed at the edge of the first rod, the edges of the others may coincide with it and the rod at the other given point. The line may be produced by fixing up other rods in the same manner. For example, having given two points, as A and B, (Fig. 137) on the line, and desiring to fix others, as C and D, between A and B, the observer stations himself at O some distance from A. Rods being held at A and B, the observer sights past

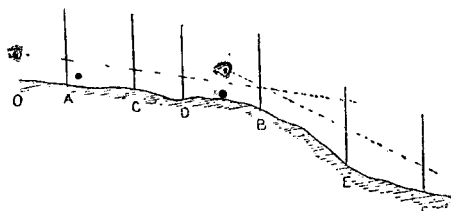


Fig. 137 :—To Range and Prolong a Line.

these and motions the rodman at C, until the rod at that point is held on line. The rod at A is then sent on to D and this point is fixed in the same way. Similarly to prolong the line AB in the direction of A B the rod at D is sent on to E, and the observer sights past rods, A and B, and motions the rod-man at E until the rod at that point is held on line. The rod at A is then sent on to

F, and the observer, moving near to B, lines F with B and E as before, and so on. The observer should be as far from the rods, and the rods themselves as far apart as may be practicable.

Method of holding the Rod.—If it is desired to sight past the rod, the rodman must, of course, stand to the side of the line of sight. Otherwise he should stand directly behind the rod and facing the observer. In either case do not grasp the rod firmly, but balance it with finger-tips using both hands. In doing this, care must be taken to fix the rods upright into the ground, and not to fix the eye too near the rod from where the observation is made. By standing 2 or 3 yds. away from the rod, a much better sight is obtained than by being very close to it.

228. Prob. II.—To Walk in a Straight Line from a Given Point to a Given Object.—It must be borne in mind that, to walk in a straight line, the surveyor must have two objects constantly in view. Before starting the surveyor should fix upon some mark, such as a stone or a tree, on the ground which may be found to be in a line with the given object. Then he should walk forward, keeping the mark and the given object strictly in line, and always selecting a fresh mark which may be in a line with the given object before reaching the intermediate mark, and so on until the object is reached.

229. Prob. III.—To Trace a Line in the Direction of Two Distant Objects.—Let two persons separate, to about 50 or 60 paces; then by alternately motioning each other to move right or left, they soon get exactly into line with the distant objects; or for greater accuracy they may hold up staves.

230. Prob. IV.—To Run a Line connecting Two Points on Opposite Sides of a Hill and Invisible from Each Other.—Let A and B (Fig. 138) be two points. Two rodmen station themselves at C and D approximately on line and in such positions that B and D are visible from C, and C and A visible from D. C lines in D between C and B. Then D lines in C between D and A, and so on, turn about, until C and D are both in line between A and B.

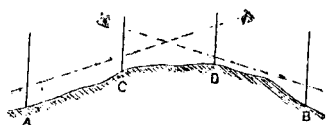


Fig. 138. To Run a Line connecting two Points invisible from each other.

D. C lines in D between C and B. Then D lines in C between D and A, and so on, turn about, until C and D are both in line between A and B.

231. Prob. V.—Ranging out a Long Line across Elevations and Depressions.—In ranging out a long line, elevations and depressions may be met with at intermediate points in it, at which places the flags showing the extremities of the line may not be visible.



Fig. 139.—Ranging Long Line across Elevations and Depressions.

Referring to Fig. 139, the flags, c, d, e, being lined in from a, upon

ranging out a long line, elevations and depressions may be met with at intermediate points in it, at which places the flags showing the extremities of the line may not be visible. To chain across these, flags should be planted on each side of the depression and on the elevation.

reaching the depression between c and d , the chain may be kept in line with d and e , although b is not visible.

232. Prob. VI. To Chain a Line between Two Fixed Points not Visible from each Other when Woods intervene, and the True Line is not to be cleared out.

(a) Range out a trial line by poles, leaving fixed points. Find the resulting error at terminus and move all points over their proportionate amount. The true line may then be obtained.

(b) To chain a line from a to c (Fig. 140) but being unable to direct the instrument (theodolite) exactly towards c on account of intervening woods, or other obstacles: first run a trial line, as $am = 2500$ ft. as nearly in the proper direction as can be guessed at. Measure mc to be $80'$ and say as $am : mc :: 100 : 3.2$. Lay off $ao = 100'$, and $os = 3.2'$ and run the final line asc . Or if mc is quite small calculate offsets like os for every $100'$ along am , and thus avoid the necessity of running a second line.

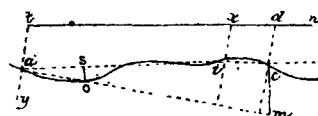


Fig. 140.—To Chain a Line with the Theodolite.
asc. Or if mc is quite small calculate offsets like os for every $100'$ along am , and thus avoid the necessity of running a second line.

233. Prob. VII. To Survey a Line the Ends of which are Visible but the Intervening Ground Difficult to measure on account of Marshes, etc.—For example, when c is visible from a (Fig. 140) but the intervening ground difficult to measure along, on account of marshes, etc., extend the side ya to good ground at t : then making the $\angle ytd = \angle yac$ by the theodolite, run the line td to that point d at which the $\angle ndc$ is found by trial $= \angle atd$. It will rarely be necessary to make more than one trial for this point d ; for, suppose it to be made at x , see where it strikes ac at i ; measure ic , and continue from x making $xd = ic$.

234. Prob. VIII. To Lay off a Perpendicular with the 100' Chain or Tape.—(a) **First Method:**—Let AD (Fig. 141) be the given line, and it is required to erect a $\perp$ at C in the direction CE . Measure off $CB = 30'$, fix one end of the chain at B , and let it be firmly held there. Fix the 90th link at C , leaving 10 links loose as shown by the dotted line CH . Take hold of the 50th link of the chain, pull taut, and fix a pin at F . The line ranged through CF will be a $\perp$ to AD at C . This is due to the fact that 30, 40, 50 represent the sides of a right-angled Δ , and in performing the above operation we make $CB = 30'$, $CF = 40'$, $BF = 50'$. In books on the subject BC is taken $40'$ instead of $30'$; but it is preferable to have a longer $\perp$, and it will be easier then to range out the $\perp$ CE which may be $100'$ or more.

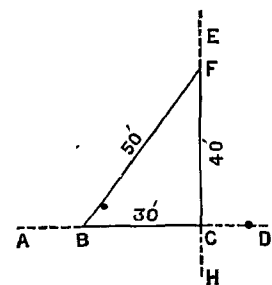


Fig. 141.—Setting out a Right Angle with the Chain.

books on the subject BC is taken $40'$ instead of $30'$; but it is preferable to have a longer $\perp$, and it will be easier then to range out the $\perp$ CE which may be $100'$ or more.

The employment of this method is limited to short $\perp$ s, as it is against all principles of sound surveying to range or to produce long lines from comparatively short ones. By the figure, the longest line from which to range out a $\perp$ to any point C cannot exceed 40' and it is not prudent to extend this towards B beyond 100' unless only approximate accuracy were demanded. This method is therefore adopted but for very insignificant distances, and should not be used for setting out lines at rt. $\angle$ s when they are to form subsidiary main lines of a survey. In a ploughed-up field upon a wet day, with the links of the chain clogged with dirt, some care is required to manage the operation successfully.

In Fig. 141 lengths of BC , CF , and FB are in the ratio of 3, 4 and 5. Similarly if they are in the ratio of 5, 12 and 13, or 7, 24 and 25, or 8, 15 and 17, or 20, 21 and 29, or 9, 40 and 41 $\perp$ may be raised from C . Any multiples of these numbers will also give right-angled Δ s. It may not be out of place here to give the following general method for finding any number of sets of whole numbers which are proportional to the sides of right-angled Δ s. Choose any two numbers m and n , m being the greater and if they are either both even; or both odd, make $x = mn$; $y = \frac{m^2 - n^2}{2}$; $z = \frac{m^2 + n^2}{2}$; but if one is even, and the other odd, multiply each of the above expressions by 2. Then $x^2 + y^2 = z^2$; and x , y and z are proportional to the three sides of a right-angled Δ , z corresponding to the hypotenuse.

(b) **2nd Method.**—If c be a point on the chain line where a $\perp$ is to be raised (Fig. 142) in the direction cf , measure equal distances cd and ce , say 25', on either side of it. Fasten the two ends of the chain at d and e on the chain line or have them held at these points. Hold the centre of the chain out to f as far as it will go from de . Then fc is $\perp$ to de .

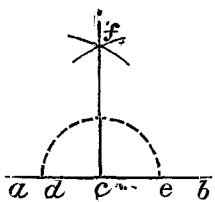


Fig. 142.—Setting out a Right Angle.

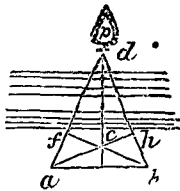
235. Prob. IX. From an External Point, to Drop a $\perp$ on a Given Line.—

(a) If the point is distant from the line less than one chain length, find two points on the line equidistant from the given external point. Mid-way between these will be the foot of the perpendicular from the given external point on the line.

(b) *The practical method of offsetting $\perp$ to a fence* or other object is to hold the ring of the tape at the point to be offsetted to. The surveyor then swings the tape horizontally until he gets the shortest measurement on the chain which he reads and reads also the point on the chain where the offset forms a right angle.

(c) If the external point is distant more than a chain-length from the given line run out another line parallel to the one given and within a chain length of the external point. A $\perp$ dropped on this 2nd line will be $\perp$ to the first line also.

236. Prob. X. To Let fall a Perpendicular upon the Line ab from an Inaccessible Point d (Fig. 143).—



From *a* and *b* lay out *ad* and *bd*. Drop $\perp$ s, *ah* and *bf*, upon the sides *ad* and *bd*. Mark their intersection *c* and *e*; *ce* produced will be $\perp$ to *ab*. The figure when plotted will give the inaccessible distance *cd* when *ab*, *af*, *bh*, *ah*, *bf* are known. If this be a signal at *d* its height may be obtained by method (Prob. XXXVII in Art. 254).

Fig. 143.—To Let fall a $\perp$ from an External Point.

237. Prob. XI. To Run a Line Parallel to Another Given Line.—

(a) Erect two $\perp$ s to the given line (Fig. 144) as far apart as may be practicable. Two points on these $\perp$ s equidistant from the given line, will determine the line required.



Fig. 144.—To Run Parallel Lines.

(b) If the parallel line is required to pass through a given point move two rods along the $\perp$ s keeping them equidistant from the given line, until the line of sight through them passes through the rod on the given point.

238. Prob. XII.—To Lay out an Angle of 60° .—The angles of an equilateral triangle are 60° . Apply this principle.

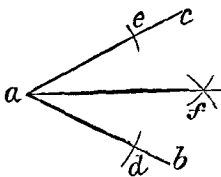


Fig. 145.—Measuring Angle on the Ground.

239. Prob. XIII.—To Measure an Angle on the Ground.—In Fig. 145 let it be desired to measure the angle *bac*. Measure equal distances, *ad* and *ae*, say 100'.

Measure *de*. Then $\angle bac = 2 \sin^{-1} \frac{de}{2ad}$,

$$\text{or } \tan \frac{1}{2} bac = \frac{de}{\sqrt{4ae^2 - de^2}}.$$

240. Prob. XIV. To Lay out any Angle on the Ground —

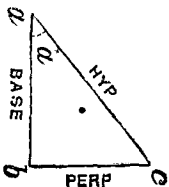


Fig. 146.—Laying out Angle on Ground.

In Fig. 146 it is desired to lay out the angle *bac* = α° . Measure any distance *ab*, say 100'. At *b* erect a $\perp$ *bc* to the line *ab* and *bc* = *ab* tan α° . *bac* is the angle required, or reverse the above operation in Fig 145. $\angle bac$ is known, assume $\alpha = 100'$ say, and compute α . Then from *a* measure off *ae* = 100'. From *e* and *d* strike arcs with radii equal to *de* and *ae* = 100' respectively giving an intersection at *d*. Then *cab* is the angle required. If *ae* is assumed not greater than 60', angles may be laid out up to 90° .

241. Prob. XV. To Avoid an Obstacle such as a House, Thorny Hedge of about a Chain-length preventing chainmen standing on Line or Planting Arrows, Swamp River,

or Lake in a Chain Line—These difficulties are easily solved by angular instruments; but when such instruments are not at hand, the chain alone may be used. Three distinct cases generally occur in practice:—*First*, those in which the obstacle can be seen over from side to side, and chained round, but not chained across. *Secondly*, those in which it can neither be seen over nor chained across, but can be chained round. *Thirdly*, those in which the obstacle can be seen over but neither be chained across nor chained round.

Case 1.—When the Obstacle can be seen over.—

(a) **By a Parallel Line** (Fig. 147).—In surveying a chain line from w to y an obstacle o is met with. To pass it, at first, plant two flags at f and j , and then lay off by the optical square or otherwise a rt. $\angle wfu$ as in Problem VIII; measure fu just to pass over the object; make $\angle fuv = 90^\circ$, measure uv so as to pass over the obstacle; make $\angle uvj = 90^\circ$; make $vj = fu$; make $\angle vjy = 90^\circ$. Then $fj = uv$; and jy is in the chain line. The operation of “squaring off” is much facilitated by having the theodo-

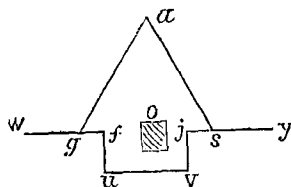


Fig. 147.—Chaining past an Obstacle.

lite or box sextant or optical square to set out the right-angles at f , u , v , and j , and thus greater accuracy is obtained. See prob. VII.

(b) **By an Equilateral Δ** (Fig. 147).—At g make $\angle fga = 60^\circ$; measure any ga ; make $\angle gas = 60^\circ$, and $as = ga$; make $\angle asj = 60^\circ$. Then $gs = ga = as$, and js continued towards y is in the chain line. The angles are set out with the theodolite or box sextant. If owing to the natural feature of the ground or to another obstacle in the vicinity of a an equilateral Δ can not be laid out, an isosceles Δ having the sides ga and as equal can always be laid out, and the distance gs may be calculated from as , ag , and the measured angles.

(c) **By any Δ** (Fig. 148).— f and j as before being points on the chain line w to y at the nearer and further sides of the obstacle o , construct a Δfsa of any form and size, so that it encloses the obstacle provided that a and s are to be ranged in a straight line with j , and that the $\angle s$ at a and s are neither to be very acute nor obtuse; measure fa , aj , js and fs .

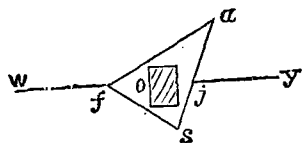


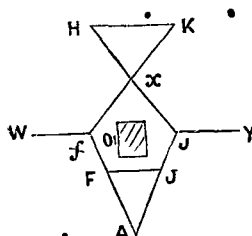
Fig. 148.—Chaining past an Obstacle.

$$\text{Then } fj = \sqrt{\left\{ \frac{fa^2 \times sj + fs^2 \times ja}{as} - aj \times js \right\}}.$$

The distance can also be obtained by plotting the Δ and the point j in its base, and measuring fj on paper with a pair of dividers. The figure of the obstacle may be obtained by offsets from the sides of the Δ .

(d) By Two Δ s as Well-conditioned as Possible.—

(1) As before f and j are points (Fig. 149) on the base line w



to y at the nearer and further sides of the obstacle o . From a convenient point A , chain the lines Af and Aj which form the sides of a ΔfAj . Lay off AF and AJ proportional to Af and Aj respectively, and measure FJ . The $\Delta s fja$ and FJA are similar.

$$\therefore fj : fA :: FJ : FA \text{ or } fj = \frac{FJ \times fA}{FA}.$$

fj can be obtained by plotting also.

Fig. 149.—Avoiding Obstacle in Chain Line.

(ii) Let it be required to pass a house o situated in the line wy (Fig. 149). At the point f range a line fK making $fx = xK$. Then from j range the line jH though the point x marking $jx = xH$. The line fj will be equal to HK . Then measure HK . $\therefore HK$ equals fj the required distance.

Case II—When the Obstacle can be Chained Round but not Chained Across nor seen over.

(a) By Parallel Lines.—In surveying from A to B (Fig. 150)

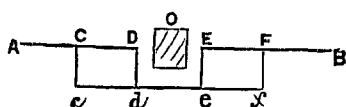


Fig. 150.—Avoiding Obstacle in a Chain Line.

an obstacle o is met with, which can not be seen over. At convenient points C and D on AB raise equal $\perp$ s Cc and Dd respectively. The line cd will be at such a distance from AD that the obstacle can be passed over. Then range and measure the straight line $cdef$. After passing the obstacle at the point e set off $\perp$ $eE = Cc$. The point E will be on the main line AB , and by sighting from E to the next point visible on the main line, the correct direction can be continued. If no point is visible the direction may be obtained by laying off from f the $\perp$ $fF = eE$. By ranging through E F the main line may be continued until one of the distant points can be seen. The points to be attended to are the exact measurement of the $\perp$ s and the ranging of the parallel line. Here $de = DE$.

(b) By Similar Δ s (Fig. 151).—From a point A , at the back

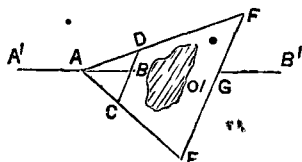


Fig. 151.—Avoiding Obstacle in a Chain Line.

of the obstacle O , range two lines AF and AE past the two sides of the obstacle; in which measure the distances AD and AC , of the two points D and C which lie in a straight line with B up to which point the main line, say, is chained. Continue chaining AF and AE and make those lengths proportional to AD and AC , so that the $\Delta s ADC$ and AFE are similar. Chain DB and BC . Chain EF in which

take any point G , dividing EF in the same ratio in which B divides CD : thus G will be a point on the main line $A'B'$ beyond the obstacle, and points still further may be found, if necessary, by a similar process. The inaccessible distance $BG = \frac{AB \times CE}{AC}$. The

form of the obstacle can be found out by offsetting from the sides of the quadrilateral $CDFE$.

Case III.—When the Obstacle can be seen over but Neither Chained Across nor Chained Round—

(a). Let AB be the chain line (fig. 152). CD is the length required across a large lake. Stand at C , and range any line CEH . From D set off $DE \perp$ to CD , cutting CE in E , and from E set off $EF \perp$ to CE , meeting the chain line at F . Chain DF and DE . Then $CD = \frac{DE^2}{DF}$.

Fig. 152.—Chaining Across a Lake or other Obstacle.

Proof—The Δ s DEF and DEC are equiangular. $\angle CDE = \angle FDE = 90^\circ$. $\angle CEF = 90^\circ = \angle DEF + \angle DFE$: but $\angle CEF = \angle CED + \angle DEF$, and consequently $\angle DEF + \angle DFE = \angle CED + \angle DEF$. Subtracting the common $\angle DEF$ from both sides of the equation, $\angle DFE = \angle CED$, and $\angle DCE = \angle DEF$. $\therefore$ the two Δ s are equiangular. By the Sixth book of Euclid we have the following proportion between the sides:— $DF : DE :: DE : CD$. Multiplying the extremes and means we get $CD = \frac{DE^2}{DF}$ = the distance required.

The inaccessible distance can be obtained quickly by an angular instrument thus:—Lay off the $\angle CDE = 90^\circ$, with a theodolite, sextant, or optical square, planting a peg E at any convenient distance DE . Also measure DE and the $\angle DEC$ by the theodolite or box sextant. Find the natural tangent of $\angle DEC$. Multiply this natural tangent by DE . The product will be CD . For example, the $\angle DEC = 53^\circ$ and $DE = 400$. Now $\text{nat. tan } 53^\circ = 1.32704$. $\therefore 1.32704 \times 400 = 530.816 \text{ ft.} = CD$.

Or lay off the $\angle CDE = 90^\circ$. From E measure the $\angle DEC$, and lay off the $\angle DEF$ equal to it, placing a peg at F in range with DC . Then is $DF = DC$.

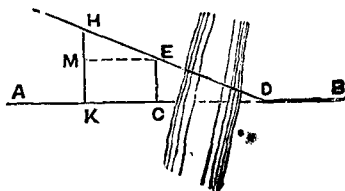


Fig. 153.—Chaining Across a River or other Obstacle.

(b) Let us suppose a river has to be crossed by the chain line AB , and CD is the distance required (Fig. 153). Chain the length KC . Range any line HED , and from the points K and C raise $\perp$ s KH and CE to the chain line AB , meeting the line DH in H and E . Chain CE and KH .

$$\therefore CD = \frac{CE \times KH}{KH - CE}$$

Proof.—Draw $EM \perp$ to HK . Then the Δ s HME and ECD are similar and equiangular, since the line HD cuts the two $\parallel$ lines HK and EC , $\angle DEC = \angle EHM$. For the same reason, since ME and KD are $\parallel$, the $\angle HEM = \angle EDC$, and as the remaining $\angle$ in each Δ is each equal to 90° , the two Δ s are equiangular. Then we have the following proportions:— $HM : ME : CE : CD$. But $HM = KH - CE$ and $ME = CK$; substituting these values in the above ratio, we have $KH - CE : CK :: CE : CD$. Multiplying the extremes and means, we obtain $(KH - CE) \times CD = CE \times CK$. $\therefore CD = \frac{CE \times CK}{KH - CE}$.

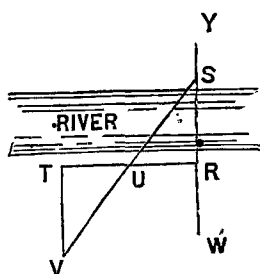
Or, with a theodolite erect $\perp$ CE and measure it and set off the $\angle CED = 60^\circ$. Then $CD = CE \tan. 60^\circ$.

(c) When the inaccessible distance is between 400' and 500' or so, it is desirable to adopt the following mechanical methods:—

(i) Hang a strong copper wire from a given height on each shore in such a manner that it assumes the catenary curve, and its lowest point just touches the surface of the water when it is quite calm. After determining the level of water, suspend the wire under precisely identical conditions on the shore, and get the horizontal distance or chord of the arc accurately measured.

The chord of the arc can be calculated without erecting the wire on shore from the formula $L = 2\sqrt{\left(\frac{s}{2}\right)^2 + \frac{4v^2}{3}}$; where $s = \frac{1}{2}$ span of the wire, $v =$ the versed sine or distance of the lowest point of the curve below the horizontal line, $L =$ whole length of the arc or wire. If the length of the wire L and the versed sine v are known, s can be determined. For example, if $v = 40'$, $L = 410.4'$; $\therefore s = 400'$.

(ii) A more direct measurement is:—A number of deal rods or long canes are threaded on a rope, and on a calm day gently floated on the water until they come nearly in the position required. They are then drawn, into a straight line, so that their ends touch each other; and thus the span between the shores is accurately measured.



(d) **By Similar Right Angled Δ s.**—Here RS is the distance required across a river in a chain line W to Y (Fig. 154). Set off $RT \perp$ to RS , and bisect RT in U . Set off $TV \perp$ to RT until V ranges in line with SU . Then $TV = RS$, and $VU = US$, for the Δ s TVU and URS are equal in all respects.

(e) **By a Rhombus.**—A T is the inaccessible distance required (Fig. 155). With a measuring tape lay down DA —the

Fig. 154.—Chaining Across length of the tape in the direction DT ; a River or other Obstacle. set off DC with another tape in any convenient direction $= DA$. Now fasten their two ends at A and C , and

when the other ends are stretched to their full lengths, and brought to B , the figure $ABCD$ will form a rhombus. Set up a mark at R where AB and TC intersect, and measure RB . Then the sides of the Δs RBC and TDC being respectively parallel, the Δs will be similar; hence, $RB : BC :: DC : DT$; or $AT = \frac{BC^2}{RB} - BC$. If the ground be a plain one, a rhombus, whose side is 100', will determine an inaccessible distance of about 900'.

(f) **By Similar Δs .**—Here GF is the distance required across a river (Fig. 156). Take any convenient point K , and set off any straight line HK and bisect it at J . Take another point F on the line GK . Make $LJ = JF$. Range I in line with HL and JG . Then $LI = FG$, and is parallel to it.

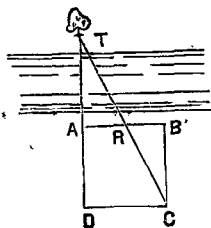


Fig. 155.—

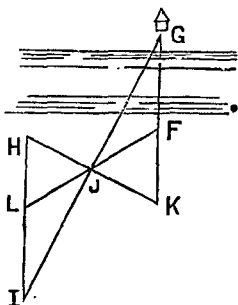


Fig. 156.—

Chaining Across Rivers or
other Obstacles.

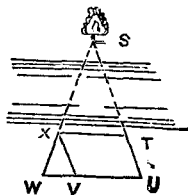


Fig. 157.—

(g) Drive two pegs T and U in range with the inaccessible object, S (Fig. 157). From T lay off any convenient length TX in any direction: From U lay off UW parallel to TX placing W in range with XS . Make $UV = TX$. Measure WV , VX and XT . Then as $WV : VX :: XT : TS$. When the figure is plotted, the $\perp$ distance of S from the line XT can be easily obtained by scale.

Instead of taking so many chain measurements, with an angular instrument, the work can be done quickly. In crossing the river, say along the chain line US , it is only necessary to run out a base, as TX , in any convenient direction along the bank, and to measure it, and to set off the $\angle s$ STX and TXS each $= 60^\circ$. The distance ST then $= XT$.

Or lay off any convenient $\angle UTX$, and set up the theodolite at X . Make the $\angle TXS = \frac{1}{2} \angle UTX$. $\therefore ST =$ the measured distance, TX , which may at the same time help to delineate the boundary of the river.

In Case III (a) (b), (d) (e), (f), (g) if the inaccessible object be a boat in the river its distance may be obtained in the same way.

242. Prob. XVI. To Find an entirely Inaccessible Distance yz , and its Direction by Similar Figures (Fig. 158).—

At any two convenient points, a and b , from each of which y and z can be seen, drive pegs. Then we have the four corners of a quadrilateral in which are given the directions of three of its sides, and two of its diagonals. These data enable us to lay out on the ground the small quadrilateral doi exactly similar to the larger one. Thus, in the line ab , plant a peg, c , and make co parallel to bz , o being at the same time in range of the diagonal az . Also from c make ci parallel to by , i being at the same time in range of ay . Then, will io be in the same direction as yz or parallel to it. Measure ac , ab and io . Then, evidently, from the principle of similar figures (Euc. VI. 2) we have

$$ac : ab :: io : yz = \frac{io \times ba}{ac}.$$

If yz were a visible line, such as a fence or boundary wall, we could from a divide it into any required portions. Thus, to plant a peg half-way between y and z , first place one half-way between i and o ; then standing at a , by means of flags, place a person in range on yz , or to find along ab a point $t \perp$ to yz at y , first make $\angle ois = 90^\circ$; and tape as ; then as $oi : as :: yz : at$.

The same results can be obtained easily by means of the theodolite thus:—Measure ab ; at a measure the angles yab and zab ; at b take the angle zba and yba . In the $\triangle yab$ we have ab and the two angles yab and yba , and from these the side, ay , can be calculated by Plane Trigonometry. Again in the $\triangle zab$ we have ab and the two angles zba and baz ; hence the side az can be calculated. Then in the $\triangle yaz$, the sides ay and az and the included angle yaz being known the side yz can be easily calculated by Plane Trigonometry, and the following rule may be adopted:—Take the angle from 180° ; the remainder will be the sum of the two unknown angles. Divide this sum by 2 and find the nat. tangent of the quotient. Then as the sum of the two given sides : their difference :: tan. of $\frac{1}{2}$ the sum of the two unknown angles : tan. of half their difference. Take from the table of Nat. tan. the angle opposite this last tangent. Add this angle to half the sum of the two unknown angles, and it will give the angle opposite the longest given side; and subtract it from the same half sum for the angle opposite the shortest given side. Having thus found the angles, find the third side.

By drawing ab on paper by a scale, and laying down the four measured angles, the distance yz may be measured upon the paper, by the same scale.

243. Prob. XVII. —To Reduce a Large Figure, $ABCDEFGH$, to a smaller one, $abcdefg$.—From a central point O draw lines to all the $\angle$ s A, B, C , etc. (Fig. 159). Join these lines by others parallel to the sides of the figure. If it should be required to enlarge

a small figure, draw, from any central point o , lines extending beyond its angles; and join these lines by others parallel to the sides of the small figure.

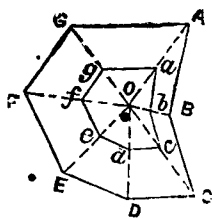


Fig. 159.—To Reduce a Larger Figure to a Smaller One.

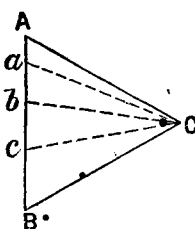


Fig. 160.—To Divide a Δr Field.

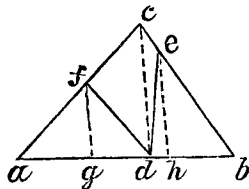


Fig. 161.—To Partition a Δr Field.

244. Prob. XVIII. To Divide a Δr Field ABC in certain Given proportions, so that a Well situated at one of its Corners may be common to all the Divisions.—Let the given proportion be as the numbers 2, 3, 4 and 5 and let the well be at C (Fig. 160).

Then as $2 + 3 + 4 + 5 = 14$: $AB :: 2 : Aa$

„ „ :: 3 : ab

„ „ :: 4 : bc

„ „ :: 5 : cB

Aa , ab , bc , and cB are the proportions of the line AB belonging to the respective Δ s. Therefore, if lines be drawn from a , b , c to C , the field will be divided in the required proportions.

245. Prob. XIX. To Partition a Δr Field, abc , in any number of equal Parts, say 3, by Fences from a Point in one of its Sides (Fig. 161).—

Let d be the given point on ab . Trisect ab in the points g and h . Join dc and draw gf and he parallel to dc , meeting ac and bc in f and e respectively. Fence along df and de . They will divide the Δ in three equal parts.

246. Prob. XX. To Cut off a Given Area by a Straight Line starting from a Given Point in the Boundary of a Survey.—Plot the survey and join the given point with one of the other corners so as to cut off nearly the required area. Compute the length and bearing of this line and the area cut off by it. Find out the difference between this area, and the area to be cut off, and the result is the area to be added, or subtracted in the form of a Δ , of which the bearings of two sides, length of one side, and the area are given, i.e., in a Δabc , one side ab , one angle at a (the difference of the bearings), and the area of the Δabc are given. The area of the Δabc being $\frac{1}{2} ca \times ab \sin a$, $ac = \frac{2 \text{ area of } \Delta abc}{ab \sin a}$. Having then the two sides ca and ab and the

included angle a , the length of the other side cb and other angles at b and c may be calculated. The bearing of the side cb is then deduced,

and the line bc run; the line ac measured as a check, as also the length cb itself.

247. Prob. XXI. To Cut off a Given Area from a Survey by a Straight Line with a Given Bearing.—Draw a line with the given bearing through one of the corners, which will cut off an area nearly equal to the given area. Compute the length of this line, and that of the part of the other side of the survey cut off by it; and also the area cut off. Find out the difference between this area and the area required to be cut off, and the result is the area to be added or deducted by a line parallel to the trial line. This area will be a trapezoid; its area, the length and the bearing of one of its parallel sides, and the bearings of the other sides are known. The dimensions of the other sides of the trapezoid may be ascertained from these data. Then measure one of the end lengths, and run the line, measuring the length of the line and the other end length as checks.

248. Prob. XXII. To Reduce any Polygon $ABCDEF$ into a $\triangle$ of the same Area, and then to find its Area.—

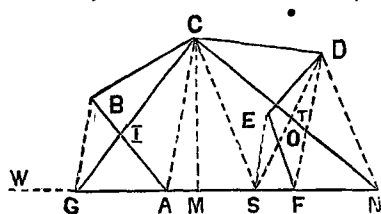


Fig. 162.—To Reduce a Polygon to a Triangle.

If we produce the side FA to W (Fig. 162), and draw BG parallel to AC , and join GC , we get equal $\triangle$ s ACB and ACG , both on the same base AC , and between the same parallels AC and BG . But the $\triangle ACI$ is common to both. Therefore, if it be taken away from both the $\triangle$ s, the remaining parts ICB and IGA are equal. Therefore, if the part

ICB be left off from the polygon, and the part IGA be taken into it, the polygon $GFEDC$ will have the same area as $ABCDEF$; but it will have but five sides, while the other has six. Again, if ES be drawn parallel to DF and DS joined, we have upon the same base ES , and between the same parallels ES and DF , the two equal $\triangle$ s ESD and ESF with EOS common to both; and consequently $\triangle$ s EOD and OSF are equal. Therefore, if OSF be left off from the polygon, and EOD taken into it, the new polygon $GSDC$ will have the same area as $GFEDC$; but it has 4 sides, while the other has 5. Finally, if GS be produced towards N , and DN drawn parallel to CS , and CN joined, we have on the same base CS , and between the same parallels CS and DN , the two equal $\triangle$ s CSN and CSD with CST common to both. Therefore, if we leave out CDT , and take in STN , we have the $\triangle GNC$ = polygon $GSDC$ or $ABCDEF$. To find the area of the $\triangle GNC$, drop a $\perp$ CM from C on GN . The area = $\frac{1}{2} GN \times CM$. This simple method is applicable to polygons of any number of sides.

249. Prob. XXIII. (a) To Reduce an Offset-piece, $abcde$, to a Right-angled $\triangle$, aen , of the same Area by an Equalising Line, en ; with a Parallel-Ruler.—(1). Draw a line $am \perp ea$ (Fig. 163). (2). Place the ruler on ca , roll it up to b , and draw $b \parallel$

ca , cutting am in o . (3). Place the ruler on od , roll it down to c and draw $cp \parallel do$, cutting am in p . (4). Place the ruler on ep , roll it up to d , and draw $dn \parallel ep$, cutting am in n ; then aen is the right-angled Δ required, and its area $= \frac{1}{2} ae \times an$.

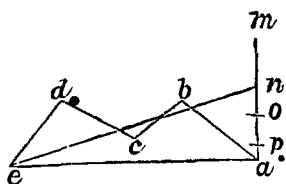


Fig. 163.—To Reduce an Offset-piece to a Right-angled Triangle.

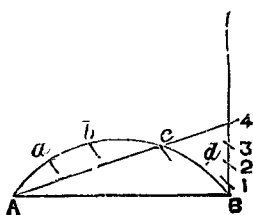


Fig. 164.—To Reduce a Curved Offset-piece to a Right-angled Triangle.

(b) If the offset-piece, $AabcdB$, be a curved one (Fig. 164), divide the curved boundary by points a, b, c, d , etc., so that the parts Aa, ab, bc, cd , etc., may be straight. Then proceed as above.

250. General Rules for Solving Problems of this Kind are as under:—Draw a temporary line, as am at right angles, or at any other angle to the chain line, as ae , (Fig. 163) of the offsets.

1. Lay the ruler from the first to the third angle, and move it parallel the 2nd angle; then make the first mark on the temporary line. 2. Lay the ruler from the first mark on the temporary line to the fourth angle, and move it parallel to the third angle; then make the second mark on the temporary line. 3. Lay the ruler from the last-named mark to the 5th angle, and move it parallel to 4th angle, then make the 3rd mark on the temporary line. 4. Lay the ruler from the last-named third mark on the temporary line to the 6th angle, and move it parallel to the 5th angle; then make the 4th mark on the temporary line. In this manner the work of casting by the parallel ruler may be conducted to any number of angles. Great care must be taken during the operation to prevent the ruler slipping; as such an accident will derange the whole of the work, if not discovered and immediately corrected.

251. Prob. XXIV. To Draw an Equalising Line through the Crooked Fence, $abcdef$, so that the Two Fields $amnf$ and $04fp$ may be Four-sided.—Fig. 165 shows a crooked fence $abcdef$, dividing the fields $aopf$ and $amnf$; now place the ruler on ac , roll it up to $b-1$, and draw $b-1$ parallel to it. Again put the ruler on $1-d$, and rule $c-2$ parallel to it. Next put the ruler on $e-2$, and draw $d-3$ parallel to it. Lastly put the ruler on $3-f$, and draw $e-4$ parallel to it. Then the line $4-f$ will be the equalising line.

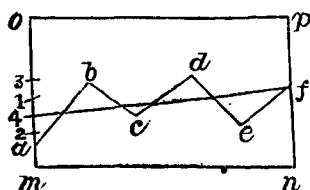


Fig. 165.—To Draw an Equalising Line through a Crooked Fence.

252. Prob. XXV. To Trace on the Ground with a Chain a $\triangle$ whose Three Sides are Given.—*1st.*—When the sides are under 100' in length.—Set off the longest side first, and drive two pegs at the ends. Then from one end as centre, and the length of either of the other sides as radius, describe an arc on the ground. With the other end as centre, and the length of the third side as radius, describe a second arc, cutting the first arc; the $\triangle$ formed by joining the point of intersection of the arcs and the ends of the base-line is the one required.

2nd.—When the sides are longer than 100'.—Here a small $\triangle$, with its sides less than the length of the 100 feet chain but proportional to the given sides, must first be laid down as in the first case. Then two of the sides must be produced to their proper lengths, and the line joining their ends will be the third side required.

253. Prob. XXVI. To Find out the Scale when it is omitted from a Map whose Area is given.—Find out the area according to any scale, and call it A . Then

$$A : \{\text{square of the scale, used in calculating } A = y^2\}$$

$$:: \text{the given area} : \{\text{the square of the scale required} = x^2\}.$$

$$\therefore \text{Scale required} = x = \sqrt{\frac{y^2 \times \text{area given}}{A}}.$$

Example.—A map, whose area is 250000 sq. chains being given, its scale omitted to be either drawn or mentioned; to find the scale.

Suppose the map was drawn to an assumed scale of 20 chains to 1 inch, and the contents thereby produced 160000 sq. chains.

$$\text{Then as } 160000 : 250000 :: 20^2 : (\text{Required Scale})^2.$$

$$\therefore \text{Required Original Scale} = \sqrt{\frac{25.0000 \times 400}{160000}} = 25 \text{ chains to an inch.}$$

254. Prob. XXVII. To Find the True Area of a Survey which was made by a Chain that is either too Long or too Short.—Let the map be plotted, and its area calculated as if the chain was of a True length, then $(\text{True Chain})^2 : (\text{Chain used})^2 :: \text{Area of map calculated} : \text{True area of Map}.$

$$\therefore \text{True area of Map} = \frac{\text{area of map} \times (\text{chain used})^2}{(\text{True chain})^2}.$$

Example:—A survey is made with a chain 3" too long or with one whose length is 42'3", and the map thereof is found to contain 920 ac. 1 rd. 20 per. Required the true area.

$$\text{As the square of } 42' = \text{square of } 504'' = 254016 \text{ sq. in.}$$

$$: \text{The square of } 42'3'' = \text{,, , } 507'' = 257049 \text{ ,, ,}$$

$$: : \text{Area of map } 920 \text{ ac. } 1 \text{ rd. } 20 \text{ per.} = 147260 \text{ perches.}$$

$$: \text{True area} = \frac{257049 \times 147260}{254016} = 149019 = 931 \text{ ac. } 1 \text{ rd. } 19 \text{ per.}$$

255. Prob. XXVIII. (a) To Find the Height of an Inaccessible Hill without the Use of an Instrument (Fig. 166).—

Drive a peg about 4' long at h , and another equally long at l where the head of a staff, fd , is in a line with the object, s , from the point, a , (the heads of these pegs being on the same level). Fix also the point, c , where the head of the staff, ce , is in a line with s from the top of any other peg, b . Measure af and bc . Set off the distance bc from f to b^1 . The $\triangle sabb^1$ and asb are similar.

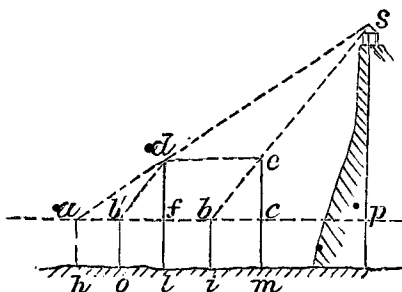


Fig. 166.—To Find the Height of an Inaccessible Hill.

$$\text{Then } \frac{ps}{df} = \frac{as}{ad} = \frac{ab}{ab^1} = \frac{hi}{ho}; \therefore ps = \frac{df \times hi}{ho};$$

$$\text{and } \frac{ap}{af} = \frac{as}{ad} = \frac{ab}{ab^1} = \frac{hi}{ho}; \therefore ap = \frac{af \times hi}{ho}.$$

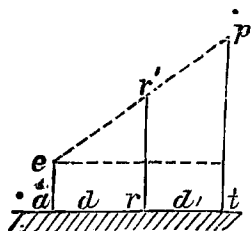


Fig. 167.—To Find the Height of an Accessible Object.

Or with an angular instrument it will be only necessary to measure $ab = hi$, and at a and b to observe the angles sap , $s\hat{b}p$.

$$\text{Then } sp = \frac{ab}{\cotan. sap - \cotan. sbp}; \text{ and } ap = ab + sp \times \cotan. sbp.$$

The height of instrument from the ground, ah , must be added to sp .

(b) **To Find the Height of an Object with the Chain only when the Base of the Object is Accessible (Fig. 167).**—Let a be the position of the observer and ae his height, and let tp be the telegraph post, the height of which is required. Range a rod, rr' , in line with a t , and fix it at a distance d from the observer till its top is in line with the observer's eye and the top of the telegraph post. Measure at . The height tp is then given by the following proportion:— $(rr' - ae) : d :: (tp - ae) : at$.

Or with an angular instrument measure the vertical angle of p at e , and also measure the distance a t . Then $t p = \tan. e \times a t + ae$.

(c) Another method is to hold a 10' rod, vertical, and to observe the extremity of the shadows of the rod and post simultaneously. Measure up the lengths of these shadows. Then, considering the ground fairly level.—shadow of the rod : shadow of the telegraph post :: length of rod : height of telegraph post.

256. Prob. XXIX. To Survey a Field by setting up the Compass at One Station.—Set up the compass as near as possible in the centre of the field, and having placed flags at every angle of it, measure the distances from the instrument to each corner, and take bearings of each of these flags. Then a plan can be easily formed of it; or the compass may be placed at one of the corners, from

whence the others are visible; then take bearings of all other corners and measure distances from this corner to all others. After plotting, when the corners are joined, a plan of the field is easily formed.

257. Prob. XXX. To Survey a Field by setting up the Compass at Two Points.—Fix two convenient points in the field, from which the corners of the field can be seen, and measure the distance between them. Take bearings of the flags planted at the corners, first, from one of the fixed points, and then from the other. The intersections of the bearings from the two fixed points will determine the corners when plotted on paper. The plan of the field is obtained when the corners are joined..

258. Prob. XXXI. Method of Surveying with Compass and Chain along a Railway, when except at the Ends of the Railway, the Compass cannot be placed beyond the Influence of the Rails.—Instead of taking one bearing at each intermediate station, take two bearings, one to the forward, and the other to the back station; then, as the deflection of the needle is constant at any place for the forward and back lines, the true interior angle between the two lines is obtained. Thus the survey work can be done and plotted by means of bearings of the initial and end lines, and of the included angles at the intermediate stations.

259. Prob. XXXII. Detection of Local Attraction.—The compass being in good order, we find that the fore-bearing and the back-bearing of a line disagree after all possible care taken in observations. It is evident that local attraction exists at one of the stations where the compass has been set up. The methods employed in detecting the error are:—(1) When the bearings of a number of connected lines of an open traverse are being found (Fig. 168).

Set up the compass at the ends A, B, C, etc., of the different lines. Take fore-bearing of AB at A, and back-bearing of AB and fore-bearing of BC from B, and so on. If the fore-bearing and back-bearing of AB do not differ by 180° , make a note of this and move the compass to C where back-bearing of BC is taken. If, now, the back-bearing and fore-bearing of BC agree, it proves that there is no local attraction at B. Therefore the disagreement from 180° of the difference of fore and back-bearings of AB was caused by local attraction at A, and the fore-bearing at A on AB must be corrected so as to differ by 180° with the back-bearing on the same line taken at B. If the fore and back-bearings on BC do not differ by 180° proceed as before until a line is reached whose fore and back-bearings do differ by 180° . In Fig. 168 let EF be the first line whose fore-bearing and back-bearing are found to differ by 180° . The back-bearing on DE is correct and the fore-bearing of DE is to be changed to give a difference of 180° . From DE we can pass to CD, and so on back to the beginning.

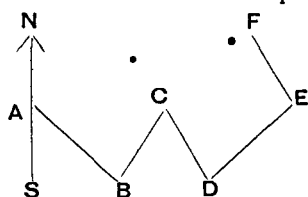


Fig. 168—Open Traverse—
Detection of Local Attraction.

Example: In Fig. 168, let the fore-bearings and back-bearings be as in the margin; The correct bearing of AB will be S $73^{\circ} 15'$ E; of BC, N. $66^{\circ} 55'$ E; etc.

	Fore-bearing.	Back-bearing.
AB	S. $76^{\circ} 15'$ E.	N. $75^{\circ} 00'$ W.
BC	N. $65^{\circ} 10'$ E.	S. $66^{\circ} 15'$ W.
CD	S. $33^{\circ} 30'$ E.	N. $32^{\circ} 15'$ W.
DE	N. $40^{\circ} 00'$ E.	S. $39^{\circ} 25'$ W.
EF	N. $45^{\circ} 20'$ W.	S. $45^{\circ} 20'$ E.

(2) The second method consists in setting the compass at a third point on the line whose fore-bearing and back-bearing do not differ by 180° , reading the bearing from that point and noting whether it agrees with one of the bearings determined before. If not, try a fourth point, and so proceed until two points are found, some distance apart on the line, from which points, the bearings of the lines agree. Guard against the existence at both points of the same conditions, as to local attraction. For instance, if the line is an iron fence, equally near to the two points at which the readings agree, the two bearings, though agreeing, may be wrong by the same amount, owing to the equal attraction of the fence in each case.

260. Prob. XXXV.—To find the Heights and Distances by means of the Box Sextant.—The height and the distance of objects, as walls or buildings, whether accessible or otherwise, may be ascertained with the sextant on level ground, by means of the following table:—*Make a mark on the object, if accessible, equal*

Multiplier.	Angle.	Divisor.	Angles.
1	$45^{\circ} 0'$	1	$45^{\circ} 0'$
2	$63^{\circ} 26'$	2	$26^{\circ} 34'$
3	$71^{\circ} 34'$	3	$18^{\circ} 26'$
4	$75^{\circ} 58'$	4	$14^{\circ} 2'$
5	$78^{\circ} 41'$	5	$11^{\circ} 19'$
6	$80^{\circ} 32'$	6	$9^{\circ} 28'$
8	$82^{\circ} 52'$	8	$7^{\circ} 8'$
10	$84^{\circ} 17'$	10	$5^{\circ} 43'$

to the height of your eye from the ground. Set the index to one of the angles in the table, and advance or go backwards from the object on level ground, until, by reflection, the top of the object is brought by the glasses to coincide with the mark. If the angle be greater than 45° , multiply the distance to the object by the corresponding figure to the angle in the table; if it be less, divide—and the product, or quotient, will be the height of the object above the mark; to which add the height of the eye. Thus let E B be a wall (Fig. 169) whose height is required: and $26^{\circ} 34'$ the angle selected. Make a mark at D equal to the height of the eye; then step back from the wall, until the top at E is brought down by the glasses to coincide with the mark: measure the distance AB, divide that distance by 2, the figure corresponding to $26^{\circ} 34'$; this will give the height DE, to which BD must be added.

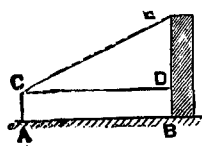


Fig. 169.—To Find Heights and Distances with Sextant.

When the Object is Inaccessible.—Set the index to the greatest of the divisor angles in the table, that the least distance from the object

will admit of, and advance or recede, till the top of it be brought down by the sextant to a level with the eye ; at this place, set up a staff, equal to the height of the eye. Then set the index to one of the lesser angles, and retire in a line from the object, till the top be brought to coincide with the staff, set up to indicate the height of the eye ; place a mark here, and measure the distance between the two marks : this, divided by the difference of the figures opposite the angles ; used, will give the height of the object above the height of the eye or mark.

For the Distance—Multiply the height of the object by the numbers

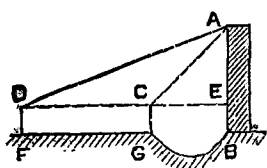


Fig. 170.—To Find Height and Distances with Sextant.

against either of the angles made use of, and the product will be the distance of the object from the place where such angle was used. Thus, let AB be a wall (Fig. 170) not to be approached nearer than C ; and it is found upon trial that this distance admits of the angle 45° being used ; assume a point E on the wall, as the height of the eye ; then the index being set to 45° , the observer must station himself, so that the glasses shall bring the top A to coincide with E. At this point, place a staff, CG, equal to the height of the eye. Now select any one of the lesser angles from the table— $18^{\circ}26'$, for instance, and retire until the point A agrees with the top of the staff CG, which occurs at F. Place a mark at F, and measure the distance from F to G ; which divided by 2, the difference of the numbers opposite to the angles used, will give AE—to which add BE=CG ; the height of the eye, and the total height AB is obtained.

For the distance—the height AE, multiplied by 3, its corresponding figure, will give the length DE ; and AB multiplied by 1, will, in like manner, give GE=A E in this instance.

Horizontal Distances, may be also ascertained by means of

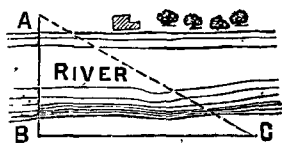


Fig. 171.—To Find Distances and B in contact : then will the distance, with the Sextant. AB, be a product or quotient of the base,

BC, according to the angle used. For instance, if the angle, $26^{\circ}34'$, be used, then must the distance BC be divided by 2.

261. Prob. XXXVI. To Find Breadth of a River with

the Sextant.—On reaching the bank of a river, standing at B, (Fig. 172), send a man with a staff to a distance which is judged to be greater than the breadth of the river. Then motion him to the right or left until he is in such a position that the reflected image of the staff is shown exactly over a tree on the other bank

(as seen directly), when the sextant indicates 90° on its arc. Now having set the sextant to 45° , walk in a straight line towards the staff until you reach a position A, where on looking through the sextant you see the reflected image of the tree shown exactly over a mark set up at B (as seen directly). Then measure the distance from A to B which is found to be say 350'; from this subtract 25' distance from the water to B, and this will give you the breadth of the river to be 325 feet.

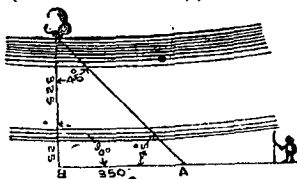


Fig. 172.—Finding Breadth of a River with Sextant.

CHAPTER IV.

SURVEYING BY TRIANGULATION WITH THE CHAIN ONLY.

262. The simplest form of surveying is that in which the chain, flag, and offset-staff or tape are the only instruments used. Surveys of small extent are very often made with the chain only, although as a rule the use of an angle-measuring instrument greatly facilitates survey operations. In the accurate survey of a long length of line or a considerable tract of country, the use of an optical instrument becomes a necessity. In surveying with the chain, and staff straight lines are measured horizontally along the surface of the ground. Surveying with the chain alone is the roughest kind of surveying, and highly accurate results cannot be expected except in the case of small areas, and where steel tape is used.

263. The Terms used in a Chain Survey are as under.—

A Station is the point where any main, base, or any other chain line or a tie-line commences or terminates. The stations may be distinguished into main stations and subsidiary stations. **Main Stations** are those selected at the principal corners of the boundary of a survey as *a, b, c, d* in Fig. 173 which represents a field.

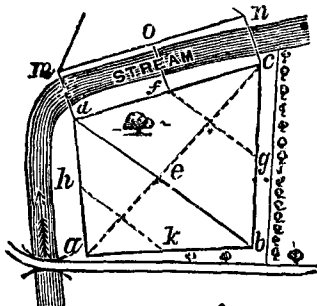


Fig. 173.—Example of Station Points, Lines, Tie-lines, etc.

Station Lines.—The straight lines joining the station points are called Station or Chain lines. The straight lines joining the main station points on the boundary are called **Main or Survey Lines** as *ab, bc, cd, da* in Fig. 173 to distinguish

them from **Subsidiary chain lines** or **Tie-lines** as *ce* and *ea* because they *tie in* the apices of the Δs , *a, c*.

The Base-Line is a line correctly measured horizontally about the centre of the field to be surveyed on which the system of Δ s and polygons forming the skeleton survey is constructed and developed. Great care must be taken to ensure the straightness of the base-line which being of the greatest practicable length should be selected over as level and open a ground as possible, for upon the accuracy and straightness of the base-line the accuracy of the whole details depends. If convenience offers, two base-lines should be laid out. The letter X is the best form for the base-lines in that case to take. It is always desirable to measure the base-line twice or thrice for the sake of obtaining an accurate base to commence with. The object in selecting the base-line in the centre of the survey is to avoid accumulation of errors in the system of Δ s laid out on it to take in the boundaries, as the liability to inaccuracy in the triangulation increases with the distance from the original base. This will be clear from Fig. 173 which represents the plan of a survey. Here *bd* is selected as a base-line through the centre of the survey.

Secondary Triangulation.—**Subsidiary Stations** are points selected as the process of chaining the main boundary or base-lines is carried on according to the nature of the country to be surveyed, so that these points may be joined by running auxiliary or tie-lines to take in the boundaries of internal hedges or compound walls of holdings, roads, etc. In the field represented by the fig. 173 when chaining *bd*, *ab*, *bc*, *cd*, *da*, it will be necessary at *e*, *k*, *g*, *f* and *h* to leave station-pegs for the purpose of either running tie-lines or subsidiary lines to pick up the boundaries of internal fences. Each station should be marked round in the rough-sketch and the field-book with a triangle, circle or oval. The subsidiary stations can be conveniently marked with *whites* (see p. 9) with their distances noted on thin papers.

A well-conditioned Δ is one in which the angles are neither very acute nor very obtuse. Δ s in which the angle at the point to be determined is less than 30° or more than 150° are said to be ill-conditioned, and are avoided by skilful surveyors.

264. Why Area must be divided into Triangles.—**The Principle on which Chain Surveying is based** is that of dividing the area to be surveyed into a certain number of Δ s, the relative position and accuracy of which can be checked by more than one independent method. Of all geometrical figures the Δ is the only one which cannot change its form without at the same time altering the length of its sides. If the length of the sides be constant, its form is immutable, or in other words, the Δ is a figure which is always determinate in shape and size, if the lengths of its sides are known, or are ascertained by measurement (Euc. 1-7). Therefore it must not be thought that the adoption of this form ensures accurate measurement, or renders errors in chaining of no consequence. **Proof or Tie-lines** are always necessary as checks.

265. Except Δ any other Rectilineal Figure does not fulfil the same Conditions.—This will be evident from the following:—In the trapezium $abcd$ (Fig. 174), if the lengths ab , bc , cd , and da be measured, and if ab , bc , cd , and da be formed of rigid bars unalterable in length but free at the joints or angles, it is clear that the points a and c being put closer together, the points d , c and b will be thrown out of their former positions, and will form a figure like $ab'c'd'$, different in shape and area from $abcd$ although the sides remain the same.

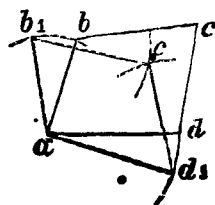


Fig. 174.—Other Rectilineal Figures not determinate in Shape and Size Except Δ .

266. **Planning a Survey.**—By this is meant the acquisition of that knowledge of the ground and its principle features which enables the surveyor to form an idea of the method upon which he intends proceeding.

267. **Preliminary Steps.—Reconnaissance and Hand-Sketch Map.**—In planning a survey, the first thing the surveyor has to do is to make a preliminary reconnaissance. Accompanied by a guide, he must walk or ride over the ground, to be surveyed and mapped, carefully noting all roads, streams, houses, the various difficulties and obstacles that may be in the way of the proposed chain lines, the principal places which may form suitable stations, and other chief features, and making a **Rough Sketch** of the area with the boundary, etc., in a note-book. Whether a map of the district is available or not, this walking or riding over the ground must not be omitted. The sizes of the angles, and the lengths of the station lines can be estimated by the eye, and transferred roughly on the paper. Where feasible, it is desirable to pace the lengths of boundaries. By doing so the surveyor not only obtains an intimate knowledge of the district, but is enabled to form an accurate idea of the difficulty of the work, the time required for its completion, the best routes for his main chain lines free from obstacles on level ground, and to draw the *hand-sketch* bearing a general resemblance to the plan of the ground. The time placed at the disposal of a surveyor always influences the degree of accuracy which he intends adhering to. When a survey is required for a building site, where every foot of frontage is of value, extreme accuracy is not only desirable but absolutely necessary. Rough sketching assists the memory in the distribution of the surface into necessary Δ s. The hand-sketch is not drawn to any scale and is usually made in a *Field-book*, *i.e.*, a book in which all minute details relating to survey operations are entered correctly.

268. **Triangulation.**—After **Reconnaissance and Sketching** it will be the object of the surveyor to select a base-line, (if convenience offers two, through as near the centre of the survey as possible, and to divide the area on both sides of it into a number of main Δ s and polygons, and to post pegs at every angle on the ground.

269. Importance of keeping a Record of Station Points.—

When the triangulation is done and pegs put in at each vertex of the Δ 's the form or position of the Δ 's is then noted on the **Rough-Sketch** previously made, and distinctive letters are written on the diagram at each angle. This arrangement admits of easy reference in the field-book, or on the ground, to any Δ or part of a Δ . It becomes easy to plot the survey, if the stations and the chain-lines are shewn on the **Rough Sketch**.

270. Points to be attended to in making the Triangulation.—(1) Station points How selected—The stations should be so selected on level ground that they will be well seen from each other, and will give a clear straight line for measurement between them.

(2) The Δ 's are to be as large, with reference to the whole area to be surveyed, as is consistent with the features of the ground. The longer the chain lines, and the fewer the Δ 's consistent with the measurement of tie-lines the better. It is evident that the more lines there are to be measured, the greater will be the opportunity for the occurrence of errors, and the difficulty of detecting their whereabouts. It is absolutely necessary that the long lines for this purpose should be made exactly straight. They must be ranged out by means of the station flags and ranging rods carefully before measurement. The use of the Line-Ranger (See p. 101, Art. 182) will make the operation easy.

(3) The sides of the larger Δ 's must pass as close as possible, and as parallel as possible, to the buildings, and roads, and boundaries, etc., to be located to render offsets shorter and to avoid labor in making subsidiary Δ 's to pick up the boundaries, etc. At the same time it would be a serious error to **spoil a Well-conditioned Δ** for the sake of running one of its sides along a fence. This should be avoided as it is a common practice, and is an example of unscientific work.

(4) **Importance of Well-conditioned Δ 's.**—In laying out Δ 's on the base-lines, it should be endeavoured to make them as nearly equiangular, and consequently equilateral, or best-conditioned as possible. This aids materially in the plotting and renders less laborious the calculation of area. This in many cases is impossible, but the principle must not be lost sight of, for the further the form of the Δ deviates from the equilateral, the greater will be the error incurred, if some of its dimensions be a little out. In an ill-conditioned Δ , the error in the position of c , will be more than double the corresponding error in the measurement of a side of the Δbcd (Fig. 173); for example, if the $\angle bcd$ is less than 30° the error in the position of c will be more than double any error in one of the lengths, dc , bc , when the Δbcd is laid out on the base-line bd . Sometimes it is impossible to avoid making angles less than 30° owing to the features of the ground, or to the limited time at the surveyor's disposal.

(5) **Selection of Tie-lines.**—The sides of the Δ s are then to be chained, and as a check a straight line is in addition measured from one of the vertices to a point in or near the middle point of the opposite side of every Δ as *ce*, *ae*. This fourth line is called the **Tie-line** or **Proof-line**. **Proof-lines** serve to rectify the other measures of the Δ , and render unnecessary to chain station-lines twice. It is recommended that this line should meet the base at right angles. Then it affords a means of calculating upon the ground the area of the Δ . Sometimes two points in or near the middle points of two of the three sides are joined as *gf*, *hk* in Fig. 173. Another object in measuring the tie-lines is to obtain chain lines running as near the various details of the survey as possible so that offsets will not be of unusual lengths. The system of tie-lines to check the measurement and plotting of the chain lines and the station points, and the system of checking a long offset by two or more oblique offsets (see p. 168, Art 293) are in accordance with the maxim in surveying, viz., *to ascertain or test by at least two independent methods the dimensions of the main lines and positions of important objects to be shown on the map where accuracy is aimed at.*

(6) Where the land is divided up into fields by walls, ditches, or hedges, the subsidiary figures are generally made to follow as closely as is convenient the lines of division between enclosures, the position of every three sides being secured by means of a check or tie-line generally made in a direction nearly perpendicular to the base of the figure under consideration from its apex.

(7) Sometimes for the purpose of a check and for the sake of the appearance of the map it will be necessary to arrange the Δ s. in a system of parallel lines preferably parallel to some prominent side of the survey (Fig. 190). The distance of the boundary from the survey lines is measured from recorded points on the chain by offsets or short lines laid perpendicularly from the chain to the boundary.

271. How Stations are marked more or less permanently on the ground have already been described in Art. 28, page 9. When the survey is of a temporary nature, such as can be executed in a single day, ranging rods or flags or whites only, may be set up to mark the station points; but when the survey is extensive, stout pegs must be driven into the ground. To identify the pegs easily a corner at the top of the pegs may be sawn off and the cut surface lettered according to the rough-sketch. In pasture land, the position of temporary stations may be marked by cutting the turf in the shape of a Δ , or a square with a small peg in the middle. For rough surveys, flags or ranging rods are set up by the pegs when the chain lines are ranged, but for very accurate work pegs should be drawn out and flags put in their places. In putting pegs in the ground specially in meadow land care should be taken that they are driven almost flush with the ground; otherwise serious injury may be done to cattle grazing thereon.

272. Marking Stations with Pegs and Stone Prisms.—In extensive surveys, marks must be left in the ground by driving

pegs for future reference with $\odot$ s, squares, or Δ s cut into the grass; but in arable lands stout pegs should be driven nearly flush with the ground level, as they are liable to be removed, or ploughed up; and when permanence is desired in villages and towns, by stones embedded below the surface, their positions being recorded by measuring their distances from at least three corners of buildings, and in cities by marking Δ s on kerb-stones at all angles.

273. Flagmen, Chainmen and Offsetman.—In a chain survey, 5 men are required to assist the surveyor—2 to carry, fix, and hold the forward and back pegs and flags, 2 to chain, and one to measure offsets. The surveyor is to range out the chain lines either by the eye or by means of the Line-Ranger, and to take offsets either by holding the tape at the chain line by the eye or by a cross staff or optical square, and to record and supervise his men generally. The chainmen should be willing, intelligent, and trustworthy. The more intelligent of the two should be selected for the follower as he has to direct the leader in line. In commencing a survey the surveyor should satisfy himself that his men are thoroughly well up in their duties. The men, *khalasis*, as they are called in this country, who have had experience at this kind of work, will run lines over all sorts of country, and will really render great help to the surveyor.

274. Scale of Plan or the Degree of Accuracy should be considered before the actual survey work is taken in hand. The $\frac{1}{80}$ th of an inch can be shown upon paper distinctly, so that, if the scale be 80' to one inch or larger, the chainage and offsets must be read accurately to the nearest foot; whereas if the scale be 400' to one inch, no distance under 5' long would be perceptible on the plan, and smaller distances than 5' need not be measured.

Even assuming the limit of accuracy to be $\frac{1}{100}$ th of 1 inch, and taking the following common scales, $\frac{1}{100}$ th of 1 inch is equal in each to the following dimensions, in feet or links:—

1 chain to 1" ($\frac{792}{1}$) = 1 link	10 feet to 1" ($\frac{120}{10}$) = $\frac{1}{10}$ ft.
2 chains to 1" ($\frac{1584}{2}$) = 2 links	20 " " 1" ($\frac{240}{20}$) = $\frac{2}{10}$ "
3 " " 1" ($\frac{2376}{3}$) = 3 "	30 " " 1" ($\frac{360}{30}$) = $\frac{3}{10}$ "
4 " " 1" ($\frac{3168}{4}$) = 4 "	40 " " 1" ($\frac{480}{40}$) = $\frac{4}{10}$ "
5 " " 1" ($\frac{3960}{5}$) = 5 "	100 " " 1" ($\frac{1200}{100}$) = 1 "
6 " " 1" ($\frac{4752}{6}$) = 6 "	150 " " 1" ($\frac{1800}{150}$) = 1.5 "
8 " " 1" ($\frac{6336}{8}$) = 8 "	200 " " 1" ($\frac{2400}{200}$) = 2 "
	400 " " 1" ($\frac{4800}{400}$) = 4 "
1 mile to 1 inch or $\frac{1}{63360}$ = 52.8 feet or 80 links.	
$\frac{1}{6}$ " " 1 " or $\frac{1}{10560}$ = 8.8 " or 13.33 "	
1 " " 2112 feet or $\frac{1}{21120}$ = 2.0833 feet or 3.15 links.	
41.667 feet to 1 inch or $\frac{1}{41667}$ = 41700 feet or .631 link.	

Thus it is clear that with a scale of 1 mile to 1 inch an error of less than 52.8 feet, i.e., $\frac{1}{100}$ th of a mile, will leave no sign of its presence, while with a scale of 10 feet to 1 inch the permissible error is less than $\frac{1}{10}$ of a foot. These considerations should lead to the

adoption of a suitable scale according to the accuracy required. For Scales in general use in India (See p. 7, Art. 23).

275. Ranging Intermediate Points in Long Lines.—In ranging out a long line, it is a good plan to plant a flag in front of every fence or hedge crossed, as it often happens in the process of chaining such lines, especially in a valley, that it is not only found impossible to command a view of the end of the line, but the hedges themselves may obscure the view also. Another object for marking the exact point of intersection is that the chainmen can see the exact place through which the chain should be dragged, for which purpose the offset-rod with a hook arrangement (Fig. 15) will facilitate getting it through.

276. Cutting down Hedges and Trees along the Line.—The surveyor should exercise prudence and caution in cutting down hedges, trees, or in causing unnecessary damage of any description along the chain line. Sometimes flags are planted at the stations without regard to their being visible from each other, and the line between them cleared of all jungle. In all large surveys, lot of line-clearing has to be done, when the country is wooded. If proper reconnaissance is made, many hedges and jungles may be left by the chain line by properly ranging it out. By proper reconnoitre, valuable or shady trees may be left outside the chain line. "Do not cut down a shady tree to save moving a line."

277. Chaining on Hilly and Sloping Grounds.—In chaining, it has been assumed that the ground has been level, or nearly so, to cause no appreciable difference between a truly horizontal measurement and that taken along the surface. It is often necessary to make some allowance for the sloping nature of the ground. The necessity for this becomes more apparent as the surface of the ground departs from the plane of the horizon, as the greater the angle of inclination, the greater will be the difference between the inclined and horizontal measurement. Even where it is so nearly level that the eye cannot detect the slope, an over-measurement of an inch or two in 100' may readily occur. It is plain that, if we measure all the undulations of the ground, we shall get greater totals than if we measure horizontal, as is *supposed* always to be done; but since few surveyors pretend to measure horizontal, until the slope becomes apparent to the eye, their lines are too long by from one to two inches in 100'. To counteract this to some extent, chains are frequently made from one to two inches longer than 100'; some surveyors use old chains which are so lengthened; and for ordinary purposes the precaution is a good one. For greater accuracy, the following are the approximate methods of finding horizontal distances along sloping sides of hills or undulating grounds:—viz, by "*correction*," by "*stepping*," and by "*Reducing the hypotenusal measure*" by calculation, having obtained the angle of slope and the difference in heights.

278. By Correction.—A method of finding the approximate horizontal distances of sloping lengths is to divide it into parts of

fairly uniform slope-lengths, to find the difference on each portion, between the horizontal length and slope-length of a chain, and, then

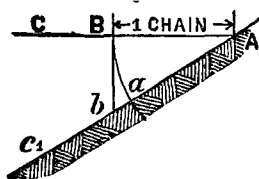


Fig. 175.—Chaining on Slope.

to add the horizontal lengths for the result. In Fig. 175, let us suppose that the chain, when stretched on the slope of a hill, extends from A to a ; but the real horizontal length extends from A to B . The point B is obtained by drawing an arc aB with Aa as radius, i.e., holding Aa horizontal. If the line Bb be projected $\perp$ to AC at B , i.e., plumbing down Bb , it will cut the inclined surface at b , so that the true distance to be measured along the slope by one chain's length is not Aa , but Ab . From the appearance of the slope the horizontal length of ab can be estimated or measured with a tape; and the next chain-length is to be measured from b , and so on, as often as may be necessary. The same process may be adopted for ascent also.

Let us suppose that AC_1 is measured on the same slope, then the correction for ab may be proportionately applied to this distance; for $AC = AC_1 - \frac{AC_1}{1 \text{ chain} + ab} \times ab$; and if $AC_1 = n$ chains; $AC = n - nab$ approximately. From this, the rule, for applying the correction to the measured distance on sloping sides of hills, is—*"multiply the correction for one chain by the number of chains and subtract it from the measured distance."*

279. By Stepping.—When the slope is not uniform, the horizontal measurement may be effected by holding that end of the chain up, on which the ground slopes, so as to keep it, as nearly as the eye can judge, horizontal, the plumb-bob fixed to the pointed end of the arrow, called *drop-arrow*, being dropped down vertically under the end of the chain on the ground below, and the operation may go on for any distance in the same manner. If the work is being done in high grass, this should be trodden down while the end of the chain is

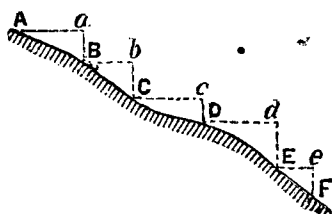


Fig. 176.—Chaining by Stepping.

being transferred to the ground by plumbing. If the drop-arrow hangs in a pile of leaves, scrape these away before setting the pin in the ground just under the end of the horizontal length. It may be explained by an example. Let AF be a line on a slope down a hill (Fig. 176). To chain from A to F , the follower holds one end of the chain or tape at A , while the leader takes a certain chain-length, and holding it horizontally as Aa , transfers the end division by a drop-arrow vertically at B . He then fixes the point on the chain at which he dropped the drop-arrow at B , and waits till the follower comes there to hold the point of the

chain. He then takes the other portion of the chain, and holds it horizontally as Bb , and drops the drop-arrow again at C . Similarly the whole length may be measured by repeating the process. In chaining up-hill as FA , the reverse process of plumbing and chaining is done. This process is called "stepping," and may be carried on by 50' or 33' chains conveniently. Chaining down-hill is the more accurate and this method should be adopted where practicable. The down-hill man generally holds his end too low. The horizontality of the chain is better estimated when viewed from the side. It may then be compared with the horizon, a horizontal line on a building, etc.

280. Reducing Hypotenusal Measure obtained by Calculation, having obtained the Difference in Height by Levelling.—

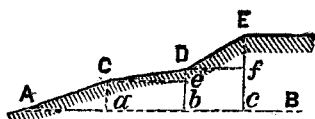


Fig. 177.—Reducing Hypotenusal Measure.

The methods above described are approximate but more accurate methods are necessary to find the true horizontal lengths of base-lines of small Trigonometrical surveys, these can be attained by levelling or by observing the angles of elevation and depression. For example, in Fig. 177, let $ACDE$ be a line on a sloping piece of ground. By levelling, the differences in heights of the points A and C , C and D , D and E are first determined, as well as the lengths of AC , CD , DE by chain measurements. Then $Aa^2 = AC^2 - Ca^2$, where Ca = difference in height between A and C ; $ab^2 = Ce^2 = CD^2 - De^2$, where De = " " C and D ; $bc^2 = Df^2 = DE^2 - Ef^2$, where Ef = " " D and E ;

$\therefore Aa + ab + bc$ = true horizontal distance Aa

$$= \sqrt{AC^2 - Ca^2} + \sqrt{CD^2 - De^2} + \sqrt{DE^2 - Ef^2}.$$

It must be understood that AC , CD , DE are short straight lengths on the sloping surface.

281. Reducing Hypotenusal Measure by Calculation, having obtained the Angle of Slope.—

If, instead of the differences in heights of the points A and C , C and D , D and E , the angles of elevation, aAC , eCD , fDE are ascertained, the problem can be solved equally easily. For Aa = true horizontal length of the sloping length $AC = AC \cdot \cos. CAa$ (Fig. 177). The angle CAa and AC being known, the true horizontal length of Aa is found. So on for other lengths.

282. Observing Angle of Slope.—

The portable instrument for this purpose is the Abney level (described under Levelling Instruments), but a useful little clinometer may be made by cutting a piece of card-board shaped into a semicircle and dividing it right and left of the centre into degrees and subdivisions of a degree, each of which may be marked with the figures given in the table below. Holding it up to the eye by one hand and sighting along the diameter of the semicircle, note, when this line cuts the vane of the staff

adjusted to the level of the observer's eye, and get the angle read by an assistant as marked by a plummet hanging from the centre of the semicircle. A good form of hanging clinometer is shown by Fig. 124. The theodolite may be used for this purpose, but it is a very heavy instrument; with the Abney Level or clinometer it is very simple to read the slope angle and to make the necessary reduction in the chainage as the work proceeds.

283. Adjusting the Allowance for Slope.—As an example, supposing the distance Ab in Fig. 175 to measure 100', and the $\angle BAb = 15^\circ$ what is the value of the correct horizontal measurement AB , and of the reduction ($Ab - AB$)? By the rule we have $AB = Ab \cos. 15^\circ = 100 \times 0.96592 = 96.592'$. The reduction, therefore, is equal to $100 - 96.592 = 3.408$. The correct distance to be recorded in the field-book from A to b is $96.592'$, but there is no necessity for noting the point b on the plan; the simplest method will be to add $3.408'$ to the $100'$ already measured and to commence the next chain's length from that point. In other words $100'$ on the horizontal measurement equals $103.408'$ on the sloping surface.

In order to save calculation, many theodolites, clinometers, and mining dials have the correction for declivity marked on the graduated arc on which the angles in a vertical plane are measured.

284 The following table gives deductions per 100' to be made from measurements taken on slope to reduce them to the horizontal measurements. This table will be useful when using a 100'-chain.

Table for Reducing Measurements on Slope to Horizontal: 100'-chain.

Angle of slope in degrees.	Deduction in feet.	Angle of slope in degrees.	Deduction in feet.	Angle of slope in degrees.	Deduction in feet.	Angle of slope in degrees.	Deduction in feet.
$\frac{1}{4}$	.001	$5\frac{1}{4}$	.420	$10\frac{1}{4}$	1.596	$15\frac{1}{4}$	3.521
$\frac{1}{2}$	.004	$5\frac{1}{2}$	.460	$10\frac{1}{2}$	1.675	$15\frac{1}{2}$	3.637
$\frac{3}{4}$	.009	$5\frac{3}{4}$	.503	$10\frac{3}{4}$	1.755	$15\frac{3}{4}$	3.754
1	.015	6	.548	11	1.837	16	3.874
$1\frac{1}{4}$	.024	$6\frac{1}{4}$	.594	$11\frac{1}{4}$	1.921	$16\frac{1}{4}$	3.995
$1\frac{1}{2}$	.034	$6\frac{1}{2}$	.643	$11\frac{1}{2}$	2.008	$16\frac{1}{2}$	4.118
$1\frac{3}{4}$	.047	$6\frac{3}{4}$	.693	$11\frac{3}{4}$	2.095	$16\frac{3}{4}$	4.243
2	.061	7	.745	12	2.185	17	4.370
$2\frac{1}{4}$	.077	$7\frac{1}{4}$	.800	$12\frac{1}{4}$	2.277	$17\frac{1}{4}$	4.498
$2\frac{1}{2}$	.095	$7\frac{1}{2}$	.856	$12\frac{1}{2}$	2.370	$17\frac{1}{2}$	4.628
$2\frac{3}{4}$	.115	$7\frac{3}{4}$	.913	$12\frac{3}{4}$	2.466	$17\frac{3}{4}$	4.760
3	.137	8	.973	13	2.563	18	4.894
$3\frac{1}{4}$	.161	$8\frac{1}{4}$	1.035	$13\frac{1}{4}$	2.662	$18\frac{1}{4}$	5.030
$3\frac{1}{2}$	.187	$8\frac{1}{2}$	1.098	$13\frac{1}{2}$	2.763	$18\frac{1}{2}$	5.168
$3\frac{3}{4}$	.214	$8\frac{3}{4}$	1.164	$13\frac{3}{4}$	2.866	$18\frac{3}{4}$	5.307
4	.244	9	1.231	14	2.970	19	5.448
$4\frac{1}{4}$	.275	$9\frac{1}{4}$	1.300	$14\frac{1}{4}$	3.077	$19\frac{1}{4}$	5.591
$4\frac{1}{2}$	.308	$9\frac{1}{2}$	1.371	$14\frac{1}{2}$	3.185	$19\frac{1}{2}$	5.736
$4\frac{3}{4}$	.343	$9\frac{3}{4}$	1.444	$14\frac{3}{4}$	3.295	$19\frac{3}{4}$	5.882
5	.381	10	1.519	15	3.407	20	6.031

285. The following table gives the difference per chain of 66' between the hypotenusal and the horizontal measurements:—

Table for Reducing Measurement on slope to Horizontal: 66'-chain.

Degrees.	Links.	Degrees.	Links.	Degrees.	Links.	Degrees.	Links.
5	0.4	14	3.0	23	7.9	32	15.2
6	0.6	15	3.4	24	8.6	33	16.1
7	0.7	16	3.9	25	9.4	34	17.1
8	0.9	17	4.4	26	10.1	35	18.1
9	1.2	18	4.9	27	10.9	36	19.1
10	1.5	19	5.4	28	11.7	37	20.1
11	1.8	20	6.0	29	12.5	38	21.2
12	2.2	21	6.6	30	13.4	39	22.3
13	2.6	22	7.3	31	14.3	40	23.4

286 Accuracy in Ordinary Chaining.—The correct method of chaining is shown by Fig. 7 (p. 17), but ordinarily chain handles are laid flat while chaining, and a pin say $\frac{1}{4}$ " thick, is stuck into the ground outside the chain handle, thereby adding the thickness of a pin to the length of every chain measured (Fig. 8). To avoid this a chain should be of the thickness of a pin less than its nominal length; otherwise there will be an error of 13.2" in a mile when using the 100'-chain. An error of this kind is, however, compensating, and in such a case the *probable error* $\sqrt{52.8 \times \frac{1}{4}} = 1.81$ ".

The error in the length of the chain itself is, however, not compensating, and if the chain were $\frac{1}{8}$ " too long or too short, the error from this cause would be $52.8 \times \frac{1}{8} = 6.6$ ". This consideration emphasises the testing of the chain and the adjustment of its length, as the greatest errors in ordinary chaining are more due to the incorrect length of the chain than to anything else.

In ordinary chaining the permissible error is 1 in 1000, and considerable care is necessary to get this degree of accuracy. An accuracy of 1 in 5000 may be obtained with the steel tape, with proper precautions in ordinary use by simply holding and stretching it by the hand as nearly horizontal as possible, and with the pull on the tape kept as nearly constant as can be estimated, the end of the tape being marked by drop-arrows on undulating ground with one or two temperature observations during the day. The tape should be tested with a standard length, and its length under the pull proposed to be used in the field and at a given temperature ascertained. The temperature at which its length is *correct* under the proposed pull is then to be ascertained, and the chainage reduced to this temperature.

A 66' steel-band having an effective section of $\frac{1}{8} \times \frac{1}{16}$ in. under a stress of 30lbs. will stretch 0.667 link per mile. With the steel-band used as an instrument of precision, there is little difficulty in measuring to 1 in 20,000, and if the measurement be of sufficient importance to warrant the delay, much greater accuracy, 1 in 60,000, may be obtained without any special apparatus beyond that already described.

287. Offsets are lateral measurements from the chain line to outlying points, such as corners of houses, fences, bends of roads, or

streams, and to all other points that are necessary to fix the details of a survey. They are either rectangular or oblique. **Rectangular or Right offsets** are measurements taken at right angles to the chain line. **Oblique offsets** are measurements taken at oblique angles to the chain line.

The process of surveying, after the necessary station lines have been laid out, consists in ascertaining the actual boundary of any field, buildings, etc., by these lateral measurements to such points to the right or left of the chain line, as may distinguish any alteration in the shape of the fence, or the angles of the buildings, etc. An offset at the end of each chain may be taken as a check on chainage. The best method of measuring **offsets** is for the offsetman to walk along the boundary, and to give a signal to the chainmen, whenever he comes to a bend or corner; the surveyor then places himself on the chain line in rectangular position with the offsetman, when the latter measuring down towards him, gives in the length of the offset in rods, and returns immediately to the boundary to take up the next bend. In taking offsets with the tape, proceed as in Prob. IX (b), p. 138. Ordinarily offsets are most rapidly and conveniently measured with a tape, and the eye may be relied on to give the right angles with sufficient precision for all practical purposes.

The offsets are often measured, if short, with an offset-staff, or if they be long, with a tape, from the chain which is allowed to lie stretched on the ground along the chain line, during its measurement, in order to enable the offsets to be taken. When the objects are situated near the chain line, it is easy to find from where the offset springs; but when the offset is a long one, it is not so easy to judge where the right angle would be. In such cases the eye is very much assisted by laying the offset-rod or tape across the chain, as nearly at right angles as can be judged; if its prolongation does not pass through the object, it must be shifted along the chain until it does.

Offsets should not exceed 50' or 60' in length, and should invariably be taken at right angles to a chain line. Rankine says—"to ensure accuracy they (offsets) should seldom exceed about 1 chain = 66' long, although offsets of two to three chains may be made to boundaries which are nearly parallel to the station line." Referring to Fig. 178, on p. 166, it will be seen that there is no necessity for taking more offsets than what are sufficient to obtain every change of direction in the fence.

288. The number of Offsets to be taken is largely dependent on the scale to which the Survey is to be plotted. For example, if the limit of accuracy is assumed to be $\frac{1}{100}$ th of 1 inch and the scale $330' = 1"$, then no distance under 3' 30" can be shown on the plan. Therefore offsets of less than 3' 30" need not be measured. If the scale of the plan is too small to admit of roads or drains, etc., to be shown except by the conventional signs in use, it is unnecessary to take offsets except to the centre of the road or drain.

289. To illustrate the Use of Offsets in ascertaining the actual boundary of any piece of ground, or any fence that may serve

as a division between different plots of land, Fig. 178 is selected. Let $ECGDB_1$ be a crooked fence bounding a field with a road running to the right, and let AN represent a chain line forming one of the sides of any of the main Δ s laid down in a survey. Now to determine the figure of the fence, chain along the straight line AN first, and when opposite each corner on the fence, note the respective distances from A on the chain line; and also measure and note the offsets or $\perp$ distances of each of the corners, aa_1 , $bb_1, cc_1, dd_1, ee_1, ff_1$, etc., from the points a, b, c, d, e, f , etc., respectively on the chain line up to the points F and G , the offsets being shown by the dotted lines. Sometimes a fence goes beyond the reach of ordinary lengths of offsets from a chain line as represented by the portion of the fence, GDB_1 . In this case, it is necessary to construct a subsidiary Δ , nDo , and then to take offsets from the sides nD and Do . The tie-line, Dm , must not be forgotten, as it serves as a check on the accuracy of the work. If there be a hut, H , its position is fixed by taking offsets to its corners. For the sake of accuracy, offsets may be taken to points, like G and E_1 , from the sides of both the main and the subsidiary Δ s. It will be seen that the fence from a_1 to b_1 , and from C to d_1 crosses the chain line diagonally. Here it will be better to measure, in addition to the right offsets aa_1, bb_1, hc, dd_1 , the distances along the fence, $-a_1x_1, x_1b_1, Cx, xd_1$; and to fix the corner g_1 , the line gg_1 should be measured as a check from a temporary station g , on the chain line. To fix the prominent corners such as c_1 , and f_1 , diagonals

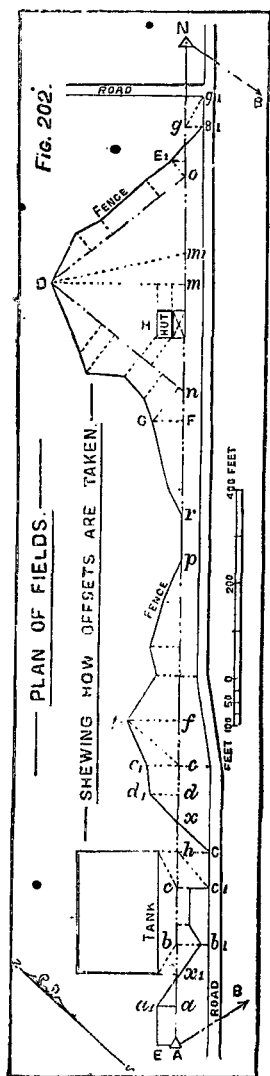


Fig. 178.—Use of Offsets.

like hc_1 and ef_1 should be measured,

290. The Field Notes corresponding to Fig. 178. would be written as in Field-Book No. 1. (Fig. 179) The measurements along the chain line AN are noted in the middle column, and those of the offsets against the distances at the side columns on the right or left, according as they are right or left offsets. All entries are begun

from the bottom of each page of the field-book and worked upwards, so that in holding up the book the distances will appear to run *from*

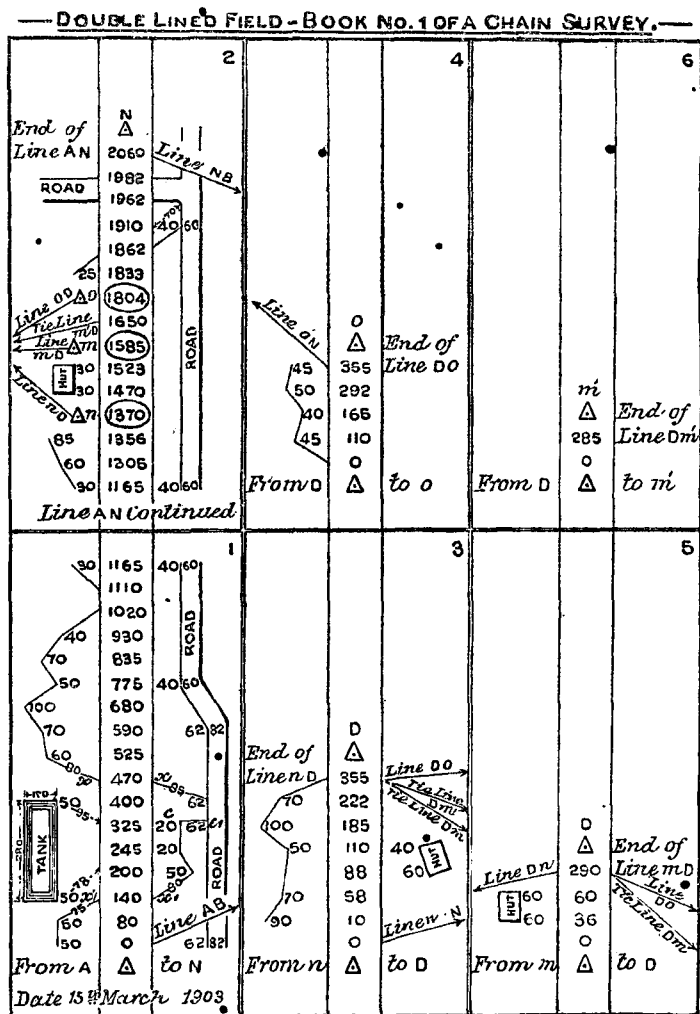


Fig. 179.—Field-Book No. 1.

the direction of the line which is being measured. Each line has a separate page. The objects offsetted to are fully described in the field-notes with sketches.

291. Plotting Offsets.—To draw the figure of the fence $ECGDB_1$ (Fig. 178), the first step to be taken is to plot the lines AN, nD, Do , on paper to a suitable scale. The most rapid way of plotting the offsets is by the use of a *plotting scale*, AB , and an *offset-*

piece, CD , (Fig. 180). The plotting scale, AB , is placed along the plotted chain-line, with its zero point opposite the beginning of the line, a . The offset-piece, CD , is slid along the plotting scale till its edge

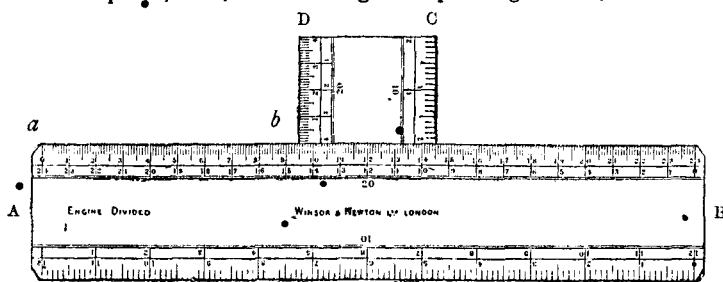


Fig. 180.—Plotting Scale and Offset-piece.

comes to a distance as b (Fig. 180) on the latter at which an offset had been taken, and the length of the offset is marked off with a fine pencil point or pricker from the offset-piece. This is then slid on to the next distance c , and the operation is repeated. The points thus obtained are joined by straight lines, and a miniature copy of the curved line is thus obtained, all operations of plotting being merely representations of measurements made on the ground.

292. Mark Intersection of Lines by Small $\odot$ s.—In plotting a survey, at all points of intersection of lines or stations, it is desirable to draw a very small circle round each; and after the principal lines have been carefully plotted, the exact length being fixed by a puncture with a very fine needle before any detail is plotted, it is absolutely necessary that these lines should be finally drawn in with lake, and on no account should a survey be plotted from pencil lines.

293. Long Offsets—Offsets longer than 50' or 60', and offsets taken to important objects may be taken with the aid of a cross-staff, or a box sextant, or an optical square. The use of the last two is limited to cases where the ground is practically level. The chain lines should be so ranged that no offsets longer than 50' or 60' will be necessary; or if the chain lines cannot be altered, subsidiary Δ s must be constructed on them, thereby preventing long offsets. Sometimes offsets of 100' to 150' are taken. This does not matter much in an open country, but in the survey of cities and towns they must be as short and as carefully taken as possible, for the frontages of houses, the areas of gardens, out-offices, and other buildings depend on the care with which the measurements are made. Long offsets are generally the result of laziness, and can be avoided by the method given below.

The Position of a Distant Object as D (Fig. 178) may be fixed by measuring a right offset Dm perpendicular to the chain line AN , its direction having been laid out $\perp$ to the chain by an optical square, or by taking two oblique offsets oD and Dn from the fixed points o and n on the chain line. The dimensions of oD , Dn , and on being known, the point D , being the vertex of the subsidiary Δ

oDn , is fixed. Care should be taken that D should form the vertex of a Δ as nearly equilateral as the eye can judge.

This process, in fact, belongs to the method of surveying by Δ s; the nearer the $\angle oDn$ approaches a right angle, the more correctly is the position of the object determined. As a check, a third oblique offset like Dm' may be taken when only oblique offsets are measured. Oblique offsets are plotted like the sides of Δ s.

295. Limit of Right Offsets.—A right offset as Dm should not exceed 100' which may be taken to be the limit to which right offsets may be taken. If the offset required becomes longer than this, then two oblique offsets, like nD and oD (Fig. 178), should be taken to the object as checks.

296. Why Points to be offsetted to should be as near as possible to the Chain Line.—Long offsets are to be avoided where

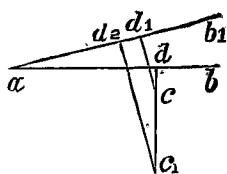


Fig. 181.—Use of Short Offsets.

much accuracy is expected, and the points to be offsetted to should be as near as possible to the chain line. This may be explained by Fig. 181. Let ab be the true chain line, and if c and c_1 be two points to which the offsets are taken from the same point d , then a slight deviation in the chained line, as ab_1 , will make the offsets from c and c_1 fall on the points d_1 and d_2 . The difference between ad and ad_1 is less than that between ad and ad_2 so that there is a greater error for a point more

distant from the line. Again any error, in the construction of the cross-staff in having the grooves or sights not at right angles, gives rise to a greater error, if the offset is a long one, than if it is a short one. Any error made in setting the cross-staff with one of the grooves or sights along the chain line, will give rise to a corresponding error in the direction of the other groove or sights, and a consequent error in the point at which an offset occurs; and this error increases, the further is the point to which an offset is taken. When offsets are measured roughly by means of an offset-rod, there is a cumulative error in measuring a long offset.

297. Advantages and Disadvantages in using Tapes and Offset-rods for Offset Measurements.—In measuring an offset, errors accumulate rapidly, even if the offset-rod is most dexterously handled. These errors are avoided by using a tape which can be stretched to the limit of 100' of an offset, but tapes are conducive to laziness and long offsets. After much use, or when wet, they elongate, or when there is wind blowing, they are shortened by sagging. It is tedious to wind it up in the case or to collect it in folds in the hand, which and, consequently, the field-book are made dirty. The figures get indistinct after continual usage, and this often leads to errors: whereas an offset-rod is a handy portable instrument which, if laid flat on the ground and not allowed to slip back when turned over end for end for measuring an offset gives the most accurate measurement. The tape is parti-

cularly convenient for the measurement of buildings, and in surveying through town.

298. Fixing Positions of Tanks, Buildings, and Banks of a Stream.

This problem is divided into two cases as the chain line being run is or is not parallel with the side of the building or tank. If the tank is parallel with the chain line, note simply the points, x_1 , and h , (Fig. 178) and the offsets to the corners of the tank at x_1 , and h . When a building is lying at an angle to the chain line, the most rapid way to survey it is to find the points l, r, s, t , on the chain line in line with the sides of the building, nm, op, no , and mp respectively (Fig. 182). The corner, o , is then fixed by the $\triangle sor$, the corner n by the $\triangle lns$, and the corner p by the $\triangle tpr$; the lines, so, ln and or , being produced coincide with the faces of the building which should then be measured all round.

Fig. 182.—Offsets to Tanks and Buildings.

This is much quicker than taking perpendicular offsets, ap, bo and cn , opposite all the angles as, shown by the dotted lines.

299. To locate a round building it is necessary to locate only its centre, taking note of the radius. In the case of a hexagonal or octagonal building, locate one of the sides. If the building is a parallelogram, locate one each of the two pairs of parallel sides, etc. If the building is very irregular it may be necessary to locate each side separately; remembering that to locate a straight line, it is necessary and sufficient to locate two of its points. In the case of an intricate building a rectangle should be set out so that it may enclose it, and the sides of that rectangle used as chain lines from which to take offsets to the various corners. To survey some buildings completely, it is necessary to have access to the inside.

300. To locate an Irregular line, as the bank of a stream, run a chain line, or a series of chain lines parallel as nearly as may be to the direction of the irregular line. Measure offsets $\perp$ to the chain line or lines to all salient corner points in the bank (Fig. 192).

301. Fixing Corners of Fields.

When commencing a chain line near the corner of a field, as b in Fig. 183, it is not enough to take one offset, Aa , on the chain line, AF ; and another offset, Am , on the chain line, AC , to obtain the corner, b , formed by the hedges; but the measurement of oblique offsets Ab, ib, mb is necessary to fix accurately the point of the intersection of the hedges. Similarly when the chain line produced crosses the hedge at g , it is not only necessary to take one offset at h to f , but the length, gf , along the hedge should be measured, so that with the length gh on the chain line the $\triangle ghf$ is formed to fix the exact position of f .

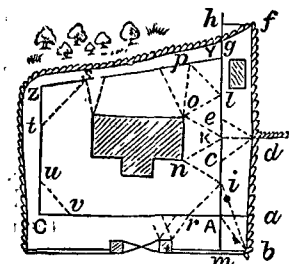


Fig. 183.—Offsets to Corners of Fields and Buildings.

302. To fix the Intersection of Hedges.—It may sometimes happen that the junction of another hedge on the other side of the hedge, *bf*, is required to be exactly fixed, for which an ordinary offset will not be enough. Suppose, the point, *d*, of the intersection of the hedges (Fig. 183) is to be determined. Construct a $\triangle ced$ on the chain line, and then measure the oblique offsets *cd* and *ed* when *Ac* and *ce* are measured in addition to the right offset *kd*. This will fix the point, *d*, exactly.

303. To fix the Angles of a Building.—Another way to fix more quickly and accurately the angles of a building from a chain line when ordinary right offsets are deemed insufficient, is to leave stations as *i*, *k* and *l* (Fig. 183), from which the oblique offsets, *in*, *nk*, *ko*, and *ol*, to the corners of the building are measured. Similarly other corners may be fixed from the other chain lines.

304. Picking up a Gate.—In picking up a gate, one of the gate-posts should be fixed accurately by means of $\triangle$ s constructed upon the chain line as in Fig. 183 and then the width of the gate is to be measured. It is better to take in both the posts in that way.

305. Footpaths and Cart-tracks.—Footpaths should be shown by a dotted line, and cart-tracks by a double dotted line. To take the latter, ascertain the average width, and take the offset to the centre; but if absolute accuracy is necessary, it may be surveyed like a road, taking offsets to both sides of this track.

306. Offsets to Footset Hedges.—When a hedge divides two properties as the hedge *bf* in Fig. 183 the centre or root of the hedge should be offsetted to, and noted in the field-book thus: I—.

307. Hedge and Ditch.—Care should be taken to show the hedges, to which field they belong, and where they change, at which point always take an offset. If there is a ditch to the hedge the ditch is always the boundary, and is noted in the field-book

ditch
thus: $\perp \overline{A} \overline{B}$; the line denotes the ditch; the letter $\perp$ shows
ditch

the side to which the hedge belongs, and on the left the hedge belongs to *A* and on the right to *B*. A post-and rail fence is shown thus: $| - | - | - | -$. Wire-fences are distinguished by a spiral line thus: $\sim \sim \sim \sim$. A paling fence is shown thus: $- - - -$, and walls or dykes by two parallel lines thus: $=====$.

308. The Double-lined Field-book is a note book in which the results obtained by measurement in the field are taken down. The size of a field-book should be $6" \times 4"$, or $8" \times 4\frac{1}{2}"$, opening lengthwise, and not at the sides like ordinary books to slip into the pocket. Every page of the book is ruled feint, with a central column of a couple of red lines $\frac{3}{4}"$ apart. The space between the red lines, is, virtually but one line representing the chain, and is required for writing the distances on the chain line, where roads or streams cross it, or whence offsets are taken to houses, fences and other details. All entries must commence at the bottom of each page of the book and proceed upwards. If the entries are made from the top down-

wards, the field-book will then show the front objects at the back and *vice versa*. The chaining and writing should be in the same direction, to avoid confusion also in noting the offsets to the boundaries, which should be noted on the right or left of the chain line, according as they are to the right or left of the surveyor, when he is proceeding to the forward flag. The offsets are written with the shape of the boundaries against the distances of the chain to which they relate, on the side columns to the right or left, according as the objects offsetted to are to the right or left of the chain line. If the chain line crosses a fence at an angle, as at x and x_1 (Fig. 178), its representation on the field-book must not be made to pass obliquely across the central column, but must arrive at one side of the column, and leave it at the other, as at x and x , and x_1 and x_1 (Field-book No. 1, Fig. 179), *i. e.*, at points exactly opposite, as it would do, were the *Central column* a mere *straight line*; for the points x , x_1 on either side of the column are actually one point where the fence crosses the chain, and if the central column is considered to have no breadth, it will be seen that these two points will fall on each other when plotted and become coincident. Whenever a fence makes a bend at right angles to the chain line, there are evidently two offsets to be entered in the same line, as at cc_1 (Fig. 178) and Field-book No. 1 (Fig. 179), *i. e.*, at 325' from ΔA . The first offset 20' is to be written against 325' to the right, and the outside offset 62' representing the total distance, to the extreme point of the fence. To ascertain accurately the position and dimensions of a tank or a building or a hut, it is only absolutely necessary to take two contiguous perpendicular offsets, one at each end, the length of the tank or hut being given by the distance along the chain line they are situated apart when it is square with the chain line. Although this is sufficient if correctly done, yet a little error made by not measuring the offsets correctly, especially if they be long ones, at right angles to the chain line, might make a serious difference in the actual length of the tank. The proper way is to take the right offsets first, and then to take an oblique offset for each angle from the chain line to the angle; and then to measure the tank all round, the figures denoting dimensions being included between arrow-heads to prevent their being confused with figures denoting offsets, which latter should be written close to the point to which they refer and, not midway between the point and the central column. This will act as a check. The junctions of the chain lines with others and their directions are indicated by arrows (Field-book No. 1) as at A , N , D , o , and n . Here the directions of AB , NB , nN , Do , and Dm , are shown by the arrows with reference to the lines AN and nD respectively. If the chain line runs along the fence as *pr* (Fig. 178), so that, in plotting, the fence will coincide with the chain line, it is shown as at chainages 1020' and 1110' of the field-book. This method may be called the *Method of Line Notes*.

The stations are expressed in the Field-books either by letters or by numerals, as $A \Delta$, $B \Delta$, or $\odot A$, $\odot B$, or ①, ②. When

a mark is left at any point of a chain line, with the intention of returning to it again, in order to measure to some other point, the point marked is called a **false station**, and is noted in the field-book as *F. S.*, or has a line drawn around it to distinguish it, or has a station mark Δ placed outside the central column to the right or left, according to the direction in which the measurement from it is to be made. Examples of these three methods are given in the margin. A false station is named by its position on the line to which it belongs, thus: "250 on 482."

From A	482	to Δ B
	250	<i>F. S.</i>
	0	.
	Δ	
From A	482	to Δ B
	(250)	.
	0	.
	Δ	.
From A	482	to Δ B
	250	Δ
	0	.
	Δ	.

The direction of a chain line may be indicated in different ways. For example:—"From Δ A go S. to Δ B" indicates that the chain line goes from the first station, A, to the second station, B, in a southerly direction.

"From ① go N. E. to ②" indicates that the chain line goes from the first station to the second station in a North-easterly direction.

"From ② range W. 40° N. to ③" indicates that the chain line goes from the 2nd station to the 3rd station in a direction which makes an angle of 40° with the westerly direction, reckoned towards the north.

"From Δ B turn left," or "from Δ B $\curvearrowleft$ or $\curvearrowright$ " indicates that on reaching the 2nd station the surveyor turns to the left, in order to proceed from the 2nd to the 3rd station. The symbol $\curvearrowright$ or $\curvearrowleft$ indicates "turn to the right." On arriving at the end of a station line the relative direction of the next line to be chained may either be stated in words as "turn to the right," or "turn to the left," or indicated by the symbols as above. Main stations should be expressed by capital letters, A, B, C, etc., and subsidiary stations by small letters, a, b, c, etc.

309. Care Necessary in keeping Field-book.—When measurements are taken in the field, all entries should be made at once in pencil, and on return from it they should be inked in the same day. Memory should never be relied upon for any thing, this procedure will lead to the detection of any error that may have been made, and can be easily corrected the next day. The pages must be numbered, before starting out. Each day's work is to be dated before commencement, and when the work is done for the day, signed and dated. Carry a small ruler, a pair of pencil-dividers, and an eraser. The pencil used should not be too soft; 4H is about right for this kind of work.

310. Importance of Neatness.—The accuracy required in a survey, not only depends upon the adoption of a suitable scale, but also on the order, system, and neatness bestowed on all steps, whether of measurement or of delineation on the part of the surveyor.

Accurate, clear and neat keeping of the field-book, *with copious notes*, is very essential, as it saves much time in plotting, and guards against errors unavoidably arising from reference to a confused field-book.

In sketching in the notes, sit down whenever practicable, directly on the line to be surveyed, facing in the direction in which it is to be run, resting the field-book on the knee in the same way. If a line or figure has to be rubbed out, do it completely, so as not to give the page a smeared appearance. Rule all straight lines with the ruler, and draw all circles with the dividers. When it comes to plotting the map, the surveyor will be amply repaid for any extra time spent in careful note-keeping. The beginners, at first, find the writing of a field-book a little puzzling. The usual error they make is to write small figures and close together; plenty of spaces should be kept between two sets of figures written vertically. Oblique offsets, small Δ s, measurements of buildings, and the like are best recorded by sketching a diagram of the lines measured, and writing their lengths along them. Every surveyor should keep his field-book so distinctly written that it may be possible for a draftsman to plot the survey from the field-book without receiving an explanation afterwards. No field-book is complete unless it contains, at the beginning of the work, an index showing on what page or pages the detailed survey of any particular line may be found.

311. Best Size of Field-book.—The size of the field-book above described is too small. A quarto size, opening lengthwise, which gives ample room for sketching in detail any features and for remarks, is preferable.

312. The Sketch Method of Field-book (about the size of a demy 4to) is one in which the Δ s, lines, offsets, together with the fences, roads, rivers, buildings, and other features of the ground surveyed, are actually hand-sketched. The measured distances are written alongside the different lines and offsets; the whole becomes a sketch plan with dimensions. This plan is suitable for a single field or a small estate; but in the case of a complicated work where hundreds of fields are to be surveyed, the first description of ruled field-book or single-lined field-book described below should be adopted. In general, it is undoubtedly the best to adopt a combination of the two methods, drawing in the first part of the field-book the sketch referred to in Arts. 267, 269 showing on this as many measurements as may be practicable, and, in the *line notes*, going more particularly into details.

313. Single-lined Field Book is preferable to the Double-lined—Instead of the ruled central column, a single red or blue line, upon which the distances measured along the chain line may be noted, is preferable. This line represents in the field-book what the chain line does in the field, and the crossing of a road or fence can be correctly shown in its proper place and direction by continuous unbroken lines. Both systems have got advantages and disadvantages. The two red lines forming a central column are almost universally used; as this system keeps the measured distances

required, each corner of the field which is to be surveyed is determined by measuring its distances from two other points. A few examples will make this clear:—

(1) **Take a Δ Field ABC.**—**Field-work:**—To survey this, measure the three sides AB , BC , CA . Chain also, as a tie-line, the distance from one of the corners, C , to some point, D , in the opposite side, at which a signal should have been left when measuring from A to B , at a known distance from A .

The **Field-notes** are given in the margin, and the student is requested to draw the simple figure from the following directions as to plotting: choose a suitable scale, then draw a line equal in length, on the chosen scale, to one of the sides, AB , for example. Take two pairs of compasses; open them to the lengths respectively of the other two sides. Put one foot of each at the ends of the first side, AB , and put their other feet together, then their point of contact will mark the point, C , of the Δ . To “prove”

the accuracy of the work, fix the point D by setting off from A the proper length, $130'$, and measure the length of the tie-line, DC . If its length on the plot agrees with its measurement on the ground, the work is correct.

(2). **A Four-Sided Field ABCD.**—**Field-work:**—Chain the four sides; chain also a diagonal, as AC , dividing the four-sided field into two Δ s. Chain also the other diagonal, DB , as a tie-line.

Plotting.—Draw the line AC , equal in length to the diagonal to any scale; on each side of it construct a Δ with the sides of the field, as stated in the previous article. To prove the accuracy of the work, measure the tie-line, BD , on the plot, and if it agrees with the measured length, the field-work and plotting are both correct.

(3). **A Many-Sided Field.**—**Field-work:**—Measure all the

sides of the field $ABCDEFG$ (Fig. 185) Measure also the diagonals, AC , CG , GD , and DF , enough to divide the field into Δ s. Select such diagonals as will divide the field into well-conditioned Δ s. Chain also diagonal, BE , as a tie-line. The figure itself is a field-book and no ruled field-book, is necessary in such cases.

Plotting.—Start with any diagonal and plot one Δ . Plot a 2nd Δ adjoining the first. Plot another adjacent Δ , and so proceed till all are constructed in their proper places. Measure also the tie-line to prove the accuracy.

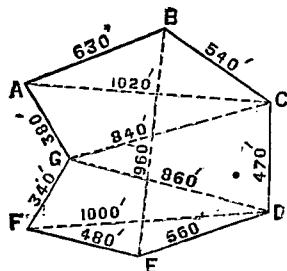


Fig. 185.—Chain Survey of a Many Sided Field.

317. Examples of Surveying by Diagonals and $\perp$ s—When the diagonals measured on the ground divide the field, its content is most easily determined by measuring the perpendiculars upon the diagonals from the opposite angular points on the ground. This is, therefore, the most rapid and easy method of surveying when the content alone is required.

(1) **A Δ r Field.**—Chain the longest side, ab , and the $\perp$, cd , let fall upon it from the opposite angle c . If obstacles prevent the measurement of the $\perp$ inside the Δ , find the point where a $\perp$ let fall from the angle, as a , to the opposite side produced, as bc , would meet it, as at e .

(2). **A Four-sided Field A B C D.**—Chain the diagonal AC . Leave marks (subsidiary stations) at the points on the diagonal at which the offsets from B and D would meet it. Measure the off-sets, and if they are long ones, measure DC and CB as checks on the offsets taken.

(3) **A Many-sided Field.**—Fig. 186 represents the field, which was surveyed by the second method, of which the field-notes are given in the margin.

When the corners can not be seen from one another or from any point within, take three or more convenient points within the field, and measure the distance from them to the corners, and thus form the different sets of Δ s. Always measure convenient tie-lines to prove the accuracy of the triangulation work.

⊙ F 100	261 105 0	
From ⊙ E		└ to ⊙ A
⊙ D 80	236 150 0	
From ⊙ C		└ to ⊙ E
⊙ B 80	342 189 150 0	180 ⊙ E
From ⊙ A		to ⊙ C

318. Examples of Surveying by Diagonals, Tie-lines, and $\perp$ s combined with the Chain only.—(a) In the simplest case, this

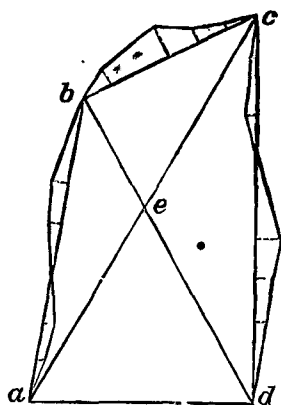


Fig. 187.—Chain Survey of a Many-sided Field.

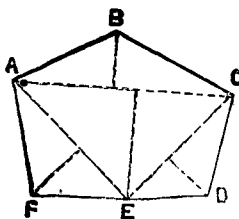


Fig. 186.—Chain Survey of a Many-sided Field.

comprises merely the accurate measurement of a quadrangular piece of ground which includes the frontage and rear of a proposed building (Fig. 187). In executing the survey, the first thing to be done is to walk over the ground and to make a rough-sketch of it; then the ground is to be divided into Δ s abd and bdc by running

the base-line, db , and stations marked upon the ground by means of pegs. The station points and lines are then marked upon the hand-sketch to facilitate the plotting operation. The base-line, db , and the chain lines ab , bc , cd , and da , are then to be ranged with ranging-rods or flags placed at a , b , c , and d , and measured carefully. The base-line, db , should pass as nearly as possible through the centre of the survey. The fences in this case are crooked, and the offsets to the crooks are taken with the tape or offset-rod as the chaining proceeds.

Field Notes.—In this case the following simple field-notes will do as the field is not covered with any other details except the boundary.

Side bc	0	300	Side cd	50	482	Base-line.	d 545 (195) b	e
	12	250		30	422		c 700 (350) a	
	30	200		37	346	Tie-line		e
	57	150			200			
	27	50			156	Side ad	a 350 d	o
Side ab	0	475	Side cd	50	482			
	20	345		30	422	Tie-line		o
	15	290		37	346			
		220			200	Side ad		o
		130			156			
		70			o			
		a			c			

Plotting.—To delineate the ground on paper, lay down on paper to scale a line representing db , and then take in the compasses the length, ba , to the same scale; and with centre, b , and radius, ba , describe an arc, and similarly with centre, d , and radius, da , describe another arc cutting the former; the intersection of these arcs gives the point a . Similarly the lengths, bc and dc , fix the point c . After this the offsets may be plotted as explained on p. 167, Art. 291.

Tie-Lines.—The measurement of the four sides and the one diagonal db is enough to plot the survey to scale, but there is no check upon the accuracy of the work. In order to effect this, the line ac , is to be measured, noting distances, ae and ec , which, when the survey is plotted, should scale the same as measured on the ground. ac is called a *tie-line*, for it “ties in” the apices, a and c , of the Δ s.

319. Example of a Tie-line serving as a subsidiary

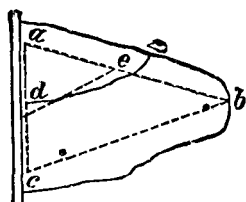


Fig. 188.—Chain Survey of Δ Field.

Line in Triangulation.—The property is made up of two fields on a road side, Fig. 188, which when taken together form a scalene Δ . The sides, ab , bc , and ca , are the main lines by the *ails*, i.e., the field boundaries. The ail, ed , divides the property, and if a subsidiary line along ed is measured, it serves the double purpose of taking up the ail, ed , by short offset measurements and of acting as a tie-line to the Δabc . For if the distances, ae on ab , and, ad on ac , be

measured as well as ed , on plotting the Δabc , the length ed should scale the same as measured on the ground.

320. Another System of Triangulation.—By Fig. 189 is represented a field, the form of which is not suited to the same system of triangulation as in Figs. 173 and 187. Those shown are enough to fix all the points. Thus taking ab for the main line and plotting it on paper, the point, c , is obtained by the intersection of the lines, ac and ec , measured before. It will be seen that it will be checked

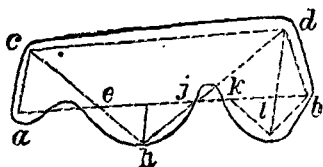


Fig. 189.—Chain Survey of a Field by Triangulation.

by another line. Having determined the point, c , the point, l , is next obtained from k and b by the intersection of the lines, kl and bl , also measured before. To find d , draw an arc with the measured radius, bd , and another with the measured radius, ld , from l , cutting the former at d . If all the measurements be correct the distance between the points, c and d , will be found equal to that measured, which will close the survey. Although not absolutely necessary, it will be better to continue the line, hj , to d which would, in case of error, determine at once which of the points, c or d , was wrong.

321. System of Triangulation dividing the Area into Tiers of Triangles parallel to some Prominent Side of the Survey.—In Fig. 190 it

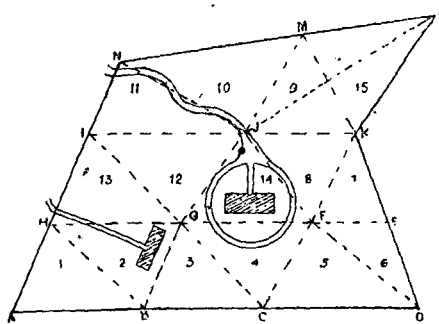


Fig. 190.—Triangulating with Tiers of Δ s.

has been advisable to divide the area into three tiers of triangles parallel to the line, AD , to show the buildings in the interior and to obtain checks. Measure all the sides of the Δ s. Now plot, beginning with the side AB , in succession around the edge of the field the triangles 1, 2, 3, and so on to 11. We have now several checks. The measured and plotted lengths of the lines HI , IG , GJ , should agree. Also, on the ground several lines in the series, as AB , BC , CD and EF , FG , GH are continuations of or parallel with each other, and should so appear on the plot.

322. General Principles to be observed in laying out the Chain Lines.—As a general principle, liable to some exceptions, the best plan is to measure a base-line, the whole length of the survey, as nearly through the centre as convenient. Upon this construct as many large Δ s as possible, taking care to tie them in to the base-line where necessary. When it is found

inconvenient and troublesome to run a base-line through the centre of the survey, the surveyor must so dispose the Δ s as to suit the ground to the best advantage. It would be also a bad policy to alter, divert, or break up a good line merely to avoid some obstacle which by a little ingenuity may be overcome as the main object in laying down the main lines is to obtain a series of well-conditioned Δ s. Within the large Δ s as many tie-lines, subsidiary Δ s and polygons as are to be measured as may be necessary to fix the positions of the several objects of the survey. This is in accordance with the principle that it is always well "to work from whole to part and not from part to whole." The method of filling in the details by offsets from chain lines with offset-rods, is also done according to the same maxim. The directions of the sides of the subsidiary Δ s must be so fixed that the offsets to be taken from them may be as short and as few in number as practicable. The surveyor should so arrange his work that he may not have to go over the same ground twice. If there are troublesome continuous obstacles, such as large hedgerows or rivers which do not allow a continuous network of Δ s being formed a connection between two Δ s having sides nearly parallel, may be formed by a long trapezoid, whose short sides are two right offsets taken across the hedgerow or river at convenient spots; the proof-line then required will be a third right offset taken between the long sides which will check the connection formed by the trapezoid. In fig. 173. $dmnc$ is a trapezoid, and dm , fo , and cn are three connecting right offsets, mn being the side of another main Δ .

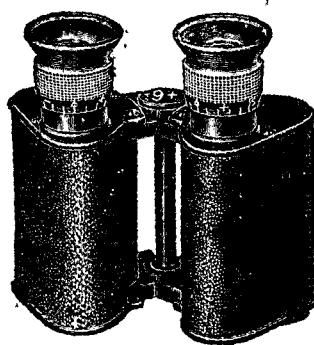


Fig. 191.—Binocular Field Glass.

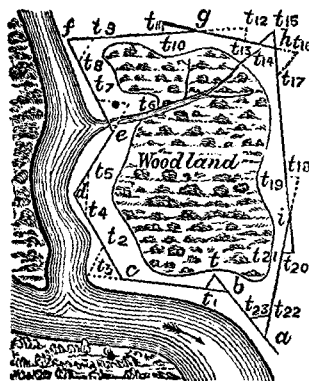


Fig. 192.—To Traverse with the Chain only.

In making a reconnaissance for a large survey a binocular telescope or a binocular field-glass (Fig. 191) or a monocular field-glass will greatly facilitate the operation. A whistle should always be used by the surveyor to call his men when they are out of reach.

323 Method of Surveying by Tie-lines or Chain-Angles, i.e., to traverse with the Chain and Staff or Tape only.—When

a meandering river does not admit of being divided into Δ s and chained across, it is better to survey with the chain in the following way (Fig. 192). Lay out and measure the lines ab, bc, cd, de , etc., and measure bl, cl , etc., in the prolongations of the lines ab, bc , etc.; and also measure the ties, ll_1, l_2l_3, l_4l_5 , etc. Then take offsets from the chain lines to the edge of the river. When plotted, these will give a plan of the left bank of the river. Points on the right bank may be obtained as explained in the problems (Chap. III). Cases might arise in which the traversing is done in the following manner:—The direction of the first line having been ranged out its length is chained. Where it is desirable to deflect the survey from the first traverse line, a line in a new required direction is ranged out, preferably from a chain end, so that the measurement may be continuous. A measurement of a convenient length is then made in the opposite direction, and the distance which separates the terminal distance measured in the opposite direction from a similar point on the first line is then measured as carefully as possible not with the chain, but with a steel tape or with a stick cut to the required length, preserved for reference, and marked; the traverse line is then continued forward until it is required again to deflect its direction, when the same process is repeated.

324. Surveying a Lake, or Woodland or other Inaccessible Areas by Chain-Angles.—Suppose, a large lake, or wood, or a plantation of thick timber trees (Fig. 192) which can not be divided up into Δ s and be crossed, is to be surveyed. Here lay out and measure the chain lines, ab, bc, cd, de , etc., round the woodland and also tie-lines ll_1, l_2l_3, l_4l_5 , etc., which determine the angles at which ab and bc , bc and cd , cd and de , de and ef , etc., meet. The measurement of one tie-line for each chain line as ll_1, l_2l_3 , etc., is enough to plot the survey, but others are necessary as checks. If it becomes inconvenient to measure the tie-lines inside the chain lines the angles, as at g, h, i , are to be measured by producing the sides and measuring the ties, $l_1l_2l_3, l_3l_4l_5, l_5l_6l_7, l_7l_8l_9$. The distances $l_1l_2, l_2l_3, l_3l_4, l_4l_5, l_5l_6, l_6l_7, l_7l_8, l_8l_9$ should also be measured as checks. With these measurements, the lines ab, bc, cd, de , etc., may be plotted at the proper angles with each other. The boundaries of the woodland are surveyed by offsets as usual from the chain lines, ab, bc, cd, de , etc.

325. Surveying a Plot of Land with a Building.—In Fig. 183 the land can not be divided up into Δ s owing to the large building in the middle of it. The chain lines AY, YZ, ZC, CA should be tied at the corners by lines ri, pl, st , and uv .

This method of surveying is liable to error. If there is a small error in measuring the tie-line, ll_1 , (Fig. 192) the direction of bc will be thrown out. Similarly other lines will be thrown out, if their ties are measured or plotted wrongly. This method is also opposed to the principle of work "from whole to part and not from part to whole," as the surveyor does his work from part to whole, *i.e.*, he fixes the situation of a part of the line, bl_3 , *i.e.*, bl_1 (Fig. 192), and from that finds out the direction of the whole line, bl_1cl_3 . This is a method which ought rarely to be

resorted to, for no time is gained by its adoption, and the chances of error are considerably multiplied, owing to the numerous additional lines to be measured; moreover it is to be observed, that angles can in general be measured in the field more correctly with an angular instrument than the length of lines with the chain, especially over uneven ground or in an enclosed country. Upon fairly level ground, owing to the unavoidable absence of angular instruments, such traversing may be resorted to; but in undulating grounds, traversing of this kind should be avoided. It may be used for taking in the position of an outlying plot of land which is easily accessible along a road when an angular instrument is not at hand. If this tie method be applied, and the angle α , corresponding to any tie set out, be required, then $\sin. \frac{\alpha}{2} = \text{Tie-line} \div 2R$; where R is the length produced, as bl in Fig. 192, and tl_1 is the tie-line, $bl = bl_1$, and $\angle lbt_1 = \alpha$.

326. Detailed Example of a Chain Survey.—Fig. 193 illustrates an example of a larger chain survey made with a 100'-chain. The ground was walked over first of all, and a rough-sketch was made in the field-book showing nearly the positions of the roads, buildings hedges, etc. This gave a good idea as to the disposition of the necessary lines which would have to be measured.

Field-work.—In this case a base-line, AB , 2860' long, has been chained through the centre of the survey. Upon it, the Δ s AFH , HCB , HBE and ADH , have been constructed, the rest of the boundaries being taken in by the Δ s, CHF and DEH . In this particular case again, the line, EH , extends to F , forming another base-line which with the base, AB , takes the form of the letter X. The side, CH , common to the Δ s, FHC and CBH , has been ranged in line with HD which forms a side common to the Δ s, HED and DHA . This line, CD , picks up the boundaries of fields or *ails* and the hedge and ditch as shown. The tie-line, aH , of the Δ AHF has been ranged in line with Hb , the tie-line of the Δ HBE . This tie-line, ab , also takes up the boundaries of the fields as shown. The line, lm , is a tie of both the Δ s, ADH and DEH intersecting the common side, DH , in n , and also forms a subsidiary chain line to take up the cross boundaries of the fields. The subsidiary chain lines, Hc , eC , gf , and ga , act also as tie-lines to the Δ s as illustrated. The lines, hi and jk , are run for the purpose of taking up the corners of the buildings, the flower beds, and walks. The subsidiary Δ AFG , has been laid out to take up the roads and field boundaries. A proof-line, Ga , has been chained to check this Δ .

It will be the best plan, to measure first, all the sides of the principal Δ s, and to plot them. When the Δ s close in properly, the subsidiary lines for which points in the base and main lines, from which it is intended to run them, were marked at the time of measuring the main lines, should be laid down and measured. Unless the subsidiary station points are so marked, a great deal of unnecessary measurements will be entailed. Thus the main and

subsidiary lines are measured in succession, and their dimensions with the additional offset measurements enable the surveyor to plot

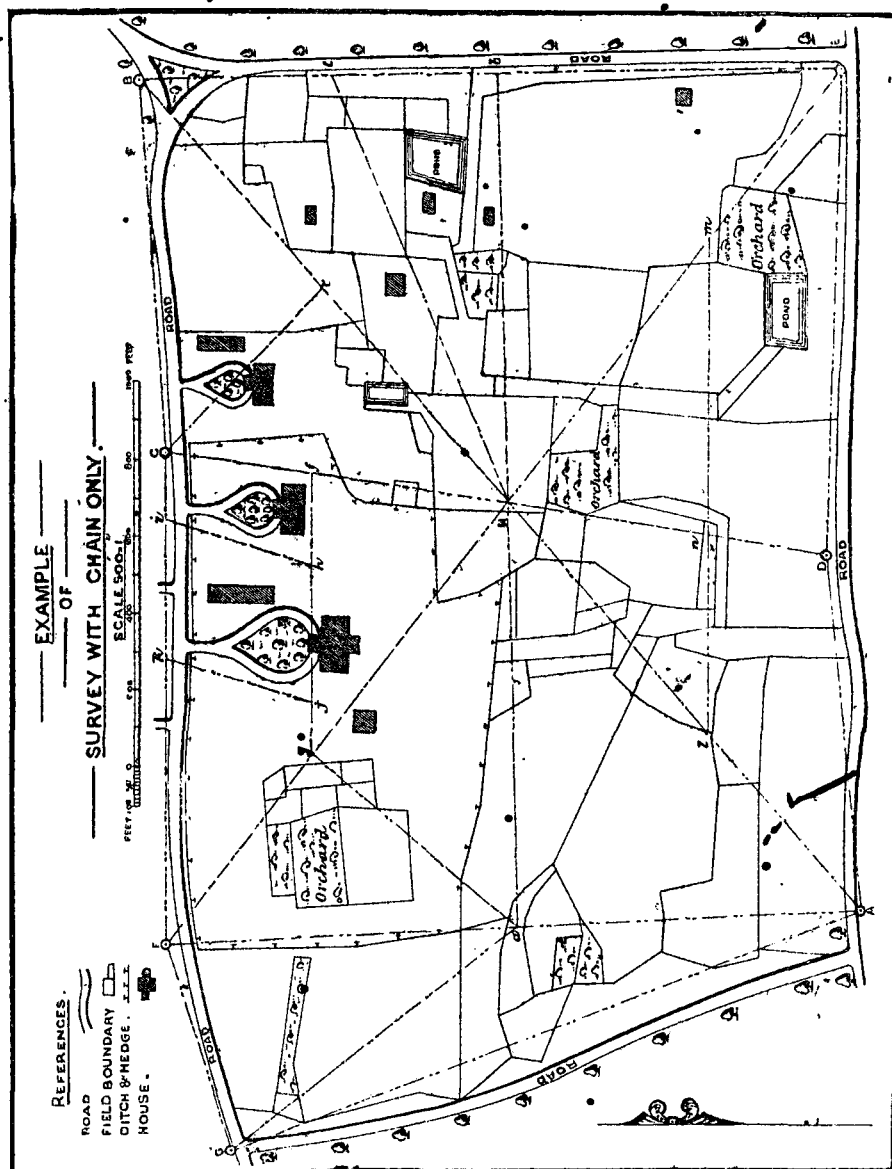


Fig. 193.—Detailed Example of a Chain Survey.

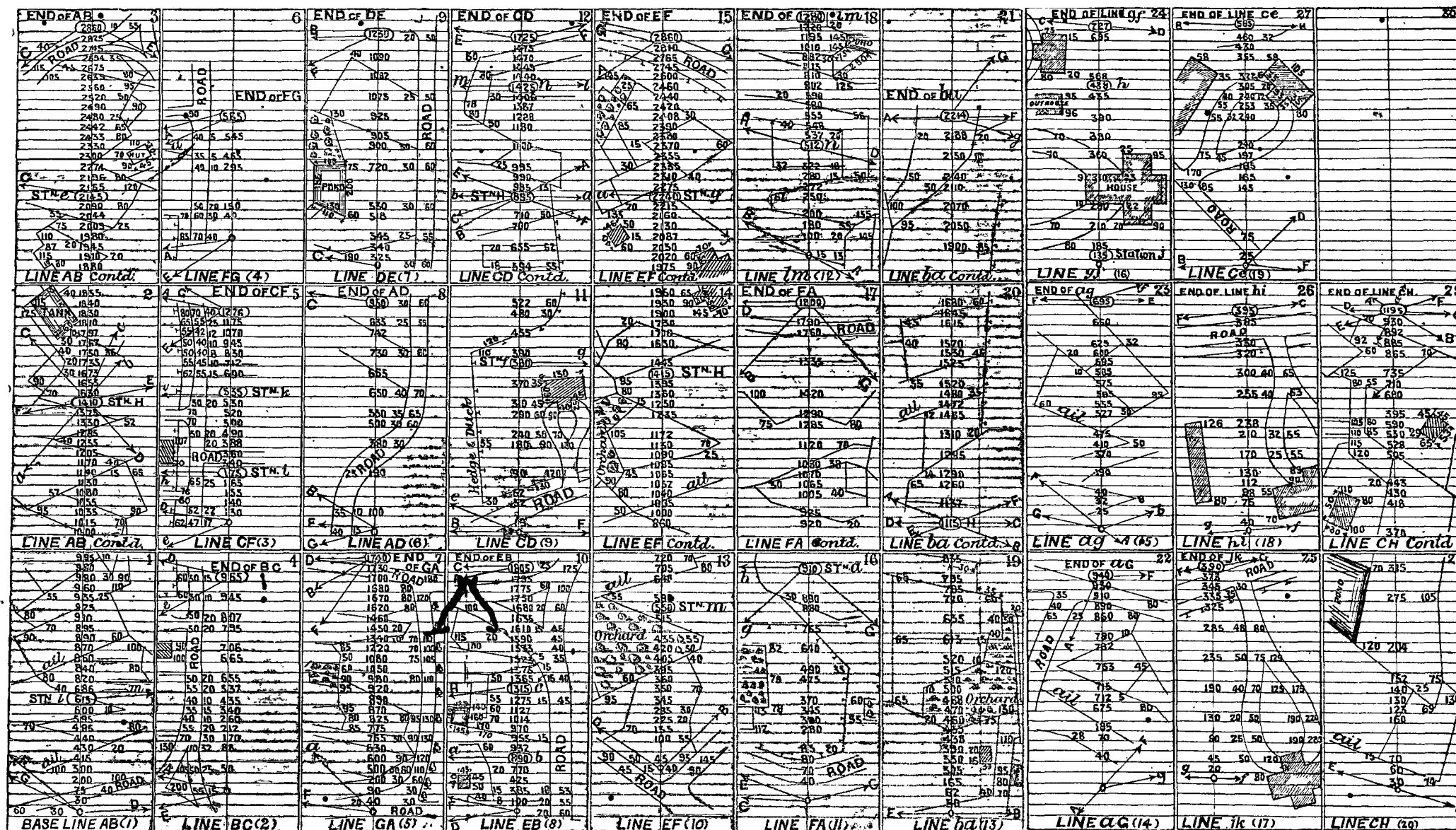
the boundaries, external and internal, and positions of houses, and to find out the area of the whole and of every part, by direct computation from the field-book.

Field-book.—The single-lined field-book No. 2 (Fig 194) gives the details of the Survey illustrated by Fig. 193, and it is recommended that the student should plot the survey from the field-book to a scale of 200 feet to 1 inch. It will be seen that a single red line to represent the chain line is used in the field-book No. 2 instead of 2 red lines to make a central column as adopted in field-book No. 1. (Fig. 179).

Referring to the field-book,* it will be gleaned that recording is begun at the foot of the page 1 with the words "Line *AB*." Then the beginning of the base-line is booked 0 (zero), while the chain line *AD* is marked to the right, and the chain lines *AF* and *AG* to the left, all diverging from the zero of the line *AB*. At zero, offsets of 30' and 60' to the left are taken to the road sides. At 30' and 75' the road is crossed; at 75' an offset of 40' is taken to the further side of the road on the right; again at 200' an offset of 100' is taken to the nearer side of the road to the right, and the road is denoted by the lines to the right and left with the word "Road" written within them.

At 300' again on the chain line an offset of 100' is taken to the *ail* to the left, which is crossed at 415' and is denoted by the lines to the right and left of 415'. At 430' the corner of the field to the right is offsetted to, the offset being 20' long. At 440' another *ail* is crossed as shown. At 495' offsets of 70' and 80' are taken to the *ails* to the left and right respectively. At 595' another *ail* is crossed as shown. At 600' another offset of 10' is taken to the corner of fields to the right. The subsidiary station, *l*, is left by means of a peg driven flush with the ground at 615', and is noted in the field-book as shown, by an ellipse drawn round the figures, 615, representing the distance of the station on the chain line, *AB*, from zero. Recording is continued till the first page of the field-book is filled up on booking the distance 995'; and then recording of the measurements on the line *AB* is continued at the foot of the 2nd page. The rest of the line, *AB*, is surveyed, and the results booked in the same manner until the station, *H*, is arrived at at 1410'. Here the diverging station lines, *HD*, *HE*, *Hb*, *Hc*, *HC*, *HF*, and *Ha*, are shown by arrows in the field-book. The measurements are booked similarly up to the station, *e*, at 2145', which is left on the ground by driving a peg, and is marked by an ellipse round the figures representing the distance on the base-line. The end of the base-line comes to 2860', and is marked "End of line *AB*," where also the lines, *CB* and *BE*, are shown converging to the end of the line *AB*. The remainder of the survey is conducted in the same way.

Plotting.—The plotting of the main Δ s, *AFH*, *HCB*, *HBE*, and *ADH*, on the base-line, *AB*, is done in the same way as explained for Fig. 187. It is manifest that, when these four Δ s are plotted, the lines, *CF* and *DE*, which make up the other two Δ s, act also



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Fig. 195.

(1) Village as surveyed. (2) Walled Village. (3) Ruined Village, as surveyed. (4) Place of Archaeological interest. (5) Scattered huts. (6) Huts temporarily occupied. (7) Deserted Village.

(8) Church. (9) Temple (9a) Pagoda. (10) Mosque. (11) Surveyed fort. (12) Conventional fort. (13) Watch-tower. (14) Battlefield (with date).

(15) Well, lined or in rock. (16) Unlined well. (17) Spring. (18) Burial ground. (19) Oil-well. (20) Mine-shaft. (21) Surveyed boundary pillar. (22) Unsurveyed boundary pillar.

(23) Karez (with depth of shaft in feet). (24) Disused Karez. (25) Pipe line. (26) Power line (27) Telegraph line. (28) Rifle Range.

(29) Lake or tank (showing limit of water). (30) Tank with embankment. (31) Tank with high embankment. (32) Lined tank. (33) Lined tank with high embankment. (34) Swamp or Marsh. (35) Salt-pans.

(36) Perennial stream. (37) Non-perennial stream. (38) Approximate course of stream. (39) Change of colour, indicating change from perennial to non-perennial water. (40) Canal with distributary and milestone. (41) Disused canal with distributary. (42) Canal generally containing no water.

(43) Navigation lock. (44) Lock or weir carrying road. (45) Lock or weir carrying foot-path. (46) Waterfall with height. (47) Rapids. (48) Aqueduct, or (if printed in black or red) Viaduct. (49) Boat or pontoon bridge.

MARINE SYMBOLS:—(50) Buoy. (51) Lightship. (52) Anchorage. (53) Lighthouse. (54) Tidal arrow. (55) Direction of flow of stream.

(56) Embankment. (57) High steep embankment. (58) Cutting. (59) Deep steep cutting. (60) Broken ground in hills. (61) Precipices. (62) Contours and form lines.

(63) Main metalled road, with milestone and bridge. (64) Main unmetalled road, with avenue of trees. (65) Cart-track. (66) Camel-track. (67) Mule-path, with pass. (68) Foot-path.

(69) Irish bridge. (70) Ford. (71) Ferry. (72) Roads in dry nala.

RAILWAYS:—(73) 5 ft. 6 ins. Gauge double line with milestone and station. (74) 5 ft. 6 ins. Gauge double line under construction. (75) 5 ft. 6 ins. Gauge single line. (76) 5 ft. 6 ins. Gauge single line under construction. (77) Other Gauges (gauge to be stated) double line. (78) Other Gauges double line under construction. (79) Other Gauges single line. (80) Other Gauges single line under construction. (81) Mineral line or Tramway.

(82) Level crossing. (83) Road over railway. (84) Road under railway. (85) Railway over railway. (86) Railway over river. (87) Bridge carrying road and railway. (88) Tunnel.

BOUNDARIES:—(89) External of India. (90) Province or State. (91) Province or State undelineated. (92) District. (93) Sub-division, Township, Taluka or Tahsil. (94) Village, with tri-junction pillar. (95) Pargana in U. P. (96) Reserved, protected or State forest.

HEIGHTS:—(97) Trigonometrical station. (98) Trigonometrical intersected point. (99) Approximate. (100) Great Trigonometrical Survey Bench Mark. (101) Canal Bench Mark. (102) Other Bench Mark. (103) Relative.

ABBREVIATIONS:—(104) Reserved Forest. (105) Protected Forest. (106) State Forest. (107) Camping ground (as surveyed). (108) Buddhist Kyaung. (109) Market or bazar (statute day). (110) Police Station. (111) Post Office. (112) Telegraph Office. (113) Combined Post and Telegraph Office. (114) Circuit House. (115) Dak Bungalow. (116) Inspection Bungalow. (117) Rest House.

TREES:—(118) Pine, fir, etc. (119) Date palm. (120) Palmyra. (121) Tamarisk. (122) Sal or Poplar. (123) Bamboo. (124) Casuarina. (125) Plantain. (126) Coconut. (127) Betel-nut. (128) Aloe or Cactus. (129) Other Trees.

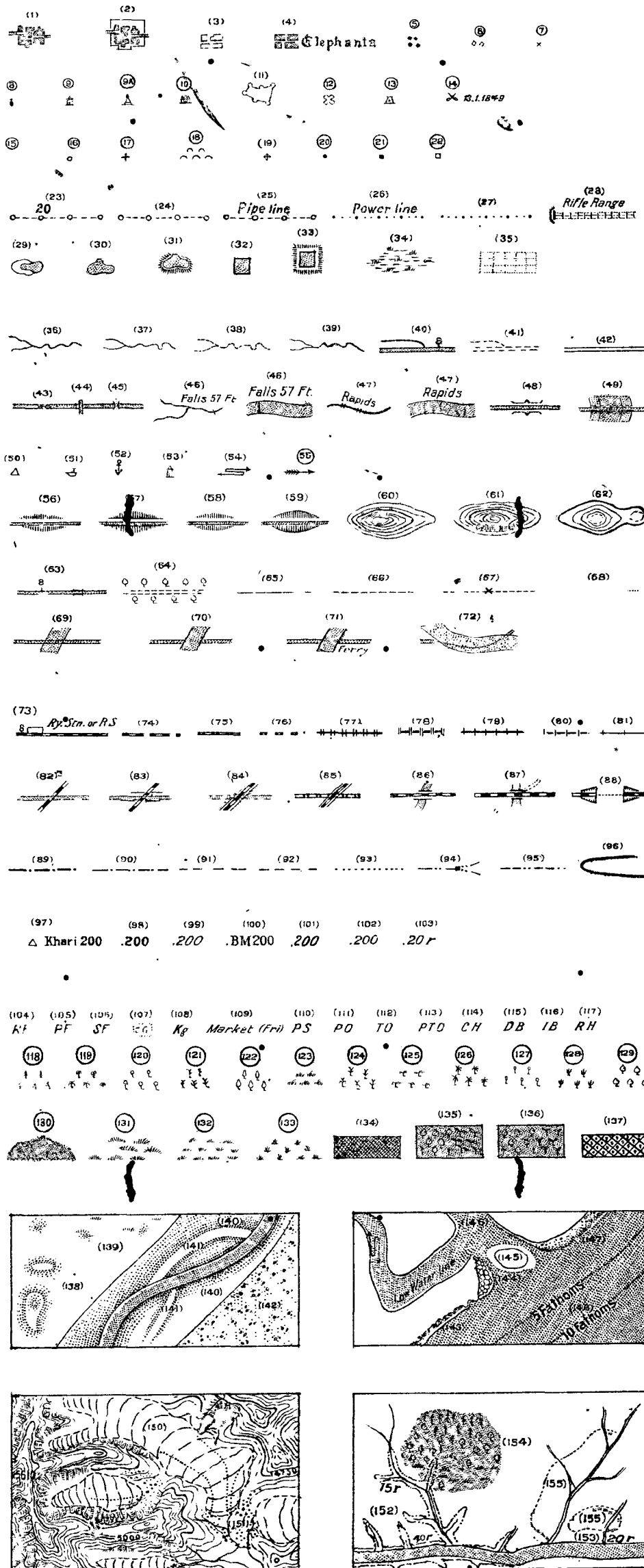
(130) Scattered trees and scrub. (131) High grass. (132) Low grass and pasture. (133) Cane brakes. (134) Betel or vine on trellis. (135) Orchard or garden. (136) Walled Orchard or garden. (137) Vegetable garden.

(138) Sand hills surveyed. (139) Sand hills, not surveyed in detail. (140) Sand in river bed. (141) Dry channel in river bed. (142) Stony waste.

FORESHORE SYMBOLS:—(143) Cliff. (144) Rocks. (145) Island. (146) Low water line. (147) Shingle and sand. (148) 5 & 10 Fathom lines.

(149) Rocky cliffs. (150) Glacier. (151) Moraine. (152) Nālas and ravines. (153) River with steep or high bank. (154) Wooded area. (155) Limit of cultivation.

*The symbols 56–59 apply also to embankments and cutting along railways.



NOTE. Dies are available for symbols which have their numbers underlined or enclosed within a circle. When the meaning of a symbol, which is not entered in the tables at the bottom of a map, is doubtful: then the description of the symbol should, if it can conveniently be done, be occasionally entered opposite the symbol on the map. This also refers to special symbols which are required in certain localities, and which are not entered on this sheet. When it is necessary to enter on a map an important symbol which can not be explained otherwise, then a special note should be entered at the bottom of the map. It is not desirable to use special symbols if it can be avoided by entering the description instead.

The colours are those used on one-inch and half-inch maps: for colours used on quarter-inch maps, see Topographical Hand-book, Chap. VI, Section III.

as tie-lines to these four Δ s, *i.e.*, the lengths, CF and DE , should scale the same as measured on the ground. For additional accuracy, each Δ in the survey has at least a tie-line so that any mistake in chaining may be located in the Δ in which it occurs.

Now commence with AB and mark on it the distances, where the offsets occur, and the field boundaries cross the base-line, in the central line of the field-book, at the same time setting off and marking the lengths of offsets by means of the same plotting scale and an offset-piece as were used for scaling the distances of the sides of the Δ s. So proceed with other lines and offsets till the plotting of all the Δ s and the offsets is completed. The line joining the ends of offsets gives the required boundary. After the offsets are plotted, the details, such as houses, tanks, roads, fences, etc., if there be any, which were offsetted to, are shown on the plan which is then fit for receiving the **conventional signs** adopted to show the topographical features of the ground, as shown in Fig. 195.

327. Care in Plotting.—In plotting a chain survey, great attention should be paid to the absolute flatness of the drawing board or table on which the paper is mounted, and to the perfect straightness of the straight-edge by which the chain lines are drawn. If the plan is to be mounted on cloth, the paper should be mounted before the plan is plotted; otherwise the mounting will alter its dimensions. Instead of straining the paper on a drawing board, it is better to pin it on the board. Each day's work should be plotted at site, or as soon as possible after the survey. The scale according to which the survey is plotted should at once be drawn on the plan as it will expand and contract along with the paper.

328. Learning Survey quickly.—For this purpose take any plan, and treat it as a piece of real ground; lay down and measure lines on it. Offsets from them to the objects are then represented, and a complete field-book is made from it. Then keeping aside the plan, plot the field-book on another sheet of paper and compare the results with the original. If there be any mistakes or omissions in the field-book, they can be then corrected. The student will thus learn the subject very easily, and be able to start with the actual survey. He is requested to draw out a field-book of the survey of Fig. 193 on a scale of 5 chains to the inch, and to enlarge it from his own field-book on the scale of 150' to one inch.

329. Where Chain Survey is recommended.—It must be borne in mind that this method of surveying is very slow but accurate, and is chiefly used for preparing plans on a large scale, such as maps of estates, or plots, not exceeding one square mile at a time, whose boundaries are to be carefully shown. When used for larger tracts, the portions thus surveyed should be fitted on to, reduced, and verified by the skeleton work or distant points of some larger survey, such as the Great Trigonometrical Survey. It is not suitable in a country which is not open and level. In an open country, when the ground to be surveyed is small in extent, a chain survey is quickly carried out by dividing up the country into Δ s,

measuring the sides of the Δ s, and offsetting from them to objects such as houses, roads, streams, etc., to be shown on the map of the country. In a hilly country chaining is tedious, and liable to error, and is, therefore, to be sparingly used; angular surveying is then preferable to chain surveying. In the survey of towns which can not be always divided into Δ s for chain survey, traversing with the compass, or with plane table and chain, or with the sextant, or theodolite, is preferable. The survey of a road, railway, canal or river is more rapidly done by traversing with an angular instrument and chain. But the great recommendation for a chain survey is that the instruments required to carry it out are very inexpensive—a chain, an offset-rod or a tape, a few flags and pegs, and some cord for making stays to the ranging-rods and tying them in bundles, and one or two spare straps being all that are absolutely necessary. But if to the chain, cross-staff, tape, offset-staff, and flags or ranging-rods, which form the equipment of a surveyor under elementary conditions, some instrument for measuring angles, such as the prismatic compass, a more advanced form of cross-staff, a box-sextant, or in large surveys a theodolite, be added, and if the optical-square be substituted for the cross-staff, the rapidity of the survey may be increased. In estate-surveys where acreage is wanted, the 66 feet or Gunter's chain should be used, and in other surveys where acreage is unimportant, the 100-feet or 50-feet chain should be used, any mileage required on any part of the plan being subsequently scaled off.

330. Clearing-up Ground after Survey.—All flags, ranging-rods, etc., should be removed, and the ground should be left as much as practicable in the same state as it was before the commencement of the survey.

CHAPTER V

SURVEYING WITH THE AID OF ANGULAR INSTRUMENTS.

Section 1. *Methods of Surveying with the Prismatic or Surveying Compass, Cross-Staff, Circumferentor.*

331. Instruments.—Detailed descriptions have already been given of the prismatic and surveyors' compasses and of the different forms of circumferentors and Mining dials in Chap. II, pp. 27—4r, and 115—125.

332. Accuracy of Compass Survey.—The compass surveys should be very limited in extent in which an error of 30 minutes, due to diurnal and annual magnetic changes, or to local attraction, shall not affect the visible accuracy of the survey when drawn to the required scale; the actual compass-bearings are then taken to 15 minutes, or more frequently to 20 minutes. For instance, if we take a measured length to be 1000 ft., and the liability to error in compass

bearing 15 minutes ; this will then, if represented by its sine at that distance, be equal to about 4.36 feet. If also the scale adopted renders such a distance* practically inappreciable, as it would be either with a scale of 500 feet to an inch or any smaller scale, lengths of this extent may be adopted (Art. 127, q.v.).

333. Methods of Compass Surveys.—This section will make the student familiar with the surveying by the compass before he is engaged with the principal surveying instrument, the theodolite; it being premised that this method of surveying will be more as an introduction than anything else to what is termed **Surveying by Traverse proper** which will be described hereafter. There are two methods of surveying with a compass; one is the method of **Traversing**, and the other, the method of **Triangulation**. *A series of right lines of known lengths, making known angles with each other, and connecting two points is called a Traverse.* When the two points coincide, it is called a **Closed Traverse**; and the lines when drawn on plan form one or more polygons, as in the case of the boundary of a plot of land, an island, or a lake. When the two points do not coincide, as in the case of a route or coast line, it is called an **Open or Unclosed Traverse**. If any line on the traverse intersected or coincided with any other line previously surveyed, the traverse would be partly closed and partly open. This system of surveying is dependent for its accuracy on an exact measurement of a system of main survey lines laid out in succession, and of the angles which each line makes with the last, with the primary line, with the true or magnetic north. For ordinary surveying and in laying out roads and railways when the absolute direction of the several lines is a matter of minor importance, the prismatic compasses (Figs. 15—27), and the circumferentors (Figs. 108—121) may be used, but much greater accuracy may be obtained by employing the theodolite to measure the angle which each line makes with the preceding one or with a meridian.

334. Method of Close Traversing with the Compass.—When survey details are only required along or near some line of route, however circuitous it may be, traversing is adopted in preference to triangulation. A compass traverse when adopted as a means of filling in details of land surveys to be plotted on a moderate scale between $\frac{1}{4800}$ th and $\frac{1}{10200}$ th should seldom exceed a mile in length, as the errors of observation in the bearings may accumulate to a great extent; and if the route consists of a large number of distances, the liability to error becomes greater; hence the need for closing on points obtained by some more accurate means, or for completing or closing the traverse by continuing it by some other return route back to the starting point, unless the lateral objects have given sufficient check on its accuracy. To illustrate the method of close traversing with the compass in finding the boundary of a field, ABCDE ... NA, lying within the roads, Fig. 196 is selected. Here, the lines AB, BC, CD, DE, etc., can be laid out with the compass in the following way:—First make a reconnaissance, and then plant pegs and flags at the station points, A, B, C, D, E, etc., all visible from each other and as near the

boundary of the field as possible. The stations should be as far apart as possible ; otherwise it would be very tedious to measure round every turn in it, and to observe its bearing. When selecting

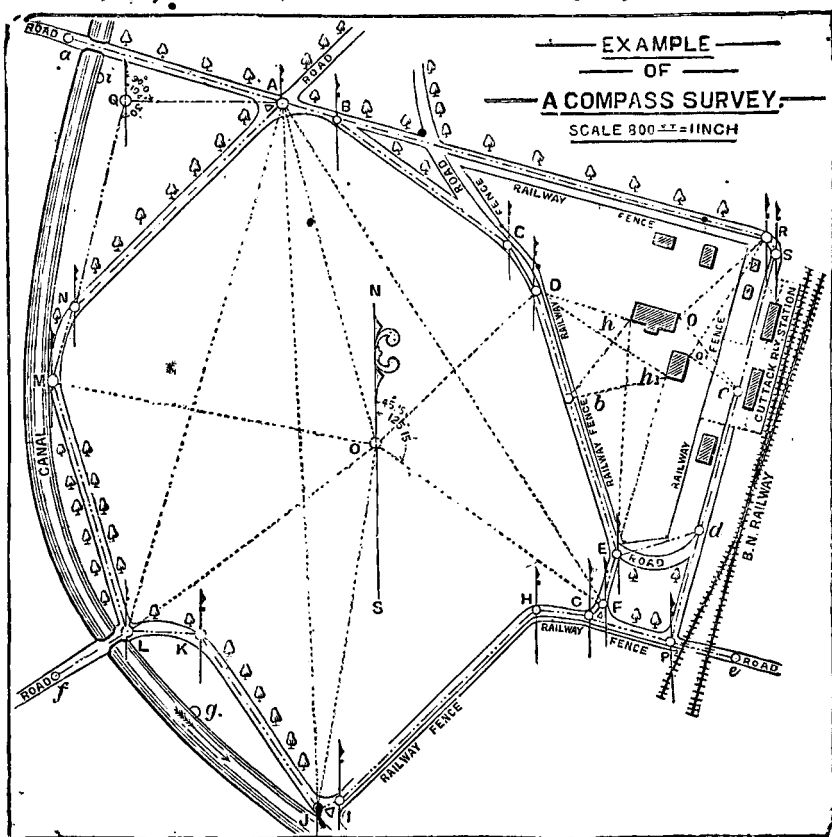


Fig. 196.—Method of Close Traversing with the Prismatic Compass.

the stations, the surveyor should bear in mind that no bend in the boundary should be more than a chain (100') distant from the nearest chain line. Having fixed the point, A, set up the compass there, take the magnetic bearings of the flags at *a*, B, F, J, L, and N, and then chain the length from A to B, taking offsets to the right and left of AB as in a chain survey. Having arrived at B, record the length, AB, in the field-book ; then set up the compass there, take the magnetic bearings of the flags at A and C, measure the length, BC, taking offsets to the right and left precisely as in a chain survey ; and so proceed with the other station lines till the whole circuit is completed. The back-bearings are taken to equalise minute errors of observation, to locate and remove them when important, and to detect magnetic deviation. Bearings must also be

taken to any conspicuous objects that are out of reach of an offset, such as the corners of the houses, h and h_1 , in the figure; readings from two stations are sufficient to fix them, but a third should be taken as a check. By this method of traversing, therefore, the formation of Δ s, and the measurement of subsidiary or tie-lines, as in a chain survey, are obviated. The bearing to a is taken in order to get in the bit of road up to the bridge, and to show the position of such road in connection with the lands surveyed, and also in case the survey has to be extended in that direction, as then the instrument would be planted at a , and the back-sight fixed on A .

335. Checks on Readings of Closed Compass Traverse.—It will be seen that at the starting point, A (Fig. 196), the *cross-bearings*, AF , AJ , and AL have been taken. The object of taking these bearings is to see, on arriving at F , for instance, and taking the back-bearing, FA , to the starting point A , if this bearing differs from the fore-bearing, AF , as taken at A , by 180° . Thus a check on the accuracy of the bearings is made on arriving at the point, F . Similarly at J the bearing JA checks the bearings up to that point. Thus any error in the bearings is discovered before finishing the whole traverse, and, if serious, is rectified before proceeding further. In fact the bearings FA , JA , and LA divide the whole traverse, as far as the bearings are concerned, into smaller closed traverses, $ABCDEFA$, $FAJIHGF$, $JALKJ$, $LANML$ (Fig. 196). The first two are six-sided and the last two are quadrilaterals. The internal angles of a six-sided figure should be equal to eight right angles, and that of a quadrilateral equal to four right angles. For the method of obtaining the interior angles from the bearings see p. 190. If, as is almost sure to be the case with this instrument, we get some few minutes, either over or under eight or four right angles, as the case may be, divide this difference amongst the bearings.

336. Checks on Plotting of Closed Compass Traverse.—A station, O , in a conspicuous position near the centre of the survey, and visible from as many stations of the circuit as possible, should be selected, and the various bearings, AO , DO , FO , JO , LO , MO (Fig. 196) taken to it. The object of taking these cross-bearings is to check the plotting, if done with a protractor, as it proceeds. For instance, the bearings of AO and DO , when plotted, give by their intersection the point O . When the work is plotted, as far as F , the bearing, FO , should pass through the same point O , and similarly at J , L , and M , the bearings, JO , LO , and MO , should all pass through the point O . In this way any serious mistake in the plotting is detected as the work goes on, and before the whole of the work is plotted. All the bearings should be plotted from the station, A , without once moving the protractor (see Fig. 200).

337. Proof of the Angular Work of a Closed Traverse.—If a traverse survey is undertaken with a compass, i.e., if from a starting point, A (Fig. 196), we measure successively several lines and their bearings till we return to the starting point, then the angular work of the polygon thus formed can be proved by the following:

"The sum of the inward angles of a polygon is equal to twice as many right angles as the figure has sides, less four right angles." Then from the following field-notes of the survey in Fig. 196, the angular work of the traverse is proved as under by the formula— $a =$ inward angle between two stations $= P - F \mp 180^\circ$; using the + sign, when $P - F < 180^\circ$, and the - sign when it is greater, where P is the observed bearing at a preceding, and F the observed bearing of a following station, the survey being executed by keeping the plot on the right-hand.

FIELD-NOTES OF THE COMPASS SURVEY IN FIG. 196.

From Station.	To Station.	Fore-Bearing.	Back-bearing.	Distances in feet.	REMARKS.
A	B	108° 30'	288° 30'	230	Fore-bearings. Back-bearings. AF = 147° 30'. FA = 327° 30' AJ = 177° 30'. JA = 357° 15' AL = 196° 15'. LA = 16° 15' AO = 164° 45'. OA = 344° 45' DO = 225° 15'. OD = 45° 15' FO = 305° 15'. OF = 125° 15' JO = 9° 15'. OJ = 189° 15' LO = 53° 15'. OL = 233° 15' MO = 101° 15'. OM = 281° 15'
B	C	126° 00'	306° 00'	870	
C	D	149° 45'	329° 45'	230	
D	E	162° 45'	342° 45'	1130	
E	F	192° 00'	12° 00'	230	
F	G	235° 15'	55° 15'	80	
G	H	278° 45'	98° 45'	230	
H	I	225° 45'	45° 45'	1130	
I	J	246° 30'	66° 30'	90	
J	K	326° 30'	146° 30'	860	
K	L	274° 15'	94° 15'	300	
L	M	343° 00'	163° 00'	1080	
M	N	18° 45'	198° 45'	330	
N	A	45° 15'	225° 15'	1210	

CALCULATION OF INWARD ANGLES.

Bearing of	Bearing of	Inward $\angle$	
AB 108° 30'	BC 126° 00' + 180° =	ABC	162° 30'
BC 126° 00'	CD 149° 45' + 180° =	" "	BCD 156° 15'
CD 149° 45'	DE 162° 45' + 180° =	" "	CDE 167° 00'
DE 162° 45'	EF 192° 00' + 180° =	" "	DEF 150° 45'
EF 192° 00'	FG 235° 15' + 180° =	" "	EFG 136° 45'
FG 235° 15'	GH 278° 45' + 180° =	" "	FGH 136° 30'
GH 278° 45'	GI 225° 45' + 180° =	" "	GHI 233° 00'
HI 225° 45'	IJ 246° 30' + 180° =	" "	HIJ 159° 15'
IJ 246° 30'	JK 326° 30' + 180° =	" "	IJK 100° 00'
JK 326° 30'	KL 274° 15' + 180° =	" "	JKL 232° 15'
KL 274° 15'	LM 343° 00' + 180° =	" "	KLM 111° 15'
LM 343° 00'	MN 18° 45' - 180° =	" "	LMN 144° 15'
MN 18° 45'	NA 45° 15' + 180° =	" "	MNA 153° 30'
NA 45° 15'	AB 108° 30' + 180° =	" "	NAB 116° 45'

$\therefore$ Sum of inward angles = 2160° .

Also by rule the sum of inward angles = $2 \times 90^\circ \times 14 - 360^\circ = 2160^\circ$;

$\therefore$ The angular work of the survey was correct.

338. Where the Land to be surveyed is covered with close detail, which is required to be plotted on a large scale, as in the case of a large village, or a small town, a Chain and Compass Survey is generally the most expeditious method by which it can be executed. The ground should be at first reconnoitred, and a hand-sketch made of it. This is to be done with a view to choosing the station points on the main-circuit. On the hand-sketch should then be marked the tie-lines, between points on the main-circuit, which will subdivide the area most conveniently, and will pass through the open ground. These subsidiary stations on the main-circuit are called tie-lines stations, and should be fixed while the boundary is being surveyed. When this is done, straight lines should be run along aligned flags from one main station to another, or to intermediate stations selected on the main-circuit. On these tie-lines also, stations should be selected to run minor tie-lines until the whole of the details is brought within reach of the ordinary length of offsets. The bearings of all main and minor tie-lines should be taken as checks on the main traverse lines. The offsets should then be taken as in a chain survey. (See pages on Cadastral Survey).

339. Method of Triangulation.—(a) When the whole of the details within an area have to be surveyed, and when, for some reason or other, it becomes impracticable to divide the area conveniently by tie-lines, it may be divided into triangles, if it can be done, as in a chain survey (see Fig. 193). In a triangulation, a base-line is to be measured as nearly through the centre of the survey as possible, and the station points are to be so selected as to form a system of Δ s constructed and developed on the base-line. The compass is then set up at each of the stations, and the bearings of all the lines meeting at each of the points are taken. The lines are also measured, as well as the offsets to noticeable objects situated on both sides of the lines as before. The object of taking bearings of all the lines is to find by the difference of bearings the angles of each of the Δ s in the system. The sum of the three angles of a Δ should amount to 180° . The readings of two angles of a Δ are absolutely necessary, and the third can be obtained by subtracting their sum from 180° . But for check, it would be better to observe all the three angles. This method should be adopted, wherever possible, as the Δ is the only safe figure to work with, and it is always determinate in shape and size, if the lengths of its sides are known.

(b) **Another Method of Triangulation** (Fig. 197) is to measure a base-line, AB, near the centre of the survey, from the ends of which most of the important points around may be visible. Bearings are then taken from the two ends of the base to all conspicuous points C, D, E, etc; and check-bearings are also taken from each of these points severally, both on all the rest that are visible, and on the ends of the base. Smaller details, such as corners of hedges, walls, and buildings, are interpolated by bearings in the same way from these points; occasionally, also, short offsets are taped to aid in completing the minor triangulation (See Art. 257, Prob. XXX). The extreme points of the first survey thus determined may act as points for extend-

ing the survey similarly in all other directions. A large extent of country may thus be surveyed in pieces, and the portions are after-

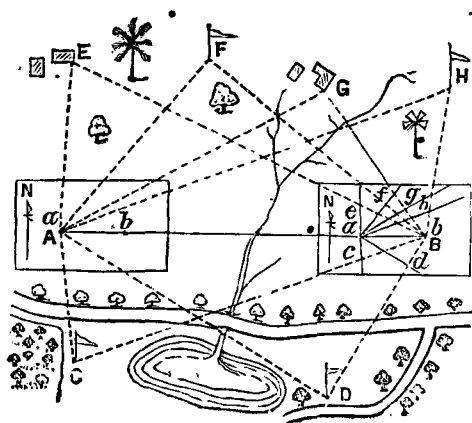


Fig. 197.—Compass Survey by Triangulation.

wards joined and fitted with care, provided that they are also fitted and adjusted to a series of well-defined and easily obtained points occurring in some Trigonometrical or good Topographical survey; but without these the junction of pieces of a compass survey can not be effected with accuracy, and the pieces are then of independent value.

340. Method of Open Traversing with the Compass.—

The compass may also be used for a road survey between two points situated apart. The bearings along the bends of the road are taken at the same time that the lengths are chained, care being taken also, to get bearings from several stations to one or two objects, likely to be visible parts of the survey. For the traverse of a road between stations, *B* and *R*, and from *R* to *G* (Fig. 196), the intermediate stations *R*, *S*, and *P*, each visible from one another, are selected and marked by pegs after making a reconnaissance of the locality. These intermediate stations should be as few in number as possible, and should be selected only to avoid intervening obstructions, so that there should be the least number of linear and angular measurements. After ranging out the line and setting up the compass at *B*, the fore-bearing of *R*, where a flag is planted, is taken from *B*, and the length, *BR*, is measured; the compass is then removed to *R*, and the fore-bearing of *S* taken; the back-bearing of *B* is also observed to test the correctness of the observation from *B*, and the distance *RS* measured. The same process is repeated till *G* is reached, whence only the back-bearing of *P* is taken. Offsets to conspicuous objects, on both sides of the line are taken in the same way as in surveying with the chain only. Bearings to conspicuous objects, such as *o*, *o*₁, may also be taken from the several stations from which they are visible as checks.

341. The Field-book for a chain and compass survey is kept as in the chain survey, the only additions being the bearings. The first of these is written in the central column above the station No. 1, and afterwards to the right or left of the ruled column, according as the new line, of which the bearing is observed, is to the right or left of the last chained line (Fig. 198). The back-bearings are entered in the

margin at the end of the line. Sometimes the fore-bearings are written above the station letters, and the bearings to conspicuous objects, to the right or left of the ruled column, according as they are taken to the right or left of the chain line.

342. Specimen of Plan and Field-book—Fig. 196 and

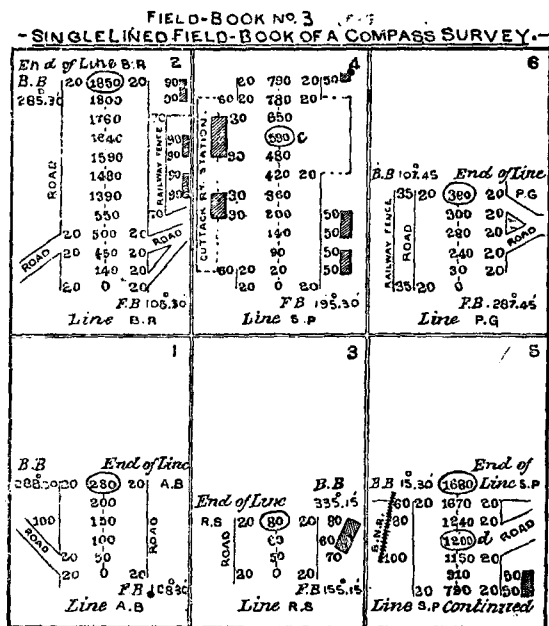


Fig. 198.—Field-book No. 3 for a Prismatic Compass Survey.

Field-book No. 3 (Fig. 198) represent a specimen of plan and field-book respectively of a compass traverse survey of a large field enclosed within roads. In the field-book, both the angular and lineal measurements of the lines from A to R and then on to G are entered.

343. Plotting the Compass Survey.—Having explained the methods of measurement of both linear and angular distances we have now to consider the means to be used for laying them down on paper. This is called *Plotting the Survey*. There are 6 methods of plotting a compass traverse survey the field-notes on p. 190 being used.

344. First Method.—*By Parallel Meridians*.—With a view to plot the survey from the field-notes, first fix a convenient point on the paper for the starting point, A, so that the finished plan of the survey may be within the paper, and as nearly as possible in the middle of it. For this purpose, a plot of the outline to a small scale should first be made, or a rough-sketch made on the ground will often suffice. Now draw a line through it to represent the magnetic meridian as AN_1 . Then place the protractor (either

semicircular or rectangular) to the right of this line (Fig. 199), the edge coinciding with it and the centre with the point A ; now mark the required number of degrees (bearing of AB) with a pointed

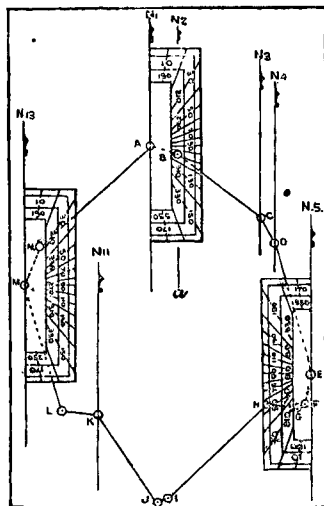


Fig. 199.—Plotting Compass Survey by Rectangular Protractor.

drawn by means of a T-square, the north and south lines on the paper being made parallel to one of the sides of the drawing-board. After the chain lines are laid down and found to be correctly plotted, the offsets can be plotted in the same way as in a chain survey to determine the roads, etc.

The plotting would be more accurate, if the line joining the 90° graduation with the centre of protractor were laid along the meridian line instead of putting the plain edge of the protractor along it. The surveyor should then bear in mind in which quadrant, the bearing is. If the bearing is in the 1st or 4th quadrant, the protractor should be adjusted north of the station with the plain edge downwards, and when it is in the 2nd or 3rd quadrant, the protractor should be set to the south of the station with the plain edge upwards.

More accurate results are obtained with a circular protractor. With this instrument, not only the bearing itself but the back-bearing on the other side also should be dotted off. Then, if the line joining the fore and back-bearing points passes through the centre previously marked on the meridian line, it is evident that the bearing is correctly plotted.

345. Convergence of Meridians.—In Fig. 199 lines are parallel to one another, but strictly speaking they are not parallel. The terrestrial meridians converge to the poles of the earth, and the magnetic meridians to the magnetic poles, but their convergency

is so small that the error in taking the meridians parallel is of no practical importance. For example, in latitude 21° the convergency of two meridians is 20 seconds for a departure of one mile.

346. The Second Method consists in drawing only one meridian line through the first station, *A*, which is so fixed that the survey may be contained within the paper, and as nearly in the middle as possible; in plotting the fore-bearing and the length of the first line *AB*, on the scale chosen; in plotting the inward $\angle ABC$ deduced from the bearings of *AB* and *BC* and the length *BC* from the same scale; and in plotting the $\angle BCD$ deduced from the bearings of *BC* and *CD* and the length *CD*; and so on.

347. The Third Method (Fig. 200) consists in marking off

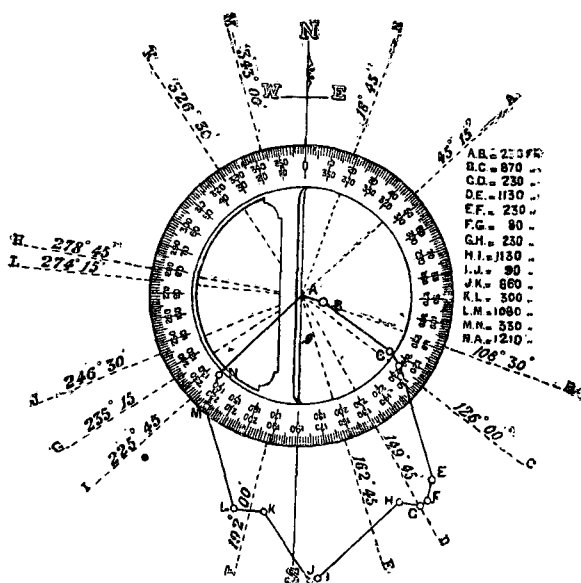


Fig. 200.—Plotting Compass Survey by Plotting a number of Bearings from the same point with a Circular Protractor.

in rotation the bearings of all the lines *AB*, *BC*, *CD*, *DE*...*NA*, at the point *A* with reference to the meridian through it with a circular card or talc protractor, fixing the point *B* by marking the distance *AB* on the scale chosen; and in drawing *BC* parallel to *AC* by means of a rolling parallel ruler, and marking its distance; and similarly for other stations. Let the bearings of the lines, *AB*, *BC*, *CD*, *DE*, etc, be represented by the directions of *AB*, *AC*, *AD*, *AE*, etc., respectively. On *AB* scale off *AB* and thus fix the point *B*; draw *BC* parallel to *AC* through *B*, scale off *BC*; and mark the point *C*. Next draw *CD* parallel to *AD*, and mark *D*. Then draw *DE* parallel to *AE*, and mark *E*. Similarly draw the other lines to complete the plot. It will be a better plan to figure on the paper by the

edge of the protractor the bearings and not the number or letter of the station, as is done, to avoid confusion. The parallel ruler should only be used on a perfectly flat drawing-board.

348. The Fourth Method.—

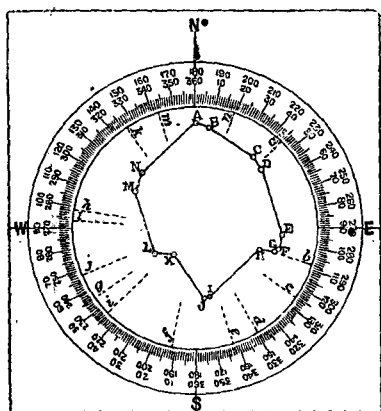


Fig. 201.—Plotting Compass Survey with a Cardboard Protractor.

The most rapid method of plotting a survey is by means of a cardboard protractor, as with this instrument there is no necessity of figuring the various bearings on the paper. It consists of a large square sheet of cardboard with a circle, 12" or 15" in diameter, cut out of it (Fig. 201). The divisions are made to read inwards of the circumference instead of outwards, as in other protractors, for the purpose of carrying on the protraction within it. The circle is divided to quarter degrees and numbered in two rows, like numbers being opposite to one another, *i.e.*, the zero of the second row begins at the 180° of the first.

To use this protractor, it is laid down with its 0 or 360° and 180° divisions coinciding with the meridian line through the point *A*, and fixed in this position with leaden weights or drawing pins to prevent its shifting. The bearing of the line, *AB*, is $108^\circ 30'$. Now take a parallel ruler and set its edge to $108^\circ 30'$ and $288^\circ 30'$ on the protractor, roll it until its edge passes through *A*, draw a line by it and then scale on it the length *AB*. The bearing of the line *BC* being 126° , set the edge of the ruler to 126° and 306° on the protractor, and roll it until its edge passes through *B*, and draw the line *BC*. Similarly, the rest of the bearings may be plotted without shifting the protractor. When the extent permitted by the space cut out in the protractor has been plotted, remove the protractor, draw a new meridian parallel to the first, and again place the protractor with its 360° and 180° divisions on this line and proceed as before. By this method much time is saved, and the plan is kept neat and clean and not covered with pencil lines. The results are very accurate on account of the large diameter of the protractor.

349. The Fifth Method (Fig. 202) consists in plotting each of the points *B, C, D, E, F, ... N*, by its rectangular co-ordinates calculated with reference to the meridian and its perpendicular through the point *A*. **Rectangular Co-ordinates** are a pair of straight lines locating any point in a plane measuring its shortest distance from two fiducial lines at right-angles to one another. This is called plotting by **Gale's Traverse**. Let *AB, BC, CD, DE, EF, ... NA* be the traverse lines, and *NS* the fiducial meridian line: Draw *AT* perpendicular to the meridian *NS*, another

fiducial line; also draw the lines Bb , Cc , Dd , Ee , etc., parallel to NS , and Bb_1 , Cc_1 , Dd_1 , Ee_1 etc., parallel to AT . Now, if the

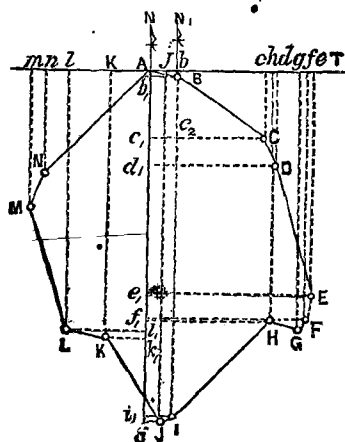


Fig. 202.—Plotting Compass Survey by Rectangular Co-ordinates. $BC \sin.CBc_2$, and $Bc_2 = BC \cos.CBc_2$. Therefore the co-ordinates of C , Ac and c_1C , are known. Therefore the point, C , can be plotted on paper independently of the point B . Similarly for other points, D , E ,... N .

350. Latitudes and Departures.—The distance that one end of a line is due north or south of the other end, is called the difference of latitude of the ends of the line, or briefly, the latitude of the line, or its *northing*, or *southing*. Similarly, the distance which one end of a line is due east or west of the other end is called the difference of longitude of the ends of the line, or the departure of the line, or its *easting*, or *westing*. “**Working a Traverse**” is the reduction of the angular survey lines, AB , BC , CD , etc., to rectangular co-ordinates of latitude and departure. The distances, Bb , Cc , Dd , Ee , etc., and their equals Ab_1 , Ac_1 , Ad_1 , Ae_1 , etc., are known in Gale’s Traverse System as **differences of latitudes** or **Northings** and **Southings** of the point A on the meridian NS (in this case all Southings), so called because they are measured in North or South directions from A ; and the distances Ab , Ac , Ad , Ae , etc., as **departures**, or **longitudes**, or **Eastings** and **Westings**, because they are measured in East and West directions. Gale’s Traverse System, therefore is a method of protracting by rectangular co-ordinates, and is applicable to any mode of angular surveying, such as Route, Railway, or Road Surveys, and the like, where every station is fixed by the distances on the meridian and the perpendicular. This method is chiefly used in surveying mines and also may be applied to surveys of towns. The computation form is a record of the position of each station independently of the plan.

351. When a Traverse closes the Following Three Con-

TRAVERSE COMPUTATION FORM.

REDUCTION OF A CLOSED COMPASS TRAVERSE SURVEY IN FIG. 196
TO A MERIDIAN PASSING THROUGH A IN FIG. 202.

1	2	3		4	5	a	6	a	7	a	8	a	9	10			
Stations.	Observed Magnetic Bearings.	Reduced Magnetic Bearings.		Distances measured in feet.	CONSECUTIVE CO-ORDINATES.								INDEPENDENT CO-ORDINATES.				
					Latitude = Col. 4 × Cos. 3.				Departure = Col. 4 × Sin. 3.								
					N.	Cor. -	S.	Cor. +	E.	Cor. -	W.	Cor. +	N. or S.	E. or W.			
A	108° 30'	S	71° 30'	E	230	..	..	72-98	41	218-11	46	..	..	73-39	S	217-65	E
B	126° 00'	S	54° 00'	E	870	..	..	511-37	1-56	703-84	1-73	..	..	586-32	S	919-76	E
C	149° 45'	S	30° 15'	E	230	..	..	198-68	41	115-87	46	..	..	785-41	S	1035-17	E
D	162° 45'	S	17° 15'	E	1130	..	..	1079-17	2-03	335-09	2-24	..	..	1866-61	S	1368-02	E
E	102° 00'	S	12° 00'	W	230	..	..	224-97	41	..	..	47-82	46	2091-99	S	1319-74	E
F	235° 15'	S	55° 15'	W	80	..	..	45-60	15	..	..	65-73	16	2137-74	S	1253-85	E
G	278° 45'	N	81° 15'	W	230	34-99	41	..	..	..	..	227-32	46	2103-16	S	1026-07	E
H	235° 45'	S	45° 45'	W	1130	..	..	788-50	2-03	..	..	809-42	2-24	2893-69	S	214-41	E
I	246° 30'	S	66° 30'	W	90	..	..	35-89	16	..	..	82-54	18	2929-74	S	131-69	E
J	326° 30'	N	33° 30'	W	860	717-14	1-54	..	..	..	..	474-66	1-71	2214-14	S	344-68	W
K	274° 15'	N	35° 45'	W	300	22-23	54	..	..	..	..	299-18	59	2192-45	S	644-45	W
L	343° 00'	N	17° 00'	W	1080	1032-81	1-94	..	..	..	..	315-76	2-14	1161-58	S	962-35	W
M	18° 45'	N	18° 45'	E	330	312-49	60	..	..	106-08	65	..	..	849-69	S	855-92	W
N	45° 15'	N	45° 15'	E	1210	851-86	2-17	..	..	859-32	2-40	..	..	00-00	..	00-00	..
Summation of Traverse— .. 8000					2971-52	-	2957-16	+	2338-81	-	2322-43	+	.				
Latitude and departure of closing error—					2957-16	7-16			2322-43	7-94							
					14-36	14-36			15-88	15-88							

ditions must be fulfilled :—*1st.* That all the interior deduced or observed angles, together with four right angles, must be equal to twice as many right angles as the figure has sides. *2nd.* That the Northings, or distances travelled North, must be equal to the Southings, or distances returned South. *3rd.* That the Eastings, or distances travelled East, must be equal to the Westings, or distances returned West. The first condition is proved in Euc. I. 32, Cor. I, and the correctness of the other conditions is self-evident for in making his tour the surveyor has to go the same distance north as he has to return south, and the same distance east as he has to return west.

352. Traverse Computation Form.—The closed traverse, (Fig. 196), is reduced to a single meridian in the form on p. 198. (1) The first column shows the names of the stations. (2) The column 2 gives the bearings observed in the field. (3) The column 3 gives the reduced bearings. To obtain these reduced values, let F be the observed bearing of a following station, and R the reduced bearing in any quadrant is thus obtained. When F lies between 0° and 90° , $R = F$ and lies *N.E.*; when F lies between 90° and 180° , $R = 180^\circ - F$ and lies *S.E.*; when F lies between 180° and 270° , $R = F - 180^\circ$ and lies *S.W.*; and when F lies between 270° and 360° , $R = 360^\circ - F$, and lies *N.W.* (4) Column 4 gives distances measured in the field. (5-8) The separate co-ordinates (latitudes and departures) in columns 5-8 are obtained thus :

Each latitude along the meridian = distance $\times$ Cos. R .

„ departure $\perp$ to „ „ = distance $\times$ Sin. R .

The computation of latitudes and departures may be facilitated by the use of Tables of Natural Sines and Cosines thus :

The latitude of AB = Nat. Cos. $71^\circ 30' \times 230'$
 $= 3173047 \times 230' = 72980'$
 „ departure „ „ = Nat. Sin. $71^\circ 30' \times 230'$
 $= 9483237 \times 230' = 218111'$.

The tedious multiplication by Natural Sines and Cosines may be avoided by using logarithms : for which see the Section on Gale's Traverse System under Surveying with the Theodolite.

353. Adjustment of Error of Closure—When the latitudes and departures of a closed traverse are calculated, the algebraic sum of the latitudes should be zero, and the algebraic sum of the departures should also be zero. Owing to unavoidable inaccuracy in chaining, the sum of the latitudes and departures will seldom be equal to zero and have to be adjusted. From the above calculations of the latitudes and departures, it will be seen that the sum of the northings is 1436 feet in excess of the southings, and the sum of the eastings is 1588 feet in excess of the westings, or in other words the error at closing is 1436 feet to the north and 1588 feet to the east of the starting point, A .

The latitudes and departures must now be corrected before plotting, so that their algebraic sums shall be equal to zero. This is done by distributing the error among the lines in proportion to their

length, the *balancing* being effected by the following proportion :—*As the sum of all the lengths is to each particular length, so is the total error, in latitude (or departure) to the correction of the corresponding latitude (or departure).* The corrections, as shown in columns, *a, a; a, a*, have been made. It may often be made by ascertaining the error per chain, without making the exact proportion.

354. The Error of closure is the length of the line joining the first and last points of the survey of a closed polygon (as plotted or computed from the field-notes). Being the hypotenuse of a right-angled Δ , of which the errors in latitude and in departure are the other sides, the length of this line is equal to the square root of the sum of the squares of those two errors. This divided by the whole perimeter gives the ratio of the error of closure. In ordinary surface-surveys the closing error should not exceed 1 in 300, or 27 links per mile. In this case, it is $21.40/8000 = 1/374$ nearly. Therefore the survey was within the allowable limit of error.

355. Independent Co-ordinates :—The separate co-ordinates after correction, are added up or subtracted in accordance with their directions being similar or different, from the starting point up to the point whose independent co-ordinates are required; and when the survey is a closed traverse round back to the starting point again where the final verification is shown by proving its co-ordinates through results of summation to be equal to zero as shown in cols. 9 and 10 in form p. 198.

356. Plotting by Rectangular Co-ordinates.—The independent co-ordinates in columns 9 and 10 are used in

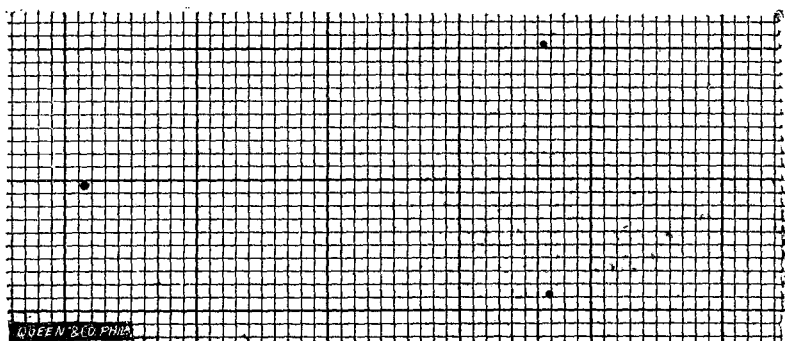


Fig. 203.—Cross-Section Paper.

plotting the survey :—Draw the meridian line, *NS* (Fig. 202), and from the starting point, *A*, fixed on it, draw a line, *AT*, perpendicular to it. Along the line, *AT*, set off with a plotting scale the distances, *Ab* = 217'65, *Ac* = 919'76, *Ad* = 1035'17, and so on, until all the independent departures are laid off. Through the points *b, c, d*, etc., draw lines parallel to the meridian, and on these lines set off the respective latitudes, *bB* = 73'39, *cC* = 586'32, *dD* =

785'51', etc. The above is the best rapid method of plotting the survey. The work can be facilitated by the use of **cross-section paper** (Fig. 203) which is divided into sections, $\frac{1}{10}$ of an inch square, heavy lines horizontal and vertical being ruled at every inch. With such paper, the use of an auxiliary scale is unnecessary.

357. Details of Traverse Work.—The above forms the skeleton of the traverse work; the filling in may be done by distances and offsets measured, or with occasional compass-bearings in the same way as by oblique offsets, or by means of the plane table.

358. Co-ordinate Protractor or Trigonometèr may be used on which the co-ordinates are at once obtained graphically. It

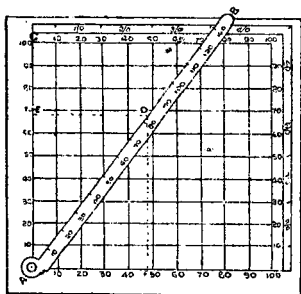


Fig. 204.—Co-ordinate Protractor or Trigonometèr.

consists of a metal or bristol-board plate, 15" square (Fig. 204), divided into 100 equal squares by horizontal and vertical lines. An arm, A B, turns upon a centre A which forms the zero of the graduations of both the protractor at one corner A, and that of the arm, A B. The arm is graduated similar to the plate to read distances from the centre outwards. On two sides angular graduations, to read minutes of arc by a vernier, are marked on the plate. By moving the arm to the given angle, the latitude is at once read off on the vertical scale, and the departure on the horizontal scale, for the given length as read on the arm. If for example, BAC is the given angle, and AD the given distance, DE and DF are the co-ordinates. The readings are correct to within 0.1 per cent. The instrument is also useful for finding the horizontal and vertical lengths of inclined chain lines.

359. The Sixth Method —Plotting by means of Chords.—A traverse may be plotted from one meridian, without a protractor, by means of a Table of Chords or a Table of Sines and a rolling parallel-ruler, say 2' long. As the chord of an angle is twice the sine of half the angle to a radius of unity, an easy means is afforded of protracting to any radius required. The magnetic bearings must first be reduced as shown before. Now take the chords corresponding to given bearings from table of chords and tabulate as under:—

Having tabulated the chords of the bearings, describe a circle with a radius of 1000 units (say 10 inches). In this way it will not be necessary to multiply the chord by the radius; the decimal point merely having to be shifted 3 places to the right. Angles can be plotted by this method as easily as with a 20-inch protractor. To plot the work, the circle is drawn in a convenient part of the paper, and the meridian line drawn through it, with the north and south ends marked on it. If the bearing is S.E. the chord is measured off from the south point, eastwards. Thus for a bearing of S. $71^{\circ} 30'$ E. (as in Fig. 196), the point at which an arc with a radius of

1168.5 units and with the south point on the circle as centre intersects the circumference of the circle is dotted with a pencil, and the

Stations.	Observed Magnetic Bearings.	Reduced Magnetic Bearings.			Chords. $r=1000$.	Distances.	REMARKS.
A	108° 30'	S	71° 30'	E	1168.5	230	
B	126° 00'	S	54° 00'	E	908.0	870	
C	149° 45'	S	30° 15'	E	521.8	230	
D	162° 45'	S	17° 15'	E	299.9	1130	
E	192° 00'	S	12° 00'	W	209.1	230	
F	235° 15'	S	55° 15'	W	927.3	80	
G	278° 45'	N	81° 15'	W	1302.2	230	
H	225° 45'	S	45° 45'	W	777.4	1130	
I	246° 30'	S	66° 30'	W	1096.6	90	
J	326° 30'	N	33° 30'	W	593.1	860	
K	274° 15'	N	85° 45'	W	1360.8	300	
L	343° 00'	N	17° 00'	W	295.6	1080	
M	18° 45'	N	18° 45'	E	325.7	330	
N	45° 15'	N	45° 15'	E	769.3	1210	

letter of the station marked. The north and south points on the circle are the zero points to measure from, and from these points the chords are measured off to the east or west as the case may be. The rest of the process is as done in *method 3*.

360. To Lay with the Scale of Chords as engraved on the rectangular protractor a given angle or bearing, say 80°, proceed as follows: With the given point where the bearing is to be plotted as centre, and radius equal to the distance from 0 to 60° on the scale describe an arc cutting the meridian line. From this intersection as centre, and the radius equal to the distance from 0 to 80° describe another arc cutting the former arc. The angle between the joining line from the given point to this second intersection of the arcs and the meridian is the bearing required. When the bearing is greater than 90° draw a East-West line through the given point and plot the excess angle over 90° from this line, and so on.

361. Plotting the Open Compass Traverse Survey.—The plotting is done exactly in the same way as the Closed Compass Traverse Survey described above.

362. Comparison of the Methods of Plotting.—Of the 6 methods described above, *the fifth* is by far the best, for a protractor is not required; and the co-ordinates which are to be calculated beforehand, can be easily plotted by the aid of a T-square and a set square, or a pair of set-squares, and a plotting scale. Each point of the traverse is independent of the other, and if in plotting one point, a mistake is made, it is not carried to the other points, as the position of each point depends on its own co-ordinates, with regard

to the meridian passing through the initial point. Plotting the polygon or other figure on paper with protractor and scale is an untrustworthy method. Hence when the survey is of any considerable size, plotting by co-ordinates is preferable, and can be done more accurately than with *any* protractor. The calculated co-ordinates are of great value for testing the accuracy of a closed traverse. Obviously, when a survey makes a complete circuit returning to the starting point, the surveyor has gone exactly the same distance north as he has gone south, and as far to the east as to the west. If his survey is correct the sum of the northings should be equal to that of the southings, and the sum of the eastings equal to the westings. *The sixth method* is the next best. No protractor is required, and a traverse may be plotted from one meridian by means of a Table of Chords, or a Table of Sines, and a 2-foot rolling parallel-ruler. In the *third method* the directions of the lines are plotted at one set-up of the protractor with reference to one meridian. This method avoids the possible error in replacing the protractor each time. The protractor, when placed correctly on the plan, is really the same as the graduated circle of the compass, and the lines marked off are the various positions of the sights of the compass. *The fourth method* saves the trouble of shifting the protractor at every bearing and also ensures greater accuracy in plotting, as a large number of bearings being laid down from one meridian, a trifling error in the direction of one does not affect the next; the accuracy of the plot depends much upon using a rule that moves actually parallel. This is the most rapid and accurate method when plotting a traverse of short lines with varying bearings. The plan is kept clean and becomes accurate on account of the large diameter of the protractor. Cardboard protractors are generally accurate, and are little liable to distortion through changes of temperature and damp; but it will be well to check a new protractor before using it. In the *first method* parallel meridians have to be drawn through all the points. The protraction of the angles, and the setting off of the distances, as well as the plotting of the meridians, are liable to error which should be obviated by plotting the meridian and other lines carefully. In the *second method* the inconvenience is that the protractor has to be used as many times as there are angles, and also there is a liability of a cumulative error in protracting the angles similar to the first method.

363. Care to be taken in Plotting.—Surveys should not be plotted on paper mounted on a board, but the sheets should, previously to use, be damped, mounted several together on a board, cut off before they are quite dry, and stacked together for some weeks in order that they may become flat and take up the surrounding hygrometrical conditions. The surveyors' semicircular protractor should be about 12" or 15" in diameter and graduated to 10 minutes. His straight-edge and triangle should be made of German silver by a skilful instrument-maker. A fine needle should be used for pricking off distances and angles; it must be held vertically; and the eye of the draftsman must be directly over it. The lead pencil should be hard (Faber's No. 4 is good for protracting), and must be kept to a

sharp point by rubbing on a fine file, after using a knife for paring off the wood. The scale should be at least as long as the longest side of the plot, and should be drawn at the edge of the same paper as the plot is drawn on. This will obviate to a considerable extent, errors arising from contraction and expansion. Unfortunately, a sheet of paper does not contract and expand in the same proportion lengthwise and crosswise, thus preventing the paper scale from being a perfect corrective. For ordinary small surveys errors from this source may be neglected. For such plots as may be protracted, divided, and computed within a time too short to admit of appreciable change, the ordinary scales of wood or ivory may be used; but satisfactory accuracy cannot be obtained with them on plots requiring several days, if the air be subject to considerable variations of temperature.

364. Rapid Method of Surveying with the Compass :—

When great speed is necessary, instead of taking magnetic bearings at all the points A, B, C, D, E , etc., magnetic bearings at B of both BA and BC , at D of both DC and DE , at F of both FE and FG , etc., (Fig. 196) may be taken, but this method will not suit the way in which the field-book No. 3 is written, and will involve the trouble of adding or subtracting 180° from every second reading before the work can be plotted. Referring to Fig. 199, the bearing of $BC = \angle N_2 BC < 180^\circ$, and of $BA = \angle N_2 BA > 180^\circ$. In order to find the bearing of AB which is $\angle N_1 AB$, 180° is to be deducted from the bearing of BA which is $(\angle N_2 Ba + \angle aBA) = (180^\circ + \angle aBA)$ for $\angle N_1 AB = \angle aBA$.

365. To Find Back-Bearings.—When the fore-bearing is given, and it is necessary to determine the back-bearing, *add* 180° to fore-bearing, if it is *less than* 180° ; and subtract 180° from it, if *more than* 180° , for the back-bearing.

Ex. 1. At N the fore-bearing of A is $45^\circ 15'$ (Fig. 196); required the back-bearing of N at A . *Ans.*— $45^\circ 15' + 180^\circ = 225^\circ 15'$.

Ex. 2. At E the fore-bearing of F is $192^\circ 00'$; required the back-bearing of E at F . *Ans.*— $192^\circ 00' - 180^\circ = 12^\circ 00'$.

366. Use of Back-Bearings.—As has been said above, the fore and back-bearings should differ by 180° . If there be a slight difference, *i.e.*, up to $15'$, then the mean of the two should be taken; but if great, the compass must be taken to be out of order, or there is some local attraction, or one of the bearings must have been read wrongly. In order to check the readings, it is better to read both the fore and back-bearings at each station. In case of local attraction, the angles between the lines must be recorded instead of the bearings, until the correspondence of the fore and back-bearings shows that there is no local attraction. The angle between any two lines will be exactly shown by the difference of the bearings in spite of any local attraction, as both the bearings will be equally affected by it. Back-bearing is to be taken when a line has to be produced in the opposite direction, or in other words, it is to be taken when the surveyor has to go in a direction just opposite to the one he has been following.

The fore-bearing and the back-bearing from the two ends of a line will not read exactly the same angle; and the difference varies with the latitude and with length of the line, but not in the same proportion either. It is too small to be detected by the needle, being only $\frac{1}{4}$ minute in a line one mile long at lat. 40° . In higher latitude it is more and in lower ones less. It is caused by the fact that meridians or north and south lines are not truly parallel but would meet at the poles if extended. On a true north and south line there is no convergence; also on a due east and west line at the equator there is no convergence.

367. Testing the Accuracy of a Plan.—The prismatic compass may be rendered very useful in testing the accuracy of a plan, the correctness of which we have any reason to doubt. Draw two or three lines across the plan, and mark on it the angles they make with each other; then in the field set out the corresponding lines with ranging-rods, take their bearings, and see if the angles between them agree with those measured upon the plan. This may also be accomplished without the rods, by drawing any line on the plan through two or three junctions of fences, setting off the corresponding line on the ground, taking the angles that adjacent fences make with this line, and then comparing these angles with those similarly made on the paper.

368. Graphic Adjustment of Closing Error.—**First Method.**—It will be seen from Fig. 196 that the protracted plot

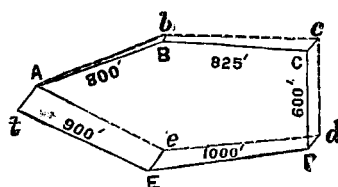


Fig. 205.—Graphic Plan.—Adjustment of Closing Error.

closes without much visible inaccuracy from the smallness of the scale, but as perfect accuracy is unattainable in any operation involving the measurements of angles and distances, errors always become apparent when we come to plot on a large scale from field-notes traverses of many sides and angles due to unavoidable inaccuracy in chaining the lines and in plotting; and since the plot of the survey and the calculations for finding the contents should be consistent within themselves, we do what is usually called *correcting the errors*. Errors are distributed all around the survey in the following way:—In Fig. 205 suppose a closed traverse is plotted to be $ABCDEA$, and suppose after very correct protraction from careful field-measurements the last line, EA , does not coincide with EA , and there is an error (for clearness the closing error has been much exaggerated in the figure) of $40'$ in the periphery of $4125'$, the distance EA being the closing error which, however, as shown in Fig. 205, is more than three times as great, proportionally to the size of the survey, as would be allowable in practice. The closing error should not exceed $\frac{1}{300}$ part of the periphery in ordinary surveys. Now to correct this error, rule through all the corners short lines parallel to EA . On each of these lines set off the distances, Bb, Cc, Dd, Ee . These distances are obtained as under:—Add all the sides together and measure EA by the scale of the plot. Then beginning at corner B , at the far end of side AB , say as the

Sum of all sides : Closing Error At :: side AB : Error for side AB .

Lay off this error from B to b . Then at corner C , say as the

Sum of all	Total closing	:	Sum of sides	:	Error for
the sides	Error At		AB and BC		side BC .

which error lay off from C to c ; and so at each of the corners; always using as the third term, the sum of the sides between the starting point and the given corner. Finally join the points A, b, c, d, e, A , and the plot is complete. The correction has evidently changed the length of every side; lengthening some, shortening others. It has also changed the angles. The new lengths and angles may, with tolerable accuracy, be found by means of the scale and protractor; and be noted on the plot instead of the old ones.

369. 2nd Method.—By means of this convenient method, calculations

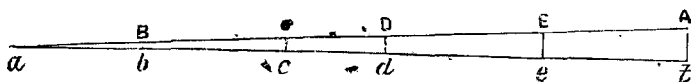


Fig. 206.—Graphic Section.—Adjustment of Closing Error.

may be avoided, and the distances bB, cC, dD, eE are found by drawing a line, at , equal to the united lengths of all the sides AB, BC, CD, DE, EA ; dividing it into distances, ab, bc, cd, de, et , equal to the respective sides of the Traverse $ABCDE$ (Fig. 205). Draw tA perpendicular to at and make tA , (Fig. 206), equal to the entire closing error in Fig. 205; join aA and draw bB, cC, dD, eE perpendiculars to at , which will give the error at each corner as in Fig. 205. The points A, b, c, d, e now being joined will be the adjusted traverse closing at A .

In mine-surveys the closing error should not be more than 1 in 1600 or 5 links per mile. In town surveys it should not exceed 1 in 5000. These also apply to theodolite surveys.

370. Surveying with the Cross-Staff.—The cross-staff shown by Fig. 19 can be used to read bearings from the magnetic north. There are graduations round the circumference; the top part slides over the under part, and may thus be set to any required angle by means of the graduations. Failing the prismatic compass, the cross-staff is more convenient than the sextant, because its construction permits of angles the apices of which are at varying levels being read in the horizontal plane, and because it is free from parallax, except when the observed point is very close to the instrument.

371. Surveying with the Miners' Dials or Circumferentors.—A traverse survey may be executed with the circumferentor or Mining dial in exactly the same manner as those already described. As the instruments are read with the aid of verniers, angles to 3 minutes are obtained. This kind of surveys are therefore more precise than that executed with the prismatic or surveyors' compass or plain

miners' dial without rack and vernier. The bearing of each line being taken independently from the magnetic meridian, the angular errors are not cumulative. In this method of surveying, the compass-box is clamped with the needle lying upon the zero or north and south line marked on the dial. The sights being then directed to the object, the bearing is read direct from the vernier to 3 minutes, or by estimation to 1 minute. This method is very quick, and gives very accurate results.

372. When Compass Survey is recommended.—Compass or Theodolite surveying, *i.e.*, angular surveying, is preferred to chain surveying in hilly countries where chaining is tedious, and is liable to error. In the survey of towns which can not always be divided into Δ s for the chain survey, traversing with the compass or plane table and chain is preferable. The survey of a road, railway, canal, or river is easily and quickly done by an angular instrument and chain; whereas to survey a small, open and level country an ordinary chain survey is the most accurate and quick method that can be adopted. The compass surveying is more convenient where the ground to be surveyed is a large open tract with a few isolated noticeable features, or where long straight lines and convenient ties or Δ s can not be made on account of obstructions, but it is more liable to error, as the compass needle does not exactly point north, and therefore cross-tie measurements are to be taken as checks to avoid doing the detailed work over again. For the survey of details in jungle or dense forest where traversing is the only method adopted, the prismatic compass is much used. In cities it is not possible to avoid local attraction, such as iron or steel pipes underground, lamp-posts, gas-pipes, iron-railings, etc., and in these localities as well as in districts abounding in deposits of iron or magnetic ore, or iron bridges, compass survey is out of the question. To make a number of observations at equidistant points along a straight line is the best method of finding out if there is any local attraction affecting the needle. In districts composed of magnetic rocks the compass can not be employed as observations show that along a straight line at equidistant points it gives varying bearings.

373. Surveying with the Pocket Compass.—A rough traverse may be made occasionally even with the pocket compass. The bearings are judged approximately by simply holding the compass in the hand so that the needle reads 360° , and estimating the bearing of the line by the eye.

374. Surveying with the Prismatic Compass and Passometer.—The passometer counts the paces; the accuracy of the measurement depends on the regularity with which the surveyor can pace. Pacing should be practised during hours of recreation. The surveyor should walk naturally without trying to step yards or any other specified distances, he is to hold the body erect and to walk at the same speed. The data of pacing should be ascertained on a road where milestones are correctly fixed. The paces vary in length according to slope; for which also a deduction has to be made for the

horizontal projection. As going uphill tends to shorten the step, and increase the number of them to the mile, the error is increased by the projection. In going downhill the contrary is the effect. The steps become longer and the distance indicated by the passometer becomes shorter and approximates more to the horizontal projection. A closed traverse along a hilly winding route of over 2 miles long had a *closing error* of $25\frac{1}{2}$ paces (Gribble, Preliminary Survey, 3rd Edition, pp. 51, 56).

375. Filling-in a Survey.—The compass is very useful for

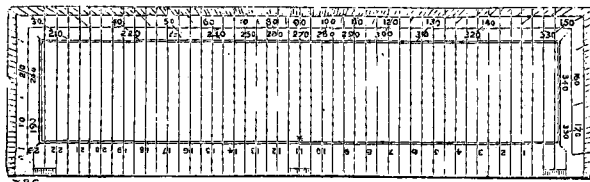


Fig. 207.—Military Protractor.

what is called filling-in a survey. For instance, the corners of the house, h , h_1 , in Fig. 196 is to be fixed. To do this, take bearings, from b and D , and for check also from C or E . By plotting the bearings the position of the corners of the house is obtained.

Usually this kind of work is done in the field on a prepared sheet of paper on which the main lines are already plotted and faint parallel lines are ruled in a direction perpendicular to the meridian at unequal distances apart, varying from a quarter to half an inch. Then as each bearing is taken, it is laid down at once on the paper by placing the military protractor (Fig. 207) at right angles to these lines so that no field-book is required. For example, it is required to take the traverse of a river embankment with a compass, the general direction of which being 140° from the meridian. The paper is prepared in the following way with the magnetic meridians:—Since the direction of the embankment is 140° , take the diagonal of the paper on the table, being the longest line, to represent its direction. The meridian should be such that this diagonal should bear 140° . Draw several parallel lines at unequal distances apart and close to each other at an angle of 50° or 130° to the diagonal at right angles to the meridian line, i.e., until the whole paper is filled. Then the paper is prepared with meridians. The section paper (Fig. 203) may be used for this purpose.

376. Finding One's Place in a Survey.—(1) If it is required to start the filling-in from a point Q (Fig. 196), and not from any of the principal stations, the first step to be taken is to fix the position of Q on the plan. To do this, take bearings of any two or more convenient stations as N and A , from Q . Then to fix the point Q , plot the back-bearings from N and A . The bearings of A and N at Q are 95° and $194^\circ 30'$ respectively. The former bearing being less than 180° , add that number of degrees to it, and

the latter being greater than 180° , subtract 180° from it: the resulting bearings are 270° and $14^\circ 30'$ respectively. Placing a protractor at *A* and *N*, plot these bearings, and their intersection gives the point *Q*. It is desirable to take another bearing to a third station as a check. This process of finding one's place in a survey is called

Interpolation. Observe that the nearer the two bearings intersect at a right angle, the more correctly is the position of the point determined. The above may be put into a rule thus:—*To find a station by observations taken to two points already known, protract from those points the reverse bearings to those observed; their intersection fixes the place sought.* This is a very useful problem.

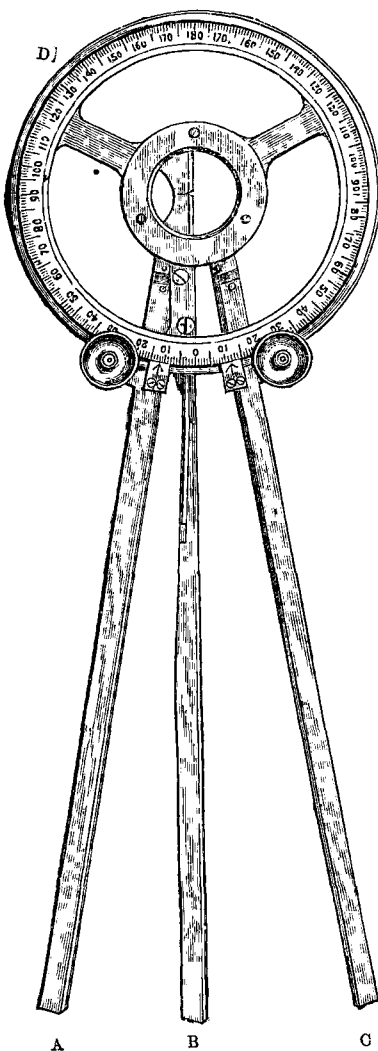


Fig. 208.—Troughton and Simm's Station-Pointer or Three-arm Protractor.

(2) **The Three-Point-Problem.**—An accurate method of fixing the position is by the angles subtended between three known objects. The position may be found with a pair of dividers and protractor, or more simply, by the aid of a protractor and a sheet of tracing paper as under:—Draw a line through the centre of the paper; place the protractor on it near to the bottom of the sheet; lay off right-hand and left-hand angles to the right and left of the centre line; rule pencil-lines, radiating from the point over which the centre of the protractor has been placed, to the points that have been laid off; then place the paper on the plan or map, and move it about until the three lines coincide with the objects taken; prick through the point that lay beneath the centre of the protractor and the observer's position is transferred to the plan. The centre object should be the nearest. The box-sextant will give the angles direct. (See Three Point Problem in the Section on Plane Table Survey which will found post.

(3) **The Three-Arm Protractor or Station-pointer** is generally used for this purpose. It has been devised to save trouble in plotting. Fig. 208 shows the

instrument. A, B, C are three arms having a common centre, marked by the notch in the figure, round which two of them, A and C , can be moved in any direction; D is a circle divided from 0° to 180° on each side of the fixed central arm, B , which is permanently set to zero. The arms, A and C , are set by means of the indicator arrows and clamp-screws shown. When angles are taken with the sextant to three fixed points forming two angles, one on each side of a central point, these two angles are laid off on each side of zero on the station-pointer, which is then laid down on the plan so that each of the three arms points to one of the fixed points. The centre of the instrument will then be at the position of the point of observation on the plan, which is then pricked through at the centre. This instrument is invaluable to hydrographers. By it they locate their position at sea from three fixed points on shore, very quickly and accurately when a great many points are to be plotted. The size of the instrument is a $6'' \odot$, divided on brass to $30'$, read by two plain indices, with arms $18''$ long, but it may be had with verniers reading to 1 minute, tangent-screw movements, and arms to lengthen up to $30''$.

377. To Find the Position of a Temple or any other Object far away from the Chain Line and to which it is not easy to chain, the surveyor should select two or more known points, either station-points or other points on the chain lines or other fixed points in the interior of the traverse, whence the temple or the object is visible, and take bearings to it from the selected points which should be so chosen, that any two of them and the temple or the object to be fixed, will form as nearly as possible an equilateral $\triangle$. When the bearings are plotted from the selected points, all should intersect at a point to fix the position of the temple or the object.

Similarly the Width of a River may be found from one of its banks by laying off a base-line on the bank in any convenient direction and parallel to it, if possible, and observing the bearings from its both ends to a flag planted on the other bank, so that the ends of the base-line and the flag will form a right-angled $\triangle$ having the right angle at the flag. A third bearing, from an intermediate point on the base-line, or from a point on the base-line produced may be taken to the flag as a check. When the base-line and the bearings are plotted, the lines representing the bearings will intersect each other, giving the point on the other bank. Now with a divider measure the width of the river from the plan.

Section 2.—Sextant Surveys.

378. Instruments.—Detailed descriptions have already been given of the sextants in Chapter II, pp. 89 to 97.

379. The surveying or box-sextant may be used by surveyors as a light substitute for a theodolite, for observing horizontal angles

in a small survey on a level country, or for observing altitudes, such as heights of buildings, and station marks; but it is not much used by them for the following reasons:—(1) When the objects observed are at different altitudes it does not read exact horizontal angles, but angles in the plane of the eye of the observer and the two objects sighted; it either becomes necessary to take altitudes of each of the objects between which the angle is required and to reduce the oblique angle to its horizontal projection (see Art. 172, p. 97), or it is necessary to adopt the mechanical means to get it as described in Art. 171 on p. 97, by means of vertical rods or plumb-lines. The mechanical methods are very troublesome on account of the additional rods or plumb-lines to be set up, while during windy weather it becomes impracticable to observe on them with precision. (2) The exact coincidence of the direct and the reflected images is necessary to obtain a correct reading; in clear weather and with well-defined objects not more than 90° apart, this is easy; but when the objects are ill-defined, the weather dull, and when the angle to be observed is between 90° and 120° , it becomes impracticable to get the reading at one observation which should be repeated to obtain the correct reading. These two objections render the instrument unsuitable in triangulation, while the difficulty in observing very obtuse angles with it, and, in fact, impossibility without a *supplementary arc*, makes it unsuitable in traversing.

380. Sextant Surveys of small extent, generally limited to a square mile of country, can under favorable circumstances be accurately made as follows:—(1) **By Triangulation.**—In surveying with the box-sextant a flat country is most suitable, as then the ordinary length of the vertical rods or flags affords sufficient means to obtain horizontal angles correctly. A base-line is chained, and a network of Δ s laid out on it by selecting suitable stations. Instead of measuring a tie-line to check each of the Δ s, as in a chain survey, all the angles may be measured with the box-sextant, and these angles act as checks on the Δ s, their sum being 180° . Except as a check the use of the box-sextant is not advisable, although, if the ground be level, it may be used in lieu of a theodolite. The Δ s should be as large and well-conditioned as possible as the nature of the country will permit. Under ordinary conditions the angle should not exceed 100° or be less than 30° ; and with a base-line of about 1000', and a scale of representation less than 400' to 1 inch, three sets of large Δ s can be laid out after starting from the base, without accumulating serious error; but so much depends on the scale adopted and the circumstances affecting the accuracy of angular observations that no hard and fast rule can be given.

Plotting Sextant Survey.—In plotting the survey done with a sextant, all the sides of all the Δ s may be calculated, and then laid down to scale, or the base-line may alone be laid down to scale, and the angles plotted with a protractor; the former method is better, the latter is more rapid; or both may be applied to verify each other. The filling in of the Δ s may be quickly done by distances and sets and compass-bearings.

(2) **By Traversing.**—The flat country is also suitable for traversing with the box-sextant, the chief difficulty in carrying out which is in observing the very *obtuse* angles. These may severally be split up into two or more acute angles by using an available lateral point or points; but a better method is to produce the traverse lines, and to observe the acute angles only. For instance, if AB, BC, CD, DE be the four lines of a traverse, the inward angles, B, C, D , being obtuse, but less than 180° , let ABb be the initial direction and AB the first chained distance, b being the position of a flag set up further on in the same direction. On arriving at B after setting up a flag at c further on, the acute angle bBc is taken, and its supplement, the inward angle ABc , booked; on chaining BC , the acute angle cCd is taken, and its supplement, the inward angle BCD , booked; and so on until the traverse is completed, except when the inward angle happens to be acute it will be so observed and recorded. This method of surveying is suited to fairly open and level country, and along rather straight roads, but not along winding routes with hedges or obstacles on both sides that impede the view; the work can be easily carried out by the aid of any conspicuous object such as a temple in the general direction forward, as the acute angles may then be observed on it, and the advanced flags, b, c, d , dispensed with; the direction of that object from the first station may then be used as the meridian of reference. In sextant traverses, checks on lateral objects are more necessary than compass traverses, where every fore-bearing is checked by a back-bearing, as no corresponding verification is possible in such sextant traverses. By the use of Hughes' Double Sextant (p. 22, Fig. 80) very obtuse angles can be observed, thereby lessening the work of erection of additional flags for observing acute angles. The details near the route traversed are obtained by means of distances and offsets, aided by compass bearings.

381. Reduction to a Single Meridian.

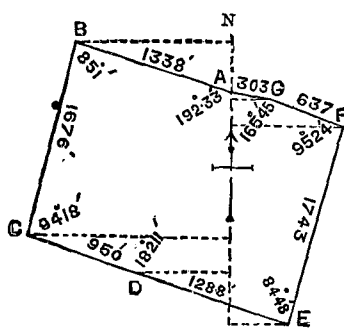


Fig. 209.—Example of a Sextant Traverse Survey.

or triangulation surveys by means of angular instruments the bearings or angles are generally laid down on paper with a large protractor; there is, however, in traverse work much risk of error from the distances being wrongly scaled or laid down, although the angles and therefore the bearings can be checked by the rule "sum of the interior angles of a rectilinear figure equals twice as many right angles as the figure has sides, less four right angles." To avoid this risk as well as to avoid protracting angles or bearings the whole of the traverse may be referred to rectangular coordinates or reduced to a convenient meridian. The distances, inward angles, and the bearing of one traverse line being observed, the res-

is reduced as in the following table of the Survey in Fig. 209. For detailed calculations see Section on Gales' Traverse System later on.

382. Plotting by Rectangular Co-ordinates.—The plotting is done in exactly the same way as described on p. 200, Art. 356, for plotting the compass traverse by rectangular co-ordinates.

Reduction of a Sextant Traverse to a Single Meridian.

1	2	3	4	5	6	7	8	9	10	11	12	
Stations.	Observed Inward Angles.	Observed Outward Angles.	Magnetic Bearings.	Reduced Bearings.	Measured Distances in feet.	Consecutive Co-ordinates.				Reduced Co-ordinates.		
						Latitudes, = Col. 6 × Cos. 5.		Departures = Col. 6 × Sin. 5.		N.		
						N.	S.	E.	W.	or		
										S.	W.	
A	85°01'		288°23'	N 71°37' W	1338	422	..	..	1270	00	00	
B	94°18'		193°24'	S 13°24' W	1676	..	1631	..	888	N 422	1270 W	
C	94°18'		107°42'	S 72°18' E	950	..	289	905	..	S 1209	1638 W	
D	182°11'	177°49'	109°53'	S 70°07' E	1288	..	488	1211	..	S 1498	753 W	
E	84°48'		14°4'	N 14°41' E	1748	1686	..	441	..	S 1936	458 E	
F	95°24'		290°05'	N 60°55' W	637	219	..	..	598	S 250	899 E	
G	165°15'		275°50'	N 84°05' W	303	31	..	..	301	S 81	301 E	
A	92°33'	167°27'	238°23'	N 71°37' W						00	00	
7	000°00'	Proof of ang. — $7 \times 180^\circ - 360^\circ = 900^\circ$				7935	2358	2358	2557	2557		

Section 3.—Surveying with the Theodolite: Closed Traverse by means of Interior Angles and Bearings.

383. Instruments.—Several forms of theodolites of horizontal circles varying from 3" to 12" in diameter have already been described in Chap. II., Arts. 106-156, 188-194

384. Ranging Lines with the Theodolite.—The theodolite is used for ranging long lines, to form a base for a system of triangulation, or for setting out a line of road, railway, or canal, as under:—Plant a ranging-rod or flag at each of the extremities of the straight line. Having driven in the first peg at the beginning of the line, set up the theodolite over it, perform the temporary adjustments and direct the cross-hairs on to the ranging-rod at the other end of the line, and clamp the instrument firmly in this position. Then plant intermediate ranging-rods between the end ones by moving them to right or left according to direction until they are bisected by the cross-hairs.

385. Pegging out the Line with the Theodolite.—First proceed as in ranging out the line. Then, it is necessary to peg out the line at every chain, hold one end of the chain on the first peg and send the leader with a ranging-rod at the other end of the chain, and direct him to move it to right or left until it is bisected by the cross-hairs. The leader then plants the rod into the ground,

and drives in a peg at the mark made by the rod. The surveyor will now direct the head of the peg to be moved by hammering to right or left until it is in the line of collimation and is exactly in the line. The leader should then put the end of the chain to the top of the peg and make a scratch on it along the chain handle with an arrow. Repeat the process as long as necessary. In a long line it will be necessary to shift the instrument ahead about every 15 or 20 chains as the rods and pegs can not be seen with distinctness at greater distances, and as then they are liable to get out of the line. In this case range in an arrow held on the top of the last peg, and with it make a point mark on the peg and over which the theodolite may be set up and the process repeated.

386. Method of Executing a Very Small Closed Theodolite Survey by Interior Angles.—When a theodolite is used, the measurement of the diagonals and tie-lines, which in a chain survey are measured only to enable the work to be plotted and checked, is unnecessary. In order to plot and check the work a sufficient number of angles may be taken with the theodolite. These angles will act as checks in the same way that the tie-lines act as checks in a chain survey. A survey of roads or fields can be made with the theodolite in the same way as that already described under compass surveying, and much more accurately as the smallest 3" transit theodolites read to one minute of a degree. As the variation of the magnetic meridian from the true meridian is different at different places, and different at the same place at different times, the error that is likely to occur from taking every angle with the magnetic meridian is also avoided. Local attraction can have no influence* if the survey is executed either by reading the interior angles with the theodolite or by taking bearings with the direction of the needle parallel to its direction at the starting point. The method of executing a small theodolite survey is as under:—In Fig. 187, (Att. 318) if the sides, ab, bc, cd , and da , of the quadrilateral, $abcd$, and the angle bad are measured the survey can be plotted. The line, ad , may be set off to scale on paper, and the $\angle bad$ protracted at the point, a , making it equal to the observed angle, and then the length of ab may be marked. Now the point, c , may be obtained, by taking the lengths, bc and dc , in the compasses, and sweeping out arcs from b and d as centres. But there is no check on the accuracy of the work. To effect this one at least of the angles, say the $\angle bcd$, must be observed; then if there be any error either in measuring or plotting any of the sides or in observing and protracting the $\angle bad$, the $\angle bcd$ when measured on the paper with a protractor will not agree with the $\angle bcd$ as observed on the ground. It is advisable to observe all the angles. The angular measurements can then be checked before leaving the field, as "*The sum of the interior angles of any rectilinear figure is equal to twice as many right angles as the figure has sides, less four right angles.*" In the present case, Fig. 187 is a quadrilateral, the sum of the interior angles should be equal to $2 \times 4 \times 90^\circ - 4 \times 90^\circ = 360^\circ$. If then the sum of the four observed

interior angles comes to 360° , or, as perfect accuracy is unattainable, to 360° plus or minus a fair allowable amount of error, the angular observations may be accepted as correct before leaving the field. When the work is plotted the accuracy is proved by measuring the angles with the protractor.

387. Field-Book.—The field-book for a Closed Theodolite Traverse is kept in the same manner as the field-book used for a chain and compass survey, except that angles are recorded in place of the bearings at the beginning of each line.

388. Example of a Small Closed Traverse Survey with the Theodolite by Interior Angles.—The theodolite is invariably used in the survey of a traverse circuit, the size of the instrument depending on the nature and extent of the survey. For the survey of boundaries of estates, covering an area of 50 to 100 square miles, the 5"-theodolite will be found suitable. For smaller areas the 4"-instrument is the most convenient and portable. In the Revenue Survey 6" and 7"-instruments are used for the survey of Main-circuits covering an area of from 100 to 300 square miles. The 7"-theodolite is also used for astronomical observations, and for subsidiary triangulation based on the sides of the G.T.S. Δ s. Whether we have to survey a traverse of a minor circuit of a square mile or less in extent, or a main-circuit covering some hundreds of these minor circuits, the principle is exactly the same.

389. Field-work.—Fig. 210 illustrates an example of a survey of this kind. The survey lines, *AB, BC, CD, DE, EF, EG, GH, HI*, and *IA* are laid down after careful reconnaissance to enclose the property and then chained. The corner-piece *DCf* is surveyed as a subsidiary Δ . The subsidiary chain lines, *bc, cd, de, ad, gh, hj, ik, Go, In, AG, pr*, etc., are run to take up the village boundaries and garden fences. In chaining along these lines, take offsets, right and left, to the boundaries of the polygon and to all adjacent objects, such as streams, roads, etc., and mark where they cross the chain lines. The angles measured are shown in Fig. 210, and are first, the interior angles of the main polygon, *ABCDEFGHIA*, which has nine sides. The accuracy of these angles can be checked before leaving the field as the sum of the interior angles should be equal to $2 \times 9 \times 90^\circ - 4 \times 90^\circ = 1260^\circ$. With a 5"-instrument reading to $30''$, the total error should not exceed 1 or 2 minutes. In the main-circuits of the Revenue Survey the correction required seldom exceeds 1' in 15 or 20 angles. The corrections are, as a rule, applied to angles formed by the shortest sides, where the chance of error in observing the angle is greater than in long distances.

390. Method of Observing Angles.—Set up the theodolite at *A*, and perform the temporary adjustments (Art. 130, p. 69). Plant a flag at the back-station, *I*, and another at the fore-station, *B*. Set the A-vernier to zero or 360° , and bisect the flag at *I*, the pole of which should be cut as near the ground, as possible: if we can get the iron spike between the wires, so much the better. Clamp the lower plate, unclamp the upper plate, turn the telescope

round to the fore-flag, B , bisect it, and clamp the upper plate. Now read off the *interior* angle IAB . Setting the instrument to

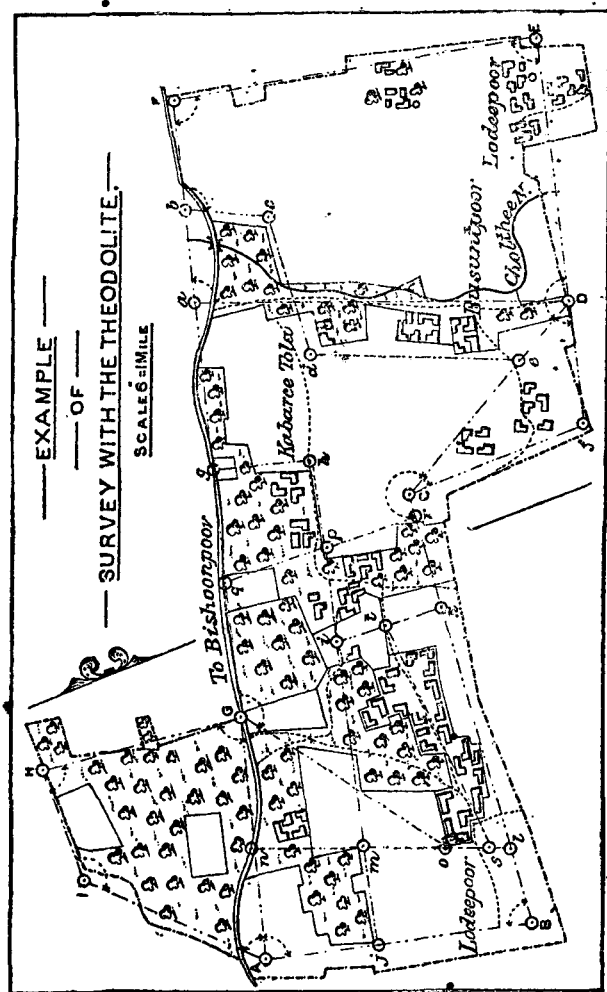


Fig. 210.—Example of a Closed Theodolite Traverse Survey.

zero again, repeat the operation, directing the telescope this time, first to the fore-flag, B , then to I . The angle now read is the *exterior* angle BAI , and the sum of the two angles should, of course, make 360° . Then set up the theodolite in succession at B, C, D, E, F, G, H, I , and observe the interior and exterior angles. As the theodolite is placed at D and C to read the angles between the main survey lines, the angles, fDC, cDc, dDE, DCf , are also observed. Similarly when the instrument is at G the angles, FGo, oGA , and

AGH, when at *A* the angle, *GAI*, are read. In measuring the angles, the telescope is first directed to the left-hand object and then to the right-hand one, so that the verniers move in the direction in which the horizontal circle is graduated. If the telescope is directed to the right-hand object first, the angle read must be subtracted from 360° in order to give the interior angle required.

391. Magnetic Bearing.—With the theodolite set up at *A* the magnetic bearing of the line, *AB*, may be taken as follows:—Set the vernier to zero, release the lower plate and turn the telescope north until the compass-needle attached to it coincides with zero on the compass-box. Clamp the limb in this position and releasing the vernier-plate, bisect the flag, *B*; clamp the vernier-plate and read off the angle which will be the magnetic bearing.

392. Plotting and Proving the Accuracy of the Linear and Angular Measurements.—Commencing with the longest side, *FG*, the sides, *FE*, *ED*, *DC*, *CB*, may be plotted by means of the distances and the interior angles. The point, *A*, is obtained by taking the lengths, *BA* and *GA*, in the compasses and by sweeping out arcs from *B* and *G* as centres, the intersection of the arcs giving the point. The accuracy of the chain measurements may now be proved by measuring with the protractor the angles, *CBA*, *BAG*, and *AGF*, which should agree with those observed in the field; or when the sides, *BC*, *CD*, *DE*, *FG*, *GA*, are plotted by means of the distances and interior angles, the distance, *AB*, should scale the same as measured on the ground. The angles, *BAG*, *ABC*, at *A* and *B*, should also measure (with the protractor) the same as observed in the field. The sides of the $\triangle DCf$ are measured in the field. When the angles *fDC* and *fCD* are plotted, the sides, *Df* and *fC*, should scale the same as measured on the ground. The polygon, *AGHI*, being a quadrilateral, should have the sum of all its interior angles equal to $2 \times 4 \times 90^\circ - 4 \times 90^\circ = 360^\circ$. It is to be plotted by means of distances and angles and checked as explained for the main polygon. The length of lines, *Da*, *nl*, *qr*, when plotted should agree with their measured lengths. These three lines thus form valuable additional checks on the survey. At *D* both the angles *CDa*, *aDE* have also been observed. The angle, *CDE*, is only necessary, but when the instrument was set up at *D*, by taking the two angles and also the angles, *FaD* and *DaG*, the angles of the polygons, *DEFa* and *DCBAGa*, may be checked separately. They being four-sided and six-sided figures the sum of their interior angles should equal 360° and 720° respectively. As to the subsidiary chain lines, *gh* and *hj*, they can be plotted by taking them in the compasses and sweeping out arcs from *g* and *j* as centres, but the work can be checked by measuring the angle *Fgh* (by means of the protractor) which should agree with that observed in the field. In fact all the angles should be observed and all the chain lines measured to render mutual checks, and by so doing the measurement of long diagonals and tie-lines which would be necessary for a chain survey is avoided. The labor necessary for chaining the diagonals and tie-lines is much more than the labor of observing these angles.

393. Plotting the Survey : Protractors.—As the angles taken by the theodolite are much more accurate than those taken by the compass, it is as well to employ a more accurate means of laying them down, and the following protractors should be used :—

One-arm Circular Protractor with Vernier.—Fig. 211 illustrates a protractor, connected with its centre by four radial

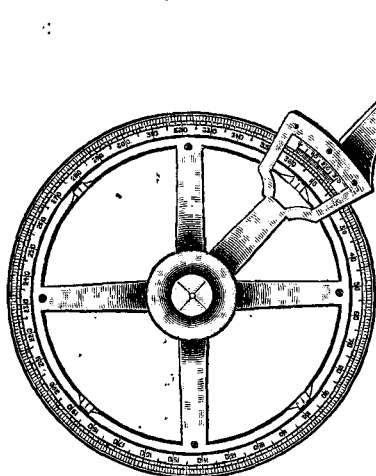


Fig. 211.—One-arm Circular Protractor with Vernier.

bars, of gun-metal or electrum, 6" in diameter. The movable arm provided with a vernier is jointed round the centre, and extends beyond the circumference of the protractor, one side of it forming a radial line from the centre. The part of the arm lying beyond the circumference remains in close contact with the paper, so that a line drawn along the straight-edge will exactly correspond with the reading of the vernier. The straight-edge has also a sharp point fixed to the end of it, which, by gentle pressure may be caused to puncture a *fine*

hole upon the drawing, which will give the exact position of the angle, according to the reading of the vernier. The point mark may be joined by a line to the central mark, after the protractor is removed. The centre of this protractor is generally made of glass, with fine lines across it; the glass, being transparent, the centre can be placed upon the point mark with ease. There is a small circle around the center with the cross-lines leading up to this circle. The eye will find the point in the center of the circle with great accuracy. With the aid of the vernier readings to 1 minute can be plotted or measured.

394. The Parallel Rule and Semicircular Protractor with Vernier-Arm.—The same construction as that of Fig. 211

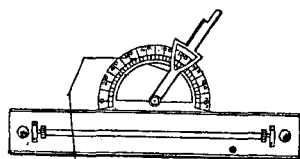


Fig. 212.—Semicircular Protractor with Single Vernier-Arm.

is also commonly applied to semicircular protractors, (Fig. 212). When this instrument is employed it is usual to lay the lower edge against a vertical or horizontal line, or against a parallel ruler placed parallel to one of these lines, and to move the arm until the proper bearing is secured, and to draw a line along its bevelled edge. A semicircular protractor fitted with a vernier may be employed with a 60° set-square and a straight-edge in the following

manner:—The arm of the protractor is set by the vernier at the required angle, and at the same time the arm-line is laid on the meridian of the plan. The set-square is then placed against the protractor by the straight-edge, and slid along the latter until it passes through the given point, when the line drawn through that point will form the required angle with the meridian.

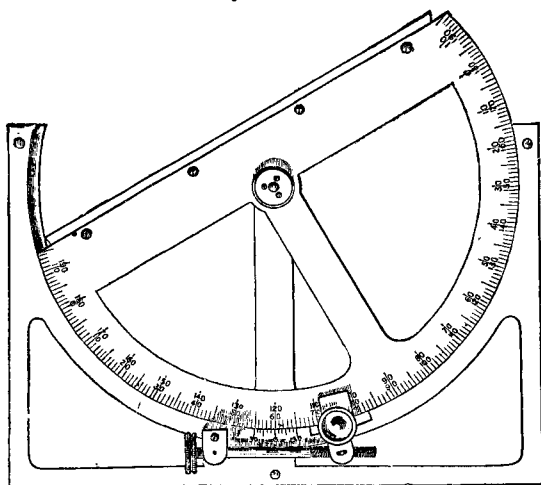


Fig. 213.—Gurley's Crozet Protractor.

and lines are produced without puncturing the paper. It is a semicircular protractor 8" in diameter of German silver, divided to $\frac{1}{2}$ degrees, reading to 1 minute by the vernier with clamp and tangent adjustment.

396. The Folding Two-Arm Protractor.—Fig. 214 shows the most perfect instrument for accurate plotting. It is generally used

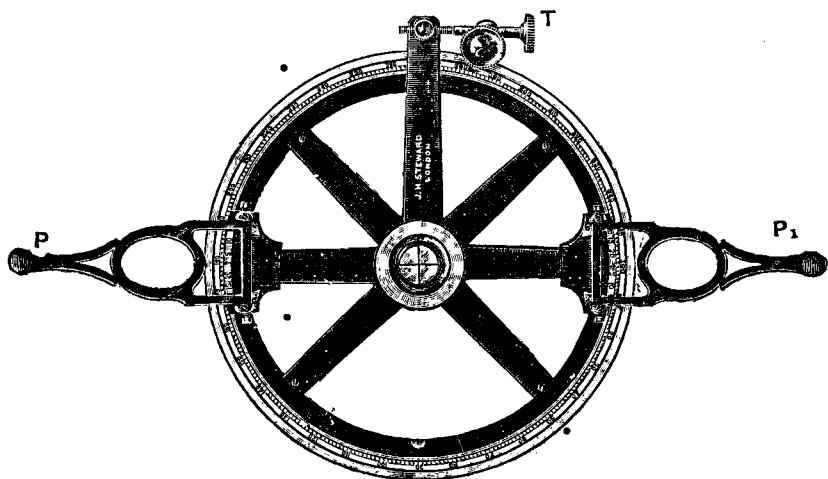


Fig. 214.—Folding Two-Arm Protractor.

to plot the work of a 5" or 6" theodolite like which it is graduated.

6" protractors read to 1 minute and 8" ones to 20" by means of the verniers. The construction of the graduated circle, verniers, and centre is similar to the one-arm protractor (Fig. 211). It has two verniers which read on to the opposite sides of the graduated circle, and serve to correct any possible inaccuracy in the construction and division of the instrument. To the vernier bars are attached the branches, P, P₁, carrying at their ends fine steel prickers whose points are kept above the surface of the paper by springs placed under their supports which give way when the branches are pressed downwards, and permit the points to make punctures on the paper. The folding arms, P, P₁, are jointed upon adjustable pivots, near the verniers, which admit of their being folded backwards over the instrument when not in use, and for packing it in its case. To the vernier bars is fixed another bar and at right angles to them. This bar carries what is termed a *Clamp and tangent-screw motion*, T. This affords means of adjusting the verniers roughly in any position by the *clamp* and then accurately adjusting them by means of the *tangent-screw*. The centre of the instrument is formed by the intersection of two lines drawn at right angles to each other on a piece of plate glass, which enables it to be placed accurately on any given point on the plan. If the instrument is in perfect adjustment, a line joining the fine pricking points would pass through the centre of the protractor, as indicated by the above-mentioned intersection of the cross-lines upon the glass, which, it may be observed, are drawn so nearly level with the under surface of the instrument as to destroy any serious amount of parallax, when setting the instrument over a point from which an angle is to be plotted.

397. Use of the Folding Arm Protractor.—In using the instrument, one of the verniers is set to zero or 360° on the graduated circle, and the other, at the division 180° exactly by means of the clamp and tangent-screw. It is then placed on the drawing paper so that the fine pricking points are on the given line and the centre of the protractor on the given point on such line from which other angular lines are to be drawn. Now the instrument is to be pressed gently down on the paper, and this will fix it in position by means of the fine points on the under side. It is now ready to lay off the given angle which is done by unclamping and turning the verniers till the 1st vernier which was set to zero reads the required angle after clamping and necessary tangent motion. Then press gently downwards the pricking points, P, P₁, which will leave punctures on the drawing at opposite sides of the circle; and the given line connecting these punctures will pass through the given point, if the instrument was first correctly set. In this way, at one setting of the protractor, a large number of angles may be laid off from the same point. As there are two verniers, one forms a check upon the other, as the reading of the 2nd vernier should always be exactly 180° more than the reading of the first.

398. Example of a Closed Traverse Survey by taking Bearings with the Transit Theodolite.—First Method.—

The method of procedure to be adopted is exactly similar to that previously laid down for the compass survey, the only difference

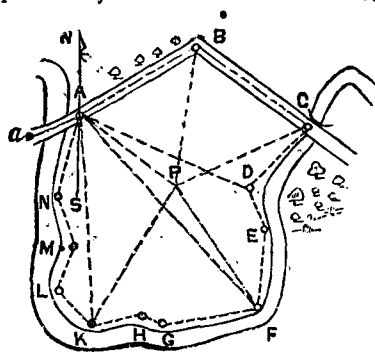


Fig. 215.—Example of a Theodolite Traverse by Bearings.

being in the method of taking the bearings. Let it be required to survey the area lying between the road and the stream (Fig. 215). The station points $a, A, B, C, D, E, \dots N$ are selected, as far apart as it is possible to see between the bends in the road and the stream, and are marked by driving in pegs. The lines, $AB, BC, CD, \dots NA$, are first to be measured, and the offsets taken and recorded in the field-book in the usual way. Next the angles are to be observed. If the instrument is set up at B and $\angle ABC$ measured between the lines, BA and BC , and then in like manner the other angles at C, D, E , etc., this would be sufficient to plot the survey; but there would be no sufficient check on the reading of the angles. The best method, and that usually adopted, is to take bearings in the following way:—

Method of taking Bearings.—Starting from a point at which observations can be made on surrounding objects, as at A , set out the lines, Aa, AN and AB , and plant the theodolite at A , and level it. Next, set the vernier to 360° or zero, and clamp the upper plate to the lower and turn the whole instrument round until the magnetic needle of the compass on the theodolite points to the zero of its graduations, i.e., until it lies over the North-and-South line, N.S., and clamp the whole instrument firm in that position, and use the tangent-screw of the lower plate to make the needle point exactly north. Now, release the vernier-plate, direct consecutively the cross-hairs on to flags at a, N, K, F, P, D and lastly on B , clamping the vernier-plate each time and bisecting the points with the intersection of the cross-hairs by the tangent-screw of the upper plate; and carefully note down at station, A , all the bearings made by the lines, Aa, AN, AK, AF, AP, AD and AB with the magnetic meridian as read each time on the vernier as was set to zero. The check on the measurements of these bearings is that the needle should read the same bearing each time on the divisions of the compass-box as the bearing already read by the vernier on the graduations of the lower plate. Having both the clamps firmly fixed the last reading being with the telescope bearing on AB , remove the instrument to B , plant it at this station, and carefully level it. Unclamp the lower plate, the vernier-plate must not be disturbed or the operation will have to be repeated. Now, turn the telescope back upon the flag at A , clamp the lower plate, and perfect the contact by the slow-motion screw and examine the verniers to see that no disturbance has taken place (Fig. 216). Then revolve the telescope vertically through

180° , so that the line of collimation is in the direction *from A to B*. The previously observed bearing of *AB* should now be indicated

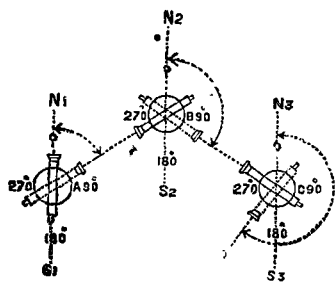


Fig. 216.—Example of a Theodolite traverse by Bearings—First Method.

by the compass needle. Again release the vernier-plate and direct the cross-hairs on to the flag at *C*; clamp the instrument; read off the bearing of *BC* on the vernier. Now check this bearing, by observing if the needle reads the same bearing as it ought to do unless the bearing of *BC* was read incorrectly. or there is some local attraction. With the instrument still clamped at the bearing of *BC*, move it to *C*, and go on in exactly the same manner, directing the cross-hairs back to *B* and then taking the bearing of *CD* and *CP*. The same process is repeated at each of the other stations until all the bearings have been taken, care being taken to check each of these bearings with those indicated by the compass needle. When the instrument was set up at *A*, the back-bearing *AN* of the last line *NA* was taken. Upon reaching the point *N*, the fore-bearing, *NA*, should differ by 180° from the back-bearing, *AN*, as read at the starting point, *A*. This proves the accuracy of the bearings taken. In practice this is seldom the case, and if there is a closing error, *i.e.*, if the bearing of *NA* does not differ by 180° from the bearing, *AN*, the amount of excess or defect must be distributed over all the bearings from the commencement. Thus if the bearing of *NA* is say 3 minutes in excess, then as there are 12 bearings, $\frac{3}{12}$ minutes or 15 seconds must be deducted from each of the bearings.

399. Checks on Reading of Closed Traverse—At the starting point, *A*, the bearings, *AD*, *AF* and *AK*, have been observed. The object of observing these bearings is to see on reaching *D* for example, and taking the back-bearing, *DA*, if this bearing differs from the fore-bearing, *AD*, by 180° . In this way a check on the accuracy of the bearings is applied on reaching *D*. Similarly at *F* the back-bearing, *FA*, checks the bearings up to this point, and at *K* the back-bearing, *KA*, checks the work up to *K*. Thus any error in the observation of bearings is detected before the whole traverse is closed and if not allowable, is rectified before proceeding further. In the case of stations which can not be seen from the starting point of the survey, check-bearings may be taken to them from the first stations from which they can be seen.

400. Checks on Plotting of Closed Traverse.—This may be done by selecting a station, *P*, in a commanding position near the centre of the survey; and by taking bearings to it from as many station points as are visible from it such as *AP*, *BP*, *CP*, *FP*, and *KP*. The object of taking these bearings is to check the plotting, if the bearings are laid off with a protractor, as it proceeds. For example, the bearings, *AP* and *BP*, intersect at the point, *P*, when plotted. When the survey is plotted as far as *C*, the bearing, *CP*,

should pass through same point, P , if there is no error in the plotting of bearings up to that point. Similarly the bearings, FP , KP , should pass through P . Thus any mistake in plotting is detected as the work proceeds, and before the plotting work is completed. Judgment is required in selecting such points as P , as they may often be very useful to chain upon in order to fill-in interior work, as fences and buildings. On an extensive traverse this should particularly be kept in sight, as it prevents the necessity of having again recourse to the instrument when filling in.

401. Second Method of Taking Bearings in Traversing.—The working meridian in the above example is made to coincide with the North-and-South line

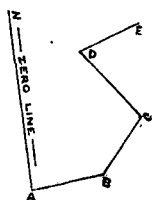


Fig. 217.—Example of a Theodolite Traverse by Bearings: 2nd Method.

merely as a matter of convenience, and any line would do as well. The following is an example: Plant the theodolite at A (Fig. 217), and perform the temporary adjustments. Clamp the vernier to zero, then unclamp the lower plate, direct the cross-hairs on the point, Z , chosen for zero, and clamp the instrument firmly by using the lower clamp and tangent-screws. Release the magnetic needle and read the bearing. Now, keeping the lower clamp fast, unclamp the upper plate, and take a round of angles to all objects the positions of which it is desired to fix only using the upper-set of screws; then bisect the next forward station B , using only the *upper clamp and tangent-screws*, each time. Note the reading of the same vernier which was set to zero, and keeping the upper plate clamped to the lower at this reading remove the theodolite to the next forward station, B , and set it up there, the *lower clamp* being released for levelling it. With the two plates still clamped together, direct the telescope back on A , using only the *lower clamp and tangent-screws*. Now, unclamp the vernier-plate, and take a round of angles as before, finishing with the angle to the next forward station, C , the angles being read from the *opposite vernier* to that previously used. When the forward angles are taken to the *right* of the zero line, *passing through a station* they will be less than 180° , when to the *left* of the zero line they will be greater than 180° . By observing this it will be easy to determine which vernier should be read. The traverse is carried out in this way to all the forward stations reading the angles alternately on the two verniers. If it is required to close the survey, as from E to Z , the vernier should be at zero when the theodolite is planted at Z , and the point, A , bisected with the cross-hairs. The accuracy of the survey may be tested at each forward station, by setting the vernier to zero, when one end of the compass needle should point to the bearing noted of the zero line. In plotting the survey all angles are plotted with reference to the zero line.

Orienting the Transit.—A transit instrument is said to be oriented when it is set up with the horizontal circle in such a position that if the vernier

is made to read Zero, the line of sight will be parallel to the meridian, true or assumed.

402. Third Method of Taking Bearings consists of observing and recording the different directions of successive portions

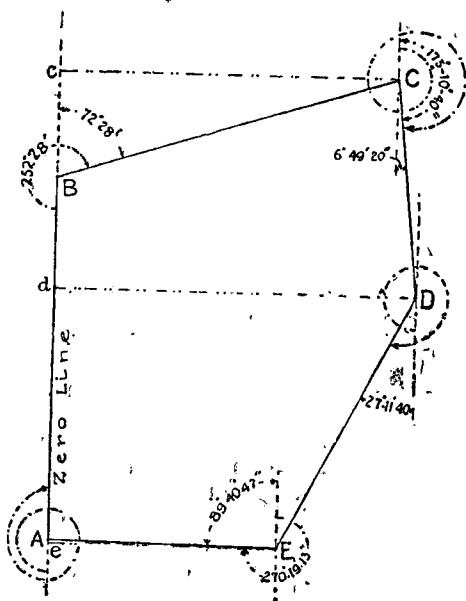


Fig. 218.—Traverse Survey by Bearings :
Third Method.

of a traverse so as to read off on the graduated circle of the theodolite at each station the angle which the traverse makes with the first line observed, which is called the *Zero-line*. Set up the instrument at some station, as B (Fig. 218); Set the vernier to Zero, sight back on A, tighten the lower clamp; *reverse the telescope*, release the vernier-plate, and sight to C, clamp the vernier-plate again, and record the reading. Remove the theodolite to C, sight back to B by the lower clamp and tangent-screws (keeping the upper set clamped at the previous reading), then clamp the lower motion; *reverse the telescope*, release the vernier-plate, b i s e c t D,

clamp the vernier-plate again, and record the reading. Then go to D; and go on as at C, and so on. The readings of the vernier gives the bearings measured to the right or "with the sun" as indicated by the arcs in Fig. 218. Keep the same side of the instrument ahead and read the *same vernier* throughout. Take the compass bearing of each line of the route for a check on the observations; for, the difference between the magnetic bearings of any two lines of route should be the same *approximately* as the angles between them measured by the theodolite. The magnetic-bearings also show whether the angles have been taken to the right or the left.

Section 4. Surveying with the Theodolite : Gale's. Traverse System — Closed and Open Traverses.

403. Gale's Traverse System.—The method of plotting points by the intersection of their computed rectangular co-ordinates is known as Gale's system (see p 196, Arts. 349-355) This is the most accurate method of plotting a survey, because the position of each station is plotted without being affected by any errors committed in plotting previous stations.

404. Demarcation and Line Clearing.—Before a theodolite traverse of an area can be commenced, the boundary of the same must be demarcated by wooden pegs, or if the demarcation is required to be of a more permanent nature, by clay cylinders of about 18" in length and 4" in diameter, or Δ r stone-prisms, embedded in the ground and within a couple of inches of the top.

¹ In demarcating and clearing the boundary, the following points should be observed :—(a) The Traverse stations must be, whenever possible, on the actual boundary and never in a cultivated field. (b) The stations should be so fixed, that no offset, from the traverse line to bends of the boundary, must exceed $2\frac{1}{2}$ chains (allowable limit in a Revenue Survey). (c) The line-clearer must see that the stations are visible from each other. (d) Bearing in mind points (a) and (b), the line-clearer should make his traverse lines about 20 chains in length: they should never exceed 40 chains. (e) In clearing a line through jungle it is only necessary to cut one sufficiently wide to permit of chaining being carried out. *In cultivation, crops should never be cut but simply bent over.* (f) A line-cutter should have *two men* to assist him in demarcating the boundary and clearing the traverse lines. He must be supplied with a Gunter's chain, a couple of bamboo pole-flags, and a few blank field-book forms, as he should, if possible, record the distances in chains and links between all stations laid down by him. (g) At each station, in addition to embedding a peg or cylinder, he will cut two shallow trenches 6" wide and 3' long indicating the directions of the forward and back stations. (h) A line-clearing squad should, in fairly open country, embed at least 15 stations, and clear the lines between, per diem. (i) The line-clearing party should be sent out 10 days ahead of the theodolite traverse surveyor, or the surveyor will be delayed in his work through an insufficient number of lines being cleared.

405. Supply of Men and Instruments.—(a) The surveyor should be supplied with a trace of the existing map, preferably that of the 4-inch revenue survey, if there be any, of the area. (b) The surveyor's squad should consist of 7 men :—2 chainmen, 3 flagmen, 1 man for theodolite and stand, 1 spare man. (c) The line-clearer must supply the surveyor with a man from his squad, who, knowing the positions of all the stations fixed, will act as the surveyor's forward flagman: this procedure adds greatly to the rapidity of the work. Similarly, the surveyor will temporarily detach a man from his squad to replace the one sent from the line-clearing party. (d) The surveyor will be supplied with the following articles :—one 5"-theodolite (with vertical arcs) and stand, 2 Gunter's chains, 3 flags, a supply of blank field-book forms (specimen given below), ink; pens, etc. (e) Two Gunter's chains are necessary, one being required as a standard chain against which the surveyor should check his "running or working chain" daily.

406. Method of Reading and Recording Angles and Distances.—The following method is usually adopted by a revenue survey party, of the Survey of India, in observing and recording the

angles, and measuring the distances between stations in the traverse survey of a village or disputed boundary:—Let A be the first station at which the surveyor sets up his theodolite (Fig. 219); B and H his

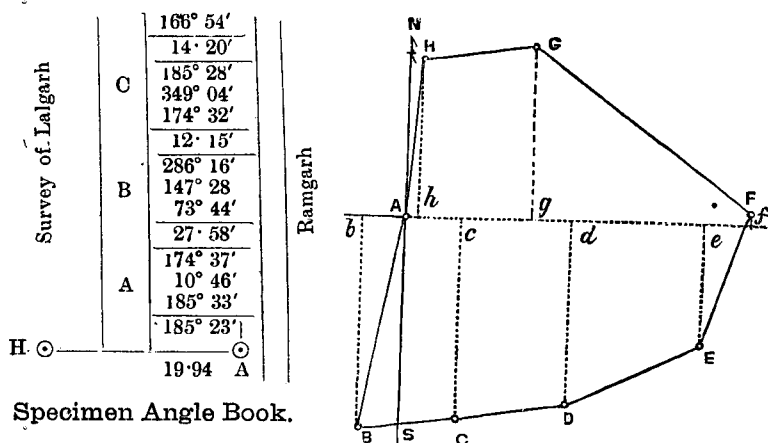


Fig. 219.—Surveying with the Theodolite, Gale's System.

forward and back stations respectively. H, of course, is the last station on the boundary of the village he is traversing. After setting up and levelling his instrument, the surveyor sets and clamps his vernier, A, to read 0 or 360°, he then by unclamping the lower plate swings his telescope round till the flag-pole, H, comes into the field of view, when he clamps the lower plate and intersects the foot of the flag, or a point, as low as possible on the pole, by means of the lower tangent-screw, having previously instructed his back flagman to place the point of the pole exactly on the centre of the station mark and to keep the staff truly perpendicular. He now unclamps the vernier-plate and, in a similar manner, intersects the pole at B, he now reads the angle HAB off his vernier, A, only, which, let us say, gives a reading of 185° 23' 0". This angle he at once enters *in ink* in his field-book (see specimen above); without touching the vernier-plate, clamp, he releases the lower plate and bisects the flag, H, again with tangent-screw of the lower plate. If the vernier, A, is now still reading 185° 23' 0", he again unclamps the vernier-plate and bisects the flag, B, when his vernier, A, should read the double of 185° 23' 0" or 370° 46' 0" or 10° 46' 0" after one complete revolution of the horizontal circle, which latter angle he records in his field-book (vide specimen). Unclamping the vernier-plate, he again sets and clamps his vernier, A, to 0°, then releasing the lower plate he bisects the flag, B, when he will release the vernier-plate only, and swing the telescope round on to flag, H, which he bisects; he has now observed the angle, BAH, the exterior angle, *i.e.*, the complement of the angle HAB, he reads his vernier, A, and finds that it gives him a reading of 174° 37', *i.e.*, 360° *minus* 185° 23'. An error of 1' or under

is allowed in the readings of these three angles. If the difference between the readings of two angles exceeds the above allowance, the observation must be repeated till the error is within the prescribed limit.

The Surveyor may now put the theodolite, into its box by himself and carry it to the next station, *B*; on no account is the theodolite to be carried on its stand from station to station. In the meantime, the tindal and his chainman will have measured the distance, *HA*, the tindal entering the same in his field-book, the surveyor will count the number of arrows in the possession of the tindal and also see that the remaining number of links is correct before entering the distance in own field-book. The tindal with his chainman will now proceed to measure the distance from *A* to *B*, the surveyor accompanying them to see that the chain is properly stretched and aligned on station, *B*. In chaining across bad ground, great care must be taken to "break chain" where necessary. The flagman at *H* will now move up to station, *A*, placing his flag over the station mark. On the surveyor reaching station, *B*, his flagman there will move on to the next station, *C*. The surveyor will again set up his theodolite at *B* centering it truly over the peg and proceed to observe the angle as before.

In chaining across water no man is to be allowed to take the chain into water beyond his depth, if it be not possible to measure the distance by means of a long rope stretched across, the method of triangulation must be resorted to for determining the distance (see Arts. 240, 242, Probs. Case III (*a* to *g*), pp. 140—145).

407. Magnetic Bearing.—While the instrument is set up at *A* the magnetic bearing of the line, *AB*, is taken (in the same way as described in Art. 398 on p. 220); the vernier, *A*, gives the bearing to be $190^{\circ} 20'$.

In the survey of a single circuit, the magnetic bearing will serve our purpose as well as the true bearing, for the calculation of the latitudes and departures and the independent co ordinates derived therefrom will be equally correct, the difference being that the traverse plot will not be in its true position with reference to the true meridian, but this displacement in the case of a single polygon will not affect its figure or area.

408. True Bearing.—In the case of a main-circuit containing a large number of minor-circuits, covering several hundred square miles, it is absolutely necessary to observe the true bearing astronomically at the first Station of the District Survey.

409. Method Observing Inward Angles.—In the traverse survey of a village boundary the surveyor should always work round left arm inwards, *i.e.*, he will observe the angle interior to the village of survey and not the exterior angle, as, in the latter case, the angle would have to be subtracted from 360° in order to obtain the interior angle which is the one used for "setting up" and computing the traverse, or in other words, in measuring the angles, the telescope is first directed to the left-hand object and then to the right-hand one, so that the verniers move in the direction in which the horizontal

limb is graduated. If the telescope is directed to the right-hand object first, the angle read must be subtracted from 360° in order to give the inward angle required. The interior angles afford a means of testing the accuracy of the angular work of a survey, as *the interior angles of the polygon together with 4 right angles should be equal to twice as many right angles as the figure has sides.*

410. A Surveyor's Daily Outturn of Work should average about 20 angles. The amount of chaining done must, however, be taken into consideration, *i.e.*, if the outturn of chaining be large, a smaller number of angles may be accepted than where the outturn of chaining is small owing to short lines between stations. As a rule, 200 to 300 chains per day is a fair amount of chaining in connection with the number of angles referred to above. Should the area under survey be much cut up by water, the outturn will be smaller, as triangulation across water delays the work. In fairly open country a good surveyor should average about 15 square miles of traverse survey per mensem.

411. The above method of surveying (Art. 406) may appear simpler, but the methods given before (p.p. 220-224) of observing the bearings of the successive lines of a traverse with some fixed meridian line are easier and better. The advantage gained by this bearing-method is that the theodolite has not to be set to zero again and again, which wears that part, and does not divide the errors of graduation; and is actually shorter in working as the setting to zero before taking every angle is avoided. When the traverse is closed by this method it is a check on the accuracy of the survey to observe before the commencement of the circuit, the bearing of some distinct object, known then as "**Referring Mark.**" On return to the point of origin the telescope when directed on to the referring mark should read the same bearing. The sum of the interior angles is not required as a check; on closing the traverse the check of the referring mark is read, and the error in that shows the error in the circuit, but if the other check is required the interior angles can be easily calculated from the bearings.

412. Geometrical Tests of the Correct Measurement of a Traverse.—There are two geometrical tests of the correct measurement of a traverse: One *angular* and the other *linear*, (Art. 351). The angular test is satisfied as under:—In Fig. 219 the circuit is an octagon, therefore, the sum of the included angles must equal $1080^\circ = [180^\circ \times 8 - 360^\circ]$ by rule in Art. 409. In the same manner *the sum of the exterior angles of a polygon, diminished by 4 right angles, being equal to twice as many right angles as the figure has sides*, in the octagon illustrated, the exterior angles should equal to 20 right angles or 1800° ; and the inward angles which can be found in each case by subtracting the observed exterior angle from 360° should be equal to 12 right angles or 1080° . If this be so, it is a proof that the angles have been correctly measured. This however will rarely occur in practice, there will always be some discrepancy, but if the difference be more than the *permissible error* in an ordinary survey (which for single observations with a 5-inch

instrument may be taken at 30" for each angle), they should be observed a second time, the final error being evenly distributed between them. If the traverse lines when plotted with a protractor do not close, the measurements must be inaccurate and should be repeated. In theodolite traverses error generally takes place in the linear measurement of distances with the chain. This is minimised to a great extent by taking the mean of two measurements taken with two chains carefully compared with standard rods, or with a steel-band. If great care is taken in chaining, the error in angular measurements will be considerably lessened.

413. Calculation of Bearings.—Next by means of the corrected angles obtain the bearings of the several sides by means of the following rule: *To the first bearing add the next inward angle. If the sum exceeds 180° , deduct that amount from it. The remainder will be the second bearing. To this, add the next inward angle, and proceed as before. When the sum of the bearing and the next inward angle is less than 180° , it must be increased by 180° . If after deducting 180° , the remainder exceeds 360° , it must be diminished by that amount to give the required bearing.* This rule may be reduced to a formula.— $F = P + a \pm 180^\circ$; using + sign, when $P + a < 180^\circ$, and - sign, when $P + a > 180^\circ$; where F = Bearing to be obtained of the following station, P = Bearing at a preceding station, and a = the observed inward angle at the station.

414. Cardinal Directions.—The deduced bearings show at once the cardinal direction of the line. If the bearing is, between 0° and 90° , the line must be in the N. E. quadrant; if between 90° and 180° , in the S. E. quadrant; if between 180° and 270° , in the S. W. quadrant; if between 270° and 360° , in the N. W. quadrant. Thus the line, AB, whose bearing is $190^\circ 20'$, is in the S. W. quadrant (Fig. 219).

415. Calculation of Reduced Bearings.—From the bearings reduced bearings, *i.e.*, angles adapted to trigonometrical work, are to be obtained.

416. Calculation of Latitudes and Departures.—From the reduced bearings and distances the traverse sheet is compiled, then the latitudes and departures are computed by multiplying the cosine of the reduced bearing by the distance for latitude, and by multiplying the sine of the reduced bearing by the distance for departure.

417. The Linear Test is this:—In an ideal closed traverse the sum of the Northings will equal the sum of the Southings, and the sum of the Eastings will equal the sum of the Westings. Thus the calculated co-ordinates are invaluable for testing the accuracy of a closed traverse. It is clear, that when a surveyor makes a complete circuit returning to the point of origin, he has gone exactly so far to the north as he has to the south, and as far to the east as to the west. Hence, if his survey is correct the algebraical sum of the X -co-ordinates and that of the Y -co-ordinates should each be $= 0$.

418. Balancing of Traverses.—In practice difference is also observed between the sums of the latitudes as well as of the

departures. These differences are unavoidable even if much attention is paid while carrying on angular and linear measurements. If there be a slight error, the latitudes and departures must be corrected before plotting, so that their sums shall be equal in each case. The error is distributed among the lines in proportion to their length, the balancing being effected according to rule (due to Bowditch) given on p. 205. On the Revenue Survey, the amount of error, allowed, is 1 link in 10 chains, additive or subtractive, from the sums of the Northings and Southings to correct the latitude and from the sums of the Eastings and Westings to correct the departure, and if this be exceeded the work is rejected. The amount of error, if within the prescribed limit, may be applied to the separate co-ordinates according to the above rule (p. 205). The corrections are entered in their respective columns headed *cor.* in the Traverse Computation Form, + or—as required. Having found the several corrections for each of the latitudes and departures, add them together severally, and see whether their total agrees with the whole error, and if so proceed to allot the corrections. If the error be an excess of Northings, subtract each correction from its collateral Northing or add it to the collateral Southing; if an excess of Easting, add to the Westing and subtract from the Easting; the sums of the corrected latitudes and departures will then be found to agree. In practice an approximation is sufficient, and when the error is below the maximum allowed, it serves all practical purposes to apply a link here and there against the separate co-ordinates of the longest lines.

419. Closing of a Circuit.—After the required corrections have been made to the angular and linear work, the circuit is said to be *closed*.

420. Procedure of the Indian Survey.—The traverses are executed in minor-circuits following the periphery of each village and in major-circuits comprising groups of several villages; the former are done with 4" to 6"-theodolites and a single chain, the latter with 7" to 10"-theodolites and a pair of chains, which are compared frequently with a standard. The main-circuits are connected with every station of the principal triangulation within reach. The meridian of origin is determined by astronomical observations; the angle at the origin between the meridian and the next station is measured, and then at each of the successive stations the angle between the immediately preceding and following stations; summing these together, the inclinations of the lines between the stations to the meridian of origin are successively determined. The rest of the operations are as tabulated on p. 232. Besides the linear and angular tests the astronomical test to be applied is this: at any station of the traverse, the azimuth of a referring mark may be determined by astronomical observations; the inclination of the line between the station and the referring mark to the meridian of the origin is given by the traverse; the two should differ by the convergency of the meridians of the station and the origin. In practice the angles of the traverse are usually adjusted to satisfy their special geometrical and astronomical tests in the first instance,

and then the co-ordinates of the stations are calculated and adjusted by corrections applied to the longest, that the angles may be least disturbed, as no further corrections are given them.

421. The Traverse Computation Form.—The annexed form of an actual village-circuit (Fig. 219) with a 5"-theodolite and chain shows the method of recording observed inward angles, recorded (observed), deduced, and reduced bearings, the measured distances, the calculated latitudes and departures of the stations with their corrections as reduced to a single meridian (p. 232, q.v.).

Col. 1.—Contains the figures or letters representing the stations of the survey taken from the Angle-book in page 226 beginning and ending with the starting point 1 or *A*.

Col. 2.—Contains the observed inward angles entered opposite the stations where they are observed. They are taken from field-book (p. 226). The space opposite letter, *A*, is left blank and the observed inward angles are entered beginning with the *second* angle at *B* between *A* and *C*, and ending with the *first* angle against *A* at the bottom of the column. Next add up all the angles ($1080^{\circ} 1'$) which in the octagon, should, by the rule in Art. 409, be $8 \times 180^{\circ} - 360^{\circ} = 1080^{\circ}$.

Col. 3.—Contains the corrections to be made in those angles so that the first condition in Art. 351, p. 195, may be fulfilled. The sum of the angles in Col. 2 is in excess by 1, therefore a *minus* correction of 1' is applied to the angle at station *C* formed by the shortest lines.

Col. 4.—Examples of the Deduction of Bearings from Observed Angles.—This column shows the cardinal directions, the observed and deduced bearings. The observed bearing of the line *AB*, $190^{\circ} 20'$, is entered opposite *B* in the column "Bearings."

The bearings of the other lines are obtained according to the rule in Art. 413 (p. 229), and placed at the end of the station lines in the same column. They are deduced as follows:—

Bearing of	<i>AB</i>	$190^{\circ} 20' +$	inward $\angle$	$73^{\circ} 44' - 180^{\circ} =$	$84^{\circ} 4'$	bearing of	<i>BC</i>
"	"	<i>BC</i>	$84^{\circ} 4' +$	"	"	$174^{\circ} 31' - 180^{\circ} =$	$78^{\circ} 35'$
"	"	<i>CD</i>	$78^{\circ} 35' +$	"	"	$166^{\circ} 54' - 180^{\circ} =$	$65^{\circ} 29'$
"	"	<i>DE</i>	$65^{\circ} 29' +$	"	"	$132^{\circ} 21' - 180^{\circ} =$	$17^{\circ} 50'$
"	"	<i>EF</i>	$17^{\circ} 50' +$	"	"	$108^{\circ} 53' + 180^{\circ} =$	$306^{\circ} 43'$
"	"	<i>FG</i>	$306^{\circ} 43' +$	"	"	$136^{\circ} 59' - 180^{\circ} =$	$263^{\circ} 42'$
"	"	<i>GH</i>	$263^{\circ} 42' +$	"	"	$101^{\circ} 15' - 180^{\circ} =$	$184^{\circ} 57'$
"	"	<i>HA</i>	$184^{\circ} 57' +$	"	"	$185^{\circ} 23' - 180^{\circ} =$	$190^{\circ} 20'$
						"	<i>AB</i>

Proof.—The deduced bearing of *AB*, $190^{\circ} 20'$, agreeing with its observed bearing shows that the calculations are correct.

Col. 5.—Examples of Reduction of Bearings.—This column contains the reduced bearings, *i.e.*, the angles adapted to trigonometrical work. In modern theodolites the circle is divided from 0° to 360° , reading clockwise; and the method of whole circle readings is adopted. As most of the trigonometrical tables only give functions of angles from 0° to 90° (Shortrede's tables which are scarce give augments from 0° to 360°) with ordinary tables to

TRAVERSE COMPUTATION FORM (Fig. 219) AS DESCRIBED IN ART. 421, P. 231.

1	2	3	4		5	6	7	a	8	a	9	a	10	a	11	12
Stations.	Observed Inward Angles.	Corrections.	Bearings.	Reduced Bearings.	Distances Measured in Chains.	DIFFERENCE OF LATITUDE AND DEPARTURE FOR EACH DISTANCE.									DIFFERENCE OF LATITUDE AND DEPARTURE OF EACH STATION REFERRED TO MERIDIAN OF FIRST STATION.	
						Latitude = Col. 6 $\times$ Cos. 5.				Departure = Col. 6 $\times$ Sin. 5.				Latitude.		Departure.
						North.	Cor. +	South.	Cor. -	East.	Cor. -	West.	Cor. +	North or South.	East or West.	
A	...	...	S	...	...	...	...	27.13	.011	...	...	4.94	.025	S	27.11	4.96
B	73°44'	...	N	190°20'	10°20'	27.58	...	...	...	...	...	...	...	S	27.11	4.96
C	174°32'	-1'	N	84°04'	84°04'	12.15	1.26	.004	...	12.08	.011	...	...	S	25.85	7.10
D	166°54'	...	N	78°35'	78°35'	14.20	2.81	.005	...	13.91	.012	...	...	S	23.04	21.00
E	132°21'	...	N	65°29'	65°29'	17.50	7.30	.007	...	16.00	.016	...	...	S	15.73	36.98
F	108°53'	...	N	17°50'	17°50'	18.06	17.19	.007	...	5.53	.016	...	...	N	1.46	42.50
G	136°59'	...	N	306°43'	53°17'	33.32	19.92	.013	...	...	...	26.70	.030	N	21.39	15.77
H	101°15'	...	N	263°42'	83°42'	14.11	...	1.55	.005	...	...	14.02	.012	N	19.85	1.73
A	185°23'	...	S	184°57'	4°57'	19.94	...	19.86	.008	...	...	1.72	.018	...	0.00	0.00
B	—	...	S	190°20'	W	...	...	...	...	...	...	...	...	...	...	...
Total,	1080°01'	-1'	...	...	...	156.86	48.48	+.036	48.54	-.024	47.52	-.055	47.38	+	...	...
8								.024	48.48		47.38		.055			
Proof of Angles, $8 \times 180 - 360^\circ = 1080^\circ$								.06	.06		.14		.14			

NOTE.—If the methods of traverse by bearings (page 220) is adopted the angles in Col. 4 are obtained at once from observations in the field, and the inward angles of Col. 2 have to be deduced if necessary from the observed bearings (p. 243, q.v.).

obtain the sine or cosine of an angle greater than 90° a preliminary calculation has to be made which is an objection to whole circle readings. When using ordinary trigonometrical tables for *bearings less than 90°* , enter tables with *bearings direct*; for *bearings greater than 90° but less than 180°* , with 180° —bearing; for *those greater than 180° but less than 270°* with bearing— 180° ; and for *those greater than 270° but less than 360°* , with 360° —bearing. Thus at station, *B*, the reduced bearing is $190^\circ 20' - 180^\circ = 10^\circ 20'$; At stations, *C, D, E, F*, the bearings and the reduced bearings are the same; at *G* the reduced bearing is $360^\circ - 306^\circ 43' = 53^\circ 17'$; at *H* the reduced bearing is $263^\circ 42' - 180^\circ = 83^\circ 42'$; at station, *A*, the reduced bearing is $184^\circ 57' - 180^\circ = 4^\circ 57'$.

Col. 6—Contains the distances as measured in the field.

Putting up.—The recording of the angles, bearings, and distances of a circuit in the Traverse Computation Form is called “putting up” a circuit.

Cols. 7 and 8.—**Example of Calculation of Latitudes**.—These columns contain the distances on the meridian between every two stations. Referring to Fig. 219, *AB* is a given distance and *BAS* ($= A B b$) its reduced bearing, *Bb* is the latitude of *B* or its distance from *A* along the meridian. By trigonometry *Bb* is found thus: *Bb* (latitude) $= AB$ (distance) $\times \cos. ABb$ (bearing),

or, $\log \text{latitude } Bb = \log \text{distance } AB + \text{Log cos. bearing } ABb$.

To find the co-ordinate (latitude, in this case, Southing) of station *B*.—

$$\log \text{distance } 27.58 = 1.4405943$$

$$\text{L. cos. } 10^\circ 20' = 9.9928984$$

$$\log 27.133 = 1.4334927$$

$\therefore$ Latitude *Bb* = 27.13 chains (Southing).

Similarly other latitudes are obtained. In making calculations with logarithms, it must be borne in mind that in tables of logarithmic sines and cosines, the logarithm of the assumed radius is 10,000,000. Hence in the preceding calculations, the log radius 10 is deducted in order to give the log. of the natural number sought.

Cols. 9 and 10.—**Example of Calculation of Departures**.

—These columns contain the departure of each station from the meridian of the preceding one. Referring to Fig. 219, *AB* is a given distance, and *BAS* or *ABb* its reduced bearing, *Ab* is the departure of *B* or its distance from *A* along the $\perp$ to the meridian. By trigonometry *Ab* is obtained thus:—

Ab (departure) $= AB$ (distance) $\times \sin. ABb$ (bearing), or,

$\log \text{departure } Ab = \log \text{distance } AB + \text{Log sin. Bearing } ABb$.

To find the co-ordinate (departure, in this case, Westing) of station *B*.—

$$\log \text{distance } 27.58 = 1.4405943$$

$$\text{L. sin. } 10^\circ 20' = 9.2537609$$

$$\log 4.9478 = 0.6943552$$

$\therefore$ Departure *Ab* = 4.94 chains (Westing).

Similarly other departures are calculated.

Col. 11.—Calculation of Difference of Latitude and Departure of each Station referred to Meridian of First Station.—Col. 11 is obtained in the following way from Columns 7 and 8 :—The difference in latitude between stations A and B will be the total southing made by the line, AB, therefore, $27.13 - .011 = 27.119$ chains is placed in column 11. Again station C is $1.26 + .004$ north of station, B, therefore, it will be $27.119 - 1.264 = 25.855$ chains south of station, A. The station, D, is $2.81 + .005$ north of station C, therefore, it will be $25.855 - 2.815 = 23.040$ chains south of station A. Similarly station, E, is 15.733 chains south of station, A. Station, F, is $17.19 + .007 = 17.197$ chains north of station, E, therefore, it will be $17.197 - 15.733 = 1.464$ chains north of A. Again, station, G, is $19.92 + .013 = 19.933$ chains north of station, F, therefore, it will be $1.464 + 19.933 = 21.397$ chains north of A. Station, H, is $1.55 - .005 = 1.545$ chains south of G, therefore, it will be $21.397 - 1.545 = 19.852$ north of A. Station, A, is $19.86 - .008 = 19.852$ chains south of station, H, therefore, it will be at $19.852 - 19.852 = 0$. The work proves itself by the final result being zero. In col. 11 the results are entered to 2 places of decimals.

Col. 12 is obtained from Cols. 9 and 10 in a manner exactly similar to Col. 11.

422. Traverse Tables:—Tables which show by inspection the amount of the latitude and departure for any bearing and distance are called Traverse Tables, because by their aid the resolution of traverses is effected without calculation, the Logarithmic Tables being dispensed with. The Traverse Tables contain the sides of a right-angled $\triangle$ whose hypotenuse is 1 and are used to calculate the *difference of latitude and departure of a traverse, the reduced bearing* to the nearest minute being known; the latitude being the side of a right-angled $\triangle$ adjacent to the reduced bearing, and the departure that opposite to it. The latitudes and departures are taken out to 4 or 5 places of decimals for a distance of 1; hence for a given distance greater than 1, the given latitude and departure are to be multiplied by the given distance in chains, feet, or any other unit, for the latitude and departure required, in corresponding chains, feet, or other unit.

The table has four columns, as given in Part. II. of this work, in the first column the degrees from 0° to 45° increasing by $1'$ are written; in the second, Lat. or N.S.; in the 3rd, Dep. or E.W. and in the 4th, the degrees from 90° to 45° . At the foot of the page in 2nd column, instead of Lat. or N. S. is written Dep. or E. W. and in the 3rd column, instead of Dep. or E. W., Lat. or N. S., so that when the angle exceeds 45° , the Lats. and Deps. are read upward from the bottom. To obtain the latitude and departure for any bearing, look for the degrees and minutes in the first column and find out the Lats. and Deps. horizontally in the 2nd and 3rd columns.

There are many other tables such as Boileau's, Gurden's, Louis', Caunt's, and Crellin's. Boileau's and Louis' tables are calculated for every minute of bearing to 5 places of decimals for

distances from 1 to 10; Gurden's for every minute of bearing to 4 decimal places for distances from 1 to 100; Caunt's for every minute of bearing to four decimal places for distances of 1 to 10; Crellin's are calculated for every minute of bearing to 4 decimal places. The Traverse Tables given in Chambers's Mathematical Tables are calculated only for every degree of bearing to one place of decimal. This though valuable for problems in navigation are useless for surveying purposes.

Example of the Use of the Traverse Tables:—What is the lat. and dep. for 27.58 chains, the reduced bearing being $10^{\circ} 20'$? Here for $10^{\circ} 20'$, we have from the Table, lat. 9838 ; dep. 1794 . Multiply these by the given distance thus:— $9838 \times 27.58 = 27133$ lat; and $1794 \times 27.58 = 4947$ dep. The same results are given before on p. 233, Art 421.

423. Placing the Traverses.—Having calculated the latitudes and departures, place them in their respective columns, North, South, East, West, as shown by the cardinal directions of the traverse lines in Col. 4. For example—Col. 4 shows that the line, AB, bears S. W., therefore the latitude is South, and the departure West, and they are entered in their respective Cols. 8 and 10.

424. Examples of Use of Boileau's or Louis' Tables.—Given the bearing of a line, N. $32^{\circ} 15'$ E., and its distance 176 links, find its latitude and departure. From the Table headed 32° , and from the section $15'$, find out the latitudes and departures severally for the *hundreds, tens and units* removing the decimal point in each case as many places to the right as the figures in each separated portion of the distance exceed those in the corresponding numbers in the tabular distance columns. The distance 176 links is split up into 100, 70, 6 and the traverses for each obtained separately, thus:—

Reduced bearing.	Distance.	Latitude.	Departure.
N. $32^{\circ} 15'$ E. ...	100	84.5727	53.3614
	70	59.2009	37.3530
	6	5.0743	3.2016
	176	148.8479	93.9160

Gurden's Tables give the latitude and departure in the above example with greater accuracy thus:—

Reduced bearing.	Distance.	Latitude.	Departure.
N. $32^{\circ} 15'$ E. ...	100	84.5728	53.3615
	76	64.2753	40.5547
	176	148.8481	93.9162

425. Traverses to be worked Twice.—In important surveys work out the traverses both by means of logarithms and Traverse Tables, so as to obtain an independent verification and check. When they agree enter them in the Traverse Computation Form.

426. Examples of Adjustment of Closing Error by Calculation.—In columns a, a, a, a, the difference between the Northings and Southings is '06, and between Eastings and Westings '14 chain. The length of the Closing Error = $\sqrt{.06^2 + .14^2} = .152$ chain. The total distance measured is 156·86 chains, so that the error does not amount to more than 1 in 1032. Therefore, the survey was within the allowable limit of error. Now these closing errors are to be apportioned according to rule in Art. 353 on p. 199 thus :

$$\left. \begin{array}{l} 156\cdot86 : .06 = 27\cdot58 : .011 \\ \quad \quad \quad = 12\cdot15 : .005 \\ \quad \quad \quad = 14\cdot20 : .005 \end{array} \right\} \text{ for the Northings and Southings,}$$

and so on.

$$\left. \begin{array}{l} \text{And } 156\cdot86 : .14 = 27\cdot58 : .025 \\ \quad \quad \quad = 12\cdot15 : .011 \\ \quad \quad \quad = 14\cdot20 : .012 \end{array} \right\} \text{ for the Eastings and Westings,}$$

and so on.

From the Form (p. 232) it will be seen that the sum of several corrections amounts to '06 = '036 + '024 chains in the Northings and Southings; and to '14 = '085 + '055 chains in the Eastings and Westings. This is the method of adjusting the error in theory, but in practice however it is not practicable, where, hundreds of circuits have to be dealt with to apportion traverse errors by the rule given on p. 199. The total error being known, it serves all practical purposes to apply a link here and there against the traverses of the longest lines. After the corrections are added or subtracted as the case may be it will be found that the Northings = the Southings, and the Eastings = the Westings; thus proving that the work is right and the 2nd geometrical test, *i.e.*, the linear is fulfilled by the corrections.

427. The Corrected Distances and Bearings.—The correction of the latitudes and departures evidently changes the bearings and distances of the lines, AB, BC, CD, etc. These may be measured with the protractor and scaled after the work has been plotted from the corrected latitudes and departures, or if greater accuracy is required, the new lengths and bearings may be calculated from the corrected latitudes and departures thus:—

$$\left. \begin{array}{l} \text{The Tangent of Angle made with} \\ \text{Meridian} \end{array} \right\} = \frac{\text{Corrected Departure.}}{\text{Corrected Latitude.}}$$

$$\text{The Corrected Distance} = \text{Latitude} \div \cos. \text{ Bearing.}$$

Thus for side, A B, we have $4\cdot965 \text{ W} \div 27\cdot119 \text{ S} = \text{natural tangent} = .18308$; opposite which in the Table of Natural Tangents is the required angle $10^\circ 23'$. The corrected bearing, therefore, is $\text{S } 190^\circ 23' \text{ W}$.

Again: for the correct length of any side, from the Table of Nat. Cosines take the cosine opposite to the angle of the corrected bearing; divide the corrected latitude (N or S) of the side by the cosine. Thus for the distance of the side, AB, we find opposite $10^\circ 23'$, the cosine .98362. And $\text{lat. } 27\cdot119 \div \cos. 10^\circ 23' = 27\cdot119 \div .98362 = 27\cdot56$ the required distance.

The following table contains all the corrections of the foregoing survey; therefore, if the bearings and distances are correctly plotted, they will close perfectly. The student is advised to practise doing this, as well as, dividing the plot into triangles, and computing the content. In this manner he will soon learn what degree of care is necessary to ensure accurate results. The above calculations are not carried to 7 decimal places but show the way in which the calculations are made.

Stations. •	Reduced Corrected Bearings.	Corrected Bearings.			Corrected Distances in Chains.
A	...	...	...	...	...
B	10°23'	S	190°23'	W	27.56
C	84°02'	N	84°02'	E	12.12
D	78°33'	N	78°33'	E	14.18
E	65°26'	N	65°26'	E	17.55
F	17°49'	N	17°49'	E	18.06
G	53°17'	N	306°43'	W	33.34
H	83°43'	N	263°43'	W	14.08
A	5°00'	S	185°00'	W	19.93

428. **Closed Traverse Survey with a Cooke's 5"-Theodolite and a 66' Steel-band.**—With a Cooke's 5"-theodolite and a 66' Steel-band the closing error in a closed traverse should not exceed 1 in 30,000. For an example of a Survey of this kind the student is referred to the paper in the *Minutes of Proceedings Inst. C. E., page 211, Vol. XCV.*, "The Practice of Surveying in the Australasian Colonies."

429. **Method of Plotting by Latitudes and Departures.**—Plotting the polygon or other figure on paper with protractor and scale is an untrustworthy method. Hence, when the survey is of large size, plotting by co-ordinates is preferable, for, the difference of latitudes and departures greatly facilitates plotting; it obviates the use of the protractor, since the points can be projected with plotting scale, T-square, set-square, and the work can be done more correctly than with the protractor. The differences of latitude and departure given in Cols 11 and 12 are used in plotting the survey in the following way:—First draw a meridian line *N.S.* (Fig. 219), and from the point, *A*, fixed on it draw a line *Af* perpendicular to the meridian. Along this line set off with a plotting scale, the distances *Ab*=4.96 chains, *Ac*=7.10 chains, *Ad*=21.00 chains, *Ae*=36.98 chains, *Af*=42.50 chains, and so on, until all the departures are laid off taking care to distinguish between Eastings and Westings. Through the points *b, c, d, e, etc.*, draw lines parallel to the meridian, and on these lines set off the respective latitudes taking care to distinguish between Northings and Southings; *v.g.*, from *b* set off *bB*=27.11 chains, from *h* set off *hH*, 19.85 chains, and so on, by the same scale. Thus the stations, *B* and *H*, and similarly all the other stations are obtained,

It should be understood that the working meridian is made to coincide with the *N.S.* line for the sake of convenience and that any line would serve the purpose. This method of plotting is by far the most perfect, and the least liable to error of any that has been contrived. It may appear to entail more labor than the ordinary method of angular protraction, or protraction by bearings, owing to the computations required, but these are easily made by the aid of the Traverse Tables, and besides the computations serve as a check on the accuracy of the work. To plot by this method the working meridian should be made parallel to the edge of the paper and this the North line generally is. Cross Section papers (Fig. 203 on p. 200) will be found very convenient for this purpose.

430. Care to be taken in Plotting : Co-ordinate Squares :—

As the paper on which the survey is plotted does not expand and contract uniformly, it will be found, if the lengths be continuously scaled off, that one sheet will not agree with another, nor the earlier measurements with the later. It is, therefore, a good plan to draw a horizontal and a vertical scale on each sheet, before the plotting work is taken in hand, and to divide the paper into squares not larger than about 3" across, the sides of which represent a given number of chains, feet, or other unit, on the scale: thus the measurements by scale can be limited to a length which cannot be greater than half the side of a single square, and the relation of

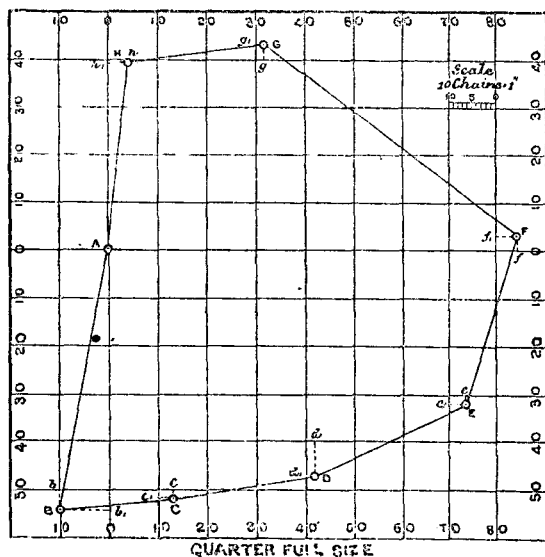


Fig. 220.—Co-ordinate Square.

adopted if the circuit be too large to admit of the co-ordinates being laid off from the initial meridian and perpendicular. To illustrate this, Fig. 220 is selected. It represents the paper ruled

every part of the plot to the scale inscribed on the paper is maintained throughout, notwithstanding a material alteration in the size and shape of the sheet. The squares must be drawn with exactitude, their dimensions, parallelism, and verticality being checked with all possible care; they should be ruled in faint carmine and any line drawn inaccurate may be easily removed with chloride of lime. This plan of dividing the paper into squares should always be

into squares of 1" sides, and if the traverse were to be plotted to a scale of 10 chains = 1", each square would therefore represent $10 \times 10 = 100$ sq. chains or 10 acres. Now, if it were required to plot the circuit in Fig. 219 to the scale of 10 chains = 1", but with all the distances in chains doubled as under :—

Distances Doubled in Chains.	DIFFERENCE OF LATITUDE AND DEPARTURE OF EACH STATION REFERRED TO MERIDIAN OF FIRST STATION DOUBLED.			
	Latitudes.		Departures.	
AB = 55'16	S	Bb = 54'22	Ab = 9'92	W
BC = 24'30	S	Cc = 51'70	Ac = 14'20	E
CD = 28'40	S	Dd = 46'08	Ad = 42'00	E
DE = 35'00	S	Ee = 31'46	Ae = 73'96	E
EF = 36'12	N	Ff = 2'92	Af = 85'00	E
FG = 66'64	N	Gg = 42'78	Ag = 31'54	E
GH = 28'22	N	Hh = 39'70	Ah = 3'46	E
HA = 39'88				

Then go on to plot the work in this way, having obtained a paper ruled into squares of 1" sides :—choose first the station point, A, at the intersection of two lines on a convenient part of the paper ; the *N. S.* line through it is, therefore, the meridian of origin, and the East and West line the initial perpendicular. Mark the ends of these lines with 0 (zero) ; at the ends of each line parallel to the meridian of origin and on either side of it, write successively 10, 20, 30, etc. ; also write successively 10, 20, 30, etc., at the ends of the lines parallel to the initial perpendicular on both sides of it ; then draw the scale of 10 chains = 1" at a spare corner of the paper which is then ready for plotting. To plot B, lay off from the given scale, bB = 4'22 (= 50 + 4'22) south of the departure (perpendicular) marked 50, and Bb₁ = 9'92 west of the meridian of origin ; for C lay off Cc = 1'70 (= 50 + 1'70) south of the departure marked 50, and Cc₁ = 4'20 (= 10 + 4'20), east of meridian marked 10 ; and so on for the rest, always scaling off from the meridian and the perpendicular nearest the station on the side of the initial meridian and perpendicular.

By this method also the area of the circuit can be taken out with facility. Count the number of squares which coincide with the figure, measure the fractional parts of squares, add the excess and deduct the deficiency, and the required area is then obtained. The only part liable to error is the detached portions of the whole area consisting of the fractional parts of the squares.

431. Co-ordinatograph.—Figs. 221 and 222 show the instrument. It is used for accurately marking the stations on survey plans and the points lying between intersections determined by co-

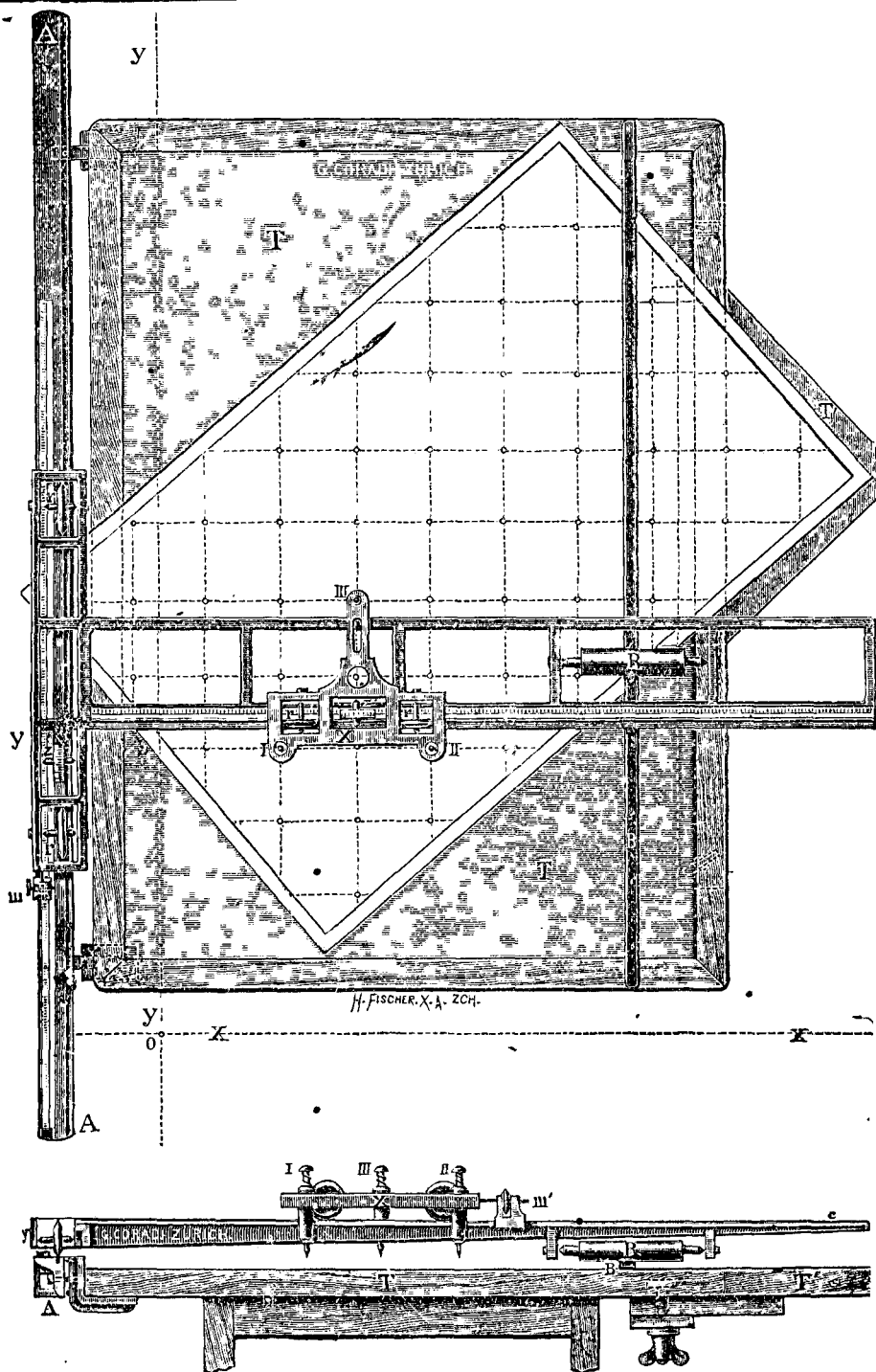
ordinates. On the board, *T*, the abscissa rule, *AA*, of 1 m 50 (60") length is fixed, in the groove of which the abscissa carriage, *P*, on rollers can be displaced 1 m (40"). This carriage rests with the cylindrical roller, *R*, as a third point of support on the board. The *P*—carriage carries the ordinate rule, in the groove of which at a true right angle to the movement of the former carriage, the *X*—carriage can be moved a distance of 0.9 metre (36"). This carriage carries 3 vertical pricking points, I, II, and III, which by means of a spring, are kept suspended in their sleeves. The points of I and II touch the same ordinate, and have a distance of 200 mm (8") in the *X*—direction. Point III has an ordinate distance of 100 mm (4") from I and II and an abscissa distance of 200 mm (8"). With the apparatus is supplied a drawing pen which can be so adjusted that it closely fits in the sleeves of the points, I, II and III, and has a free fall, so that its writing plane is either parallel to the ordinates or to the abscissae, the line which it draws passes exactly through the stations, so that the latter need not be marked by pricking. The two scales have graduations for 2 ratios (1 : 1000) and 1 : 500 (1" = 100' and 1" = 50'), or any other desired ratios. Each scale is provided with accurate tothing in which measuring wheels engage which carry a graduated circle on which the subdivisions of a metre or inch can be read in sevenfold enlargement so that, for instance, an actual displacement of the carriages of $\frac{1}{10}$ mm appears on the graduated circles of the measuring wheels as $\frac{3}{4}$ mm. It is, therefore, also possible to mark with the greatest accuracy the measurements in the middle of the board without a magnifying glass (for instance, on the scale 1 : 1000 as far as 2 to 4 cm). The numbering is marked on a special band wound on rollers at both ends of the scales. The graduation of the bands can, by turning these rollers, be made to directly coincide with the co-ordinate figures of the respective plan. With the instrument described plans of 1 metre (40") long and 66 cm (26") wide can be worked on in any desired direction.

432. Sub-traverses in Fig. 219 and Detail Measurements.—

After the traverse of the boundary is plotted, the interior details are surveyed. In order to do so stations for sub-traverses are fixed by the line-clearer on the traverse lines. In laying down the stations for sub-traverses the line-clearer must see that the sub-traverses are never at a greater distance than 40 chains or half-a mile apart. Each sub-traverse line must start from a station on the boundary or another sub-traverse closing similarly on a station on the boundary or some sub-traverse (Fig. 223). Sub-traverse stations must always be on the divisions (*merks* or *ails*) between fields, and not

Fig. 223.—Sub-traverses.

in a field. All streams that are unfordable and over three chains in width, running through a village or along a boundary, must have traverse lines along both banks at a distance of about one chain from



FIGS. 221, 222. — Coradi's Co-ordinatograph.

either bank. All large village sites, if required to be surveyed in detail, must have sub-traverses running round and through them. In a similar manner tracts of jungle and large tanks should be encircled by sub-traverses. Roads are also surveyed by sub-traverses but other details such as fences, houses, small tanks, etc., are filled in by the aid of the plane table (see *Section on Plane Table Survey* later on) or prismatic compass, or with the chain, and other measurements by the introduction of subsidiary lines or Δ s with offsets as described under chain surveying.

433. Calculation of Area of a Closed Traverse from the Traverse Computation Form.—When the rectangular co-ordinates (latitudes and departures) are calculated from the lengths and bearings, the area in any circuit is expressed by the formula:—

$$\text{Twice Area} = (x_1 + x_2) (y_3 - y_1) + (x_2 + x_3) (y_3 - y_2) + (x_3 + x_4) (y_4 - y_3) + (x_4 + x_5) (y_5 - y_4) + \text{etc.}$$

where the rectangular co-ordinates of the angular points, *A, B, C, D, E*, etc., are $x_1, y_1; x_2, y_2; x_3, y_3; x_4, y_4; x_5, y_5$; etc., respectively. The co-ordinates should be taken with their proper algebraical signs. The North latitudes and East departures are to be +, and South latitudes and West departures —; the *y* co-ordinates being latitudes, and *x* co-ordinates departures.

Ex. By substituting the figures of the rectangular co-ordinates in Fig. 219. and Traverse Computation Form on p. 232 in the above formula we get :

$$\begin{aligned} \text{Twice Area} &= (0 + 4.96) (27.11 - 0) + (-4.96 + 7.10) (-23.85 + 27.11) + (7.10 + 21.06) (-23.04 + 25.85) + (21.06 + 36.98) (-15.73 + 23.04) + (36.98 + 42.50) (1.46 + 15.73) + (42.50 + 15.77) (21.39 + 1.46) + (15.77 + 1.73) (19.85 - 21.39) + (1.73 + 0) (0 - 19.85) \\ &= 134.46 + 2.69 + 78.96 + 423.83 + 1366.26 + 1161.32 - 26.95 - 31.34 = 3167.62 - 61.29 = 3106.23 \text{ sq. chains} = 310.623 \text{ acres nearly.} \\ \therefore \text{The area, is } 155.311 \text{ acres nearly} \end{aligned}$$

434. The "Universal Theorem" for finding the area of a closed traverse by means of the rectangular co-ordinates is the

Stations.	Corrected Latitudes.		Corrected Departures.		Total Latitudes.	Adjacent Departures.	Double Areas in Square Chains.
	N+	S-	E+	W-			
A		27.110		4.965			
B	1.264	..	12.069	..	-27.119	+ 7.104	192.653
C	2.815	..	13.898	..	-25.855	+25.967	671.376
D	7.307	..	15.984	..	-23.040	+29.882	688.481
E	17.197	..	5.514	..	-15.733	+21.498	338.228
F	19.933	..		26.730	+ 1.464	-21.216	31.060
G	..	1.545	..	14.032	+21.397	-40.762	872.180
H	..	19.852	..	1.738	+19.852	-15.770	313.066
..	..	..	..	..	..	..	3107.04

following:—Multiply the total latitude of each station by the sum of the departures of the two adjacent lines; the algebraical sum of

these products is double the area. The total latitude is obtained as in column 11 of the Traverse Computation Form on p. 232. The adjacent departures are obtained by adding the departures of the two lines, one on each side of the station. An example is given in the Table on p. 241. The figures in latitudes and departures are taken from the Computation Form on p. 232.

The area, therefore, is 1553.52 square chains or 155.352 acres. As the difference between the two calculations is below .03 acre which is due to the last calculation being made to 3 places of decimals the area may be accepted.

As an additional check triangulate the polygon by Prob. XXII, p. 147, and then find its area, the result should agree to within 1 per cent. of the area as obtained by multiplication. Another accurate method is to reduce the polygon into Δ s, calculate their areas separately by means of the two sides and the included angle, and to sum them up for the area of the whole figure, when any Δ falls without the figure it being excluded. In Fig. 219 the polygon *ABCDEFGH* may be divided into Δ s *ABC, CDE, ACE, EFG, AEG, GHA*. If the polygon is a small one or in the case of a closed traverse made with either a compass or sextant, its area may be computed by means of the acre-comb or computing scale or planimeter.

435. Calculation of Offsets.—Having found the area of the traverse without offsets, calculate the areas of offsets to the right and left of all the survey lines by the talc-square, add together all those outside the polygon and also those inside the polygon; then subtract the lesser sum from the greater, the difference is the total area of the offsets additive or subtractive, as the case may be, to the area of the polygon already found for the area of the survey.

436 The meridian distance of a point is its perpendicular distance from the *reference meridian* which in Fig. 218 is BA.

The meridian distance of a course is the meridian distance of the middle point of that course; hence, *the double meridian distance of a course* is equal to the sum of the meridian distances to the extremities of that course. The D.M.D.'s of the two courses adjacent to the *reference meridian* are evidently equal to their respective departures. The D.M.D. of any other course is equal to the D.M.D. of the preceding course plus the departure of that course plus the departure of the course itself, easterly departures being counted positively and westerly departures negatively. *A check on the computation of the D.M.D.'s* is that, when computed continuously in either direction and from any corner, the numerical value of the *D.M.D.* of the last course must equal its departure. This is a very important check, as it proves the accuracy of all the D. M. D.'s.

437. Traverse Table for Survey by Bearings from an Actual or Assumed Meridian (Fig. 218).—In the following Traverse Computation Form the bearings of the various courses or lines are obtained in accordance with instructions in Art. 402, p. 224. The latitudes and departures are calculated and balanced as shown. The area is found by the following rule: *Twice the area of the figure is equal to the algebraic sum of the products of the double meridian distances of the several courses into the corresponding latitudes*, north latitudes being reckoned positively and south latitudes

**Traverse Computation Form for Computing Areas from Bearings (and Distances of the Sides)
measured from the Line joining the Second and First Station as the Zero-Line, i.e., the
Assumed Meridian (Fig. 218).**

Stations.	Observed Bearings.	Cardinal Directions.	Reduced Bearings. θ	Distances Measured. feet. = D	DIFFERENCE OF LATITUDES AND DEPARTURES OF EACH DISTANCE.										Co-ordinates Referred to Meridian of A Station.				Double Meridian Distance. D.M.D.	Double Area + or - 1 Sq. Ft.						
					LATITUDES.					DEPARTURES.					Latitude.	N. or S.	Departure.	E. or W.								
					Log. D + L. Cos θ - 10	North.	Cor. +	South.	Cor. +	Log D. + L. sin θ - 10	East.	Cor. +	West.	Cor. -												
A	0° 0' 0"	N.	0° 0' 0"	1500.0	3.15625	1500.0	0.3				3.15625					1499.7	N	0.0		6.0	0.0					
B	72° 28' 0"	N.E.	72° 28' 0"	1433.0	1.47894 2.63519	431.7	0.3				1.97934 3.13539	1365.8	0.2			1981.1	N	1366.0	E	1366.0	+ 589292.40					
C	173° 10' 40"	S.E.	6° 49' 20"	804.5	2.93158 1.99692 2.94850			888.2	0.2		2.95158 1.07444 2.02602	106.2	0.1			1042.7	N	1472.3	E	2538.3	- 2521545.72					
D	207° 11' 40"	S.W.	27° 11' 40"	1177.5	3.07096 1.94914 3.02010			1047.3	0.4		3.07096 1.65092 2.73085			538.1	0.1	5.0	S	934.3	E	2406.6	- 2521894.82					
E	270° 10' 10"	N.W.	89° 40' 40"	934.5	2.97058 3.74991 0.72049			5.2	0.2		2.97058 1.99999 2.97057			934.5	0.2	0.0		0.0		934.3	+ 4671.5					
				5939.5	1935.5			1936.9	0.8	1975.5	0.6	1472.0	0.8	1472.6	0.3					2	- 4448076.65					
					Error in Lat. = 1.4, "										Error in Dep. = .6										43560 Area Sq. ft. = 2224488.32	
					Error of Closure = $\sqrt{1.4^2 + 0.6^2} = 1.5$ in 3973.										" in Acres = 51.069											

negatively, and the tract being kept on the left in making the survey.

If the tract be kept on the right in the survey, as in Fig. 218, then the numerical value of the result is the same, but it comes out with a negative sign.

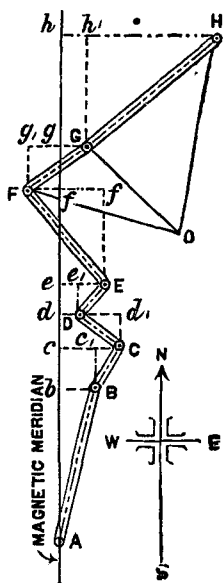


Fig. 224.—Open or Unclosed Traverse Survey.

438. Example of an Open (Unclosed) Traverse Survey.—Let the Fig. 224, represent the survey of a portion of the road between the points, *A* and *H*. The station points are chosen as far apart as it is possible to see between the bends in the road, and are marked by driving in pegs. The station lines, *AB*, *BC*, *CD*, *DE*, *EF*, *FG*, *GH*, are first measured, and offsets taken and recorded in the usual way. The bearings are taken in exactly the same manner, as described for the Closed Traverse Survey (Fig. 215). The lengths and bearings are as under:—

Line.	Length.	Bearing.
<i>AB</i> ...	1350'	14°
<i>BC</i> ...	382'	31°
<i>CD</i> ...	450'	315° 42'
<i>DE</i> ...	314'	37° 15'
<i>EF</i> ...	1000'	319° 30'
<i>FG</i> ...	636'	52°
<i>GH</i> ...	1427'	49°

439. Plotting by Distances and Bearings with a $\odot$ or Protractor.—In plotting theodolite surveys, $\odot$ or protractors as described in Figs. 211-214 should be used instead of a $\odot$ or a rectangular protractor as described for plotting traverse lines under compass survey. To plot the work rule a line on the paper representing the magnetic meridian through the station, *A*, and place a $\odot$ or protractor (Fig. 214) with its centre at *A*, 360° and 180° divisions coinciding with this line. The bearing of *AB* is laid off and the distance, *AB*, is marked off by the scale along the line so protracted. Rule another line representing the magnetic meridian through the point, *B*, and parallel to the first meridian line through *A*, place the protractor on it and lay off the bearing of *BC* and length, *BC*, to same scale which was used in setting off *AB*, similarly other lines are plotted.

440. Plotting by Latitudes and Departures.—The most accurate method of plotting each point *B*, *C*, *D*, etc.; is to calculate the latitudes and departures of each of the lines. The calculations are made as described under Gale's Traverse (pp. 229-234, Arts. 415-425) and are tabulated as follows on p. 245:—

To Plot the Survey, mark the distances, *Ab*, *Ac*, *Ad*, *Ae*, ... *Ah*, on *Ah* by means of a plotting scale and draw the perpendiculars from *b*, *c*, *d*, *e*, *f*, *g*, *h* and with the offset-piece scale off *Bb*, *Cc*, *Dd*, *Ee*, *Ff*, *Gg* and *Hh* and the points *B*, *C*, *D*, *E*, *F*, *G*, *H* are obtained.

OPEN TRAVERSE TABLE, FIG. 224

Line.	Length.	Bearing.	Latitude = Length $\times$ Cosine Bearing.	Departure = Length $\times$ Sine Bearing.	TRAVERSES OF EACH STATION REDUCED TO FIRST STATION.	
					Latitude.	Departure.
AB	1350'	14°00'	+1309.3 = Ab	+328.9 = Bb	+1309.3 = Ab	+328.9 = Bb
BC	382'	31°00'	+327.4 = Bc'	+196.7 = Ce'	+1636.7 = Ac	+525.6 = Cc
CD	450'	315°42'	+262.6 = Cd'	-365.4 = Dd'	+1899.3 = Ad	+160.2 = Dd
DE	314'	37°15'	+249.9 = De'	+190.1 = Ee'	+2149.2 = Ae	+350.3 = Ee
EF	1000'	319°30'	+760.4 = Ef'	-649.4 = Ff'	+2909.6 = Af	-299.1 = Ff
FG	636'	52°00'	+391.6 = Fg'	+501 = Gg'	+3301.2 = Ag	+201.9 = Gg
GH	1427'	49°00'	+936.2 = Gh'	+1076 = Hh'	+4237.4 = Ah	+1277.9 = Hh
North or East is shown +			+4237.4 = Ah	+1277.9 = Hh	South or West is shown -	

441. Checks on Open Traverse by Bearings to Lateral Objects.—In an ideal polygon the sum of the northings will equal the sum of the southings, and sum of the eastings will equal the sum of the westings. In an open or unclosed traverse no such check on the accuracy of the readings and measurements can be obtained, the utmost care is therefore necessary both in observation and measurement, and angles should be taken to distant objects, such as well-defined peaks, or a large tree, which are visible from as many stations as may be, which will serve as checks on the accuracy of both the angular and linear measurements. For example, in Fig. 224, the object, *O*, being visible from both *F* and *G*, the bearings, *FO* and *GO*, are taken. From the length, *FG*, and the angles, *OFG* and *FGO*, as deduced from the bearings, the length, *GO*, is calculated. On arriving at *H*, *O* being again visible, the bearing, *HO*, is taken. From the bearings, *GO* and *HO*, the angles, *OGH* and *OHG*, are found out, and from these and the length, *GO*, the distance, *GH*, is calculated. This calculated length of *GH* should then agree with the actual measured length. In a long traverse no opportunity of making this test should be lost.

In a long traverse for a line of railway, say 50 to 100 miles or more in length, through an unmapped country, the bearings are checked by taking a sun observation daily, and from it calculating the true bearing of the line then being run. The error is then adjusted over the lines as far as the last bearing where a sun observation was taken. The survey, instead of being commenced with the magnetic bearing of the first line, is commenced by finding from a sun observation the true bearing of this line, and the true bearings are worked to throughout.

442. Checks on Calculation and Plotting of An Open Traverse.—Ordinarily as a check on the accuracy of the calculation and plotting, the lengths of the lines *AB*, *BC*, *CD*, *etc.*, should when measured with a scale, agree with the lengths measured on the ground. When the relative positions of the beginning and end of the traverse, as *A* and *H*, are known, (Fig. 224) the two points, *A* and *H*, being first plotted, the traverse lines should fit in exactly between.

Section. 5.—Traversing with the Theodolite, such as are likely to be undertaken by Civil Court Surveyors, i.e., Amins.

443. Traverses with the Theodolite, such as are likely to be carried out by the Civil Court surveyors in this country, may be divided as under:—(i) The survey of a portion of land which has already been surveyed by professional surveyors. (ii) The survey of an area which was not surveyed before and of which no maps exist. (iii) Where a boundary is under dispute, and it is required to lay down the correct boundary as shown on the map.

(i) **Old Traverse Stations and Trijunctions:**—Where a professional survey has already taken place, old traverse stations, more specially village trijunctions, are generally to be found either: (a) On the boundary of the land to be resurveyed, or that of the adjoining villages. (b) At some distance from the area of re-survey.

(a) To find the old stations, the surveyor should get the necessary data from the traverse “set up” of the old survey, in which, he finds, for example, the distance, AX , (Fig. 225) to be 18.20 chains, and the traverse line, AM , on an adjoining village boundary to be 17.50 chains. On arriving at the trijunction, A , he begins to find out the station, X or M , along the boundaries, AX or AM . The villagers will render often great assistance in finding out the approximate positions of traverse stations, if a long interval of time has not elapsed since the previous survey. On fixing one of the stations, the surveyor can start the traverse of the boundary, XAB , irrespective of whether his forestation, B , as laid down by his line-

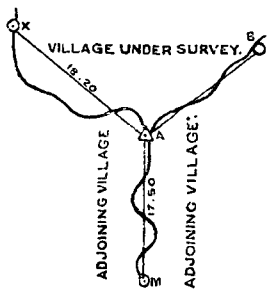


Fig. 225.—To Find Old Stations in Traversing by Civil Court Surveyors.

clearer, coincides with the next old traverse station. On completion of his traverse all round the area the work can be “set up” and computed and the stations plotted for the detail survey.

(b) When old traverse stations or trijunctions can not be found on the boundary for the area for survey or that of an adjoining village (this is always the case in “diara lands”), the surveyor must search for a station on the high ground above the line of riverain action. Having obtained one and its adjoining station, he will proceed to run a connecting traverse from this old station to some station on the boundary of the area for re-survey (Fig. 226), the angles of this connecting traverse should be observed and booked in exactly the same way as laid down on p. 232. By connecting the re-survey with all old traverse stations the new surveyor’s completion will be in the same geographical position as the old, and on a comparison being made with the latter any changes or discrepancies in the boundaries, etc., will at once be apparent.

(ii) **The Survey of An Area of which no Map exists :—**
The procedure is exactly the same as laid down on pp. 225-242.

(iii) **Where a Boundary is in Dispute and it is required**

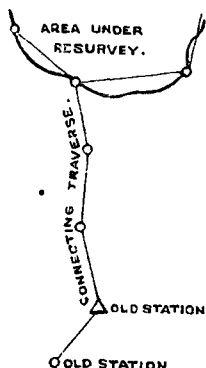


Fig. 226. —To Find Old Station in Traversing by Civil Court Amin.

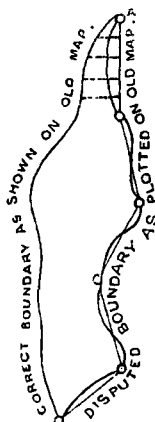


Fig. 227. — To Lay out Correct Boundary.

of the map the lengths of these offsets and their distances along the traverse lines, the surveyor has only now to lay off these offsets on the ground to obtain the correct boundary.

to lay down on the Ground the Correct Boundary as shown on the Map :—This is a case that often occurs in land disputes. The surveyor should go on to execute the traverse of the boundary under dispute, starting from and closing on an old traverse station, plot and draw in the same, on the same scale as the map ; the boundary as then shown must be transferred to the original map and the differences between it and the correct boundary on the map measured by offsets from the traverse lines (Fig. 227).

Having found from the scale

Section 6. —Trigonometrical Surveying or Surveying by Triangulation with the Theodolite.

444. An extensive survey made over large portions of the surface of the earth, either for the purpose of ascertaining the exact position of the principal places of a country, or of determining the dimensions and figure of the earth, is usually termed a *Trigonometrical Survey*. This branch of surveying is termed *Geodesy*. For this purpose a country is first divided into a number of large Δ s, whose sides are usually from 10 to 20 miles in length ; but sometimes they extend to 50 or 60 miles or even to 100 miles in length. A line situated in a level tract of country forming one of the sides of a principal Δ , called a *Base line*, is measured with extreme care and all the angles of the Δ s are then carefully observed. The distances of these stations from the ends of the base and from each other are then calculated, and laid down upon paper, forming so many fresh bases from whence other trigonometrical points are determined until the entire tract of country to be surveyed is covered with a network of Δ s. These Δ s may be said to form a species of polyhedron, circumscribing a portion of the earth, and they are reduced to others on its surface at the level of the sea, by supposing $\perp$ s to be drawn from each station to the surface. The latitudes

and longitudes of the different stations are then determined; and also their heights, and the angles which the sides of the Δ make with the meridional line. The great Δ s into which the country is first divided are termed *Primary* or *Principal* Δ s. They are afterwards subdivided into smaller ones, called *Secondary* Δ s with sides 10 to 12 miles, and these again are split up into others of still smaller sizes, called *Tertiary triangles*, with sides 3 to 4 miles in open country, and 1 to 2 miles in closed country, until the required degree of minuteness in the survey is obtained. A few sides of the principal Δ s may be measured in parts of the survey far distant from the original base, in order to check the correctness of the whole work by comparing their measured lengths with those determined by calculation; these are called *Bases of Verification*. In selecting trigonometrical stations the following two points should be attended to:—1st. The angles should have such a magnitude, that any small inevitable errors in the observations shall produce the least effect on the sides to be calculated. 2nd. The stations should all be distinctly visible from each other. The most advantageous conditions of a Δ are that its sides should be as nearly equal as possible.

Thus in the Ordnance Survey of the United Kingdom two principal bases were measured by means of General Colby's compensated bars, one on the Salisbury plain, 6.97 miles long, the other near and partly across Lough Foyle 7.89 miles long. The length of the latter base was calculated by a triangulation carried from the Salisbury Plain base, and the difference between the computed and measured length of the Lough Foyle base was only 5" on account of which a correction of $2\frac{1}{2}"$ was made.

The triangulation consisted in a network of Δ s over the entire country, the primary Δ s having sides 30 to 111 miles in length. The angles were measured with 4 large theodolites: two 3' in diameter, one 2', and one 18". By means of the Secondary triangulation the long sides of the principal triangulation were reduced to lengths of 5 miles. The angles were measured with a 12" theodolite. The 5-mile sides of the secondary triangulation were reduced by means of the tertiary triangulation to lengths of 1 mile or less. In towns the station points were sometimes within as short a distance as one furlong. The angles were measured with 7" and 5" theodolites. The traversing was done to close on the tertiary Δ s, and the filling-in and detail closed on the traversing. The area of each field was computed from the plotted detail.

In the Survey of India a corresponding network of primary Δ s was actually begun, but that system was abandoned in favor of chains of primary Δ s on various meridians and parallels of latitude, forming a vast quadrilateral reticulation, while secondary triangulation closing on it were reduced to minor Δ s and details of the Topographical and Revenue Surveys were afterwards undertaken according to requirements. The angles of the Δ s were restricted to 30° and 90° , the average side length was 30 miles in hilly country to 11 in the plains; the longest principal side was 62.7 miles, though

in the Secondary triangulation to the Himalyan peaks there were sides exceeding 200 miles. The angles of the principal Δ s were read with 3' and 2' instruments with 5 equidistant microscopes.

In such surveys, the spheroidal form of the earth's surface has to be taken into account, and the amount of accuracy needed is much greater than that required for any ordinary topographical survey. The former are beyond the scope of this work. Triangulation surveys, as ordinarily undertaken for topographical purposes, are based on the same principles, operations, and modes of procedure as were originally adopted in the Great Trigonometrical Surveys of large countries. In this chapter the effect of the spheroidal form of the earth is ignored, and it will not be out of place to show when it will be necessary to consider the earth as a sphere and to what extent a survey may be executed without considering its sphericity. From Rankine's Civil Engineering the following dimensions are taken :—

Earth's mean radius =	20,889,000	feet.
Arc subtended by 1° =	364,582	"
" " " $1'$ =	6,076.36	"

From these figures it is found that in working from a fixed meridian, when the departures East or West have summed up to 6076 feet, the direction of the meridian will be inclined to the meridian of origin by $1'$.

445. The Principal Points of difference in modes of procedure between the topographical surveys and G. T. Surveys are the following :—(1) In topographical surveys the astronomical observations of latitude and longitude are very often dispensed with. In most cases, the surveys end at or include some known station of the G. T. S. of the country, where latitude and longitude are known. Astronomical observations may be required when these are not known. (2) The sphericity of the earth is either ignored, all Δ s being treated as plane or Legendre's method is adopted for treating large Δ s as plane after applying a correction to the angles observed. (3) The country is divided into a network of triangles instead of being divided into parallelograms by meridional and latitudinal series of Δ s. (4) 4" to 8"-theodolites are used in observing the angles which read to $1'$, $2'$, $30''$, $10''$, instead of large theodolites reading to hundredths of a second. The 12"-theodolites of the present day, with micrometer adjustments to assist in the reading of minute subdivisions of angular graduation, is found to be equal to the old 24" or even to the 36" instruments of 30 years ago. (5) The measurement of base-lines is executed with the ordinary chain, without using troughs, trestles, compensating bars, etc., needed in the G. T. S. (6) The scale of operation and the amount of accuracy are smaller, the appliances portable, the work rapid with less number of men.

446. Reconnaissance.—The same remarks as to reconnaissance of ground, as made in Art. 267 on p. 156, apply to theodolite triangulation or traverse surveys.

447. Selection of Base-Lines.—After reconnaissance the first step to be taken in the trigonometrical survey of a tract of country is the careful selection of an appropriate site for the measurement of a base-line. The ground selected for the base should be as smooth and nearly level as possible, and the ends of the base must be visible from the stations in the neighbourhood; hence an elevated plateau is an advantageous position for a base-line of an extensive survey. The ground should be moderately soft and capable of being marked without chiselling at each chain length. It should situate as near the centre of the survey as possible, but this is not absolutely necessary. The base must bear a fair proportion in length to the distance of the points, it is desired, to fix, and must be so situated with regard to those points as to give well-conditioned Δ s. Each extremity of the base should be marked by means of some permanent object, such as a stone sunk into the ground or pile driven deep and hidden from casual observation. In the top surface of the stone, a hole about 4" deep should be made to mark the true end of the line, and to serve for the reception of a flag-pole. When marked in this way stations could be readily found afterwards.

448. Measurement of Base-Lines with Steel-Bands.—Greatest care should be taken in measuring a base-line as any error in its measurement is *multiplied* throughout the whole work. For ordinary practical purposes in trigonometrical surveys of moderate extent a steel-band may be used in its measurement. Tension handle should be furnished to the steel-band so as to regulate the tension on it, and the length of the band for a given tension should be determined. Observations for temperature should also be taken at each stretch of the band to allow for expansion of the metal. A thin plain glass thermometer of the *clinical* form is the most sensitive. When it is used it is placed upon the ground by the side of the chain. The thermometer is carried in a wooden pull-off case lined with India-rubber. The co-efficient of expansion for steel between 32° and 212° Fahr. is about $\frac{1}{100000}12$, which is less than $\frac{1}{100000}01$ per degree per chain. Temperature corrections should therefore be made upon very exact work, and appreciable only when long bands of 5 to 10 chains are used. Each measurement should be on the actual slope of the ground, and the total distance reduced to its horizontal projection by levelling a careful section along the whole base as described in Art. 280, p. 162; or by reducing the hypotenusal measure by calculation having obtained the angle of slope as stated in Art. 282, p. 162.

By using steel-band an accuracy of 1 in 300,000 may be attained and if the most favorable times are chosen, an accuracy of 1 in 1,000,000. It is probable, therefore, that measurements, even including primary base-lines will yet be made by the steel-tape. The tape used may be 300' to 500' long, a usual length is 300', and the area of cross-section of the tape from $\frac{2}{10000}$ and $\frac{3}{10000}$ sq. in.; a tape $\frac{1}{10}$ " wide and $\frac{3}{1000}$ " thick is often used. For an accuracy of 1 in 5000 the tape may be used in all kinds of weather, held and stretched by hand, the horizontal position and amount of pull by chainmen kept

as constant as can be estimated. The temperature may be estimated, or read from a thermometer carried along for the purpose. On uneven ground the end marks are given by plumb-line.

449. Method of Determining the Horizontal Equivalent of a Base-line by means of Observations of the Angles of Elevations and Depressions of Sloping Lengths by the Aid of the Theodolite.—The theodolite, being planted at one extremity of the base-line, the angle of inclination of the ground as far as it is uniform, is observed, by sending a staff with a vane set to the height of the instrument, to the point where the inclination changes. The distance between these points is then measured carefully both forwards and backwards with the chain compared with a standard or steel tape. The mean of the measurements should be taken for its true length. The theodolite is then removed to the point, where the staff was held, which is called the 2nd station, and the angle of inclination of the portion of the base-line already measured again observed, as well as that of the next portion, up to the 3rd station. The length between the 2nd and 3rd station is then twice chained as before, and in this way the whole length of the base-line is measured as well as the inclination of the ground with a view to reduce it to one horizontal length between the two ends of the base. The reduction is made either by reading on the back of the vertical $\odot$, the number of links which have to be subtracted from *each chain* to give the horizontal measurement of the base, or if these figures are not given at the back of the vertical $\odot$, by calculating the horizontal distance with the mean of the observed angle of slope and the measured distance as the hypotenuse of a right-angled $\triangle$ as described on p. 162.

450. Another Method of Measuring a Base-line :—Mark the extremities of the base by metal dots on blocks of stone let into the ground, clear the line to be measured of all obstructions. The line between the dots is then carefully ranged by means of a theodolite planted at one end with ranging-rods in a vertical plane passing through the other end, the positions of the rods should be checked by a similar theodolite observation from the other end. The positions of the rods are then marked by stout pegs of sâle or teak, $2\frac{1}{2}'' \times 2\frac{1}{2}''$ in section. The pegs should be driven 10' apart and their tops are then to be brought to the same level by means of a levelling instrument. A pair of well-seasoned teak rods $10' \times 2'' \times 1\frac{1}{2}''$ are then placed upon the tops of the pegs, by butting the end of one rod against the end of another, and the line is measured by means of these rods in the same way as the steel standard rods are used in testing the chain.

451. An Expeditious Method is to use one rod 10' long well-trussed and furnished with points, forming a large beam compass, to mark the exact length from peg to peg. The base-line should be measured backwards and forwards a couple of times and then the mean taken for the true length. The length of the base-line should be from 1800' to about half-a-mile. A base-line may be measured with rapidity and fair accuracy along the rails of a straight

portion of a railway and it is better to use a short base-line carefully measured than to employ a longer one which has to pass over broken ground, and which may not be laid out with the same care. (q.v. Art. 481.)

452. Base of Verification.—The last $\triangle$ of the system should be so chosen that one of its sides may be measured. This line is called *base of verification*, one base-line can then be calculated from the other, and the calculated and measured results compared. Thus the accuracy of the survey may be checked.

453. Prolonging a Base-line.—It sometimes happens that the level ground being of limited extent is not suited to the measurement of the whole of a base-line; in this case a portion only of the base-line is measured and which is then prolonged by ranging lines in continuation of it, at one or both ends, until a suitable length is obtained to give well-conditioned angles from each of its ends to

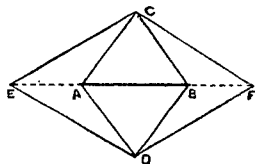


Fig. 228.—Prolonging a Base-line.

the points to be fixed. The lengths of the additional lines are calculated from the measured base and the observed angles, as under:—1st. When the measured base, AB , can be conveniently prolonged in both directions towards E and F , select two temporary station points, C and D (Fig. 228), so that the resulting $\triangle$ s, ACB and ADB , may be well-conditioned. Observe all the angles of these two $\triangle$ s and calculate the side, CD , through each $\triangle$, thus verifying the result; then select two points, E and F , in the prolongation of AB , so that the $\triangle$ s CDE and CDF may be well-conditioned. Observe all the angles in these two $\triangle$ s, and calculate EF twice through the separate $\triangle$ s.

2nd. When the prolongation can only be conveniently effected in one direction, as towards F (Fig. 228) a corresponding method can be adopted which differs only in being one-sided. In selecting points, C and D , rather move towards F , and observing all the angles, calculate BC and BD ; then, choosing F , so that CDF may be well-conditioned, and observing all the angles, compute BF both in the $\triangle BCF$ and in $\triangle BDF$, thus verifying the result.

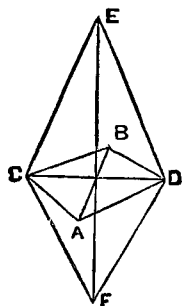


Fig. 229.—Extension of Triangulation from Measured Base.

454. Extension of the Triangulation from the Measured Base.—A mode of connecting a comparatively short base-line with the sides of large $\triangle$ s, without prolonging it, or introducing ill-conditioned $\triangle$ s by continually increasing the sides of the $\triangle$ is as under:—In Fig. 229, let AB be the measured base, C and D the nearest stations so selected on opposite sides of the base that the $\triangle$ s, ABC and ABD , are well-conditioned. All the angles of the $\triangle$ s, ABC , ABD

are taken, and the sides, AC , BC , AD , BD , calculated from these and the measured base, AB . Then in the $\triangle ADC$, the two sides, AD and AC , and the included angle will give the side, CD . Similarly CD may be calculated from BC and BD , and the included angle, and the mean of the results may be taken as the length of CD . Taking CD as a new base, choose a pair of stations, E and F , on opposite sides of the base, and as far from each other as is consistent with making CDE and CDF well-conditioned $\triangle$ s. The angles of the $\triangle$ s, CDE and CDF , are then observed and EF calculated in exactly the same manner. If EF is of the required length, the triangulation may now be proceeded with, and the $\triangle$ s laid out from EF . If not, the process is repeated until a distance sufficiently long is obtained.

455. Reduction of Base to Mean Sea-Level.—In order to

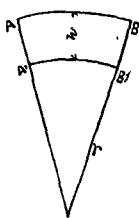


Fig. 230.—Reduction of Base to Mean Sea-Level.

compare together, and connect bases measured at different elevations in distant parts of the country it is required that they be referred to a common elevation. For this common standard, the mean level of the sea, is naturally the most suitable. A base measured at an elevated plain is thus reduced to its proper measure at the level of the sea: In Fig. 230, let $AB=b$ the measured base, $A_1B_1=b_1$ the base reduced to mean sea-level, h =height of measured base above mean sea-level, r =radius of the earth at mean sea-level = 20,890,592 ft. say. Therefore $r+h$ = radius at the level of the measured base. Then because similar arcs are in the same ratio as their radii we have, $r+h : r :: b : b_1$

which gives $b_1 = \frac{rb}{r+h} = b \left(1 + \frac{h}{r} \right)^{-1}$ which expanded by common division becomes $b \left(1 - \frac{h}{r} + \frac{h^2}{r^2} - \frac{h^3}{r^3} + \text{etc.} \right)$

$$\therefore b_1 = b - \frac{bh}{r} + \frac{bh^2}{r^2} - \frac{bh^3}{r^3} + \text{etc.}$$

Subtracting both sides of the equation from b we have

$$b - b_1 = \frac{bh}{r} - \frac{bh^2}{r^2} + \frac{bh^3}{r^3} - \text{etc.}$$

which is the required expression for the reduction of the measured line to the level of the sea. As h is very small compared with r it is usually near enough to use the first term only, then

$$b - b_1 = \frac{bh}{r}.$$

As an example to illustrate this the following is taken:—

Measured length of Base-line	=	$b = 24000'$	log	4.3802112
Mean height of the base above Sea-level,	$h = 1000'$		"	3.0000000
Mean radius of the earth	=	$r = 20,890,592'$	ar. cò.	8.6800493

Required Correction	=	$1.149'$	log	0.0602605
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The correction deducted from the measured base will give its length at the sea-level.

In reducing the base to mean sea-level, if the surface slope is uniform h may be taken as the height of the lower end of the base. On broken ground the base may be split up into several parts and the reduction to mean sea-level made for each separately, different values of h being used; otherwise the mean height of the base above sea-level may be used. For small bases of about half-a mile this reduction will not be of any practical importance.

456. Selection of Stations.—The choice of the station-points for the network of Δ s to be spread over a tract of country will necessarily be in accordance with the size of the Δ s adopted as most suitable to the conditions and extent of the tract.

The station points should be so chosen that the flag at each one may be visible from the neighbouring stations and that all three angles of every Δ may be measured; for this reason elevated points on open ground are to be preferred. If a spire or tower is made one of the triangulation points and forms a good signal point without affording facilities for setting up an instrument and observing angles from it, a satellite station of observation near the former must be employed, and angles taken there reduced by calculation to the point observed (q.v. Art. 477). This, however, should be avoided whenever possible. Two satellite stations or two inaccessible points are not to be used in any one Δ .

457. The Size of the Δ s to be adopted must depend on the scale and on plotting, the length measured with the chain not exceeding that which can be detected in plotting. With a scale of $200' = 1''$ or $\frac{1}{2400}$ it has already been shown that $2'$ is the smallest dimension that can be plotted, and as the probable chainage error is 1 per 1000, the length to be chained or computed within any Δ should not exceed $2400'$. With a scale of $\frac{1}{10000}$ the length to be computed might be increased to $10,000'$ without detectable error in plotting.

458. Well-conditioned Δ s.—A right angle is the angle least liable to error, but the error tends to increase slowly as the angle becomes more obtuse, and more rapidly as it becomes more acute. For this reason the angles of every Δ should be as regular as possible; none should be less than 30° or more than 120° , as a point is not definitely defined if the lines fixing it meet at a very obtuse or acute angles. A small angle may be measured as correctly as a large one, but a given error, say $1''$, in a small angle, gives a much greater error in the resulting computed distance than an error of $1''$ in a large angle. The distances are proportional to the sines of the angles, and the differences of the sines of small angles near 0° are very much greater than the sines of large angles near 90° , which are nearly zero; the error in distance due to a given error in a small angle is, therefore, much greater than the error in distance due to the same error in a larger angle, and each angle, therefore, should approximate as nearly as may be to 60° , i.e., the trigonometrical stations must be selected with a view to the formation of well-conditioned Δ s. With angles ranging from 25° to 155° , the rate of error is always less than 1 in 1000 which may be taken as the limit of error.

459. The Points to be Particularly Noted in the Selection of Station Points are: (1) that the Δ s are well-conditioned; (2) the amount of jungle clearing necessary to make the stations selected visible from each other; (3) height and convenience of access to stations, (4) liability to movement of stations during the progress of survey; (5) that the lines should be rejected in which prominent buildings intervene; (6) that when jungle or undergrowth obstructs the line between stations, if stations can be raised by erecting wooden scaffold so as to clear the greater part of the obstruction and the remainder cleared at a small cost.

460. Demarcation of the Survey Points.—The triangular points may be marked by means of wooden pegs 1' long and 4" thick of sál or teak, driven into the ground. In these, holes should be bored to a depth 4" or 5" for the reception of flag-poles. The theodolite may be placed over the hole after the pole has been removed.

461. Best Form of Signals.—The best description of signal is a straight bamboo 15' or 16' long with a red and white flag attached to its top. A strip of white cloth wrapped near the bottom of the pole will admit of the pointings being made so low down that errors arising from disturbance of the pole by the wind will be inappreciable.

462. Observation of Angles.—A method of *repetition* or *reiteration* should be adopted to observe the angles with the greatest accuracy. The method of Repetition is described in Art. 150, p. 85.

463. Method of Reiteration.—This method consists in clamping the A-vernier to zero of the vernier-plate, directing the cross-hairs at a selected flag, bisecting it, reading all the verniers, releasing the vernier-plate, bisecting each of the flags in succession, and returning to the selected flag, the angles read by each vernier being booked in each case. Then, if there be 2 or 4 verniers make the A-vernier, read 90° , if 2 verniers make it read 120° and repeat the operation; in the same way with the A-vernier at 180° and 270° when the instrument has 2 or 4 verniers, or at 240° when it has 3. Next the telescope must be transited and the whole process repeated. This system may be further extended by bringing each of the vernier to zero successively. This is the most correct method of observing an angle but this degree of refinement is requisite only in extensive operations.

464. The Horizontal Angle-Book is usually $10'' \times 8''$ in size, opening length-wise. The entries are made from the top to the bottom begun at the first page.

Ordinary Methods.—When great accuracy is not needed the above laborious methods, are unnecessary for a survey by triangulation of limited area. It is sufficient that two readings of each object should be booked one being taken with *face right* and one with *face left*. An example is given as under on p. 256:—

465. Vertical Angle Book.—When necessary the vertical angles of neighbouring stations as observed in the field are entered in a book called the *Vertical Angle-Book* (q.v., p. 256.)

HORIZONTAL ANGLE-BOOK.

Horizontal Angles, taken at S, Keshpur village station, on the 25th January, 1905, with Cooke and Son's 5"—Theodolite, No. 41 (reading to 30 seconds).

Description of Station.—The station is situated at the corner of Sibsagar tank, 65' from a Mango tree to its east, and 57' from a Tamarind tree to its south. The Keshpur village is about $\frac{1}{4}$ mile north of the station.

Observer :—M. N. Ray.

Objects observed to.	Face of Theodolite.	VERNIERS OF HORIZONTAL CIRCLE.			Means of Vernier Readings.	Angles.	REMARKS.
		A	B	C			
A Flag	L.	0° 00' 00"	01' 00"	00' 00"	0° 00' 20"	56° 16' 40"	
B "	"	56° 17' 30"	17' 30"	17' 00"	56° 17' 00"	83° 36' 50"	
C "	"	139° 54' 00"	53' 30"	54' 00"	139° 53' 50"	...	
C "	R.	319° 54' 00"	55' 00"	54' 30"	319° 54' 30"	83° 36' 50"	
B "	"	238° 17' 30"	18' 00"	17' 30"	238° 17' 40"	50° 15' 30"	
A "	"	180° 01' 30"	02' 30"	0' 30"	180° 02' 10"	...	

Abstract of Mean Angles.

Objects.		Mean Angles,
A Flag and B Flag		56° 16' 05"
B Flag and C Flag		83° 36' 50"

VERTICAL ANGLE-BOOK.

Vertical Angles taken at T, *Chatia Hill* Station, on the 10th February, 1905, with Messrs T. Cooke and Son's 5"—Theodolite, No. 41 (reading to 30 seconds).

Observer.—R. N. Ray.

Objects Observed to	Face of Theodolite.	VERTICAL VERNIER.		MEANS OF VERNIER READINGS.		HEIGHT OF		REMARKS.
		A	B	One Face	Both Faces.	Eye.	Object.	
A flag ..	L	E 4° 37' 30"	E 37' 30"	E 4° 37' 30"	E 4° 38' 00"	5ft.	0 feet.	Bottom.
B "	R	E 4° 38' 30"	E 38' 30"	E 4° 38' 30"				
B Mango tree ..	R	D 3° 50' 00"	D 49' 30"	D 3° 49' 45"	D 3° 50' 15"	5ft.	0 feet.	Bottom.
B Mango tree ..	L	D 3° 51' 00"	D 50' 30"	D 3° 50' 45"				
C Flag ..	L	E 0° 02' 00"	E 02' 30"	E 0° 02' 15"	E 0° 02' 30"	5ft.	0 feet.	Bottom.
" "	R	E 0° 02' 30"	E 03' 00"	E 0° 02' 45"				

Abstract of Mean Angles.

Objects.	Angles.	HEIGHT OF	
		Eye.	Object.
A Flag ..	E 4° 38' 00"	5'	0'
B Mango tree ..	D 3° 50' 15"	5'	0'
C Flag ..	E 0° 02' 30"	5'	0'

466. Example of a Small Trigonometric Survey:
Field-Work.—Fig. 231 illustrates a survey of this kind. The base-line, ab , is first measured after careful selection upon a fairly level ground, in the tract to be surveyed, and is so situated with regard to other stations as to give well-conditioned Δ s. **Form I** shows the method of booking the measurements with a chain corrected before and after measurement by means of a steel-tape or level-staves laid end to end, and of the angles of elevation and depression of sloping lengths with a theodolite fixed up at intervals in the line at changes of slope. The instrument should be set up to the same height every time and the readings of vertical angles should be taken to a mark on a rod equal to the height of the telescope axis for determining the horizontal equivalent of the base-line:—*

Form II is used for the calculation of the horizontal distance of the base-line, from mean measured distances and the inclination of the ground deduced from the entries in **Form I**. The relative altitudes of the different positions of the theodolite are obtained, but this is not a necessity.

467. Triangulating.—Having selected and measured the base, go on with the triangulation, measuring the angles at every station. For example, in Fig. 231, set up the theodolite at a , and measure the two angles dac , cab ; next go to b , and measure the two angles abc , cbe ; now remove the theodolite to c , and observe the five angles dcf , fce , ecb , bca , acd ; similarly observe the angles at e , f , h , g , and d .

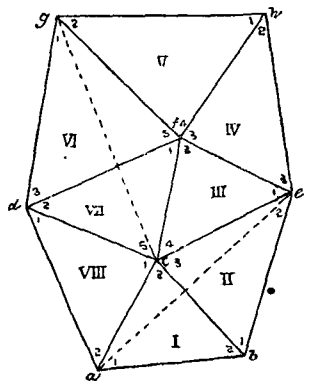


Fig. 231.—Example of a Small Trigonometric Survey.

In observing the angles, the cross-hairs are first directed to the left-hand object and then to the right-hand one, so that the verniers move in the direction in which the horizontal limb is graduated. If the telescope is directed to the right-hand object first the angle observed must be deducted from 360° to get the inward angle required, (p. 83, Art. 144, q. v.)

468. Rule for Correcting the Angles of Polygons when Carrying a Network of Triangulation across a District.—(1) Take out the log sines of all the angles marked 1 and 2 (Fig. 231, Southern pentagon). (2) The sum of the first set should be equal to that of the second;* but if not, (3) take the difference of the sums of these two sets of Log. sines and divide by 2 (call this A).

* Note—If straight lines be drawn from any point either within or without a polygon to all the angular points, the continued products of the sines of the alternate angles, made by the sides of the polygon and the lines so drawn, will be equal. For,
 $\frac{ca}{cd} = \frac{\text{Sin. } b_2}{\text{Sin. } a_1}; \frac{ce}{cf} = \frac{\text{Sin. } f_2}{\text{Sin. } e_1}; \frac{cb}{ce} = \frac{\text{Sin. } e_2}{\text{Sin. } b_1}; \frac{cf}{dc} = \frac{\text{Sin. } d_2}{\text{Sin. } f_1}; \frac{dc}{ac} = \frac{\text{Sin. } a_2}{\text{Sin. } d_1}$ (Fig. 231).
 But, $ac \times ce \times cb \times cf \times dc = cb \times cf \times ce \times dc \times ac$.
 $\therefore \text{Sin. } b_2 \times \text{Sin. } f_2 \times \text{Sin. } e_2 \times \text{Sin. } d_2 \times \text{Sin. } a_2 = \text{Sin. } a_1 \times \text{Sin. } e_1 \times \text{Sin. } b_1 \times \text{Sin. } f_1 \times \text{Sin. } d_1$.

FORM I.—FIELD-BOOK.—Measurement of Base-Line for a Trigonometric Survey, January 15th, 1905.

Stations.	Chain Measurements in Feet.		Angles of Inclination.			Remarks.
	Forward and Back	Means	Forward and Back.		Means.	
			A	B		
1	304·85	304·84	E. 0°08'00"	E. 08' 30"	E. 0°08'30"	West End of Base, a.
	304·83		D. 0°08'30"	D. 03' 00"		
	620·72		D. 0°26'00"	D. 26' 30"		
2	620·62	620·67	E. 0°27'00"	E. 27' 00"	D. 0°26'37"	Length of Chain before Measurement of Base= 99 65' Do. after=100·44' Mean =100'·05'
	567·15		E. 1°01'00"	E. 01' 30"		
3	567·23	567·19	D. 1°00'00"	D. 01' 00"	E. 1°00'52"	
	425·00		D. 0°47'00"	D. 47' 00"		
4	425·12	425·06	E. 0°46'30"	E. 47' 00"	D. 0°46'52"	
	286·63		E. 0°21'00"	E. 21' 00"		
5	286·51	286·57	D. 0°20'30"	D. 20' 30"	East End of Base, b.	

FORM II.—CALCULATION BOOK.—Reduction of Base-Line.

Stations.	Measured Distances in feet.	Inclination of Ground.	Logarithmic Computations.		Horizontal distances in feet.	Relative altitudes in feet.	Reduced levels.	REMARKS.
1	304·84	E. 0°08'30"	L. cos.	9·9999987	304·839	E. 0°754	100·000	a
			log	2·4840720				
			Sum	2·4840707				
			L. tan.	7·3931459				
				1·8772166				
2	620·67	D. 0°26'37"	L. cos.	9·9999870	620·651	D. 4°805	100·754	
			log	2·7928008				
			Sum	2·7928478				
			L. tan.	7·8888876				
				0·6817354				
3	567·19	E. 1°00'52"	L. cos.	9·9999148	567·078	E. 10°041	95·940	
			log	2·7537286				
			Sum	2·7536434				
			L. tan.	8·2481568				
				1·0017992				
4	425·06	D. 0°46'52"	L. cos.	9·9999597	425·020	D. 5°795	105·990	
			log	2·6284502				
			Sum	2·6284099				
			L. tan.	8·1946170				
				0·7630269				
5	286·57	E. 0°20'45"	L. cos.	9·9999921	286·564	E. 1°730	100·195	
			log	2·4572307				
			Sum	2·4572228				
			L. tan.	7·7807495				
				0·2379723				
∴ Total Horizontal Distance = 2204'152.								

(4) Take also from the Log. book the difference of the Log. sines for minutes of these angles and total them up (call this B). (5) Then, as B is to A, so is each log. difference per minute to the correction required to the log sines of the angles of the polygon, to be added or subtracted, as the case may be. (6) The new angles derived from these corrected Log. sines will be the corrected angles of the polygon to be used for computation of the sides.

469. The Following is the Record of the Observed and Corrected Angles of the Southern Pentagon (Fig. 231):—

FORM III (a)—ANGLE-BOOK.

Observed Angles.		Corrn.	Corrected Angles.	Corrn.	Corrected Angles.
a_1	57° 27' 40"	+ 4"	57° 27' 44"		
b_2	51° 46' 08"	+ 4"	51° 46' 12"		
c_2	70° 46' 00"	+ 4"	70° 46' 04"		
	179° 59' 48"	+12"	180° 00' 00"		
b_{1*}	57° 49' 30"	+ 6"	57° 49' 36"	- 1"	57° 49' 35"
c_3	75° 33' 30"	+ 6"	75° 33' 36"	+ 1"	75° 33' 37"
e_2	46° 36' 42"	+ 6"	46° 36' 48"		46° 36' 48"
	179° 59' 42"	+18"	180° 00' 00"		180° 00' 00"
e_1	51° 40' 33"	+ 6"	51° 40' 39"		
f_2	75° 19' 30"	+ 6"	75° 19' 36"		
c_4	52° 59' 40"	+ 5"	52° 59' 45"		
	179° 59' 43"	+17"	180° 00' 00"		
f_1	55° 24' 00"	- 1"	55° 23' 59"	- 1"	55° 23' 58"
d_2	46° 05' 25"	- 2"	46° 05' 23"		46° 05' 23"
c_5	78° 30' 40"	- 2"	78° 30' 38"	+ 1"	78° 30' 39"
	180° 00' 05"	- 5"	180° 00' 00"		180° 00' 00"
d_1	44° 06' 38"			0"	44° 06' 38"
a_2	53° 43' 28"			- 1"	53° 43' 27"
c_1	82° 09' 54"			+ 1"	82° 09' 55"
	180° 00' 00"				180° 00' 00"
Central Angles at c.		Corrn.	Corrected Angles.	Log. Sines.	
c_1	82° 09' 54"	+ 1"	80° 09' 55"	9.9959269	
c_2	70° 46' 04"	0"	70° 46' 04"	9.9750599	
c_3	75° 33' 36"	+ 1"	75° 33' 37"	9.9860594	
c_4	52° 59' 45"	0"	52° 59' 45"	9.9023248	
c_5	78° 30' 30"	+ 1"	78° 30' 39"	9.9912094	
	359° 59' 57"	+ 3"	360° 00' 00"		

FORM IV (a).—CORRECTION TABLE ACCORDING TO THE RULES ON Pp. 257-258, Art. 468.

	Angles.	Log. Sines	Differences.	Corrn.	Cor. log. Sines.	Cor. Angles.
a_1	57°27'44"	9.9258466	+ 806	+1.01	9.9258467	57°27'44".06
b_1	57°49'35"	9.9275954	+ 795	+1.00	9.9275955	57°49'35".09
e_1	51°40'39"	9.8940112	+ 998	+1.25	9.8946113	51°40'39".07
f_1	55°23'58"	9.9154689	+ 872	+1.10	9.9154690	55°23'58".08
d_1	44°06'38"	9.8426373	+1303	+1.64	9.8426375	44°06'38".08
		9.5061594	4774(B)	+6	9.5061600	
a_2	53°43'27"	9.9064309	- 927	-1.19	9.9064307.81	53°43'26".92
b_2	51°46'12"	9.8951644	- 995	-1.28	9.8951642.72	51°46'11".94
e_2	46°36'48"	9.8613758	-1194	-1.54	9.8613756.46	46°36'47".91
f_2	75°19'36"	9.9855997	- 331	-0.43	9.9855996.57	75°19'35".93
d_2	46°05'23"	9.8575898	-1216	-1.56	9.8575896.44	46°05'22".92
		9.5061606	4664(B)	-6	9.5061600	
Difference =		12;		Half Difference = .6(A)		

470. The Following are the Observed and Corrected Angles of the Northern Pentagon (Fig. 231) :—

. FORM III (b).—ANGLE-BOOK.

Observed Angles.		Corrn.	Corrected Angles.	Corrn.	Corrected Angles.
e^1			51° 40' 39"		
f^2			75° 19' 36"		
c^4			52° 59' 45"		
			180° 00' 00"		
f^1					55° 23' 58"
d^2					46° 05' 23"
c^3					78° 30' 39"
					180° 00' 00"
d^3	55° 56' 05"	- 4"	55° 56' 01"	- 1'	55° 56' 00"
f^5	71° 44' 55"	- 3"	71° 44' 52"	+ 1"	71° 46' 46"
g^1	52° 19' 10"	- 3"	52° 19' 07"		52° 19' 07"
	180° 00' 10"	- 10"	180° 00' 00"		180° 00' 00"
g^2	47° 17' 30"	- 4"	47° 17' 26"		47° 17' 26"
f^4	75° 46' 55"	- 4"	75° 46' 51"	+ 1"	75° 46' 52"
h^1	56° 55' 43"	- 3"	56° 55' 43"	- 1"	56° 55' 42"
	180° 00' 11"	- 11"	180° 0' 0"		180° 00' 00"
h^2	42° 25' 20"	+ 5"	42° 25' 25"		42° 25' 25"
f^3	81° 44' 35"	+ 5"	81° 44' 40"	+ 1"	81° 44' 41"
e^3	55° 49' 50"	+ 5"	55° 49' 55"	- 1"	55° 49' 54"
	179° 59' 45"	+ 15"	180° 00' 00"		180° 00' 00"
Central Angles at f		Corrn.	Corrected Angles.	Log. Sines.	
f^1	55° 23' 58"	0	55° 23' 58"		9.9154689
f^2	75° 19' 36"	0	75° 19' 36"		9.9855996
f^3	81° 44' 40"	+ 1"	81° 44' 41"		9.9954764
f^4	75° 46' 51"	+ 1"	75° 46' 52"		9.9864870
f^5	71° 44' 52"	+ 1"	71° 44' 53"		9.9775811
	359° 59' 57"	+ 3"	360° 00' 00"		

FORM IV (b).—CORRECTION TABLE ACCORDING TO THE RULES ON Pp. 257-258, Art. 468.

	Angles.	Log. Sines.	Differences.	Corrn.	Cor. log. Sines.	Cor. Angles.
c_1	52° 59' 45"	9.9023248	+ 952	+ 3.16	9.9023251	52° 59' 45" .20
d_2	46° 05' 23"	9.8575898	+ 1216	+ 4.03	9.8575902	46° 05' 23" .20
g_1	52° 19' 07"	9.8984082	+ 976	+ 3.24	9.8984085	52° 19' 07" .20
h_1	56° 55' 42"	9.9232381	+ 823	+ 2.73	9.9232384	56° 55' 42" .20
e_1	55° 49' 54"	9.9177108	+ 858	+ 2.84	9.9177111	55° 49' 54" .20
		9.4992717	4825(B)	+ 16	9.4992733	
e_1	51° 40' 39"	9.8946112	- 998	- 3.43	9.8946109	51° 40' 38" .80
c_5	78° 30' 39"	9.9912094	- 257	- 0.88	9.9912093	78° 30' 38" .80
d_3	55° 56' 00"	9.9182329	- 854	- 2.93	9.9182326	55° 55' 59" .80
g_2	47° 17' 26"	9.8661708	- 1166	- 4.01	9.8661704	47° 17' 25" .80
h_2	42° 25' 25"	9.8290506	- 1382	- 4.75	9.8290501	42° 25' 24" .80
		9.4992749	4657(B)	- 16	9.4992733	
Difference =		32 ;	Half difference = 16(A)			

FORM V.—MEAN ANGLE-BOOK.

Angles.	Log. Sine in South Pentagon.	Log. Sine in North Pentagon.	Mean Value of log. Sine.	• Mean Angles.
f_1	9.9154690	9.9154689	9.9154689.5	55° 23' 58" 04
f_2	9.9855996.57	9.9855996	9.9855996.28	75° 19' 35" 96
e_1	9.8946113	9.8946109	9.8946111	51° 40' 38" 93
d_2	9.8575896.44	9.8575902	9.8575899.22	46° 05' 23" 06
c_4	9.9023248	9.9023251	9.9023249.5	52° 59' 45" 10
c_5	9.9912094	9.9912093	9.9912093.5	78° 30' 38" 90

Now, since the angles d_2 , e_1 , f_1 , f_2 , c_4 , c_5 , of the two Δ s common to both pentagons are equally probable from whichever pentagon they are obtained, their means may be taken as their final values as above in Form V.

471. Calculating the Sides of the Δ s.—To find the sides of the Δ s when the base and three angles are given, the following ordinary *Sine formula* is used :—
$$\frac{\sin. A}{a} = \frac{\sin. B}{b} = \frac{\sin. C}{c} \quad \dots (1)$$

If c represents the base-line, $a = \frac{c. \sin. A}{\sin. C}$; or, using logarithms, $\log a = \log. c + L. \sin. A - L. \sin. C. \quad \dots (2)$

Instead of deducting $L. \sin. C$, $L. \operatorname{cosec}. C$ may be added; the formula then comes to—

$$\log. a = \log. c + L. \sin. A + L. \operatorname{cosec}. C - 20. \quad \dots (3)$$

472. The Sides are Calculated in the following way :—

To find side, bc , in ΔI —
 $ab : bc = \sin. acb : \sin. cab.$
 $\log. bc = \log ab + L. \sin. cab - L. \sin. acb$
 $\log. (ab) \ 2205.254 = 3.3434587$
 $+ L. \sin. 57^\circ 27' 44''.06 = 9.9258467$
 $- L. \sin. 70^\circ 46' 04'' = 9.9750599$
 $\log. bc = 3.2942455$
 $\therefore bc = 1968.998 \text{ feet.}$

If instead of deducting the sine of the angle opposite to the given side, its cosecant is added, the result will be as under :—

$\log. (ab) \ 2205.254 = 3.3434587$
 $+ L. \sin. 57^\circ 27' 44''.06 = 9.9258467$
 $+ L. \operatorname{cosec}. 70^\circ 46' 04'' = 10.0249401$
 $\log. bc = 3.2942455$
 $\therefore bc = 1968.998.$

Same result as before.

To find side, ac , in ΔI —
 $ab : ac = \sin. acb : \sin. abc.$
 $\log. (ab) \ 2205.254 = 3.3434587$
 $+ L. \sin. 51^\circ 46' 11''.94 = 9.8951643$
 $+ L. \operatorname{cosec}. 70^\circ 46' 04'' = 10.0249401.$
 $\log. ac = 3.2635631$
 $\therefore ac = 1834.691.$

To find side, bc , in ΔII . —
 $\log bc = 3.2942455$
 $+ L. \sin. 75^\circ 33' 37'' = 9.9860594$
 $+ L. \operatorname{cosec}. 46^\circ 36' 47''.91 = 10.1386243$
 $\log. bc = 3.4189292$
 $\therefore bc = 2623.791.'$

To find side, ce , in ΔII . —
 $\log bc = 3.2942455$
 $+ L. \sin. 57^\circ 49' 35''.09 = 9.9275955$
 $+ L. \operatorname{cosec}. 46^\circ 36' 47''.91 = 10.1386243$
 $\log ce = 3.3604653$
 $\therefore ce = 2293.323.$

To find side, fe , in ΔIII . —
 $\log ce = 3.3604653$
 $+ L. \sin. 52^\circ 59' 45''.10 = 9.9023250$
 $+ L. \operatorname{cosec}. 77^\circ 19' 35''.96 = 10.0144004$
 $\log. fe = 3.2771907$
 $\therefore fe = 1893.174'.$

To find side, fc , in ΔIII . —
 $\log ce = 3.3604653$
 $+ L. \sin. 51^\circ 40' 38''.93 = 9.8946111$
 $+ L. \operatorname{cosec}. 75^\circ 19' 35''.96 = 10.0144004$
 $\log. fc = 3.2694768$
 $\therefore fc = 1859.845'$

To find side, eh , in $\triangle IV$.—

$$\begin{aligned} & \log. fe = 3.2771907 \\ & + \text{L. sin. } 81^{\circ}44'41'' = 9.9954764 \\ & + \text{L. cosec. } 42^{\circ}25'24''.80 = 10.1709499 \\ & \log. eh = 3.4436171 \\ & \therefore eh = 2777.263'. \end{aligned}$$

To find side, fh , in $\triangle IV$.—

$$\begin{aligned} & \log. fe = 3.2771907 \\ & + \text{L. sin. } 55^{\circ}49'54''.20 = 9.9177111 \\ & + \text{L. cosec. } 42^{\circ}25'24''.80 = 10.1709499 \\ & \log. fh = 3.3658517 \\ & \therefore fh = 2321.944'. \end{aligned}$$

To find side, gh , in $\triangle V$.—

$$\begin{aligned} & \log. fh = 3.3658517 \\ & + \text{L. sin. } 75^{\circ}46'52'' = 9.9864870 \\ & + \text{L. cosec. } 47^{\circ}17'25''.80 = 10.1338296 \\ & \log. gh = 3.4861683 \\ & \therefore gh = 3063.151''. \end{aligned}$$

To find side, fg , in $\triangle V$.—

$$\begin{aligned} & \log. fh = 3.3658517 \\ & + \text{L. sin. } 56^{\circ}55'42''.20 = 9.9232384 \\ & + \text{L. cosec. } 47^{\circ}17'25''.80 = 10.1338296 \\ & \log. fg = 3.4229197 \\ & \therefore fg = 2648.010'. \end{aligned}$$

To find side, dg , in $\triangle VI$.—

$$\begin{aligned} & \log. fg = 3.4229197 \\ & + \text{L. sin. } 71^{\circ}44'53'' = 9.9775811 \\ & + \text{L. cosec. } 53^{\circ}55'59''.80 = 10.0817674 \\ & \log. dg = 3.4822682 \\ & \therefore dg = 3035.765'. \end{aligned}$$

To find side, df , in $\triangle VI$.—

$$\begin{aligned} & \log. fg = 3.4229197 \\ & + \text{L. sin. } 52^{\circ}19'07''.20 = 9.8984085 \\ & + \text{L. cosec. } 55^{\circ}55'59''.80 = 10.0817674 \\ & \log. df = 3.4030956 \\ & \therefore df = 2529.854'. \end{aligned}$$

To find side, dc , in $\triangle VII$.—

$$\begin{aligned} & \log. df = 3.4030956 \\ & + \text{L. sin. } 55^{\circ}23'58''.04 = 9.9154689 \\ & + \text{L. cosec. } 78^{\circ}30'38''.80 = 10.0087906 \\ & \log. dc = 3.3273551 \\ & \therefore dc = 2124.981'. \end{aligned}$$

To find side, fc , in $\triangle VII$.—

$$\begin{aligned} & \log. df = 3.4030956 \\ & + \text{L. sin. } 46^{\circ}05'23''.06 = 9.8575899 \\ & + \text{L. cosec. } 78^{\circ}30'38''.80 = 10.0087906 \\ & \log. fc = 3.2694761 \\ & \therefore fc = 1859.842'. \end{aligned}$$

To find side, ad , in $\triangle VIII$.—

$$\begin{aligned} & \log. dc = 3.3273551 \\ & + \text{L. sin. } 82^{\circ}09'55'' = 9.9959269 \\ & + \text{L. cosec. } 53^{\circ}43'26''.89 = 10.0935692 \\ & \log. ad = 3.4168512 \\ & \therefore ad = 2611.266'. \end{aligned}$$

To find side, ac , in $\triangle VIII$.—

$$\begin{aligned} & \log. dc = 3.3273551 \\ & + \text{L. sin. } 44^{\circ}06'38''.08 = 9.8426375 \\ & \text{L. cosec. } 53^{\circ}43'26''.89 = 10.0935692 \\ & \log. ac = 3.2635618 \\ & \therefore ac = 1834.686 \text{ feet.} \end{aligned}$$

FORM VI.—COMPUTATION OF AREA OF $\triangle abc$.

Distances in feet.		Area Computation.
2205.254	ab	Formula is— Area = $\frac{1}{2} a. b. \sin. C$
1968.998	bc	L. sin. C = 9.9750599 log bc = 3.2942455
1834.691	ac	log ac = 3.2635631 co. cor. log = 5.3609121
Double area = 78.3034 = 1.8937806		
Area = 39.152 acres.		

The calculations on pp. 261. 262 may be tabulated as in Form VII (p. 263) which shows how the entries are made for all the $\triangle$ s (Fig 231).

473. Area Computation.—

Form VI shows the method of calculating the area of $\triangle$ s. In the area computation the reason for deducting 20 from the index, is because the tabular L sine of C is taken, which is too much by 10, and because the complement of the constant log. 4.6390879 is 5.3609121, to which 10 is added making it 5.3609121. A total of 20 therefore has to be deducted from the sum in the area computation. To get

acres add 5.3609121 the complement of the constant log for feet (4.6390879) to the sum of L. sin. C, log. bc , and log ac , since 43560 sq. ft. = 1 acre, and 4.6390879 is the log. of 43,560, and to divide by a quantity is the same thing as subtracting its logarithm or adding its log's complement, if then 5.3609121 the complement of 4.6390379 (constant log.) is added

to the sum of $L. \sin. C$, $\log. bc$, $\log. ac$, and the corresponding number of the result 1.8937806 be obtained, this corresponding number will be double the area in acres. When the measurements are in links the co. con. $\log. = 5.0000000$.

474. Table of Distances.—The sides of the Δ s are now calculated. The side fc is calculated from both the Δ s III and VII, and the side ac from Δ s I and VIII and the means of their

FORM VII.—Computation of Minor Secondary and Tertiary Triangles.

Δ No. I.		Log (ab) 2205.254 (Log. Feet = 3.3434 587.						Δ No. 1.	
Instrument used.	Stations.	Angles Observed.	Apport. of Error.	Angles for Computation.	Log. Sines.	Distances in		Side.	
						Log Feet.	Feet.		
	b2	51° 46' 08"	+ 4"	51° 46' 12"	+ 9.895 165	3 263 563	1834.6	ac	
	a1	57 27 40	+ 4	57 27 44	+ 9.925 847	3 294 245	1968.9	bc	
	c2	70 46 00	+ 4	70 46 04	+ 10.024 940				
		179 59 48	+ 12	180 00 00					
Δ No. II.				Log. Feet =	3.294 245	Δ No. I.			
	b1	57 49 30	+ 6	57 49 36	9.927 597	3 360 466	2293.8	ce	
	c3	75 33 30	+ 6	75 33 36	9.986 059	3 418 928	2623.8	be	
	e2	46 36 42	+ 6	46 36 48	10.138 624				
		179 59 42	+ 18	180 00 00					
Δ No. III.				Log. Feet =	3.360 466	Δ No. II.			
	e1	51 40 33	+ 5	51 40 38	9.894 609	3 269 476	1859.8	ef	
	c4	52 59 40	+ 6	52 59 46	9.902 326	3 277 193	1895.2	fe	
	f2	75 19 30	+ 6	75 19 36	10.014 401				
		179 59 48	+ 17	180 00 00					
Δ No. IV.				Log. Feet =	3.277 193	Δ No. III.			
	e3	55 49 50	+ 5	55 49 55	9.917 718	3 365 807	2321.7	hf	
	f3	81 44 37	+ 5	81 44 40	9.995 476	3 443 570	2776.9	eh	
	h2	42 25 20	+ 5	42 25 25	10.170 901				
		179 59 45	+ 15	180 00 00					
Δ No. V.				Log. Feet =	3.365 807	Δ No. IV.			
	h1	56 55 46	- 3	56 55 48	9.923 240	3 422 876	2647.7	fg	
	f4	75 46 55	- 4	75 46 51	9.986 486	3 486 122	3063.5	hg	
	g2	47 17 30	- 4	47 17 26	10.133 829				
		180 00 11	- 11	180 0 0					
Δ No. VI.				Log. Feet =	3.422 876	Δ No. V.			
	g1	55 19 10	- 3	52 19 07	9.898 408	3 408 050	2529.6	jd	
	f5	71 44 55	- 3	71 44 52	9.977 580	3 482 212	3035.6	dg	
	d3	55 56 05	- 4	55 56 01	10.081 766				
		180 00 10	- 10	180 00 00					
Δ No. VII.				Log. Feet =	3.403 050	Δ No. VI.			
	d2	46 05 25	- 1	46 05 24	9.857 591	3 269 482	1859.7	cf	
	f1	55 24 00	- 2	55 23 58	9.915 469	3 327 310	2124.8	dc	
	c5	78 30 40	- 2	78 30 38	10.008 791				
		180 00 05	- 5	180 00 00					
Δ No. VIII.				Log. Feet =	3.327 810	Δ No. VIII.			
	d1	44 06 38	..	44 06 38	9.842 637	3 263 515	1834.5	ac	
	c1	82 09 54	..	82 09 54	9.995 927	3 416 805	2610.9	ad	
	a2	53 43 28	..	53 43 28	10.093 568				
		180 00 100	..	180 00 00					

different values are taken as the true values. The observed angles being horizontal angles, and the base-line reduced to its true horizontal length, the solution of the Δ s gives the true horizontal distances, between the stations. All the computed sides are now booked in Form VII, p. 263.

475. Computation of Minor Secondary and Tertiary Triangles:—Form VII (p. 263.) is for the computation of the sides of Δ s from a given base. The Δ s are to be written clockwise taking the base first. In Fig. 231, abc is the first Δ to be solved of which the side ab and the 3 angles are known. It is to be written as bac . The interior angles of each Δ must equal 180° , and any error is to be distributed equally in whole seconds and not proportionally to the size of angles, any residue again equally to the larger angles. In this class of minor triangulation so much refinement in calculation as shown in Arts. 469-470, pp. 259-260, need not be made. Having obtained the corrected angles take out the Log. sines of the *first* and *second* angles and the Log. cosec. of the *third*, and add the logs of 1st and 3rd to the log. base to obtain the log. side of the 1st, and add the logs of the 2nd and 3rd to the log. base to obtain the log. side of the 2nd. Finally take out the anti-logs in feet. In this Form the side appears opposite or on the same line as the angle, i.e., bc appears on the same line as angle a , and so on.

476. Calculation of the Third Side from Two Sides and the Included Angle:—In some cases the two sides of a Δ and the angle contained between them may be known, it is necessary to find the third side and the other two angles. For instance, in Fig. 231 it is required to find the distance of the station c from the station, a . Now, from the sides, ba , be , and the included angle, abe , we can get the angles, abe , bae , to be $31^\circ 42' 10'' \cdot 393$ and $38^\circ 42' 2'' \cdot 577$ respectively from the following formulas:—

$$\tan. \frac{A-B}{2} = \frac{a-b}{a+b} \cot. \frac{C}{2} \dots \dots \dots (4)$$

$$A = \frac{1}{2} (A+B) \frac{1}{2} + (A-B); B = \frac{1}{2} (A+B) - \frac{1}{2} (A-B) \dots \dots \dots (5)$$

or, using logarithms,—

$$\begin{aligned} \text{L. tan. } \frac{A-B}{2} &= 10 + \log. (a-b) + \text{L. cot. } \frac{C}{2} - 10 - \log. (a+b), \\ &= \log. (a-b) + \text{L. cot. } \frac{C}{2} + \text{Ar. Co. log. } (a+b) \dots \dots (6) \end{aligned}$$

The side, ac , is then obtained to be $3953 \cdot 312$ from the formula—

$$\frac{a}{\sin. A} = \frac{c}{\sin. C}; \text{ or by using logarithms—}$$

$$\log. a = \log. c + \text{L. coses. } C - 20 \dots \dots \dots (3)$$

The calculations may be booked as in Form VIII, p. 265.

477. Clearing Obstacle on the Line between two Stations not visible from Each Other:—When two stations d and g (Fig. 231) are not visible from each other on account of undergrowth, or jungle between d and g , the direction of dg may be obtained as under:—Select two points visible from each other, suppose the points e and f are such that they are visible from each

FORM VIII.—CALCULATION BOOK—FOR THE SOLUTION OF TRIANGLES

When 2 Sides and the Included Angle are only given.

Stations of Triangle.	Data.	Formula— $\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$	Logarithmic Computation.		Value of $\frac{A-B}{2}$	Value of $\frac{A+B}{2}$	Values of A and B.	
a b c	ab = 2205.254 be = 2623.791 $\angle abe = 109^{\circ}35'47''-080$	be + ab = 4829.045 be - ab = 418.537 $\angle \frac{abe}{2} = 54^{\circ}47'53''-515$	Ar. Co. log. L. cot. L. tan.	6.8161388 2.6217839 9.8484783 8.7863510	 30.29.56''-092	90.00.00'' 54.47.53''-515 85.12.6''-485	A ($\angle bae$) = 38.42.09''-577 B ($\angle bea$) = 81.42.10''-393	
Stations of Triangle.	Deduced Angles.	Correc- tions.	Angles for Calculation.	Logarithmic Computation.			Deduced Sides in feet.	
e b a	31.42.10''-393 109.35.47''-080 88.42.02''-577 180.00.00''	0'' 0'' 0'' 0''	31.42.10''-393 70.24.12''-970 88.42.02''-577	log. L. cosec. L. sin. L. sin.	3.3434587 10.2794153 9.9740871 9.7960553	 8.5960611 8.4189293	2205.254 3953.812 2623.791	ab ae be

AREA COMPUTATION OF TRIANGLE *bae*.

Formula—Area = $\frac{1}{2} a.b. \sin. C$	L. sin. b log. ab.	9.9740871 3.3434581	∴ area of $\triangle bae$ = 62.568 acres.
Note :— $\angle b = \angle C$ in formula	log. be. co. con. log.	3.4189293 5.3609121	
and $a = be, b = ab.$	2 area = 125.137 acres.	2.0973872	

other, and from each of which both d and g are visible. Observe the two angles dgc and gcf at the point c , and the two angles f_5 and f_1 at f , and then find the distance, cf . Compute the side df from the $\triangle dcf$, and fg from $\triangle cfg$. Then in the $\triangle dfg$, the two sides, df and fg , and the included angle, dfg , are given and the other angles can then be calculated. On finding these, the line, gd , may be cleared either from d or g , its direction being set off by means of the angles cdg and cgd . A pocket sextant or a prismatic compass and a field-glass will facilitate operations. The calculations are to be done as in Forms VI and VIII.

478. Reduction of Angles to the Centre of Station.—It sometimes happens, when church spires, temples, towers, tombs, pagodas, or other remarkable objects are selected as stations, for the continuation of triangulation, that the theodolite cannot be placed immediately over the point occupied by the axis of the signal. In this case the instrument must be removed to some other convenient station, called a *Satellite or Supplementary Station*, and in America as an *Eccentric station*, near it, and a reduction is then applied to the observed angle in order to obtain the true angle at the centre. A satellite is not of common occurrence. With the best laid out triangulation it is never used, and with tertiary or minor triangulation its occurrence is rare, as it can be easily avoided with little care. It is most likely to occur in a city survey where a steeple may be intersected from other stations,

(a) 1st Solution.—Let A (Fig. 232) be the centre of the true station observed from the stations, B and C , O the place of the centre of the instrument or satellite station, and BAC the required angle. The point O should be selected so that all the flags such as B and C are visible from it. Now set the vernier to zero after the theodolite is levelled at O , and turn the lower plate until the zero line agrees with the magnetic north, and book the bearings of all three stations of the $\triangle ABC$, and also carefully measure the distance, OA on the ground. In the $\triangle ABC$ the $\angle s$ ABC and BCA and the side BC are known. It is necessary to

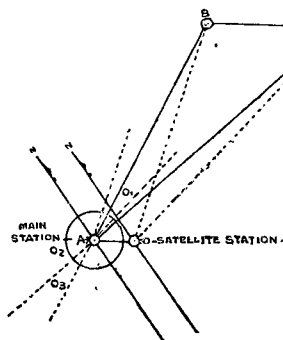


Fig. 232.—Reduction of Angles to Center of Station.

get the angular correction, $+$ or $-$, to be applied to $\angle BOC$ to reduce it to $\angle BAC$, or in other words, the corrections, $\angle ABO$ and $\angle ACO$ are to be applied to the bearings of the lines, OB and OC , to obtain the bearings of the lines, AB and AC , respectively. The difference between the last two bearings gives the reduced angle BAC , with which and the already given data of the $\triangle ABC$, the sides, AB and AC , may be accurately computed in the ordinary way. The position of the satellite may be any of the four— O , O_1 , O_2 , O_3 —shown in the figure, and it would there-

fore be necessary to make a rough plot of the survey to determine how the signs + or - are to be applied to the corrections. The $\angle BOC$ will be less than, equal to, or greater than the $\angle BAC$ according as O lie without the circle passing through B, C, A ; upon its circumference, or within it.

FORM IX.—CALCULATION BOOK FOR REDUCTION OF ANGLES TO CENTRE OF STATION (Fig. 232).

Stations.	Observed Angles.	Reduced Angles for Calculation.	Logarithmic Computation.				Distances in feet.		Remarks.
			log.						
A	20°46'37"	20°46'37"	L. cosec.	3·3473800	10·4501013	3·3473800	2225	BC	$\angle A = 180 - (44°06'38'' + 115°06'45'') = 20°46'37''$
B	44°06'38"	44°06'38"	L. sin.	9·9568771	9·8426373	10·4501013			
C	115°06'45"	64°53'15"	L. sin.			9·8426373			
	180°00'00"			3·7513084		3·6400686	4365·848 5679·477	AB AC	

FROM X.—CALCULATION BOOK (Fig. 232).

Satellite Station.	Stations Observed.	Observed Angles.	Value of Correction + or -.	Corrected Angles.
O Near Temple A (Fig. 232). Distance from Centre of temple to Satellite Station = 38°5'	A	294° 03' 00"		
	B	52° 10' 25"	+0° 26' 44"	52° 37' 09"
	C	73° 08' 30"	+0° 15' 16"	73° 23' 46"

Calculations for the Values of Differences.

Stations.	Data. Distance from Centro. Length of side obtained. Angle opposite ditto.	Logarithmic Calculation.		Differences.
B	38·50 feet. 4365·848 " 118°07'25" "	log. Ar. Co. L. sin.	1·5854607 4·3599314 9·9454354	
		L. sin.	7·8908275	0° 26' 44"
C	38·50 feet. 5679·477 " 139°05'30" "	log. Ar. Co. L. sin.	1·5854607 4·2456916 9·8161422	
		L. sin.	7·6472945	0° 15' 16"

Example.—In the $\triangle ABC$, $\angle BCA = 44°06'38''$, $\angle CBA = 115°06'45''$, and side $BC = 2225'$; therefore, approximately, $AB = 4365·848'$ and $AC = 5679·477'$ (by Eqn. 3, Art. 471, p. 261). Again, the bearings of the lines OB, OC and OA were taken to be $52°10'25''$, $73°08'30''$, and $294°03'00''$ respectively; and the side AO measured with a steel tape to be $38·50'$. Now in the $\triangle BOA$, the $\angle BOA = (360° - 294°03'00'') + 52°10'25'' = 65°57'00'' + 52°10'25'' = 118°07'25''$, and the sides BA and AO are known; therefore the $\angle OBA = 0°26'44''$ which applied with the + sign to the bearing of OB , gives the bearing $AB = 52°37'09''$. Similarly, in the $\triangle COA$, the $\angle COA = 65°57'00'' + 73°08'30'' = 139°05'30''$, and as the sides CA and AO are known, the $\angle ACO = 0°15'16''$

which applied with the + sign to the bearing, OC , gives the bearing $AC = 73^{\circ}23'46''$. Finally the $\angle BAC = 73^{\circ}23'46'' - 52^{\circ}37'09'' = 20^{\circ}48'37''$. This result also shows that the observation of the angles at B and C was correct. The calculations are shown in the Forms IX and X, p. 267. In case a station needed a satellite station the reduction of angles to centre of station should first be made before the ordinary calculations for the horizontal distances are taken in hand.

478 (b). 2nd Solution.—In Fig. 233, let A be the true station and O the satellite station. The line BC is a side of the system whose length is already found. From the measured angles at C and B the approximate value of the $\angle C$ is found, and the lengths of the sides b and c computed. At O the $\angle COB$ is measured, and also the $\angle AOB = a$ is measured. The distance $AO = r$ is also taped. Since the exterior angle at the intersection D , as CDB , is equal to the sum of the opposite interior angles, we have

$$A + y = O + x, \text{ or } A = O + x - y \quad \dots (1)$$

Fig. 233.—Reduction of Angles to Centre of Station.

In the $\triangle COA$, the sides, b , r , and the $\angle COA$ known, whence $\sin x = r \cdot \sin(O + a)/b$; and $\sin y = r \cdot \sin a/c$. $\dots (2)$

Since x and y are very small angles, their sines are proportional to their arcs, hence, $\sin x = x \sin 1''$ where x is in seconds; similarly, $\sin y = y \sin 1''$ and equations (2) become—

$$x = \frac{r \cdot \sin(O + a)}{b \cdot \sin 1''}; \text{ and } y = \frac{r \cdot \sin a}{c \cdot \sin 1''} \quad \dots (3)$$

Substituting these values in (1) we have—

$$A = O + \frac{r}{\sin 1''} \left(\frac{\sin(O + a)}{b} - \frac{\sin a}{c} \right) \quad \dots (4)$$

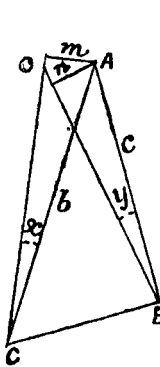


Fig. 234.—Reduction of Angles to Centre of Station.

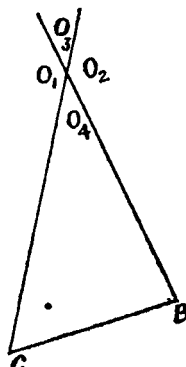


Fig. 235.—Satellite Station's Positions.

where the correction to O is given in seconds of arc. The signs of the trigonometric functions of the angle a must be attended to. $\text{Log. } \sin 1'' = 4.6855749$.

478 (c) 3rd Solution.—Measure perpendiculars from A upon CO and BO (Fig. 234) calling them m and n respectively. Then from equation (1) above, $A = O + (x - y)$; but since the $\angle$ s, x and y are very small, their sines are equal to their arcs, and we have, in seconds of arc $x = m/b \sin 1''$ and $y = n/c \sin 1''$;

whence

$$A = O + \frac{1}{\sin 1''} \left(\frac{m}{b} - \frac{n}{c} \right) \quad \dots (5)$$

There are four cases corresponding to the 4 positions of O (Fig. 235). For these several cases we have—

$$\left. \begin{aligned} A &= O_1 + \frac{1}{\sin. 1''} \left(\frac{m}{b} - \frac{n}{c} \right); & A &= O_3 + \frac{1}{\sin. 1''} \left(\frac{m}{b} + \frac{n}{c} \right) \\ A &= O_2 - \frac{1}{\sin. 1''} \left(\frac{m}{b} - \frac{n}{c} \right); & A &= O_4 - \frac{1}{\sin. 1''} \left(\frac{m}{b} + \frac{n}{c} \right) \end{aligned} \right\} \dots (6)$$

479. Sighting to Centre of Tower.—If the centre of the tower be not visible and cannot be measured to as in Fig. 236, plant the theodolite at E , and direct the telescope so that its line of sight is tangent to the tower at F and G . Measure on these tangents equal distances, Ee and Ef , and direct the telescope to m the centre of the line ef . The telescope will then be directed to the centre of the tower. The distance $EA = Eh +$ radius of the tower. The radius of the tower can be obtained from its measured circumference.

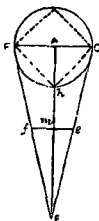


Fig. 236.—Sighting to Centre of Tower.

480. Plotting.—The Δ s are to be laid down on paper, very carefully, as in a chain survey, if necessary by the aid of the beam compasses, taking the mean length of the sides from the table of distances in Form VII, p. 263 (Fig. 231).

481. Filling in Detail.—This plot forms the skeleton triangulation (Fig. 231) and the interior details must be filled in, first by traversing the roads and other conspicuous lines with the theodolite or circumferenter. In small surveys, hedges, buildings, streams, etc. are fitted on to the triangulation by direct chained triangulation and sometimes by chained distances and offsets, or by sketching by means of the prismatic compass or by means of the plane table as described in the next Section on Plane Table Survey (q.v.).

482. Method of Interpolation.*—The following solution of the problem to find the positions of two points by observations from them to two known points is due to Major R. A. Wahab, R. E., and will be found of especial use in trans-frontier reconnaissance where a surveyor is working in a valley with fixed but inaccessible points on either flank. Let A and B be the two known points, and X and Y the points whose positions are to be determined (Figs. 237, 238). From the Δ s, BXY and AXY , $BX = XY \times \sin. BYX \operatorname{cosec}. XBY$; and $AX = XY \sin. AYX \operatorname{cosec}. XAY$; hence, $BX/AX = \sin. BYX \sin. XAY \operatorname{cosec}. XBY \operatorname{cosec}. AYX$. Thus in the ΔABX the ratio BX/AX is determined, and the included angle AXB is given by observation and the side AB is known, so that the sides AX and BX may be determined as follows:—Let $\tan x = BX/AX \sin. AXB$; then if BN is drawn $\perp$ to AX , $AM \parallel$ to BN , and $BM \parallel$ to AX , then $BX/AX \sin. AXB = BN/AX = AM/AX = \tan. AXM$; $\therefore$ the $\angle AXM = x$.

$$\begin{aligned} \text{Now } \sin. AXB \sin. x \operatorname{cosec}. (AXB - x) &= BX/AM \sin. AXB \\ &= BX/AB \times \sin. AXB / \cos. BAX; \text{ but } BX/AB \sin. AXB = \sin. BAX; \end{aligned}$$

$\therefore \tan. BAX = \sin. ARX \sin. x \operatorname{cosec}. (AXB - x)$,
 and $AX = AB \sin. ABX \operatorname{cosec}. AXB$, and $BX = AB \sin. BAX \operatorname{cosec}. AXB$;
 and XY is determined by the solution of the $\triangle AXY$.

The following is a specimen of the computation Form :—

	BYX	$114^{\circ}30'25''$	L. sin.	$\overline{1.9589989}$
	XAY	$24^{\circ}02'25''$	„ sin.	1.6099984
	XYB	$19^{\circ}20'17''$	„ cosec.	0.4799868
	AYX	$55^{\circ}24'43''$	„ cosec.	0.0844658
	AXB	$54^{\circ}23'34''$	„ sin.	$\overline{1.9101052}$
	x	$47^{\circ}52'06''$	„ tan.	0.0435551
	AXB	...	„ sin.	$\overline{1.9101052}$
	x	...	„ „	$\overline{1.8701728}$
	$AXB - x$	$6^{\circ}31'28''$	„ cosec.	0.9445181
	BAX	$79^{\circ}19'39''$	„ tan.	0.7247961

Figs. 237, 238.—Method of Interpolation.

Sum of BAX and AXB	$133^{\circ}43'13''$			
1. ABX	$46^{\circ}16'47''$	„ sin.		$\overline{1.8589716}$
2. BAX	...	„ „		$\overline{1.9924218}$
3. AXB	...	„ cosec.		0.0898948
4. „ AB	...	„ „		4.9061884
Sum of 1, 3 and 4.	...	log. AX		4.8550548
„ 2, 3 and 4.	...	„ BX		4.9885050

483. The Three—Point Problem :—Pothénot's Method.—

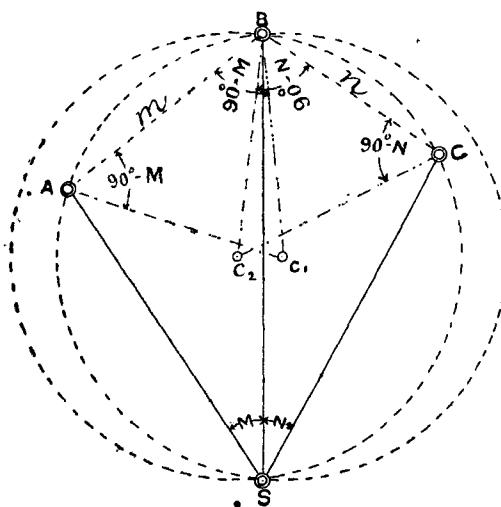


Fig. 239.—The 3-Point Problem using 2 circles.

*The Determination of the Position of an occupied point by observing upon Three fixed and Known Points :—*Let A, B, C be the three points, the positions of which are known. A fourth undetermined point, S, may have been occupied from which the horizontal $\angle s, ASB = \angle M$ and $BSC = \angle N$, may have been observed. The position of S with reference to A, B, C may then be ascertained as under :—

(1. a). **Graphic Solution using the "2-Circle Problem".**—The best method is that which locates the position of the 4th point S (Fig. 239), as the intersection of two circles, one passing through A and B and having all angles of circumference $= \angle ASB =$

$\angle M$ over AB that may be drawn over the line, AB , as chord, the other circle passing through B and C and having over BC as chord all angles of circumference equal to $\angle BSC = \angle N$. From the plotted $\triangle$ side, AB , we lay off at A and B the $\angle$ s, BAC_1 , and ABC_1 , each equal to $\frac{1}{2}(180^\circ - 2 \angle ASB) = 90^\circ - \angle M$, and about the point c_1 thus obtained, a circle ABS is described with radius $c_1A = c_1B$. The observed $\angle ASB = \angle M$ will then be an angle of circumference over AB , and the point, S , will be located somewhere on the arc over the chord AB . By means of the $\angle BSC = \angle N$, a second circle BCS is described over the $\triangle$ side BC about c_2 as centre with radius $c_2B = c_2C$. The observed second $\angle BSC = \angle N$ will be an angle of circumference over the chord BC , hence the point, S , will be situated also upon the arc over the chord BC and the true position of S is at the point of intersection, S , of the two circles.

483 (1 b). Bohnenberger and Bessel's Method.—If a circle

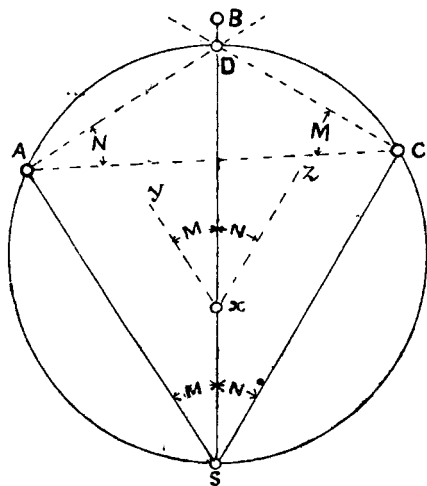


Fig. 240.—The 3-Point Problem.

is described through two of the 3 given points, A and B (Fig. 240), and the station, S , as the 3rd point, then $\angle ASB = \angle ACB = \angle M$; and $\angle BSC = \angle BAC = \angle N$ being angles of circumference upon the same arcs, AD and DC , respectively. Hence, if we lay off observed horizontal $\angle M$ on the base-line, AC , at C , and the other horizontal $\angle N$ on AC at A , the point of intersection, D , of their convergent sides, CD and AD , will be on the line connecting the 3rd point B and the station S . After having thus found the direction of the line, DB , the position of S on the line, DB , may be found as follows: At any point x on DB the observed $\angle$ s, M and N , are laid off to either side of DB as shown. Lines AS and CS drawn through A and C parallel to xy and xz respectively will locate the position of the station S (on DB) with reference to the 3 points A , B and C . This construction is only recommended when BD is sufficiently long to admit of a correct prolongation of its direction towards S .

483 (2 a). Analytical Solutions.—In Figs. 241-243, if the distances, AS and CS , are known, by swinging out arcs of these lengths from A and C their intersection will give, S , the point desired. The distances, m and n , and the $\angle B$ are known from previous work.

$$\widehat{SB} = m \sin. x / \sin. M = n \sin. y / \sin. N \quad \dots \quad \dots \quad (1)$$

$$x + B + y + M + N = 2 \times 180^\circ = 360^\circ \quad \dots \dots (2)$$

$$\text{and } x + y = 360^\circ - (B + M + N) = R \quad \dots \dots (3)$$

FIG. 241.

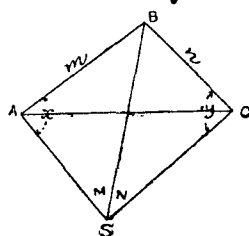


FIG. 242.

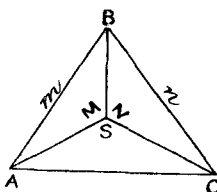
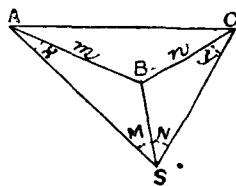


FIG. 243.



Figs. 241–243.—The Analytical Solutions of the 3-Point Problem.

$$\therefore y = R - x, \text{ and } \sin. y = \sin. R \cos. x - \cos. R \sin. x. \dots (4) \text{ \& } (5)$$

Substituting this value of $\sin. y$ in (i) we have

$$\frac{m \sin. x}{\sin. M} = \frac{n (\sin. R \cos. x - \cos. R \sin. x)}{\sin. N}; \quad \text{from which}$$

$$\text{after reduction, } \cot. x = \cot. R \left(\frac{m \sin. N}{n \sin. M \cos. R} + 1 \right) \quad \dots (6)$$

Ex : Given : n Log. 4.4766465 | Observed Angle $M = 107^\circ 29' 10''$

 " m " 4.7187414 | " " $N = 42^\circ 49' 14''$

 " $\angle B$ " $158^\circ 38' 38''$ | Find $\angle x$.

	1st Term.	2nd Term.
$\angle B = 158^\circ 38' 38''$		
$\angle M = 107^\circ 29' 10''$ cosec.	0.0205473	
$\angle N = 42^\circ 29' 14''$ sin.	9.8323201	
$\angle R = \angle B + \angle M + \angle N = 308^\circ 57' 2''$ cosec.	0.1091942	
m ... Log.	4.7187414	
n ... A. C. of Log.	5.5233535	
	0.2041565	—cot. 9076028
	+ 1.6001344	+ 0.8083564
	+ 0.8083564	
$\cot. x = + 2.4084908$... Log	0.3817450	
		$x = 22^\circ 32' 54''$

The value of $\cot. y$ can be found in the same way. With x and y known, the distances AS and BS may be computed by the law of Sines (Art. 471, p. 261). The method fails when S lies on the $\odot$ ce of the circle passed through A, B, C.

483 (2. b). The lengths of the straight lines which join 3 points, A, B, C, are known (Figs. 241–243); at any point, S, in the same plane as A, B, C, the $\angle$ s, ASB and BSC, are observed; it is required to fix S, i.e., to find the distance of S from each of the points A, B, C. Let the $\angle$ ASB be denoted by $\angle M$, $\angle$ BSC by $\angle N$, $\angle$ SAB by x and $\angle$ SCB by y ; then M and N are known, and when x and y are found the required distances, SA, SC, SB, can be found; we have $B + M + N + x + y = 360^\circ$. Let $B + M + N = P$. Then $x + y = 360^\circ - P$.

Ex : *Computation of the Position of a Station from Observations to Three Known Points :—*

Station of Observation, S.....Case I.....(vide note at foot).

Given Stations A, B, C ; $\angle B = 61^\circ 51' 57''$

Given sides ; Log. AB (or $m = 3080'$) = 3.488551 ; Log. BC (or $n = 3586'$) = 3.554610.

Observed Angles at S between A and B or $\angle M = 27^\circ 44'$; between B and C or $\angle N = 25^\circ 40'$. Find the distances AS, BS, CS.

r s'	Angles.		Loga- rithms.	Stations.	Angles.	Log. Sines.	Log. feet.	Feet.	Sides.
B or $(360^\circ - B)^*$	$61^\circ 51' 57''$				Log. (m) 3080	3.488551			
$\angle M$	$27^\circ 44' 00''$	Log. sin. M	9.667786	A (x)	„ $112^\circ 23' 34''$	9.965952	3.786717	6119.5	BS
$\angle N$	$25^\circ 40' 00''$	„ cosec. N	10.363377	B(Sup)	„ $39^\circ 52' 21''$	9.806925	3.627690	4243.2	AS
Sum = P	$115^\circ 15' 57''$	„ n	3.554610	S (M)	„ $27^\circ 44' 00''$	10.332214			
$360^\circ - P = x + y$	$244^\circ 44' 3''$	Ar. Co. log. m	6.511449						
θ	$51^\circ 21' 37''$	Log. tan. θ	10.097222		Log. (n) 3586	3.554610			
$\theta - 45^\circ$	$6^\circ 21' 37''$	„ tan. $(\theta - 45^\circ)$	9.047142	C (y)	„ $132^\circ 20' 29''$	9.868729	3.786716	6119.5	BS
$\frac{1}{2}(x + y)$	$122^\circ 22' 1\frac{1}{2}''$	„ tan. $\frac{1}{2}(x + y)$	10.198038	B(Sup)	„ $21^\circ 59' 31''$	9.573424	3.491411	3100.4	CS
$\frac{1}{2}(x - y)^{\dagger}$	$-9^\circ 58' 27\frac{1}{2}''$	„ tan. $\frac{1}{2}(x - y)$	1.245180	S (N)	„ $25^\circ 40' 00''$	10.363377			
x	$112^\circ 23' 34''$								
y	$132^\circ 20' 29''$								

* In Case III, $360^\circ - B$ must be used instead of B.

$\dagger$ When $\frac{1}{2}(x + y) < 90$, $\frac{1}{2}(x - y)$ has the same sign as $\theta - 45^\circ$. When $\frac{1}{2}(x + y) > 90$, $\frac{1}{2}(x - y)$ has the opposite sign to $\theta - 45^\circ$. The sign for $\frac{1}{2}(x + y)$ is always +. If $B + M + N = 180^\circ$, the problem is insoluble.

From $\triangle ASB$, $BS = m \sin. x / \sin. M$; from $\triangle SBC$, $SB = n \sin. y / \sin. N$. $\therefore \frac{m \sin. x}{\sin. M} = \frac{n \sin. y}{\sin. N}$; $\therefore \frac{\sin. x}{\sin. y} = \frac{n \sin. M}{m \sin. N}$.

Now assume $\tan. \theta = n \sin. M / m \sin. N$, then the value of θ can be found by the Logarithmic Tables; thus $\tan. \theta = \sin. x / \sin. y$;

$$\therefore \frac{\sin. x - \sin. y}{\sin. x + \sin. y} = \frac{\tan. \theta - 1}{\tan. \theta + 1} = \tan. (\theta - 45^\circ); \text{ but}$$

$$\frac{\sin. x - \sin. y}{\sin. x + \sin. y} = \frac{\tan. \frac{1}{2} (x - y)}{\tan. \frac{1}{2} (x + y)}; \therefore \tan. (\theta - 45^\circ) = \frac{\tan. \frac{1}{2} (x - y)}{\tan. \frac{1}{2} (x + y)};$$

$\therefore \tan. \frac{1}{2} (x - y) = \tan. \frac{1}{2} (x + y) \times \tan. (\theta - 45^\circ)$. From this $x - y$ is determined, since $x + y$ is known, thus x and y can be found.

484. River Survey by Triangulation.—The triangulation system can best be applied to the survey of a river. The vertices of the $\triangle$ s are taken on the bank wherever practicable. Any convenient side may be chosen as a base-line. In the case shown by Fig. 244, it would be best, probably, to measure the line 1—3 as the base. Having measured this, as



Fig. 244.—River Survey by Triangulation.

well as all the angles of the various $\triangle$ s, the other sides can be computed. Taking the $\triangle$, 1-2-3, first, and knowing all its angles and the side, 1—3, compute the sides, 1—2 and 2—3. Next in the $\triangle$, 2—3—4, knowing all the angles and side, 2—3, the sides, 2—4 and 3—4, can be computed, and so on throughout the chain. The base-line 1—3 is to be selected on a suitable piece of level ground and its length determined. For a check, measure another line, as 6—8, at the other end of the survey. The work will be checked when the measured and computed lengths of 6—8 will agree. The angles may be measured by the method of repetition. The following are the recorded angles at station 3. The procedure is to start with the line of sight on one point, as 1, and then direct it successively to the points 2, 4, 5 and 1 again. This is known as “closing the horizon.”

Stations observed.	Vernier A.	Vernier B.	Average Reading.	Total Angle.	Mean Angle.	Vertical Angle.
1	0° 00' 00"	0° 00' 30"	0° 00' 15"	0° 00' 03"	0° 00' 05"	0° 40' 00"
2	187 30 00	31 00	187 30 30	62 30 10	62 30 05	1° 25 30
4	376 30 00	31 00	376 30 30	125 30 10	63 00 00	1° 20 00
5	556 30 00	29 30	556 29 45	185 29 55	59 59 45	0 47 30
1	1080 00 00	1079 59 30	1079 59 45	359 59 55	174 30 00	
				359 59 55		

Before any of the computations are taken in hand, the angles must be balanced so that the condition is satisfied that the sum of the angles of each $\triangle$ equals 180° . In practice this condition is not fulfilled. In any $\triangle$, if all the angles have been observed with equal care, one third of the total correction should be applied to each. With the method of repetition and the ordinary theodolite the total error in each $\triangle$ should not exceed one minute. Form XI

below will be suitable for use in reducing the observations, (See also Form VII, p. 263).

Note that, starting with the base-line, it will not be necessary to look up any of the anti-logs except for the purpose of a check. For, having balanced all the Δ s and knowing the bearing of the base-line, from the corrected angles, the bearings of the lines down one side of the chain, across, up the other side, and across again may be found. Knowing the bearings and the logs of the lengths of these sides, these values can be substituted in the Form on p. 243 and the survey plotted and area calculated by the method of latitudes and departures. The bearing of the base-line is to be observed during field-work. The calculation of the bearings from the angles may be checked by noting, when the base-line is again reached after going round the entire outside boundary, whether the bearing then found agrees with the bearing assumed at the start. Note also that the algebraic sums of the latitudes and departures should each be zero, without further balancing.

Measurement of Base-Line.—The base-line may be measured with 500 feet steel-bands, hung clear of the ground, the depth of the catenary and level of end being measured, and corrections made for inclination, temperature, catenary, stretch of tape (q.v. pp. 250-251).

Form XI.—Computation of Lines in Plané Triangulation.
(Fig. 244).

	Logs.	Antilogs.		Logs.	Antilogs.
1—3			4—5		
sin. 1—3—2			sin. 4—5—6		
sin. 1—2—3			sin. 4—6—5		
1—2			4—6		
sin. 2—1—3			sin. 5—4—6		
sin. 1—3—2			sin. 4—5—6		
2—3			5—6		
sin. 2—3—4			sin. 5—6—7		
sin. 2—4—3			sin. 5—7—6		
2—4			5—7		
sin. 3—2—4			sin. 6—5—7		
sin. 2—3—4			sin. 5—6—7		
3—4			6—7		
sin. 3—4—5			sin. 6—7—8		
sin. 3—5—4			sin. 6—8—7		
3—5			6—8		
sin. 4—3—5			sin. 7—6—8		
sin. 3—4—5			sin. 6—7—8		
4—5			7—8		

Details may be filled in by running compass or plane table traverses between stations. In Fig. 244, if an accurate map of the river is desired, compass or plane table traverses may be run between adjacent stations, as 2—4, 4—6, etc., and offsets taken to the bends in the bank as often as may be necessary.

485. X-Y-Z Surveying.—It will be a good practice to compute the co-ordinates of a number of points referred to some fixed point and line as the origin and axis of X. For example, take a base-line AC, (Fig. 245) and any number of other stations, B, D, E, etc. The base-line should situate on high level ground and be as long as practicable up to about a quarter of a mile. The other stations may be any conspicuous points, as the tops of temples, small hills, etc.

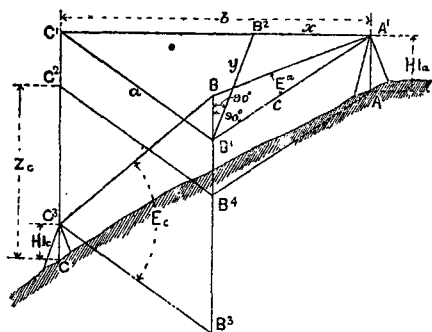


Fig. 245.—x-y-z Surveying.

Plant the theodolite at A and C, observe and book the horizontal angles made with the base-line, AC, by the lines running from each of those points to the various outside stations, observing and booking the angles of elevation and depression in each case. In Fig. 245 B is an outside station. AC is the base, with C lower than A by the depth $CC^1 = 9'$. The line $AC^1 (= 686.12')$ is therefore horizontal and equal to the reduced length of the base. $H1a = AA^1$ is the height of the instrument at A, while $H1c$ is the corresponding height at C. Ea and Ec the angles of elevation or depression of the station from points A and C. Zc is the elevation of C with respect to A. A^1, C^1, B^1 are in the same horizontal plane, and the horizontal angles taken at A and C are $C^1A^1B^1$ and $A^1C^1B^1$. Having these angles and the length $b = AC^1 = A^1C^1$, compute a and c by the law of sines (equation 3, p.). Having a and c compute $B^1B = z$ by the formulas:

$$z = a \tan. B^1C^1B + Ccr - (C^1C - H1c) - Hs$$

$$= a \tan. Ec - Zc + H1c + Ccr - Hs \quad \dots \quad (7)$$

and
$$z = c \tan. B^1A^1B + Ccr + H1a - Hs$$

$$= c \tan. Ea + Ccr + H1a - Hs \quad \dots \quad (8)$$

Form XII.—Calculations of Sides of a Triangle (Fig. 245).

Triangle.	ABC	ADC
A	107° 41'	72° 34'
B	8° 12'	4° 46'
C	64° 07'	102° 40'
Check	180° 00'	180° 00'
a	4583.3	7877.6
Log a	3.66117	3.89639
L. sin. A	9.97898	9.97958
Log b (686.12)	2.83640	2.83640
Co-log sin. B	0.84579	1.08041
L. sin. C	9.95409	9.98930
Log. c	3.63628	3.90611
c	4328.0	8055.9

where Ccr denotes correction for curvature and refraction and Hs the height of the signal above ground at the stations. The agreement of these results will check the work both instrumental and numerical, as B^1C^1B and B^1A^1B are independent angles. There will evidently be two formulas for computing each of the distances $B^1B = z$ and $A^1B^1 = x$, namely

$$y = c \sin. C^1A^1B^1 \quad \dots (9)$$

$$y = a \sin. A^1C^1B^1 \quad \dots (10)$$

and $x = c \cos. C^1A^1B^1 \quad \dots (11)$

$$x = b - \cos. A^1C^1B^1 \quad \dots (12)$$

**FORM XIII.—Calculations of
X-Y-Z. (Fig. 245).**

Co-ordinates.	Station B	Station D.
x	-1314.6	2413.5
Log. x	3.71880	3.38265
L. cos. A	9.48250	9.47654
Log. c	3.63628	3.90611
L. sin. A	9.97898	9.97958
Log. y	3.61526	3.88569
y	4123.4	7685.8
x	-1314.6	2413.5
b	686.12	686.12
a cos. C	2000.7	1727.4
Log. (a cos C)	3.30119	3.23739
Log. cos. C	9.64002	9.34100
Log. a	3.66117	3.89639
Log. sin. C	9.95409	9.98930
Log. y	3.61526	3.88569
y	4123.4	7685.8
Log. c	3.63628	3.90611
Log. tan. Ea	8.39577	8.12651
Log. (c tan Ea)	2.03205	2.03262
c tan. Ea	+107.69	+107.80
Ccr.	+ .39	+ 1.33
H Ia	+4.91	+ 4.91
	112.99	114.04
Ha.	-16.00	-21.00
z	96.99	93.04
Log a	3.66117	3.89639
L. tan. Ec	8.37496	8.14045
Log a tan. Ec	2.03613	2.03684
a tan. Ec	+108.67	+108.85
Ccr.	+ .43	+ 1.28
H. Ic	+ 4.66	+ 4.66
	113.76	114.79
Ha.	-16.00	-21.00
	97.76	93.79
zc	.75	- .75
z	97.01	93.04
Mean of z	97.00	93.04

If the results agree in these last cases the numerical work only will be checked. The angles may be observed by the method of repetition.

For station B, Ea and Ec were found to be $1^{\circ}25'30''$ and $1^{\circ}21'30''$ respectively, and for station D, $0^{\circ}46'00''$ and $0^{\circ}47'30''$. Forms XII and XIII may be used for the reduction of the field-notes. Form XII is used in finding the lengths a and c (see Forms VI and XI) and Form XIII for finding x, y, z .

486. American Practice in Triangulation: The Following Instructions of the Mississippi River Commission for Secondary and Tertiary Triangulations, which are the results of many years' experience, obtained by the author from Messrs W. & L. E. Gurley, Troy, N. Y., U.S.A., through the courtesy of a United States Engineer officer, will be found useful:—

Appendix C₃.—Instructions for Secondary Triangulation.

Locating Station.—In locating stations it is desirable to fix them at such points as give good conditioned triangles. The smallest angle in any triangle should never be less the 30° and but few of these should be permitted to enter into the system. The triangles should lie in such a way that pointings can be made from any station to the stations immediately above and below on the same side of the river. That is, blind lines should always be avoided. Other things being equal, stations should be set where they can be readily found and where they will not be disturbed.

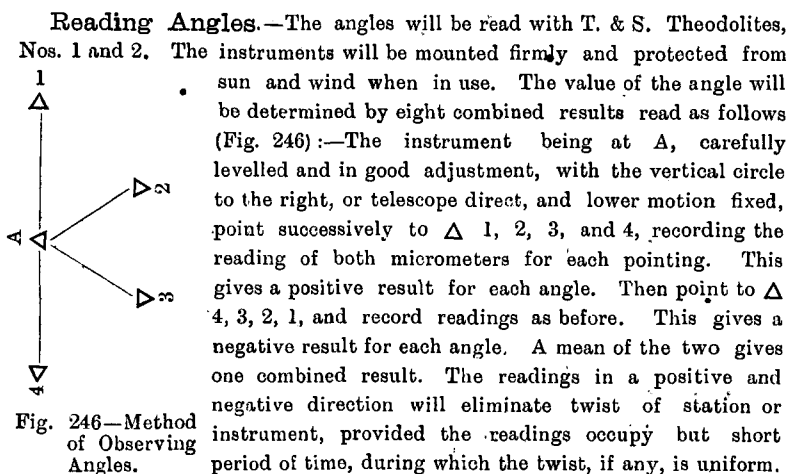


Fig. 246—Method of Observing Angles.

For the next combined result. The telescope will now be reversed, that is, revolved through, leaving the pivots in the same wyres and the whole will be revolved 180° in azimuth. The vertical circle will then be on the left; the limb will be shifted $22\frac{1}{2}^\circ$, and the stations will be read forward and back as before. The notes for this series will be headed circle left. Reversing the telescope will eliminate errors of collimation, small level errors, and inequality of pivots. Shifting the limb so as to read the angles at equal intervals around the circle will eliminate periodic errors and errors of graduation.

The same programme is followed until all the results are obtained—the limb being shifted and the telescope reversed after each combined result. The micrometers should be adjusted so the run will be nearly zero. This should, however, be tested at the beginning of each day's work, and entered in the note-book.

Closing Triangles.—The error in closing a triangle should rarely reach and never exceed 6 seconds, and the average closure should be much below this. This will require great care in the centering of instruments and targets. A discrepancy of one-third of an inch will give an error of a second in a distance of 1 mile. A transparent cloth, phaseless target will be used, the size varying with the length of triangle sides.

Base-Lines.—Base-lines will be measured at intervals of about 75 miles. This will be done with the 300-foot steel tape. The line should be carefully staked out, and its grade determined instrumentally. Supporting stakes will be driven at intervals of 30 feet. The stakes marking the extremities of each tape will be firmly set and free from any disturbing influence due to tension of tape or otherwise. On these stakes strips of zinc will be fastened and remain until the whole measurement is completed. The temperature of the tape will be determined by three thermometers placed near the ends and in the middle of the tape. They will be attached

to suitable supports and placed with their bulbs near the tape when measurements are being taken. Observers must be careful to keep sufficiently far away so as not to affect the thermometers.

The tape will be suspended in hooks at intervals of 30 feet, and attached in such a way that it may swing freely and eliminate friction as far as practicable. The tension of the tape will be kept uniform while measuring by attaching a weight of 16 pounds. The extremity of each tape length will be marked on the zinc strip with a fine line and suitably numbered. The preservation of these strips furnishes a ready means of comparison of each tape length at any future time.

The line should be measured two or more times, with a discrepancy when reduced of not more than 1 in 250,000. This can readily be done if measurements are made on cloudy days or at night.

Observations for Azimuth.—The azimuth of each base-line will be determined by observing, with a triangulation instrument two closely circum-polar stars at elongation on two different nights. The instrument and light should preferably be at the extremities of the base or a triangle side. The following order of observing will be used.

FIRST.	SECOND.	THIRD.	FOURTH.
Circle right. Point to light. Point to star and note time. Read level direct and reverse. Point to star and note time. Point to light.	Shift limb 45° Circle left. Point to light. Point to star and note time. Read level direct and reverse. Point to star and note time. Point to light.	Shift limb 45° Circle left. Point to light. Point to star and note time. Read level direct and reverse. Point to star and note time. Point to light.	Shift limb 45° Circle right. Point to light. Point to star and note time. Read level direct and reverse. Point to star and note time. Point to light.

On the second night repeat this programme, starting with a reading of limb 45° greater than the last reading of the previous evening. It will probably be found most convenient in these observations to use Polaris, δ Ursæ Minoris, λ Ursæ Minoris, and β Cephei. The time will be determined by observing the meridian passage of high and low stars.

Stone Line Bench-Marks.—At intervals of about 3. miles along the river, lines of pipe and tile marks will be set for future surveys. These lines will be numbered and located about as shown on maps on file in this office. The marks nearest the river will be far enough back to be safe from erosion for many years; the others will be half a mile further back. In cases where the bluffs are near the river the rear marks may be omitted. The marks will preferably be placed at property corners, along public roads, or on property lines, in places where they can be readily found and where they will not be liable to disturbance. It is desirable to determine the azimuth and distance between the successive marks on the same line when

practicable. The marks should also be as nearly in a line as the conditions of location above named will admit. The marks will be connected directly with the secondary triangulation, where practicable, by 3 pointings from 2 or more secondary stations, and an equal number from the point to be located to 2 stations that will give a fairly good triangle. Where the points cannot be located directly from the secondary work, a tertiary system may be used, starting and closing on a secondary line. In this work the angles may be read with a good 10-second transit, and the triangles should close within 15 seconds. A steel tape or chain may also be used, where desirable, in locating the point which is farthest from the river.

Cutting Timber.—Cutting timber to clear the lines of sight or for material with which to build stations should be avoided as far as practicable. Where cutting is necessary, a strict account must be kept of the number of trees cut, their size, and kind of timber.

Description of Stations.—A minute description of each station will be made and entered in note-book kept for that purpose. This description will be complete for each station, and will show what the Geodetic point is and how marked. Its location with reference to surrounding objects will be shown by an accurate sketch giving azimuth and distance to bearing trees, houses, or other prominent objects.

A similar record will also be kept of the stoneline marks.

Tertiary Triangulation.

Triangles.—Where the secondary stations are more than 3 miles apart a tertiary system will be carried giving points on either bank at intervals of a mile or less. This system will begin on a triangle side of the secondary system, or a carefully measured base, and all of the available secondary stations, will be used in the tertiary chain. The tertiary work will also close on a line of known length as a check on its accuracy. The discrepancy should not exceed one in 3,000. The system should be laid out and the angles read in advance of the topographers, so that the azimuths and lengths of sides can be used in checking stadia work.

Stations.—The station point may be marked by a pole 2" in diameter stuck into the ground, and bearing a red and white flag to distinguish it from the ordinary sounding flags. A strip of white cloth wrapped near the bottom of the pole will admit of the pointings being made so low down that errors arising from disturbance of the pole by the wind will be inappreciable.

Instruments Stations.—For observing the instrument may be placed on the ordinary tripod centered over the hole after the pole has been removed.

Observing Angles.—The angles should be read with a 10-second instrument in good adjustment, and should be repeated at least 3 times on different parts of the circle to check errors of reading. It is desirable to have the first series read on azimuth. Having pointed to the first station, read to all of the others in succession. Pointings should also be made to

all of the sounding flags in the vicinity, as well as prominent objects on land, such as chimneys, houses, etc., the location of which will serve to check the topographical work.

For the second series slip the lower limb 60° and read to the stations in the opposite order from the first series. Slip the limb the same amount again and read the third series.

Marking Stations.—The river ends of the stone lines will be made points in the tertiary system, and whenever practicable the stones should also be located trigonometrically. Tertiary points which are likely to remain undisturbed for some time should be plainly marked with strong stakes, 2 ft. high, the number of the point, the initials of the observer, and date being marked on it with red chalk.

Section VII.—Surveying with the Plane Table.

487. Orienting the Plane Table.—The plane table is said to be oriented when, a line being drawn on it to represent a line on the ground, the line on the plane table lies in the same vertical plane with the line on the ground, and any one point in the one is situated directly over the corresponding point in the other. Take for instance, a closed traverse, a plot of which is drawn on the table. For the table to be oriented at one corner of the traverse it would have to be set up at that corner with the corresponding point on the plot directly over it. The two lines intersecting at that corner would be vertically below the corresponding ones on the plot, and all the other lines of the plot would be parallel and proportional to the corresponding lines on the ground. Or suppose a point from which a number of points are radiating. For the table to be oriented at that point, the corresponding point on the plot must be directly over it, and every one of the lines

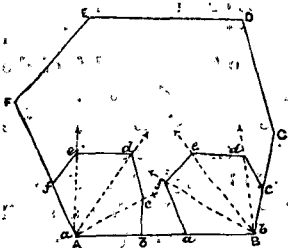


Fig. 247.—Orienting the Plane Table.

on the plot must be found in the same vertical plane with the corresponding line on the ground. From this the method of orienting will easily be understood. The table is first set up with the point on the plot, and the line or lines running from it over the corresponding point and line or lines on the ground. Calling the point *A* and the further extremity of a line running from it *B* (Fig. 247), let the plotted line be *ab*. The edge of the alidade is next brought into coincidence with *ab*, and an observation taken to determine how far and to which side the line of sight misses *B*. The table is then revolved on its vertical axis, the tripod leg moved, etc., until *a* being directly over *A*, and the edge of the rule coinciding with *ab*, the line of sight is found directed to *B*. The operation is often difficult except when *a* lies in the vertical axis of the table. The plumbing arrangement, shown in Figs. 38 and 44, is so arranged that if the end of the upper arm is brought in contact with a point on the paper, the plumb-bob will hang over a point

exactly below on the ground. A shifting centre would be convenient here (Art. 110, p. 56). The following will make *orienting* more clear.

488. The Method of Plane-tabling is based on the following Principle:—In Fig. 247, let the plot *abcdef* be the representation on paper mounted on a table of the ground *ABCD EF*; if the table be so placed that the point, *a*, on paper is just over the point, *A*, on the ground represented by it, and if the directions of *af* and *ab* are made to coincide with those of *AF* and *AB*, represented by them by the edge of the alidade, then all the points on the paper will be symmetrically situated with the points they represent on the ground. The line *ab* will be parallel and proportional to *AB*, *cd* to *CD*, *ac* to *AC*, *ad* to *AD*, *cb* to *CB*, *bd* to *BD*, and so on. The point *c* will be on the line joining *A* to *C*, *d* on the line *AD*, *e* on *AE*. Now if the table be moved parallel to itself and transferred to *B* till *b* coincides with *B*; then *ba* will lie in the direction *BA*, *bc* in *BC*, and as before, all other points on the plan with respect to *b* will be symmetrically situated to all other points on the ground with respect to *B*. The case will be similar if the table be moved parallel to itself to points *C*, *D*, *E*, *F* on the ground respectively.

489. Orientation by the Trough Compass.—The table may also be transferred parallel to itself by moving the table until the rectangular compass needle, *declinator*, points north when placed along the meridian line drawn beforehand or by setting the compass attached to the table to the same reading at each station, but it is a slow and inaccurate method. The magnetic meridian is roughly determined at any well-determined station, when the table is perfectly oriented by the use of the declinoire itself, the meridian line being drawn upon the sheet along the straight-edge of the box when the needle points to 0° . Or the table may be oriented by making the straight-edge of the box coincide with one of the meridians of the projection and then turning the board until the needle points to the right or left of the zero, according to the amount and direction of the magnetic deviation.

All of this and future discussion assumes the line of sight and the edge of the ruler to lie in the same vertical plane. This is the strict requirement, but if the line of sight and the ruler lie in vertical planes parallel and about an inch or an inch and a half apart, or even making with each other a small angle, the error introduced will generally be inappreciable. Usually they are parallel. The purpose of the plane-table is to furnish a means of making a map at once in the field.

490. The General Use of the Plane Table is best explained by descriptions of its use in special cases. These are divided into two classes as the *point sought* to be plotted is found (i) directly at, or (ii) removed from the station occupied by the instrument.

491. Problems of the First Class.—The position on the table of the station at which the table is set up (always should be leveled with a $\odot$ level on the ruler) can be found, and the table oriented in many ways;—

491. Prob. 1. When the Station has been fixed by Two Rays from the Ends of a Base or from other Stations, all that has to be done is to stick a pin in the table at the station mark, lay the alidade against it and against the mark on the table showing the most distant station from which a ray has been drawn, turn the table until the sights are in a line, *e.g.*, D is fixed by rays from the ends of the base, AB . Then place the alidade against pin at d and move the table until the sights are in a line with DA , clamp the table, which is then oriented (Fig. 247).

491. Prob. 2. Location By Re-section, i.e., To Find the Position of an Unknown Point when only One Ray has been drawn to the Station from a Fixed Point.—Plant the table over the station to be fixed, say E (Fig. 247), and place the alidade along the ray that has been drawn, say ae , revolve the table until the sights align on A , clamp the table, which will then be oriented. Stick a pin in at d on the table and turn the alidade about until it is aligned on D , and draw a ray which will intersect the line already drawn at e on the table, the position required. This latter operation is called **resection**. The 2nd station should be so selected that a pointing to which is as nearly 90° from the orienting line as possible. Repeating the last operation with other fixed stations will, if the rays intersect, give certainty to the new position. This method should always be used and is always preferable.

491. Prob. 3. The "Two-point Problem."—(a) **To Locate a Point not plotted on the Table and from which are visible only Two Plotted Points.**—The occasion may arise where it is

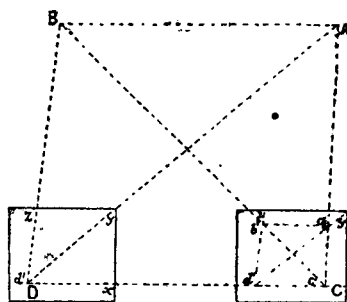


Fig. 248.—Two-Point Problem.

desirable to place the table in position at a given point, from which only two determined points are visible. This may be done by the following methods: Let the two visible points be A and B , plotted at a and b . C being the point to be located (Fig. 248), select a fourth point, D , such that the angles CAD and CBD will lie between 30° and 120° . Set up over D , plot it on the paper at d' , and draw rays, $d'x$, $d'y$, $d'z$, in the directions of C , A , and B . On $d'x$ take a point c' such that the distance $d'c'$, to the scale to which a and b have been plotted, equals the estimated distance from D to C . Next, assuming C to be correctly plotted, at c' , set up at C , and orient by sighting to D . Draw lines, $c'a'$, in the direction of A , and $c'b'$ in the direction of B , intersecting the lines, $d'y$ and $d'z$, in the points a' and b' . Join a' and b' . The quadrilateral $a'b'c'd'$ is similar to $ABCD$. Construct on the tracing paper another similar quadrilateral $abcd$, having the side, ab , equal to the line, ab , as plotted on the plane table map. Place the tracing paper on the table so that the points a and b on it coincide with a and b on the plot beneath. By pricking

through the point b , its location on the plot is finally obtained. By making the construction on paper the correctness of this method can easily be seen. If an assistant with a stadia-rod can be sent to A and B , with the table set up at C , the distances, AC and BC , can be determined. Then by swinging arcs from a and b on the plot, c may be located at once.

491. Prob. 3b. Two-Point Problem.—One method possesses the virtue of requiring no linear measurements and demonstrates in a very satisfactory manner the effectiveness of the table in determining position by resection. (Figs 249, 250, 251, 252): *Two points, A and B , not conveniently accessible, being given, by their projections a and b , to put the plane table in position, at a third point, C* (The Capital letters refer to points on the ground, and the small ones to their corresponding projections).

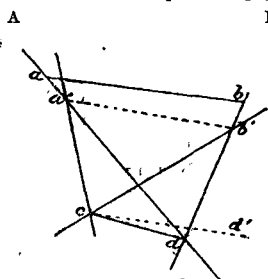


Fig. 249.—Two-Point Problem.

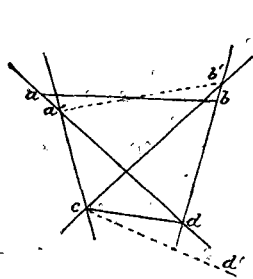


Fig. 250.—Two-Point Problem.

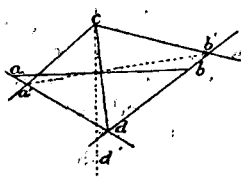


Fig. 251.—Two-Point Problem.

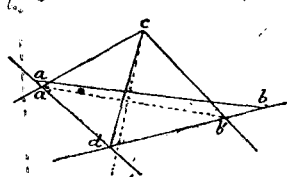


Fig. 252.—Two-Point Problem.

Select a fourth point, D , so that the intersections from C and D upon A and B make sufficiently large angles for good determinations. Put the table approximately in position at D , by estimation or by compass, and draw the lines, Aad and Bbd , intersecting at d ; through d draw a line directed to C . Then set up at C , and assuming the point c on the line dC , at an estimated distance from d , and putting the table in a position parallel to that which is occupied at D , by means of the line cd , draw the lines from c to A , and from c to B . These will intersect the lines dA and dB at points, a' and b' , which form with c and d a quadrilateral similar to the true one, but erroneous in size and position.

The angles which the lines, ab and $a'b'$, make with each other

491. Prob. 4b The 3-Point Problem.—Methods without a Compass when the Point is Visible from Three Stations already fixed on the Paper.—Set the table up at the point whose

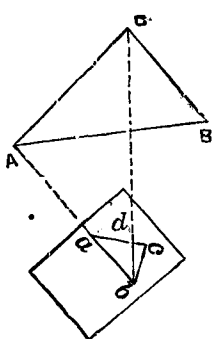


Fig. 255.

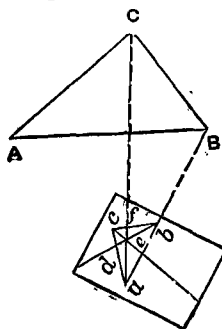


Fig. 256.

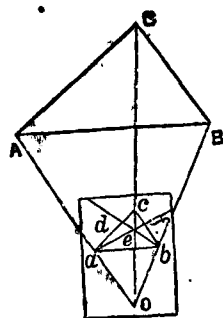


Fig. 257.

Figs. 255–257.—The Three-Point Problem.

position is required. Put the alidade along ba (Fig. 255), move the table horizontally till the point A is bisected; clamp it and draw a line along the alidade from b to C , cutting ac in d . Unclamp the table, place the alidade along ab (Fig. 256), and move the table till it bisects the station point, B . Then clamp the table and draw the line afC , intersecting the previous line bd in e . Put the alidade along ec and turn the table by unclamping it till C is bisected (Fig. 257). The direction of the table is now correct. To fix the unknown point O , draw the lines, AaO , CcO and BbO ; and their point of intersection is the point required. Lines are drawn both from A and B for check, but the point can be found by intersecting either of the points A or B . When the point C , with regard to the point O , is situated on the other side of AB or below it, the lines, af and bd , will intersect on the opposite side of ab to that on which c is; if the point O be situated within the $\triangle ABC$, these lines, af and bd , will diverge instead of converge, in which case they must

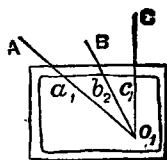


Fig. 258.

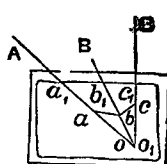


Fig. 259.—Good.

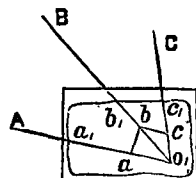


Fig. 260.—Bad.

Figs. 258–260.—Three-Point Problems.

be prolonged in the opposite direction until they intersect for the point e . As the accuracy of the result depends on the length of ce , this method should seldom be used. The above method of fixing

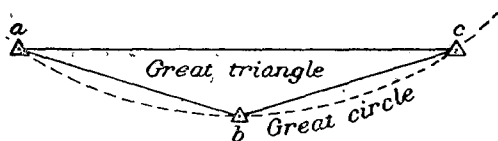
an unknown point is only applied to reconnaissance, and is hardly admissible in accurate surveying.

491. Prob. 4c. The 3—Point Problem.—The Quickest and Easiest method is to set up the table at the point whose position is required, and to stretch a piece of tracing paper on it (Fig. 258). From any point o_1 , where a pin is driven, draw the rays, o_1a_1 , o_1b_1 , o_1c_1 , to the three visible points A, B, C . Now slide the tracing paper over the table (Fig. 259) until the rays o_1a_1 , o_1b_1 , o_1c_1 , pass through the points a, b, c , already plotted. Pin through o_1 , which is then the position, O , of the station at which the table is set up. This method will not give accurate results, if the intermediate point, B , is more distant than either A or C , as in (Fig. 260.). In this case it will be found that the point, o , might be considerably to the right or left of its proper position, and the lines, o_1a_1 , o_1b_1 , o_1c_1 , still pass through the points a, b, c .

This method also fails when the point of observation is situated on the $\odot$ ce of a $\odot$ described about the three known points. Points whose relative positions are as shown in Figs. 258 and 259 should therefore be chosen.

This method is less exact and is not so convenient as the others previously or hereafter described and is impracticable when the wind blows.

491. Prob. 4d. The Three-point Problem.—A subordinate



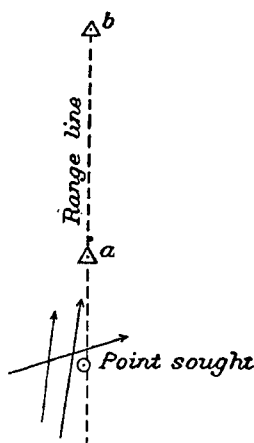
station is located at any desired place where a good view of the surrounding features can be obtained. If the position of this point has not been previously determined it is now effected by means of the resection of lines from three fixed points.

When the table is imperfectly oriented, the lines drawn

Fig. 261.—Three-point Problem.

from the three projected points, when sighting on the corresponding actual points, will not intersect at one point unless all four are on the $\odot$ ce of a circle (q.v., Fig. 263, Indeterminate position). Except in this case, two of the lines will be parallel, intersected by a third (q.v., Fig. 264, Station on range line between two fixed points, and Fig. 262, Station on prolongation of range line), or they will form a small triangle called the *triangle of error* (Figs. 261, 263, 265, 266). The solution of the three-point problem determines the location of the station occupied and orients the table simultaneously. *The relative positions of the three fixed points with reference to the new station have an important bearing on the strength of its determination.*

In the following statements in regard to the different groupings of points met in practice, the term *fixed points* means points



already determined and plotted on the sheet; the *great Δ* is one formed by the three fixed points, and the *great circle* is the circle passing through them.

When the new station is outside the great $\odot$, the strength for determination of a position will be weak when the middle point as seen from the new station is the farthest of the three and the angles are small (q.v., Fig. 263). If the new station is located outside the circle, and some distance below it, the angles are small and the determination correspondingly weak. The

Fig. 262.—Three-point Problem.

determination increases in strength for given angles as the middle point approaches the new station (Fig. 261).

When one angle is small or 0° (points in range), the determination will be strong, provided the two points making the small angle or range are not too near each other when compared to the distances to the new station and to the third point; provided also the angle to the third point is not too small (Fig. 262).

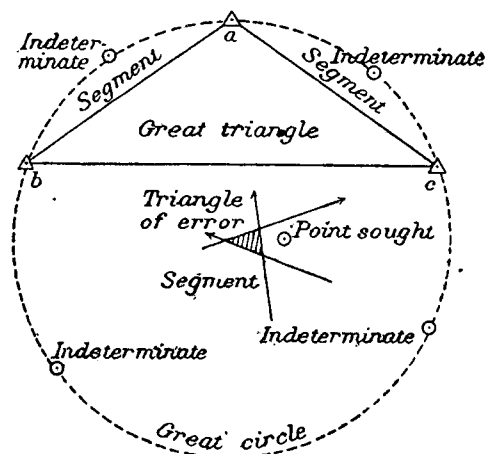


Fig. 263.—Three-point Problem.

of gravity of the great triangle (Figs. 263, 264, 265). There are a number of graphic solutions.

Lehmann's Method is the simplest and applies under all conditions. The directions are stated in the following rules,

RULE I.—The point sought is always distant from each of the three lines drawn from the three fixed points in proportion to

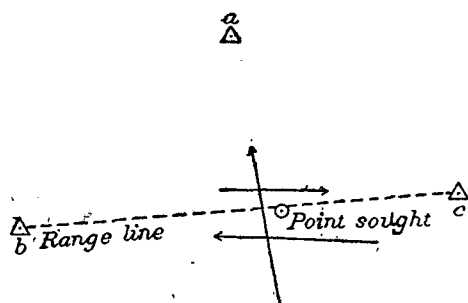


Fig. 264.—Three-point Problem.

it will always be found on the corresponding side of each of the lines drawn from the fixed points, i.e., if it is on the right side of one line, it is on the right side of each one of the other two, and if on the left side of one, it is on the left side of each one of the other two.

The simplest case for the application of this rule occurs when

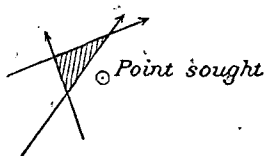
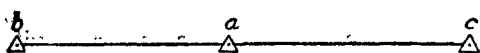


Fig. 266.—Three-point Problem.

locating the point sought with reference to the lines from the fixed points:

RULE 2.—When the point sought is without the great circle it is always on the same side of the line from the most distant point as the intersection of the other two lines (Fig. 261).

RULE 3.—When the point sought falls within either of the three segments of the great circle formed by the sides of the great triangle the line drawn from the middle point lies between the point sought and the intersection of the other two lines (Figs. 263, 264, 266).

* *Demonstration.*—A, B, C, (Fig. 267) are projections of the three signals from which it is desired to determine by resection the position of a fourth point, D. The table being out of position to the right, the triangle of error formed by the three lines from A, B, C is ab, ac, bc . The true point occupied lies at D, being at the intersection of the circles $AB\ ab; AC\ ac; BC\ bc$. Now, if perpendiculars be drawn from D to the lines drawn from A, B, C, we shall have— $Da; Db; DA; DB$ or $Db; Dc; DB; DC$.

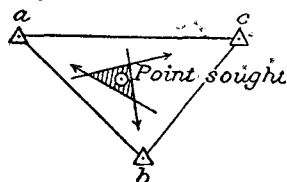


Fig. 265.—Three-point Problem.

the distances of the corresponding actual points from the station occupied,* and

the station to be determined is within the triangle formed by the three fixed points; the point sought must then be within the triangle of error to satisfy the conditions (Fig. 265). Although Rule I is sufficient in itself for the solution of the problem, there are two subordinate rules, which assist in properly

Fig. 266 : When the three fixed points are in a straight line. In this case the points are considered as being in the circumference of a circle of infinite diameter and the point sought always lying in one of the segments of the great circle. The treatment of this case is then identical with that of Fig. 263.

The preceding cases are all examples of the conditions which may occur when the table is deflected to the right. By turning the printed side of the illustration to the light and looking at the figures through the paper, they will appear reversed, and they will then be examples of conditions which may occur when the table is deflected to the left.

Repetition.—When the true point has been estimated and

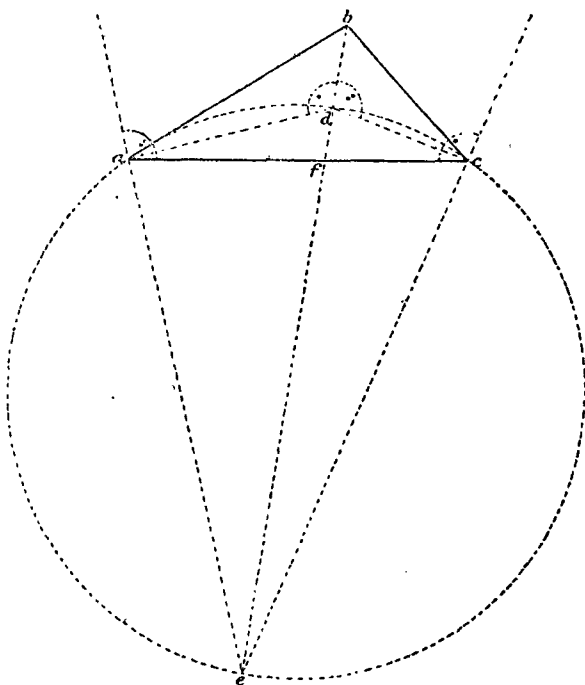


Fig. 268.—Bessel's Solution of Three-point Problem.

marked on the sheet in accordance with the foregoing rules a new orientation is made. If the lines from the three stations now intersect at that point, it proves the estimate to have been correct and the position is determined. If a new triangle of error is formed, it indicates an erroneous estimate, and the operation must be repeated.

491. Prob. 4e. Bessel's Method by Inscribed Quadrilateral is the simplest method by construction. The objection to it

arises from the fact that in practice the intersection of the construction lines often falls beyond the limit of the board.

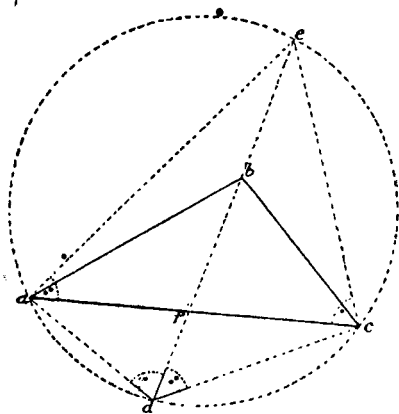


Fig. 269.—Bessel's Solution of Three-Point Problem.

on the sheet representing the signals, A, B, C, on the ground. The table is set up at the point to be determined, d , and levelled. The alidade is set upon the line, ca , and a directed, by revolving the table to its corresponding signal, A, and the table clamped; then,

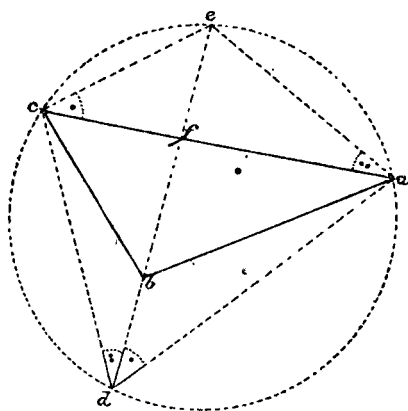


Fig. 270.—Bessels Solution of Three-point Problem.

Set the alidade upon the line, be , direct b to the signal, B, by revolving the table, and the table will be *in position*. Clamp the table, centre the alidade upon a , direct the telescope to the signal, A, and draw along the rule the line, ad . This will intersect the line, be , at the point sought. Resection upon C, centering the alidade on c in the same manner as upon A, will verify its position.

By this method a quadrilateral is constructed with all the angles in the circumference of a circle, one diagonal of which passes through the middle one of the three fixed points and the point sought. On this line the alidade is set, the telescope directed to the middle point, and the table is *in position*. Resection upon the extreme points intersects in this line and determines the position of the point sought.

Figs. 268, 269, 270 and 271: Let a, b, c be the points with the alidade centering on c , the middle signal, B, is sighted with the telescope and the line, ce , drawn along the edge of the rule. The alidade is then set upon the line, ac , and the telescope directed to the signal, C, by revolving the table, and the table clamped. Then, with the alidade centering on a , the telescope is directed to the middle signal, B, and the line, ae , is drawn along the edge of the rule. The point e (the intersection of these two lines) will be in the line passing through the middle point and the point sought.

The opposite angles of the quadrilateral, $adce$, being supplementary, $\angle ace$ and $\angle ade$ are subtended by the same chord ae .

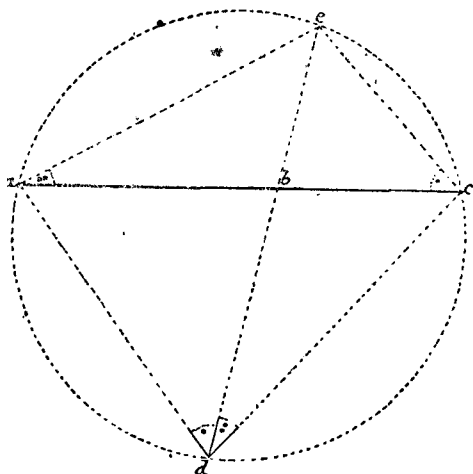


Fig. 271.—Bessel's Solution of the Three-point Problem.

can be contracted or extended, as may be desirable, by drawing a line parallel to the one joining the two extreme points, and terminated by those joining the extremes with the middle point. The graphic solution can then proceed in the same manner as that described for an original triangle.

and $\angle cae$ and $\angle cde$ are subtended by the same chord, ce : hence, the intersection of ae and ce at e must fall on the line db ; or, the segments of two intersecting chords in a circle being reciprocally proportional, the triangles, adf and cef , are similar, and the triangles, cdf and acf , are similar, and d , f and e must be in a right line passing through b .

In using this method, the triangle formed by the three fixed points

492. Problems of the Second Class are the following:—

Prob. 1. The Method of Radiation with the Plane Table from a Station within the Field (Fig. 272).—Set the instrument

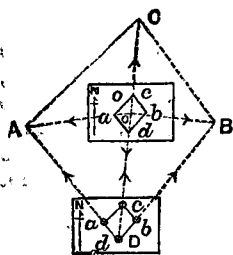


Fig. 272 — Planetabbling
by Radiation.

Fig. 272—¹—~~Illustration~~ ^{by} Radiation. Let A, C, B, D , be some of the vast number of points to be fixed on a plan. Select any station, D , from which other stations can be seen. Set the table up at D , and drive a pin, d , on the paper to represent, D , on the ground, taking care in selecting d , that, when the whole survey is executed, it may be entirely on the paper. Orient the table horizontally, and clamp by screwing the butterfly-nut underneath it, put the alidade against the

at any point from which are visible all the points desired to be plotted. Assuming any point on the paper, keep the edge of the rule in contact with it while the line of sight is directed in succession to each of the desired points. A line being drawn along the rule in each case from the assumed point towards the desired point, and the proper distance, determined by direct measurement or by stadia methods, scaled off, the points are fixed on the paper. By joining these by lines, a plot of the entire tract is obtained.

pin, d , direct it towards A, C, B , in succession, and draw pencil lines along the alidade, representing lines joining D with A, C, B . Measure the lines, AD, CD, DB , on the ground with a chain or a tape, or by stadia methods, and plot these on the paper to any scale. When the points, thus fixed, are joined, the figure formed will be a plan of the ground $ACBD$ which may also be prepared by planting the table on any point, O , inside the field, drawing rays oA, oB, oC , and oD , and measuring and plotting these lengths to scale. This method is often used for the interior filling in of a survey.

492. Prob. 2. Location by Resection with the Plane table with or without a Compass (Fig. 273).—Set up the table at C ,

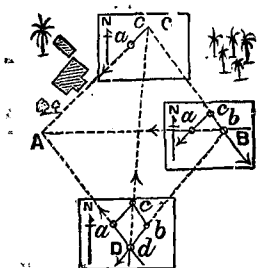


Fig. 273.—Planetabling by Resection.

taking c on the paper to represent C on the ground, and draw a line showing the magnetic meridian along the trough-compass; direct the sights to A and B , and draw lines representing CA and CB along the alidade; measure and plot CA only to scale as ca . Move the table on to B , and placing the alidade along bc , turn the table round till the point C is bisected by the sights. The needle, now, will point north, if the compass-box is placed along the line representing the magnetic meridian, and if the operation is correctly done. Now, place the alidade against the pin at a , direct the sights on A , and draw a line along the alidade; it will intersect bc . The point of intersection of these lines fixes the point, B , on the table at b . Again bisect D , draw bd , remove the table to D , and sight back to B , along the line db . Check the position of the table by the compass as before. To fix the point D on the table, place the alidade against the pin, a , and bisect A ; or against the pin, c and bisect C ; and draw the lines Aad or Ccd . The point of intersection of these lines with bd gives the position of the station D on the paper at d . So on for other stations in the field.

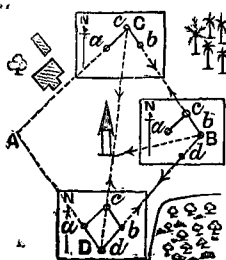


Fig. 274.—Plane-tableing by Traversing.

Prob. 3. Method of Traversing or Progression with the Plane table with or without a Compass (Fig. 274).—Set

up the table at C , draw the line showing the magnetic meridian, clamp the table; direct the alidade to A and B , and bisect them; draw lines CaA and CbB ; measure CA and CB on the ground, or find their lengths by stadia methods plot ca and cb to scale on the paper. Now, move the table on to B , unclamp butterfly-nut, orient, and take backsight of C along bc . If the operation is correct, then, if the compass-box is placed alongside the north line, the needle will point north. After this, clamp the table, intersect D , and draw the line BdD .

Measure BD and plot bd . Again move the table to D , and repeat the same process. If now the alidade be put against the pin at a , it will intersect A , and the magnetic needle will point north. But if the traverse does not close, then some error must have been made in the measurement or sighting; and, if the average error does not exceed 1 in 500 in the linear measurement or 15 minutes in every angular one, then this error is allowable in this class of surveys, and should be distributed. If more than this, then the work must be re-done from the last point backward.

In the methods described above, all the stations should be accessible points; in the method of intersection described below, this condition is not obligatory.

492. Prob. 4. The Method of Intersections, i.e., from any Two Points to fix a Series of Others with or without a

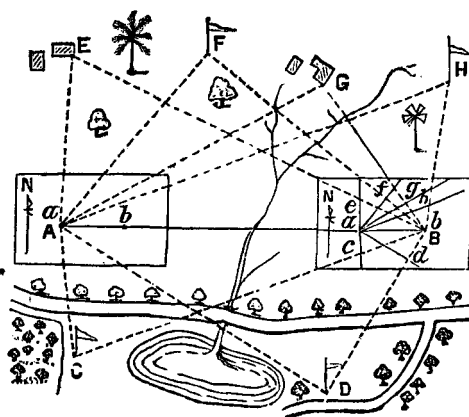


Fig. 275.—Planetabling by the Method of Intersections.

Compass.—Set up at a point from which all the other points are visible, and rule indefinite lines towards them, but without scaling off the distances, except in the case of one, which may be either the distance to one of the points to be plotted, or to any other conveniently assumed or fixed point. Next move the instrument to this point, orient it there, and draw lines towards the other points as before. The intersections of these lines with those drawn from the station first occupied will locate the points on the plot. The intersections, if possible, should be at angles between 30° to 150° . As a check the table can now be moved to a third station to intersect the lines first drawn. If the lengths of the lines are found by stadia methods, a third check can be had by comparing these with the lengths as scaled from the plot, e.g., select a base-line, AB (Fig 275), find its length by measuring it on the ground, or by stadia method; then plot on the plane table a straight line, ab , to represent, AB , on a suitable scale. The plane table is then set up and levelled at the station, A , which is just below the point, a , on the paper; a pin is then driven upright, at a , and the alidade is put against its left on the table so that its bevelled edge will coincide with the line ab . When this is to be done, the table is to be turned until the sights of the alidade are in a straight line with the station B , and the clamp screwed so as to keep it steady. In this position of the table, with the alidade touching the pin at a , observe through the sights

the directions of all important visible objects whose positions relatively to the base-line AB are to be fixed, such as C, E, F, G, H, D , etc., and draw pencil lines along the edge of the alidade to rerepresent the directions of those objects as aC, aE, aF, aG, aH, aD , etc., on the paper. The table is now to be removed to station B , and the pin from a to b which will be exactly above the point, B , on the ground; the alidade coinciding with ba , the table is to be adjusted and clamped when the sights are in a straight line with A . In this position of the table, again with the alidade touching the pin b , observe through the sights all the same important objects, and draw pencil lines along the edge of the alidade to represent the directions of those objects as bC, bE, bF, bG, bH, bD , etc., cutting the lines first drawn, i.e., aC, aE, aF, aG, aH , and aD in c, e, f, g, h , and d , which are the representations of C, E, F, G, H , and D on the paper. Similarly the table may be removed to C , and the pin to c . Then the alidade coinciding with either ca or cb and touching the pin at c , it may be clamped, and then the position of other objects in the field may be fixed. The survey may then be extended in any direction by intersecting rays from any pair of points now fixed on the paper. Afterwards all the details are filled in by sketching. The method is expeditious and simple, the work is cheaply executed, and the trouble often experienced in plotting from assistants' badly kept field-books is avoided.

When a proposed line of communication is ranged out by a theodolite or compass and the line pegged out, the plane table may be used for filling in the details on both sides of the line by setting the table according to the *method of traversing* along the line from peg to peg. While carrying on a traverse, offsets may be taken to bends of roads, corners of houses, tanks, etc., if there be any; besides, any remarkable objects can be fixed by their directions being taken from two or more stations, such as a spire which has been fixed by intersections from stations D, B, C (Fig. 274).

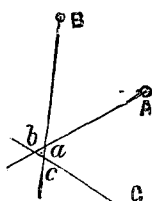


Fig. 276.—Triangle of Error.

It should be remembered that generally in plotting, when a point has been fixed by intersections from three or more stations, if the lines drawn from them do not exactly meet in a point, but form a little Δ as abc in Fig. 276; it is usual to prick a hole in the centre of this little Δ to mark the intersected point, if the error is not very much, and the point is not of sufficient importance to re-do the work. (q.v., Art. 491, Prob. 4d).

492. Prob. 5. The Method of Radio-Progression.—The method of traversing requires the instrument to be oriented at each station occupied. In practice this will be found tedious. In the method now to be described, by a *combination of the methods of radiation and traversing*, this difficulty is avoided. Fig. 277 represents a tract, $ABCDE$, to be plotted by the method of radio-progression. The instrument is first set up at any corner as A , with the *centre of the table*, which should be marked by a small plate of brass, let in flush, and having a pin-hole to indicate its exact intersection by the vertical axis of the instrument, directly over the station occupied. The centre of the table being indicated by p , the lines, $p(ea)$ and $p(ab)$,

are next drawn in the directions, EA and AB, and equal to scale to those lines. Moving the instrument to B, it is set up with its centre

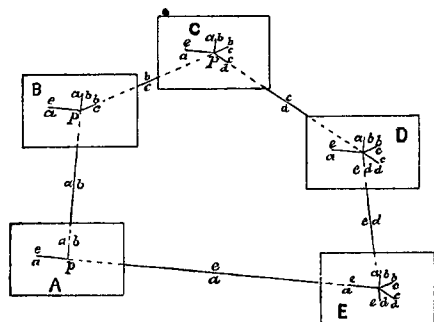


Fig. 277.—Planetabing by the Method of Radio-Progression.

over that point, the lines already drawn are brought parallel to the corresponding ones on the ground, and the line, $p(bc)$, drawn in a similar manner, and so on, occupying each station. When the last station is occupied, a check is had by the coincidence of the line drawn in the direction of the first station, with that previously drawn from the first station in the same direction. Indeed, a check of this kind can be had at any time during the progress of the work by sighting across the tract from any station, X , to any other station, Y , from which a sight has previously been taken to X . All the stations having been occupied, there will appear on the table a series of lines radiating from the centre, p , and numbered (ab), (bc), (cd), etc. Then by starting at any point and drawing a line, ab , equal and parallel to $p(ab)$, from b drawing a line, bc , equal and parallel to $p(bc)$, etc., a plot of the tract will be made. Note that the lines should all be drawn in the directions from p towards the lettered ends. As a final check, the finished plot will close.

493. Filling in Details with the Plane Table and Chain or Telescopic Alidade—Let us take the Fig. 210 to illustrate how a plane table can be used for filling in the interior details of a theodolite survey, assuming that B , H , E are the three theodolite stations fixed either by the Gale's Traverse System, or by Triangulation, and that they have been plotted on the table. Plant the table at B , and if the station, H , be, not visible from B , turn the table north until the magnetic needle of the compass coincides with zero; clamp the table, and drive a pin in at B , against which lay the alidade and turn it round until it cuts the flag at A ; draw the ray, BA , measure its distance and scale it off on the line BA , on the table. Keeping the table still clamped, set the alidade against the pin at B , direct it to a flag at C , measure and plot the distance, BC . The stations A , and C are now fixed with reference to B . Orient the table in succession at C , f , D , and if our survey be correct the next line from D will run into E , and the distance found when plotted will also fall exactly on E , this checks all the measurements between B and E . Start again from E , close on the point H , and finally on A . If the fixed point H can be seen from B , the use of the trough compass may be dispensed with, and the alidade may be aligned on the line BH , and the table clamped. The points C , f , D , etc., may now be fixed as before.

While traversing the boundary, take offsets to all adjacent objects and interpolate all conspicuous ones. Then run lines along *Da*, *bc*, *cd*, *de*, *gh*, *hj*, *lm*, *oG*, *AG*, etc., through the interior of the circuit, closing as often as opportunity offers on the fixed or intermediate stations on the boundary, and take offsets on the adjacent details in the usual way.

The Offsets in a Plane Table Survey need not be right offsets, for when the table is set up at a place, the alidade may be directed to any object obliquely from the survey-line, the distance from it being ascertained in the same direction on the ground and set off on the corresponding pencil ray on the table.

Scale.—As the drawing paper mounted on the table is liable to be affected by the changes which take place in the hygrometric state of the air the working scale should be drawn on it. The scale will thus expand and contract with the paper giving the measurements always true (q.v., p. 302, Art. 502).

494. The Advantages of a Plane Table Survey are: (*a*) With the plane table, the work is done in the field, and the various objects are directly plotted off from nature on the paper. This method ensures much accuracy and saves much time. (*b*) The map being directly plotted off on the spot any errors liable to arise can be eliminated on the spot or any omissions can be easily detected and corrected; (*c*) By the aid of a skilful surveyor a complete method of triangulation can be adopted producing results of an accuracy, even over an extended area, almost equal to that obtained with good theodolites; its horizontal angles being fixed by rays aided if necessary by the telescopic alidade and independent of magnetic variation. (*d*) When the country is very magnetic the needle becomes untrustworthy, and the plane table may have to be used for the whole of the field-work. (*e*) The plane table is employed for filling in the details of a survey by setting it up at the most prominent points and sketching by the eye what is not necessary to be more accurately determined thus turning the paper on the table into a representation of the ground to be surveyed. This sketching is the real use of the instrument. For representing broken grounds at the head of streams, and undulating country, or any such work where very accurate dimensions are not wanted the plane table is more expeditious than a compass. In hilly countries, therefore, it is peculiarly adapted for the filling in. It is useful for the preliminary survey of a tract of country through which the practicability of laying a line of communication is proposed to be ascertained in which despatch is of more consequence than strict accuracy. (*f*) When the scale adopted is small, and numerous details, are to be filled in a plane table is more expeditious than a compass. There may be mistakes in making and recording angular observations with the theodolite or compass. There may be mistakes in plotting also; the protractors may be indifferently graduated and the north lines may not be correctly drawn on the table giving the proper angles. All these errors are next door to impossible with the Plane Table.

495. The Disadvantages of a Plane Table Survey are :

(a) The plane table occupies time in mapping during daylight which might be wholly given to field-work, and the mapping done either at night, or when the sun is too hot, or when too wet to go out. (b) The usefulness of a plane table survey is greatly impaired in a wet climate as it is impossible to work with it in wet or even showery weather. (c) It has the disadvantage of being with difficulty checked when carrying forward a continuous traverse, especially so, when the traverse lines have to be short, as in a crooked road with high hedges. There are generally some salient points on which rays can be taken as independent checks, but without them every mistake in angle is perpetuated. (d) When the scale adopted is large and when surveying along a road or river, or in route surveying of any extent a compass or theodolite survey is preferable. In the case of a continuous traverse, the compass has the advantage of giving the magnetic bearing every time independently of the previous work. (e) The plane-table being of limited size often changing of the paper is necessary to carry on the survey. (f) When the plane-table is used as a universal instrument, a stadia telescope is fixed to the alidade, which has to be carried in a separate case. The class of work performed by it under these circumstances answers to the telemetric survey with the transit, but when passing rapidly through a country, it is much more cumbersome to carry a heavy plane table with a telescopic alidade than to shoulder a theodolite.

496. The Use of the Plane Table is liable to the Following Errors :—

(1) The surface of the table may be warped or uneven and not a perfect plane. This may be tested by a straight-edge applied in all directions. If the surface proves warped or uneven, it must be made perfectly flat and smooth, or a new table is to be supplied. (2) The edge of the ruler should be a mathematical straight line. Test the condition by placing the ruler on the table and drawing a line along its edge, then reverse the ruler end for end, place it in coincidence with the ends of the line, and draw another line along its edge. If the two lines coincide, the edge is straight, except in the very improbable case where are found a concavity and a convexity of exactly the same shape and size and at the same distance from the ends. (3) Error in placing the table centrally over each station, so that the point on the ground is exactly below the point on the table. (4) Error due to the table not being adjusted horizontally. (5) Error in setting up the table parallel to itself at each station. (6) Error in using the alidade, inaccurate intersections, and rays. (7) Error in the construction of the compass. (8) Error in mapping. Or in other words **the Main Practical Points** to be observed in using the table are to set it up firmly upon the tripod; to level it perfectly true with a circular bubble (q.v., Levelling Instruments); to be very exact with its *setting in meridian*; to keep the pencil finely pointed; to draw the rays with the utmost fineness, and above all things not to get hurried. A hard pencil is to be used for marking sight-lines, while a softer one is to be used for the delineation of actual objects and details.

497. The Following Checks should be adopted in a Plane Table Survey:—(a) From each station the back flag should be sighted as it is not safe to depend entirely on the compass. (b) To avoid accumulation of errors the preceding stations should be sighted from every third, or fourth station. (c) To check a work, the table should be set up at random, and other stations observed. If they are bisected by the sights, it is well, if not, the work needs correction. (d) The distance between any two stations, not consecutive may be measured on the ground, or obtained by the Stadia method, and compared with the plotted distance on the plan.

498. Shifting a Paper on the Table.—When one sheet is full, and it is required to replace it by a new one, to continue the survey, it may be done in the following way:—Draw a line through the furthest point fixed from the last station. Remove the sheet from the table and fix another on, drawing a line upon it in a part most convenient for the work; then cut the sheet just removed, by the line drawn on it; apply this edge to the line on the new sheet, and as they lie in that position, continue the last station line on the new paper, and prick through the positions of as many stations that have been fixed on the old sheet as you conveniently can. If the positions of three fixed points are thus transferred to the new sheet, the place of a new station can be found (q.v., Arts. 491, 492). On each new sheet place the compass, and turn the table horizontally untill the needle points north, and then draw a line which will represent magnetic north, unless the needle is deflected by the influence of local attraction.

499. To Join the Sheets together, and thus form one map, place the edge of the sheet that has been cut accurately against the line drawn on the new sheet and with the aid of the ruler see that the line projected on the new sheet from the last station (on the sheet that has been removed) is an exact continuation of the corresponding line on that sheet.

500. Mounting Paper on the Table.—Before use, the table is mounted with paper in the following way: A thin strip of long-cloth or muslin, about 3" wide, is washed, dried and pasted with fine rice-paste all round the table, to the extent of one-third of the breadth of the cloth, the remaining two-thirds overlapping the board. Cut the paper to $\frac{1}{8}$ of an inch less than the size of the table all round, and then moisten it on its reverse side by means of a wet sponge, stretch it tightly over the table, and paste its edges on the pasted strip of muslin, and smooth it out with a piece of dry cloth, pasting the overlapping edges of the muslin underneath the table. The paper is then allowed to dry in the shade for 2 or 3 days on the table. The paper on drying will then stretch smoothly over the surface of the table.

This method of mounting answers well for tables of larger size, 30" x 24," in extensive surveys, where it is an object to have the paper evenly stretched for several days at a time, but in using smaller size, 15" x 12", for the survey of small estates pasting may be dispensed with; and the following plan may be adopted: cut the

paper so as to have an overlap of 2" all round the surface of the table, damp the paper, turn down the edges along the sides of the board, and fasten them to the table underneath by small drawing pins.

501. Deflection of Long Lines.—In adjusting lines of intersection upon a point or object from a series of stations, when these lines do not coincide in one point, as they are usually derived from signals at unequal distances, the error should not be divided equally among them, but in proportion to their lengths if the discrepancies are not eliminated by the rules for distortion errors given later.

It should be borne in mind that very short lines from a determined point—as, for instance, to the corners of a fenced road, where the table occupies the center of the intersection of two roads—may be taken with no apparent error when the table is deflected to some extent from its true azimuth, but that in this case a prolonged line will be considerably out at its further extremity. A long line should never be obtained by the prolongation of a short one from a back station where there is no small check line, or some other point in that prolongation already fixed. It will be apparent that the more nearly at right it angles intersecting lines cross each other the more clearly the point will be defined; acute intersections, as far as possible, should be avoided, and, even when they are crossed by a third line at a satisfactory angle, a fourth line, or an accurate rod reading from a well-determined point, is advisable if within reach. Sometimes a position is established by measuring along the estimated direction from a near-by fixed point, and then orienting by this assumed position and a distant point. This method should be used with caution, but is generally reliable for rodding the detail in the vicinity.

502. Distortion Errors.—The distortion of a plane-table sheet

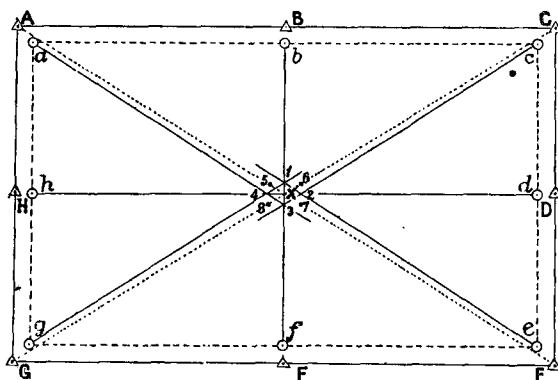


Fig. 278.—Showing Effect of Distortion Error in a Planetable Sheet.

destroys the perfect proportions which exist between the fixed points and their plotted representatives on the sheet. Fig. 278 illustrates the effect distortion would have in the determination of a point. *A, B, C, etc.*, are plotted in their true relations. After the sheet has contracted, *a, b, c, etc.*, represent the relations those points have assumed.

The paper contracts at a uniform but different rate in each direction.

The plane-table is supposed to be at *X*, the exact center of the figure, and it is required to determine the position by the distorted

points a , b , c , etc. By reversing the telescope, we immediately ascertain that we are directly on the line, HD . Reversal will also show that we are on the lines, AE , CG , and BF . But the distortion is not apparent until the telescope is pointed at the signals, and the lines are drawn on the sheet. Then if we orient by the line, HD , we shall produce the figure of the diagram, giving five determinations, 1, 2, 3, 4, and X , each made with four well-conditioned points. Any one of these conditions would be considered satisfactory if we had not the other points to show that something was wrong. To orient by the line, BF , will produce the same result. But if we take the diagonal, AE , we shall have two positions at 5 and 7, formed by the intersection of the diagonal points, with the lines from the other points running wild. Using the diagonal, CG , would give two points at 6 and 8, with the lines from the other points running wild as before.

Position by Compromise.—There is no question that out of the nine positions developed by these settings, that at X is the only true compromise. When the sheet is distorted, all positions are compromises; and X is the true compromise in this case, for it is on the lines CG , AE , etc. : a below and e above, the line connecting A and E , by equal qualities. A line drawn through the distorted points, a and e , must pass through the middle point X . The positions 5, 6, 7 and 8 can not be true, because lines forming them will not pass through the opposite points when extended, which we know to be the condition that must be filled.

Rules:—(1) A station made with three points that are on the lines of contraction, the resecting lines forming nearly right angles at their intersection, will give the true position in relation to all points in the sheet (as h , b , d).

(2) A similar condition of right-angular intersection at the station, but the lines forming diagonals to the lines of contraction, will give the worst possible position for the station (as a , c , and e).

(3) A station made with three points on one of the lines of contraction will give the correct orientation of the table (a , h , and c) but not the correct position.

(4) In estimating errors of the point due to distortion, those situated on the lines of contraction require no allowance, however distant.

Application.—If the change in the sheet due to contraction or expansion gives the same percentage of the units of length, both lengthwise and transverse of the sheet, the points are still in their true relative position, and the projection is practically as good as when laid on the paper, but is on a slightly altered scale. When the percentage of change in the units of length is greater in one direction than the other, the sheet and projection are distorted; and to make a station by the three-point problem, the change of scale in each direction must be allowed for. The difficulty in making such allowances is not great, if the principal effects of distortion in the sheet are borne in mind. It would not be per-

missible, even were it practicable, to make new points on the sheet, as this would destroy the geographic position. It is necessary, therefore, to assume the new points by estimation, applying the percentage of change to the distances measured between the points on the lines of change—that is, on lines parallel to the edges of the sheet. If the point occupied and the point sighted to are on a line parallel, or nearly so, to one edge of the sheet, its movement from the distortion can only be along that line. When the position of the point sighted to is found situated to one side of the line parallel to the edge of the sheet, the distortion will also affect it in the direction at right angles to that edge, and the effect of the distortion will be most apparent when the angle of deflection is 45° and the position at as great a distance from the point occupied as the paper will permit. As the angle of deflection increases above 45° the effect becomes less and disappears at 90° , when the position will fall again in a line parallel to an edge of the sheet.

Referring to Fig. 278 to make a station with the three points, a, b, c : If the sheet were not distorted, the station would be at X ; A, B , and C , being the true positions plotted when the projection was drawn. But the sheet having contracted, a, b , and c , show the relative positions of these points; hence we make such allowance for the contraction derived from measuring the unit of length that we can place or imagine a and c to be where they belong, at A and C ; b requires no change, as it is on a line parallel to the edge of the sheet. To locate A we must know the distances (approximately) h to a and h to X , which, multiplied by the percentages of contraction (in this case), will give the distance of A above and to the left of a . The same process locates C . If the station were to be made with the points, a, c , and e , all three points would have to be imagined in a new position by the same process that A has been located. Stations made in this way will be good for all local sketching within an area that the contraction of the sheet is inappreciable; but to take cuts on distant objects from such a station the orientation of the table must be changed. If an object is somewhere near the direction of a and the table at the *compromise station* X , the table must be oriented by a and X , imaginary position A being discarded. The same processes apply to all positions on the sheet for the station occupied.

Section 8.—Town and City Surveying.

503. City Surveying.—City Surveying differs from ordinary theodolite surveying only in the precision necessary to be attained. As the area to be surveyed cannot be divided into Δ s owing to the obstruction presented by buildings, traversing is usually the only method adopted for a city survey. Triangulation would not only be expensive, but possible only from the tops of houses, and the points thus fixed would have to be transferred to the Ground level.

504. Scales for Town and City Maps.—The value of lands in towns and cities is much more than that in villages, and farms in rural and sparsely populated areas. Hence the ordinary methods, as detailed in Sections 3 and 4, where an accuracy of $\frac{1}{360}$ and $\frac{1}{1000}$

is demanded are inadequate to the needs of the city surveyor. The accuracy must increase with the value of property. As *cities grow*, the value of property in crowded parts of cities and commercial centres increases with gradual annual increase in ground and house rents, and therefore in such cities and towns the highest class of surveying as regards accuracy is a necessity. The survey map will then settle all disputes and prevent litigation; and this can only be prevented if the maps can show up to 1 inch which will require a scale of 16.5 ft. to the inch. Such large scale maps would no doubt be very expensive but as the town lands are growing in importance the adoption of such a scale will be necessary in the near future. At present in a town like Cuttack the scale of $1\frac{1}{2}$ chains to the inch is adopted for the cadastral maps in which an error one foot is permissible and unless all measurements are printed along the lines of the plot in such a scale the error cannot be eliminated. If the scale be a large one, say 16.5 ft. to the inch = 320" to the mile = $\frac{1}{4}$ chain (Gunter) to the inch, the map should be plotted accurately to all measurements up to one inch from which areas and details can be easily found.

505. Precision Necessary.—The Engineer and Surveyor's judgment will have to be exercised to decide, first, the precision necessary; and second, the methods to be attained for its attainment; 3rd, the cost of the survey which should commensurate with the accuracy required.

In closed theodolite traverses the required precision will have been attained when on the closing error of a traverse being distributed no station, according to scale, will contain any plottable error. On scales from 16.5 to 50 ft. to the inch values of tenth of a foot may be considered plottable, or at any rate values to this accuracy, or a little greater accuracy, should be secured in the calculations, in which case all measurements, latitudes and departures should be taken to 2 places of decimals of a foot. Tape measures of lengths are to be taken to the part of an inch.

In small town work the closing error should average about 1 in 5000. With a steel tape levelled by the eye and held over a line of stakes previously aligned by eye the difference between any two measurements should not be greater than $1/5000$ part of the whole length. To secure this precision, however, the ends of the tape and all points of breaking chain should be carefully transferred to the top of the stakes by means of a plumb-bob. This method of linear measurement is probably the best to be used in general theodolite surveying. For work of a higher degree of precision requiring a still closer agreement between any two separate determination of the same line all measurements should be made with the tape at, or reduced to the same temperature, pull, and perfect horizontality. Again, if the *absolute* length of the line is required, there must be known the temperature and pull for which the tape is standard.

For medium-sized city-work, the error of closure should be within $1/10,000$. For example, in a circuit of 10,000 feet, if a traverse is found to be 0.6 ft. in error in latitude and 0.8 ft. error in

departure, error of closure equals $(0.6^2 + 0.8^2) \frac{1}{2} / 10000 = 1 / 10000$. When such an error is distributed through 50 stations of an average distance of 200' apart, the plottable error might be considered to be *nil*.

For Large Cities the error of closure should be within $1 / 17000$. Beyond this limit the cost of the survey would increase rapidly.

The Distribution of error in precise work should be done as in Rule given in Art. 353, p. 199. The slide rule will afford means for calculating these errors quickly and accurately.

506. The angular correction to the traverse should depend

on the closure of angles of a polygon and the excess or defect distributed equally. Then arises the question of azimuths to control the angular error. As both azimuths and the total of the interior angles of the polygon must rule the angular correction, a careful selection of the azimuthal stations must be made, and the angular correction though several circuits distributed before any balancing could be done. The probable angular error per station would be equal to the total error divided by the square root of the number of stations.

507. Origin.—

One origin for an ordinary-sized city will do. The linear measurements should be made with a precision of $1 / 17030$. The corresponding precision

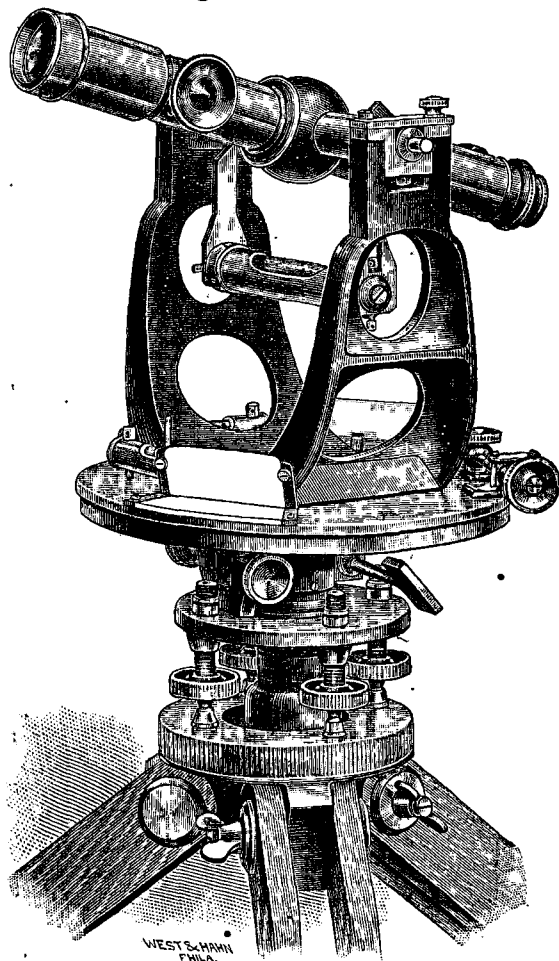


Fig. 279.—Young Sons' and City Transit.

in angular measurements will be obtained with a horizontal circle reading to 12 seconds. This being an inconvenient graduation, a vernier reading to 10 seconds is probably the best for the purpose.

508. Theodolites.—The *Tránsit* is used exclusively, but the common pattern may be modified with advantage. The magnetic needle being never precise should be discarded in a city surveyor's transit. The verniers can then be placed under the eye, the bubbles can be removed from the standards and placed upon the alidade and the standards themselves can be more firmly braced. The result will be a steady and convenient instrument. The adjustable tripod head and levelling attachment are more convenient. For topographic work, the vertical circle or a sector and stadia-wires are necessary. The thermometer needed for making proper correction for temperature may be fixed to one of the standards facing the eyepiece of the telescope. To protect the bubble-tube a guard sufficiently deep may be provided while the bulb made open on the side towards the observer. Figs. 49 (with micrometer reading), 56 (8" ones), 98, 100, 101, 279 show some good forms of the theodolites which may be used in city surveying.

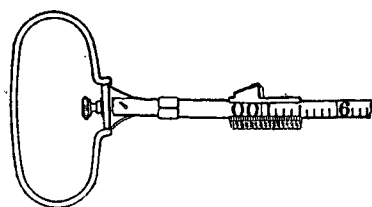
509. Young & Sons City-Transit.—The instrument (Fig. 279) has the horizontal limb $6\frac{1}{2}$ " in diameter with verniers reading to $10''$, and has no compass. The upper frame mounting the telescope is in U-shaped piece provided with ribs and which rests directly on the top flange of the inner centre. On the top of each bearing of telescope axis is mounted a cap with an adjusting screw. These caps are so arranged as to exclude dust. The instrument has single spring tangent-screws for the upper and lower plates, shifting centre, and split leg tripod. It has a striding level suspended from cross-axis of telescope to test the motion of telescope in a vertical plane.

510. Instruments of Precision in Linear Measurement: Steel Tapes with Thermometer Scale.—Fig. 280 shows the end



Fig. 280.—Steel Tape with Thermometer Scale.

of 100 foot tape with thermometer scale. Steel tapes with this scale should be employed as a means of obtaining additional accuracy and uniformity in measurement. This scale, which is numbered to correspond to the Fahrenheit thermometer, takes



the place of the terminal 50 foot or 100 foot mark. In the United States at 62° F. (in India 85°), at which the steel tapes are standard, thermometer reading will coincide with the terminal mark. At any other temperature it will be before or beyond it: For instance, at 80° the terminal mark will be at the graduation numbered 80° on the thermometer scale, at 20° it will be at the graduation marked 20° , etc. Fig. 280 will show how important it is for exact measuring, to make this correction for temperature, as the variation

in 100 feet between $90^{\circ}+$ and $20^{\circ}-$ is over 0.06 feet. The figure "9" in the cut is the 9th $\frac{1}{10}$ of the last foot of a 100 foot tape. Some tapes have temperature scales at their middle points, etc.

511. Compensatory Handles.—Fig. 281 shows the compensatory handle for a 100 ft. tape and serves the same purpose as the temperature scale.

512. Tension Handles for Steel Tapes.—The handle shown in Fig. 282 has an attached spring balance by which the pull can be measured. These handles enable the user to apply exactly the tension at which the tape is standard.



Fig. 282:—
Tension Handle
and Steel Tape.

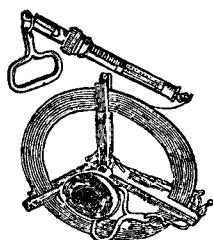


Fig. 283.—City
Engineers' Steel
Tapes.

513. Keuffel & Esser's City Engineers' Steel Tapes.—

This tape (Fig. 283) is $\frac{3}{8}$ " wide with improved spring balance, adjustable for temperature, with level and thermometer, two nickel-plated handles on folding brass reel. The spring balance consists of two telescoping brass tubes connected by a strong spring; the inner tube carries the spirit-level and tension mark,

and the outer one carries thermometer, which is protected by a revolving semi-tubular cover. A clamping ring encircles the outer tube: in it is cut a V-shaped groove representing the end mark of the measure. The spring balance up to the groove in the ring is included in the measure. On the outer tube is engraved the temperature scale, which compensates for expansion and contraction and is marked with the corresponding degrees Fahrenheit. Correction for temperature, i.e., allowance for contraction and expansion is made by adjusting the clamping ring on the temperature scale to the degree indicated by the thermometer. The starting point is marked by another V-shaped groove in a brass plate at the other end of the tape. There are no intermediate graduations on this tape, and the tension and temperature corrections apply to its entire length only. These tapes are made of the best and toughest flexible steel-ribbon, carefully tempered to prevent breaking or kinking. They can be furnished in any length up to 1000 ft. without joints, and either $\frac{3}{8}$ or $\frac{1}{2}$ in wide. Two strong nickel plated handles accompany each tape. These tapes are graduated according to the standard and are correct at 62° Fahr. (in the United States). The temperature and the tension at which the tape will agree with the standard of the country should be determined before being used.

To use this tape, adjust the clamping ring according to the temperature as read in the thermometer, then bring the V-shaped zero groove in the brass lug at the other end of the line exactly over the starting point by means of a suspended plumb-bob; pull the

telescoping handle until the tension mark coincides, and bring the tape into a horizontal plane by means of the spirit-level. A second plumb-bob suspended from the V-shaped groove on the spring balance with then indicate the terminal point on the ground.

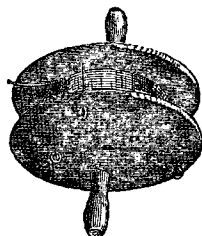
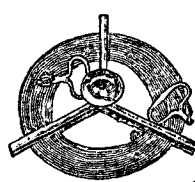
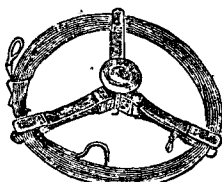


Fig. 284.—City Steel Tape.

514. Berger & Sons' Steel Tape for City Use.—These tapes are made in lengths from (Fig. 284) 100 to 1000 ft. with graduations at every 10', the last 10' graduated in single feet, and the last foot into 10ths. A clamping handle is furnished to attach to the tape at any desired length, if shorter measures

than the whole length are intended to be made.

515 Folding Hardwood Reel.—Fig. 285 shows the folding reel,



Figs. 285-286.—Folding Reel for 100—500ft. Steel Tapes.

Figs. 287-288.—Folding Reel for 100—200 ft. Seal Tapes.

with nickel-plated trimmings, for 100 to 500 tapes. Fig. 286 shows the folding brass reel with hardwood knob for 100 to 200 ft. tapes.

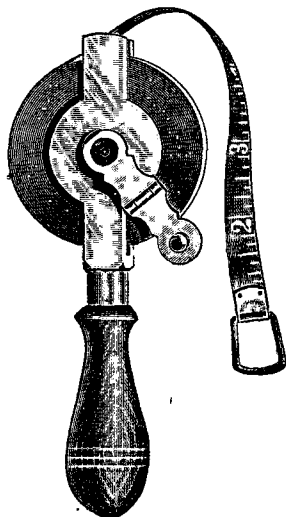


Fig. 289.—Steel Tape for Offsetting.

516. Invar, an alloy of nickel and steel, characterised by an extremely small coefficient of thermal expansion; it is specially useful in the construction of pendulums and of geodetic measuring apparatus, in fact in all devices where it is an advantage to avoid temperature compensation. The name was chosen as expressing the invariability of its dimensions with heat.

517. Invar Bands, for checking base-lines, or taking measurements where very great accuracy is required, or for use as standards. These are generally made in lengths of 100', 200', 300', 400', 500', $\frac{1}{8}$ " or $\frac{1}{4}$ " wide, and are divided at each 100' by means of a fine line engraved on a strip of silver which is soldered and riveted to the band. Each is in one piece, and is provided with German silver handles not included in the measurement, and is wound upon a large metal reel. The invar used for *standard band* has been seasoned or aged by the National Physical Laboratory,

London; it is practically unaffected by changes of temperature, and may be relied upon as giving the utmost accuracy obtainable.

The plains of North India have been traversed with invar metal (crinoline) chain in 5 sections of 66', i.e., 330' long in preference to expensive triangulation. For accurate work over long distances these chains are useful.

A 50' tape is best fitted to city surveying where there are appreciable grades. For cities without grades a 100' tape will be found useful. For measuring long lines a 300' or 400' tape is the best. A 50' or 100' tape, $\frac{1}{4}$ " wide, marked feet, inches and $\frac{1}{8}$ of an inch (Fig. 289) is useful for off-setting, etc.

518. Errors in 100 Ft. Steel tape.—Each of the following in a 100' steel tape length will give an error of 1/10,000:—(1) Length of tape differing by $\frac{1}{8}$ from the standard. (2) Error in reading, marking or plumbing $\frac{1}{8}$ ". (3) Error in sag or the centre of the tape being 8" lower than the ends. (4) Temperature correction for every 16° (approx.). (5) Tape stretched loosely so that the centre is 8" out of line. (6) The end of tape being deflected by 16". (7) For every 16lbs pull. (8) Tape not horizontal or one end 16" above or below the other.

The above values are either reciprocals or multiples of 8 and as such are nearly correct.

(1) is *cumulative* + or - and the constant is to be applied to correct it.

(4) " " Do " "

(3), (5), (6), (8) are *cumulative* +, and can be avoided and in flat areas inappreciable, (7) is *compensating*, and (2) is a mistake.

519. Method of using the Tape in City Surveying.—For precision of 1/1000 necessary for a city survey the following procedure should be adopted. The tape should be allowed to rest on stakes carefully ranged in with a theodolite and set on a uniform grade. The pull is measured with spring balances and is the same at which the tape, by experiment when supported in the same way, has previously been found to be standard. The temperature of the tape is measured by one or more thermometers attached to it or the stakes on which it rests and should be known closely. The lines when measured are reduced to the horizontal and to the standard temperature to avoid the effect of wind. Measurements of this nature should be made in calm weather and in cloudy days.

For careful work a *Standard of Length*, with which the tapes to be used and can be often compared, at mean standard temperature measuring 100' should be set up at the City Town Hall or Municipal Office. The best way to obtain such a standard is to send a 100' steel tape to the Survey of India where it should be tested and returned marked with temperature and pull at which it is of standard length, both when supported throughout its entire length, and when supported only at the ends. Having obtained such a standard, it is to be preserved permanently by nailing down on a smooth floor

two strips of zinc, stretching the standard tape along the floor between them at the standard tension, and cutting two fine lines just opposite the tape extremities on the strips of zinc. The temperature of the tape during this operation should, of course, be standard. Having laid down these marks, to compare any other tape with the standard, stretch the tape between the marks at the standard temperature and pull, and note the difference of length which can be measured by an auxiliary wood scale. If it is difficult to have the tape to be compared at standard temperature, allowance for the difference can be made from the fact that the coefficient of linear expansion for steel is about 0.0000065. To determine the pull at which a tape, when supported in any particular manner, as every 50', every 25', etc., will be standard, arrange a line of supports, the required distance apart, on the floor between the zinc plates. Next fasten one end of the tape directly over the fine line on one end of the plates. This can be done with the aid of a plumb-bob. Next, allowing the tape to rest on the supports, the reading of a spring balance attached at the other end and pulled just enough to allow the plumb-bob, fastened there to hang over the other mark, will give the tension required.

520. Selection of Stations.—The first thing to do is to fix the main-circuit stations and the station lines by permanent marks, known as monuments. Stone monuments of about $2\frac{1}{2}$ feet long and 6 inches square serve this purpose well. The top can be made circular in cross-section, about 5 ins. diameter, the exact point being marked by a drilled hole. Station prisms are generally placed at the intersection of the side, and not the centre, lines of streets. In American cities the present practice is to set them in the street 6 feet from the side lines. In streets, lined with kerb and channel stones, the marks may be made on the kerbs. The main traverse lines should be as long as possible as then the compensating errors in tape lengths will permit of the measure being nearer the truth.

In small cities the traverse lines should be laid out as nearly in the centre of the streets as possible and ordinary steel tape may be used in their measurement. The station-points may be marked at the junctions of streets by stone prisms or iron spikes, the station being exactly marked by drilling a hole in the stone or by punching a dot on the spike.

521. The station marks should be fixed by three measurements taken to corners of houses on both sides of the station-line with a steel tape. Two measurements are required to fix the position of the mark and another to check it. By the aid of these measurements the station-points can be easily found out, or if required to be refixed, in case the stone being covered up, or the spike being knocked out.

522. Details in a city survey need not be referred to Geodetic survey. It is of no practical use to join the traverse co-ordinates to adjacent Geodetic Marks. A small scale index map can be made and fitted on to 2 or 3' data common to both,

523. Traversing and Offsetting.—After the selection of the station lines angular observations are made as described before for traversing. They are next measured with steel tapes, and offsets taken to the corners of the streets, and buildings, etc., as usual. Property belonging to public bodies, such as the Harbour, and Port trust, Railway, Municipality and of all owners of land should be measured along their boundaries and where insufficient monuments exist or where the boundary is disputed should be noted and settled. The survey of the traverse lines and the front outlines of the streets may be carried out during the night in busy thoroughfares or with the aid of flaring paraffin lamps, the verniers being read with a candle or a small hand lamp, or when the traffic is suspended, and the interior details may be filled in during the day. Tacheometric planetabling is eminently fitted for city-work as Stadía measurements can be taken without interfering with the traffic. All corners of houses at street junctions should be surveyed by subsidiary Δ s constructed on the station lines, or by the aid of an optical square, as right offsets are not deemed sufficiently accurate for fixing such points. Offsets should be taken both to the main frontages of the buildings and to any railing or compound wall, etc., that may be in front of them. When this is done the work should be plotted on paper pasted beforehand on a plane table which should then be used in surveying the outlines of buildings, gardens, street gullies, man-holes, hydrants, lamp posts, tram rails, etc., and indicated on the map in conventional colors distinguishing those below or above ground with those at ground level, and the details filled in on it, each measurement being directly plotted to scale on the spot as soon as taken.

524. Laying Out a Town Site.—In laying out a town site or dividing any land into blocks, it will be necessary first to make or obtain a map showing the boundary of the tract. Then decide on the plan for subdividing the land and draw in the split lines showing the centre lines of the proposed streets on the map. Having done this, go round the boundary lines, setting stone prisms and pegs to be located there. The centre and outer lines of the proposed streets, from one marked point to another, should be aligned last. In preparing a map of this kind no shade lines for blocks, buildings, etc., be allowed as confusion would arise as to whether the inner or outer edge of the shade line is the measured limit.

525. Provisions for Growth.—Cities grow; therefore for future extension of the town or city provision must be made from before.

City Levelling should be *levelling of precision* (q. v., Chap. on Levelling).

526. Datum and Permanent Bench-marks.—Level values referred to a permanent datum are needed where the city authorities are likely to require them. High-water marks indicating the danger of overflow, the lowest available outlet for a drainage system in a flat country; the average sea, lake or river levels as affecting commerce,

are often chosen. The datum selected has its value accurately ascertained and marked by a monument as enduring as the granite hills; a block of masonry with a single durable capstone firmly bolted to its place and bearing the datum, or a known relation to it, definitely marked and secured from abrasion, is needed and possible for all cities. In order to secure accuracy and uniformity in elevations throughout a city, permanent bench-marks (q. v., Chap. Levelling) are established by running lines of levels radiating from the *permanent datum* and checking the work by cross-lines at intervals.

528. Location of Streets.—When the plotting is done the most practical and economical system of grades may be secured for the location of streets. Grades are established from levels taken along the centre lines of the streets to be graded. The contour map of the district also helps towards solutions of this kind. The widths of streets and side-walks and up to which limit they are to be paved or otherwise should be given.

529. Drainage Systems.—A well-devised sewer system is a necessity. The demarcation of the areas and slopes of the whole drainage area to be sewered, if storm water is to be provided for, should be carefully made on the city plots so that houses and streets will be kept well-drained. This will give an idea of the work required of the mains at the point of discharge. Each sub-district or a combination of some of them will solve its own problem. Borings along the lines of proposed work to ascertain the nature of the soil and the depth of the bed rock are necessary in order to enable the contractors to bid intelligently. This kind of under-ground topography may sometimes change the location fixed by surface indications.

530. Watersupply.—The surveyor has to furnish the distance and elevation of the source of supply, the topography of the district through which the water-mains must pass, suitable sites for service reservoirs, with their relation in distance and elevation to all points to be supplied. The maps showing these should be referred to the Datum established for the city.

531. The Contour Maps should be drawn in sections of sufficient scale for a working plan and so arranged that when adjacent sections are placed side by side the contour lines will be continuous. If the contours of the natural surface are drawn in Indian ink and the contours showing the changes made by different kinds of public work in some color the map may give a large amount of information distinctly. The sites of the towns may be best tackled by laying out street and block lines according to the undulations of the ground. The making of all ground available and slightly points accessible, and at the same time so locating the streets as to secure economical grades is facilitated by the use of contour maps.

532. The Improvements of streets involve.—(1) The estimate of Earthwork in grading and shaping of the street. (2) The location of improvements along the lines of streets. City laws prescribe a certain width of side-walks and roadway for each width

of street; (3) The position of monuments, which will be disturbed during the progress of the work, is preserved by witness-pegs driven beyond the limits of disturbance from which the monuments of the new grade are replaced.

Section 9: Cadastral Survey.

(1) Traverse Section: Field-Season.

533. The Cadastral Survey is a field-by-field survey within a village (Revenue Mauza), or an estate undertaken by Government to ascertain the position, area, and quality of each field. It gives the data on which a settlement is to be based and is necessary for a settlement of land revenue as well as for recording existing rents and fixing fair rents. Survey includes *measurement* and *identification* of boundaries and all other operations antecedent to and connected with Survey.

534. Scale adopted for Cadastral Survey.—For cadastral or field-by-field survey of cultivation in villages the scale used is ordinarily $16'' = 1$ mile, and the chain used is Gunter's chain. For large village sites, the scale of $32'' = 1$ mile is adopted, and for towns; $64'' = 1$ mile. The principle is precisely similar whatever be the scale adopted for survey except that the measurements have to be made more detailed, the larger the scale of survey. All maps should be plotted according to scale.

535. Sections of the Settlement Survey.—The work of the survey is divided into two sections—the *Traverse* and the *Cadastral*.

The **Traverse Section** comprises the measurement of the angular and linear distances of the boundaries of villages or Estates, with the theodolite and chain, and furnishes the skeleton plots of the polygons laid out by them for the detailed field-work to follow, while the **Cadastral Section** comprises the planetabling of the detailed topography within the village or estate on the plotted skeleton sheets.

536. Traverse and Field-Surveyors, Inspectors, Head-Inspectors, etc.—The surveyors employed on the traverse work are called *Traverse surveyors* who take only angular and lineal measurements of the traverse lines, taking no offsets; the latter being left to field-surveyors to do afterwards; and those employed in planetabling are called the *detailed* or *field-surveyors*. These have their inspectors to *partial* or check their work, while a superior officer is placed in charge of each of the sections with a third officer in complete charge of the combined work.

537. Main-Circuits, Sub-Circuits and Village-Circuits.—A district under survey is first divided into convenient blocks of about 20 or 25 square miles, called *Main-circuits*, following fiscal boundaries, such as parganas, thanas, tehsils, appertaining to the district and then traversed. These main-circuits are then subdivided into smaller blocks, called *Sub-circuits*, containing an area of about 20 or 25 square miles following village boundaries and traversed. The sub-circuits are again split up into inside *village circuits* following the boundaries of villages and then traversed.

538. Index Plots.—When executing the traverse work the traverse surveyors should make rough angle plots of their work on a scale of 2"=1 mile; these form an index to the villages and facilitate office work.

539. Village Traverse Plots.—After making village traverses their plots are made to be supplied to the field surveyors later on, on sheets of Imperial paper divided by blue faint lines into squares on the 16 inch to-a-mile scale containing $2\frac{1}{4}$ acres to the square inch to facilitate plotting.

540. The Stage in a Cadastral Survey are the following:—(1) Identification of Lands and Demarcation, (2) Traversing, (3) Survey of the actual boundaries, (4) Detail surveying and record writing, (5) Computation of areas, (6) Attestation, (7) Preparation of Fair copies, (8) Final Publication, (9) Civil cases.

541. Indentification of Lands and Demarcation of Boundaries: May to October.—This stage of the work consists in the erection of earthen mounds about 3 feet in height at all salient angles of the boundary and where the boundary is a long straight line, mounds should be placed at intervals along the line about 600 feet apart and the planting of bamboos at spots here and there along the boundaries of the "Revenue Mauzas," in the presence of all parties concerned. Where there is a dispute as to the position of the boundary, mounds and bamboos will be placed in both positions. The officers who carry out this work are "Demarcation, kanungos and Amins." They do their work partly before the heavy rains and partly afterwards. As a Revenue Survey map of every village exists, the Civil authorities will arrange, before the arrival of the traverse party, for the demarcation of all village boundaries within the tract to be traversed with the help of Zamindars. Demarcation should be taken up simultaneously for all villages coming under survey in the following season, and should be completed early in October, as traverse parties usually start work between the 15th and 30th October.

542. Demarcation of Alluvial tracts should take place just at the commencement of the survey season as the boundary marks in such tracts are especially liable to destruction, and the field details to disturbance and change.

543. Defective Demarcation.—When the demarcation of village boundaries is found to be defective, the surveyor should note and report the matter to the Camp office and rectify any defects with the help of the villagers and Zamindars.

544. Line Clearing and Stone Embedding.—The same remarks as to demarcating and line clearing as made on p. 225, Art. 404, apply to cadastral surveys. Distinct sets of men are told off for the main-circuit, sub-circuit and village-circuits. Tri-junctions of village boundaries are usually marked permanently before the survey by the Civil authorities, either with stones or masonry pillars, but the other adjoining bends are only temporarily marked by them with bamboos, heaps of earth, or pegs, and other permanent

marks that may be required being left to be put down by the regular Survey Party. Should the offsets to the boundary between two stations exceed $2\frac{1}{2}$ chains on each side of the line, intermediate stations must be planted in such a way that the offsets on each line may be brought within the above limit.

545. Boundary Marks are either temporary or permanent. Temporary marks, such as, heaps of earth, bamboo pegs, are, at the time of demarcation, to be erected at every bend and turn of the boundary by the Zamindars and head raiyats, and in case they fail to comply, such marks should be erected in the first instance at the cost of Government. Under the law, occupants of lands can be required to maintain such marks until the survey operations have been concluded, or until the permanent boundary marks have been erected.

546. Rough Circuit Maps.—Rough circuit maps, previously supplied, and the mounds and bamboos erected in advance at every bend in the boundary form the line-cutter's guide in clearing the traverse lines.

547. Theodolite Stations to be marked with Stones.—All theodolite stations required for the preservation of the boundaries for future surveys, and for the maintenance of maps, will be permanently marked with roughly dressed stones; the exact number will vary according to the nature of the country, *but in all cases at least two contiguous stations should be permanently marked so as to give a reference line from which the intermediate temporary stations can at any time be relaid correctly.* Tri-junctions of villages will be always theodolite stations and they should have the side stations permanently marked. For other adjoining stations (Jawabs) the marks should be of such a nature that they may last till the detailed surveyors completed their work, after which no special care for their preservation is necessary. When a road forms the boundary, stones are to be embedded flush with the road surface, and on one side of it. When a theodolite station is near a pucca building, or masonry well, etc., it should be permanently marked and its distance and bearing from the corner of the building, etc., should be booked.

548. Description of Permanent Station Marks and the Manner of Fixing them.—Permanent boundary marks are usually (i) Δ r stones erected to show the tri-junction points of the true boundary lines of 3 villages; (ii) square stones for special purposes. Δ r stones which would be useless for domestic purposes, of 6" sides having their tops cut into Δ r prisms of 5" sides, 2' long, weighing from 20 to 30 seers, are to be used for tri-junctions. Square stone prisms of 1' side, 2' long having their tops cut into cubes of 5" sides, weighing about $1\frac{1}{2}$ mds. should be embedded in for special reasons by the order of the settlement officer to separate the surveyed portion from the unsurveyed portion pending decision of a boundary dispute. Square stones should be erected ordinarily at each bend of the boundary. In case of long straight lines they should be placed at intervals of 220 yds. Where stones are not easily available vitrified pottery cylinders, filled with charcoal, if

possible, may also be used, especially for other important theodolite stations along the boundary; they should be of 5" or 6" internal diameter and be 16" to 2' long. Long vitrified bricks, 12" × 4" × 3," may be used. They are less liable to be broken than the cylinders and can be easily made. No boundary marks should be ordinarily buried on roads, railway or canal banks. If, however, it is unavoidable to do so the *Stones and baked clay-cylinders are to be sunk flush with the ground* and after all angular observations are made and booked, mounds of earth 3' high, or heaps of small stones should then be raised over the marks to ensure protection.

It will be found convenient to dig holes or short trenches round pillars or other boundary marks, so as to enable them to be more easily identified, heaping the excavated earth for the marks. The embedding of the stones or cylinders may be done either by the line-clearing squads or by others following the line-cutters according to circumstances. Stones or cylinders are to be collected at, and distributed from convenient centres, one being located about the middle of every 30 square miles of country.

549. The Permanent Boundary Marks at Tri-junctions or on boundaries which have been disputed are to be shown with their appropriate symbols, as well as other permanent marks erected at traverse stations, as may be found necessary, are to be shown on the village maps and traces.

550. Use of Charcoal and Shurki.—A little charcoal or *shurki* (brick-dust) may with advantage be put inside each cylinder or buried around each stone as it enables the stations to be more easily identified, should the stone be dug up and shifted by interested parties, or have been otherwise lost.

551. Existing Boundary Marks and Rocks in situ to be made Stations.—Whenever a boundary is disputed all bends of the decreed boundary, after settlement of the disputes, are marked by stones by the Settlement Department; these or some of them, should be made theodolite stations. Also when a boundary-mark is available it should be adopted as a theodolite station in preference to embedding another mark near it. Rock *in situ* to be always used as stations where available. The mark $\triangle$ should be chiselled on them, and a description recorded in the field-book of the position of the station with reference to neighbouring objects. For stations not permanently marked, stout pegs may be driven.

552. Imbedding Stone-prisms or Pipes before Observation.—Before angular measurements are taken with the theodolite, survey stations should invariably be marked by embedding stones or pipes at the spots. If, however, owing to any unforeseen circumstances it becomes impracticable to embed the pipe before observation, a wooden peg should be driven into the ground at the proper place and observed over; six smaller pegs should then be driven into the ground around the station peg and about 2' away from it so that the intersection of the lines joining the opposite pegs marks the point of observation. Then when the pipes or stones are available,

the traverse surveyor may replace the central wooden pegs by permanent ones. The work should be carefully done and tested by the aid of the six pegs, the point of intersection of the lines joining the opposite pegs showing the final position of the embedded pipe.

553. Booking Stations in Field-books.—In the field-books permanently marked stations should be shown by symbols Δ and $\odot$ according as a stone or a pipe has been employed. The temporarily marked stations may be shown by $\times$. Tri-junction platforms shown by $\square$. The surveyor must be careful to enter the words "masonry pillar" whenever there is one on the ground. See field-book Fig. 184, p. 175.

554. Main Traverse Circuits.—Main-circuits follow fiscal boundaries, such as, thana or tehsil boundaries. If the pargana or thana be too large they may follow mauza boundaries in part. Generally a main-circuit would contain from 50 to 350 villages. The main-circuit should be surveyed very carefully with the best angular instruments, best tindel and chainmen being employed for chaining. The main traverse circuit system is employed with a view to avoid accumulated small errors, as well as to secure a basis of traverse lines on which observations for azimuth are taken and on which the interior village boundary work may be closed. The whole of the demarcated area of country to be traversed in one season is divided into circuits before taking the field and convenient portions of each traverse allotted to various surveyors to be commenced and terminated at the same time as far as possible.

555. Measurement of Traverse Lines.—All measurements of lines must be made by two independent parties. One party will precede the surveyors' measuring party and will give the surveyor the results of measurement of each line, which will then be compared by the surveyor himself. If there be a discrepancy of 1 link in 5 chains remeasurements by both the chains should be taken. Also when the traverse proof has not been established in the first instance, every endeavour should be made by careful re-chaining to eliminate the errors which can be effected by triangulating bad lines on sloping ground from measured bases, or the subtense bar may be used.

The same remarks as to use of chain as made on pages 16-21 apply to cadastral surveys. Gunter's chain is used in chain measurements. It is better to have the second chain of a different length from the Gunter's chain. This may either be a Gunter's chain with a few links removed, or it may be a 100' chain. To convert feet into links see Table II in Part II of this work.

556. Measurement of Traverses over uneven Ground.—The same remarks as to chaining on sloping ground as made on pp. 160-164, Arts. 277-285, apply to cadastral surveys. The table for reducing hypotenusal to base measurements will be found convenient. See Table III in Part II of this work.

557. The Great Trigonometrical Survey Stations which are within reach should always be connected by the main-circuit surveyor either by separate traverses, or through village traverses, or by triangulation.

558. Conspicuous Objects in Exterior Topography to be Intersected.—All conspicuous objects exterior of the district boundary within a reasonable distance from it should always be fixed by intersections by the traverse surveyors, and a topographic survey made of an overlapping strip varying in width from $\frac{1}{4}$ to 1 mile according to the nature of the country, the wider strip being taken when the details are few.

559. River Banks to be Surveyed.—Main-circuit traverse lines should never, as a rule, be run along the banks of a river. The river circuit should form a line of villages on either bank beyond the flood line. The masonry pillars at tri-junctions of such lines of villages being invariably adopted as theodolite stations to afford permanent marks for future connection.

560. Villages and interlaced Portions of Other Districts or States falling within main-circuits should always be surveyed.

561. Conspicuous objects should be observed to and fixed from two or more stations while traversing, such as, G. T. S. points, temples, large trees, etc. If the conspicuous objects are invisible from the traverse stations their positions may be fixed by means of a system of triangulation based on one of the traverse lines by observing the various angles of the Δ s so formed.

The latitude and longitude of prominent points on the sea-coast should be determined to fix their positions. All points so fixed should be shown on the index and other maps.

562. Main-Circuits how 'put up' and proved.—Each main-circuit surveyor on completion of his portion of the traverse will forward it to the Camp Officer to be put up, traversed and proved by him (p. 323, q.v.).

563. Testing the Length of Chains.—Surveyors are generally given three chains: one a standard chain 66' long which will not be used for chaining, and two, one a 66', and another a 100', for the double measurements required. Both these should be tested and if necessary corrected, with the standard chain every morning before starting for the day's work, or the surveyor may keep his chains of true length by laying down a standard line on level ground in the village under survey and comparing his chains daily with it before going to work. When the surveyors come into Head quarters camp they should invariably test their standard chain with the true length of a Gunter's chain marked out on the ground by means of steel standard bars supplied to every survey party. Inspecting officers especially should test the amin's chain and see that he is observing this rule; and also every chain should be tested before issue at the head quarters of the cadastral camp. In any case care should be taken that distances from the 50th link to either end of chain are equal.

564. Instruments to be kept in Order.—The surveyors are to keep all the instruments in a proper and efficient state and to see that they are in perfect adjustment. A theodolite should be carefully cleaned and dusted with a fine camel-hair brush before use every

day, and on the close of a field-season, the graduated silver limb should be gently washed with soap and water, and then cleaned with a piece of old silk, some fine olive oil, and a little lamp black freshly made and free from grit—the best oil for axis is goose or duck fat.

565. Booking the Entries.—All records in the angle-books and field-books should be in ink only. No erasures should be allowed. If any corrections become necessary, they should be made in ink as they are more reliable. Stylographic pens should be used for this purpose.

566. Starting Points of a Traverse : Area under Survey to be to the Left.—In executing the traverse survey of a main or sub-circuit or of a village-circuit, work should commence, if possible from a tri-junction point (a station that is common to 3 parganas, sub-circuits or village traverses) on the northwest of the main-circuit or sub-circuit or village or some point on the boundary which should be exactly marked permanently so as to be easily discovered by the cadastral amin and should proceed in a southwest direction round the main or sub-circuit or village so as to keep it on the left-hand. In the case of two adjoining villages bordering on the sea start should be made from a similarly situated bi-junction point.

567. Signals and the Method of their Use.—The best description of signal is a thin straight bamboo about 20' long surmounted by a red and white flag and shod with an iron spike. In sighting on to a point on which a flag-pole is being held, always take the iron-shod point at the lower end of the pole, or, if that is not visible, owing to undergrowth or other causes see that the pole is carefully plumbed, and sight on to the lowest visible part of the pole set upright on the centre of the station mark, otherwise the angular work will not be correct. Plumbing can be easily done by means of a fine string of about the length of the bamboo and attached to its top with a broken stone or brick or other heavy object as a plumb-bob suspended at the lower end of it. The pole will be vertical when the string will coincide with the pole. This is troublesome and inconvenient; it is quite sufficient for the flagman to place the ironspike of the bamboo on the centre of the station peg, keeping the latter between his feet and holding the bamboo, with both hands at about the height of his chest, and in a line with his nose, his front being turned towards the surveyor.

568. Observations of Angle.—All the inward angles of the polygons for main, and village traverses need be observed with *face left* of the instrument only. The system to be pursued is to record all three verniers before commencing, as well as, for each observation in succession after measuring the angle.

569. Specimen Main-Circuit Field-books.—On pp. 321-322 are given extracts from a Main-Circuit Field-Book in which change of face is not used, repeated angles having been considered sufficient as adopted by the Survey of India Department.

570. Sub-Circuit and Village-Circuit Field-Books.—The entries in the angle-book are made from the bottom upwards, as in

Survey of India Department.

MAIN-CIRCUIT FIELD-BOOK.

Page 36.

Names of adjoining Villages.	STATIONS.		Face & Zero.	VERNIER READINGS.			Means.	Mean Angle.	DISTANCES.		Angle of Elevation or Depression.	Names of adjoining Villages.	REMARKS.
	of observation.	Observed.		A	B	C			Chs.	Lks.			
			° ' "	° ' "	° ' "	° ' "	° ' "	° ' "					
	X ^o	Forward .. } Forward .. } Back .. }	182 01 40 {	99 24 00 281 25 00 103 26 00	24 00 25 00 26 00	24 00 25 00 26 00	99 24 00 281 25 00 103 26 00	177 59 00 177 59 00 ..	5 5 5	86 51 51			
	W ^o	Forward .. } Forward .. } Back .. }	265 17 20 {	352 29 00 257 46 20 163 03 40	29 00 46 20 03 40	29 00 46 20 03 40	352 29 00 257 46 20 163 03 40	94 42 40 94 42 40 ..	3 3 3	49 28 28			
	V ^o	Forward .. } Forward .. } Back .. }	57 38 20 {	249 11 40 306 50 00 4 28 40	11 40 50 00 28 40	11 40 50 00 28 40	249 11 40 306 50 00 4 28 40	302 21 40 302 21 20 ..			74		
	U ^o	Forward .. } Forward .. } Back .. }	123 40 00 {	332 21 00 96 00 40 219 40 00	21 00 00 40 40 00	21 00 00 40 40 00	332 21 00 96 00 40 219 40 00	236 20 20 236 20 40 ..	2 1 1	04 90 90			
	T ^o	Forward .. } Forward .. } Back .. }	282 40 00 {	273 27 20 196 06 20 118 45 20	27 20 06 20 45 20	27 20 06 20 45 20	273 27 20 196 06 20 118 45 20	77 21 00 77 21 00 ..	11 10 10	37 60 60			* by chain of 93 links.
	S ^o	Forward .. } Forward .. } Back .. }											

NOTES.—If the two first angles differ from each other more than 1', third angle should be observed, and if necessary a fourth or more. The angle of Elevation or Depression need not be recorded if less than 3°. Ordinarily the observations need only be on one face.

the forms given. At the commencement of a survey the name of the sub-circuit or village under survey, date of survey, and the name of the surveyor should be booked at the bottom of the first page. Above this a diagram is to be drawn and the angles made by the forward line with the two back (villages) traverse lines on either side are also to be recorded, as well as, the linear measure

Survey of India Department.

MAIN-CIRCUIT FIELD-BOOK.

Page 37.

Names of adjoining Village.	Of obser- vation.	STATIONS.		Face & Zero.	VERNIER READINGS.			Means.	Mean Angle.	DISTANCES.		Angle of Eleva- tion or Depres- sion.	Names of adjoin- ing Villages.	REMARKS.
		Observed.			A	B	C			Chs.	Lks.			
E	10	Forward Back		208 41 40	238 50 20 89 32 00 299 13 40	50 20 32 00 13 40	50 20 32 00 13 40	239 50 20 89 32 00 299 13 40	150 18 20 150 18 20 ..	7 6 6	20 71 70 1/2			
D	10	Forward Back		274 20 40	87 53 00 2 14 40 276 36 20	53 00 14 40 36 20	53 00 14 40 36 20	87 53 00 2 14 40 276 36 20	85 38 20 85 38 20 ..	3 3 3	23 02 02			
C	10	Forward Back		188 47 40	163 14 00 349 02 00 172 50 00	14 00 02 00 50 00	14 00 02 00 50 00	163 14 00 349 02 00 172 50 00	176 12 00 176 12 00 ..	5 5 5	61 26 36			
B	10	Forward Back		138 33 20	270 41 20 49 14 00 187 47 00	41 20 14 00 47 00	41 20 14 00 47 00	270 41 20 49 14 00 187 47 00	221 27 20 221 27 00 ..	10 9 9	41 71 71			
A	10	Forward Back		187 46 20	227 59 00 56 45 20 243 32 00	59 00 45 20 32 00	59 00 42 20 32 00	227 59 00 55 45 20 243 32 00	172 13 40 172 13 20 ..	7 6 6	24 75 75			
Z	9	Forward Back		184 57 00	133 19 40 318 16 20 143 13 20	19 40 10 20 13 20	19 40 16 20 13 20	133 19 40 318 16 20 143 13 20	175 03 20 175 08 00 ..	4 4 4	67 39 89			
Y	9	Forward Back		177 13 00	236 30 40 33 33 40 230 47 00	30 40 33 40 47 00	30 40 33 40 47 00	236 30 40 33 33 40 230 47 00	182 47 00 182 46 40 ..	4 4 4	50 23 22			
				0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0					

DISTRICT.....Contd.

SURVEY OF MAIN-CIRCUIT No.

ements of all the 3 lines diverging from the starting point. By means of the angular and linear measurements thus recorded, it will be easy to determine in the office the starting point of a traverse. When a tri-junction of the village under survey is reached, a line is drawn across the side of the page in the anglebook to the right or left as the case may be, and the name of the next village recorded. Again

The following is the traverse "put up" of the above field-book in pages 321-22:—

AREA OF MAIN TRAVERSE CIRCUIT No.

DISTRICT SURVEYED IN 19 — .

Origin of Surveys ☉.....				DISTANCES ON THE												Sums of Successive pair of Co-ordinates.		PRODUCTS.				Offsets.	
ADJOINING NAMES OF				Observed Angles.	Bearings.	Distances.		Meridian.		Perpendicular.		Meridian.	Perpendicular.										
Mn. Ct.	Villages.		Stations.					Chs.	Lks.	Chs.	Lks.	Chs.	Lks.	Chs.	Lks.	Chs.	Lks.	Chs.	Lks.	North N × E or S × W	South N × W or S × E		
	Exterior.	Interior.		North.	South.	East.	West.															of each Station from the Origin of Survey.	
				o	'	Cor.	Chs.	Lks.	Chs.	Lks.	Chs.	Lks.	Chs.	Lks.	Chs.	Lks.	Acres.	Dec.	Acres.	Dec.	Acres.	Dec.	
No. 1.				o	"	"																	
Observed Az :				97	49	53	T ₉	77	21	278	24	10	60										
Convergency W. +				34	14		U ₉	236	20	175	45	1	90										
Working Bearing...				98	24	07	U ₉	302	21	232	05	7	74										
							W ₉	94	43	354	26	3	28										
							X ₉	177	59	269	09	5	51										
							Y ₉	182	47	267	08	4	22										
							Z ₁₀	175	03	269	55	4	39										
							A ₁₀	172	13	264	58	6	75										
No. 2.							B ₁₀	221	7	257	11	9	71										
Observed Az :				19	53	50	C ₁₀	176	12	298	38	5	26										
Convergency W. +				34	28		D ₁₀	85	38	294	50	3	02										
Working Bearing ..				20	28	18	E ₁₀	150	18	200	28	6	70										
										170	46												

when the traverse closes either on the starting point, or a station of another traverse, a diagram is to be drawn in the angle-book and the angular and linear measurements of the connecting traverse lines converging to it are to be duly booked therein.

Specimen of a Sub-Circuit Field-book.

Ballau	G	18-88	Hudha	Sunarya Kusal	P	38-40	Buzurg
		108 01 30	251 58 30			111 40 00	248 30 00
Orha	F	234 18 00	251 58 30	O	70°01'00"	248 20 30	
		342 19 00	251 59 00		118 40 30	248 20 00	
		90 20 00	Maufi		230 20 30	248 20 00	
		54-40	186 23 00		10-49	Altara	
E	173 37 00	93 01 00	186 22 30	N	255 88 30	197 40 00	
		266 38 30	186 23 00		57 58 00	197 40 30	
		80 15 30	19-02		220 18 30	197 40 00	
		285 50 30	74 09 30		26-65		
D	218 35 00	144 25 00	74 10 00	An	81 24 30	115 35 30	
		70 15 00	74 10 00		825 40 00	115 35 30	
		7-64	151 33 00		210 14 00	115 35 00	
		208 27 00	151 33 30		16-61	167 00 30	
C	211 43 00	60 10 00	151 33 30	M	192 59 30	167 01 30	
		18-33	219 04 30		174 44 30	167 01 30	
		140 55 30	219 05 00		7 43 00	167 01 30	
		128 10 00	Tur. rab		200 42 00	Muhtara	
B	269 05 00	50 00 00	219 05 00	L	24-98	188 33 00	
		25-58	249 57 00		171 27 00	188 33 30	
		110 03 00	249 57 00		207 24 30	188 33 00	
		179 53 30	Intermediate Entries Omitted.		18 51 00		
A	289 57 00	40 00 00	46-15	I	190 18 00		
		209 26 00	209 26 00		32-56		
		331 08 30	150 34 00		34-10	Chandraur.	
		180 34 00	150 34 30		210 23 30	149 36 30	
Z	30 00 00	22-57	234 28 00	K	79 17 00	149 36 00	
		271 04 30	125 32 00		289 41 00	149 36 00	
		145 32 00	125 32 30		140 05 00	Badausa.	
		20 00 00	148 28 00		42-93	Khas.	
Y	29-85	211 32 00	148 27 30	J	206 22 00	153 37 30	
		306 56 00	148 28 30		77 18 00	153 37 00	
		158 28 30	226 10 00		983 41 00	153 37 00	
		10 00 00	226 10 00		130 04 00		
X	21-01	133 50 00	226 10 00	H	6-87		
		92 20 00	226 10 00		114 18 30	245 41 30	
		226 10 00	226 10 00		251 26 00	245 42 00	
		00 00 00	226 10 00		5 44 00	245 42 00	
W	47-02	47-02	305°55'00"	Ballau	120 02 00		
		79°45'30"	54 05 00		80-85		
		NAGWARA.				222 58 30	137 01 30
		Commenced from Main Circuit No. 1.				24 07 00	137 01 00
		Survey of Sub-circuit No. 1 by				247 06 00	137 01 00
		M. Roy, 10th Dec., 1905.				110 05 00	137 01 00
						39-16	
						179 43 30	180 16 30
						100 34 00	180 16 00
						280 18 00	180 16 00
						100 02 00	180 16 00

Commenced from Main Circuit No. 1.
Survey of Sub-circuit No. 1 by
M. Roy, 10th Dec., 1905.

Sub-Circuit Field-book—contd.

571. **Example of a Field-book of Sub-Circuit.**—The above are extracts from a field-book to illustrate how angular and linear measurements are recorded for a sub-circuit by the Survey of India Department (p. 325, q.v.).

CLOSED ON MAIN CIRCUIT No. I.

17°25'00"		
X 47.02	W	87.41 Y
96° 50' 00"		280° 35' 00"
166 20 00		158 50 00
263 10 00		79 25 00
Nagwari	W	263 10 00
		174 02 30
		77 12 00
		340 23 00
		9° 58
	Z	100 08 30
		130 00 30
		230 09 30
		330 18 30
		14° 62
Y	Y	148 00 30
		22 20 00
		171 21 00
		320 22 00
		23° 97
	X	202 52 00
		204 34 30
		107 26 00
		310 18 00
		38° 10
W	W	187 01 00
		286 10 00
		113 12 00
		300 14 00
		24° 97
	V	254 53 30
		140 16 00
		35 10 00
		290 04 00
		15° 60
U	U	199 55 00
		240 18 30
		80 13 00
		280 08 30
		12° 88
	T	164 02 00
		41 01 00
		206 02 30
		270 04 30
		23° 21
S	S	111 09 30
		35 59 30
		148 09 00
		260 19 00
		18° 11
	R	259 21 30
		93 30 00
		351 52 00
		250 14 00
		37° 48
Sunarya Kusal	Q	182 55 00
		234 28 30
		57 23 30
		240 18 30

SUB-CIRCUIT FIELD-BOOK.—

Contd.

SPECIMEN OF
VILLAGE CIRCUIT FIELD-BOOK.

Closed on Sub-circuit No. 1-		
21.01	X.	47.02 W
138° 11' 00"		
57° 53' 00"		
Survey of Mouza Orha	X	221 49 00
		276 22 00
		138 11 00
		15° 13
		254° 38 00
	f	290 42 00
		145 21 00
		29° 32
		181 59 00
		350 02 00
e	e	178 01 00
		38° 19
		163 12 00
		33 34 00
		190 47 00
	d	40° 42
		255 20 00
		209 20 00
		104 40 00
		20° 79
b	b	197 14 00
		325 32 00
		162 46 00
		31° 65
		166 14 00
	a	27 33 00
		193 46 00
		24° 83
		265° 14 00
		189 32 00
G	G	94 46 00
		54° 40
		24° 83
		157° 13' 00"
		94° 10' 00"
	H	54° 40
		108° 01' 00"
		11° 58
		54° 40
		11° 58

Commenced from S. C. No. 1
Survey of Mauza Orha.

572. **Traverse Computation Form for a Sub-Circuit**—On pp. 327-329 are given full traverse tables for the sub-circuit of which only extracts are given above of the field-book as kept by the Survey of India Department. The student will find the plotting of the traverse lines interesting.

SUB-CIRCUIT TRAVERSE TABLE.

Page

Mauza Sub-division No. 1. No. District

CHAP. V]

SUB-CIRCUIT TRAVERSE TABLE.

327

Names of Adjoining Villages.		Station Letters.	Observed.		Bearings.	Distances.	DISTANCES ON THE																Sums of each successive pair of Co-ordinates.		PRODUCTS.				Offsets.	
Exterior.	Interior.		°	'			Chs.	Lks.	MERIDIAN.			PERPENDICULAR.			MERIDIAN.		PERPENDICULAR.		Chs.	Lks.	Acres.	Dec.	Acres.	Dec.	+	-				
									North.	South.	East.	West.	Of each station from the first in the Series																	
													Chs.	Lks.	Cors.	Chs.	Lks.	Cors.									Chs.	Lks.		
Turrah.	Nagwan.	Ballen.	Orha.	Badan Khana.	Chan-daur.	Banai.	Panhar.	Nadan man	Bagha	Chaunsar	Tindura																			
W	X	226	10	105	50	47	02	...	...	12	83	45	24	1	...	...	...	...	...	...	...	...	...	...	...					
Y	Y	148	28	152	00	21	01	...	...	18	55	9	86	...	...	...	...	...	...	...	...	...	...	...	...					
Z	Z	125	32	120	28	29	85	...	...	15	14	25	73	1	...	...	...	...	...	...	...	...	...	...	...					
A	A	150	34	66	00	22	57	9	18	...	...	20	62	1	...	...	...	...	...	...	...	...	...	...	...					
B	B	249	57	36	34	46	15	37	07	1	...	27	49	1	...	...	...	...	...	...	...	...	...	...	...					
C	C	219	05	106	31	25	58	...	...	7	27	24	52	1	...	...	...	...	...	...	...	...	...	...	...					
D	D	151	33	145	36	18	33	...	...	15	12	10	36	...	...	...	...	...	...	...	...	...	...	...	...					
E	E	74	10	117	09	7	64	...	...	3	49	6	80	...	...	...	...	...	...	...	...	...	...	...	...					
F	F	186	23	11	18	19	02	18	65	...	...	3	73	...	...	...	...	...	...	...	...	...	...	...	...					
G	G	251	59	17	41	54	40	51	83	1	...	16	52	...	...	...	...	...	...	...	...	...	...	...	...					
H	H	180	16	89	40	18	88	...	...	18	88	39	16	...	...	...	...	...	...	...	...	...	...	...	...					
I	I	137	01	89	56	39	16	...	...	39	16	22	18	1	...	...	...	...	...	...	...	...	...	...	...					
J	J	245	42	46	57	30	35	20	72	...	...	6	34	...	...	...	...	...	...	...	...	...	...	...	...					
K	K	153	37	112	39	6	87	...	...	2	65	42	84	1	...	...	...	...	...	...	...	...	...	...	...					
L	L	149	36	86	16	42	93	...	...	2	80	28	23	1	...	...	...	...	...	...	...	...	...	...	...					
M	M	141	01	55	52	34	10	19	13	...	...	15	33	...	...	...	...	...	...	...	...	...	...	...	...					
N	N	240	44	16	53	52	79	50	51	1	...	37	37	1	...	...	...	...	...	...	...	...	...	...	...					
O	O	140	53	77	37	38	26	8	21	...	...	12	37	...	...	...	...	...	...	...	...	...	...	...	...					
P	P	147	46	38	30	19	87	15	55	...	...	2	60	...	...	...	...	...	...	...	...	...	...	...	...					
Q	Q	188	59	6	16	23	80	23	66	...	...	11	70	...	...	...	...	...	...	...	...	...	...	...	...					
R	R	286	44	15	15	44	49	42	92	1	...	39	92	1	...	...	...	...	...	...	...	...	...	...	...					
S	S	151	08	121	59	47	06	...	...	24	93	13	00	...	...	...	...	...	...	...	...	...	...	...	...					
T	T	143	51	93	07	13	02	...	...	0	71	8	65	...	...	...	...	...	...	...	...	...	...	...	...					
U	U	157	46	56	58	10	32	5	63	...	...	19	22	...	...	...	...	...	...	...	...	...	...	...	...					
V	V	128	20	34	44	33	74	27	73	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
W	W	240	01	343	03	12	92	12	36	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
X	X	106	02	43	04	26	60	19	43	...	...	18	16	...	...	...	...	...	...	...	...	...	...	...	...					
Y	Y	218	06	329	06	40	24	34	53	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
Z	Z	181	53	7	12	46	33	45	96	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
A	A	109	07	9	05	14	25	14	07	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
B	B	189	45	298	12	38	58	18	23	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
C	C	218	28	307	57	29	10	17	90	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
D	D	108	08	346	25	15	39	14	96	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
E	E	264	47	274	33	22	20	1	76	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
F	F	186	00	359	20	11	93	11	93	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
G	G	128	08	5	20	49	59	49	38	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
H	H	242	55	313	28	28	44	19	57	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
K	K	87	16	16	23	61	36	58	87	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...					
652	70	9	100	69	1	556	80	10	127	91	5																			

Set up and Proved by—

Examined and passed by—

SUB-CIRCUIT TRAVERSE TABLE,—contd.

Page

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Mauza

No.

District

Names of Adjoining Villages.		Station Letters.	Observed.		Angles.		Bearings.		Distances.		DISTANCES ON THE																Sums of each successive pair of Co-ordinates.		PRODUCTS.				Offsets.	
Exterior.	Interior.		°	Crs.	°	Crs.	Chs.	Lks.	MERIDIAN.			PERPENDICULAR.			MERT-DIAN		PERPEN-DICULAR.		Chs.	Lks.	Chs.	Lks.	Acres.	Dec.	Acres.	Dec.	+	-						
									North.	South.	East.	West.	Of each station from the first in the Series.																					
													Chs.	Lks.	Crs.	Chs.	Lks.	Crs.											Chs.	Lks.	Chs.	Lks.	Chs.	Lks.
Main-Circuit No. 3. Augasi		Omwar	J2	180	06	283	39	68	06	16	06	...	...	...	...	66	14	2	N 567	97	E 362	92	E 791	96	1271	89	...	...	19-23	...				
			I	114	20	283	45	77	05	18	31	...	...	...	...	74	84	2	586	23	288	10	651	02	1192	02	...	...	...	10-85				
			H	241	58	214	05	24	76	...	...	19	49	...	...	15	27	...	566	79	272	83	560	93	...	...	1093	25	...	0-16				
			G	255	15	280	03	29	41	5	13	...	...	...	...	28	96	1	571	92	243	88	516	71	265	07	...	...	...	4-05				
			F	106	26	355	18	22	61	22	53	...	...	...	...	1	85	...	594	45	242	03	485	91	1094	76	...	...	2-36	...				
			E	185	39	281	44	48	71	9	91	...	...	...	...	47	68	1	604	36	194	36	436	39	432	46	...	...	6-96	...				
			D	318	30	287	23	45	24	13	52	...	...	...	...	43	17	1	617	88	151	20	345	56	467	20	...	...	...	3-65				
			C	95	21	64	53	28	87	11	80	...	...	...	...	26	35	1	629	68	177	56	328	76	387	94	...	...	...	...				
			B	178	05	343	14	62	47	59	15	1	...	...	...	...	20	10	1	688	82	157	47	335	03	1981	37	...	...	12-30	...			
			A	169	40	339	19	60	89	56	97	1	...	...	...	...	21	51	1	745	78	135	97	293	44	1671	43	...	...	...	9-43			
			Z	167	21	329	59	59	49	50	98	1	...	...	...	...	30	65	1	796	75	105	33	241	30	1229	91	...	...	...	8-53			
		Y	130	09	316	19	24	95	18	04	...	...	...	...	17	23	...	814	79	88	10	193	43	348	95	...	...	4-07	...					
		X	161	59	266	28	21	88	...	...	1	35	...	...	21	84	1	813	44	66	27	154	37	...	...	20	84	0-18	...					
		W	207	34	248	27	68	32	...	...	25	09	...	...	63	54	2	788	35	2	75	E 69	02	...	...	173	17	12-69	...					
		V	187	13	276	01	60	46	6	34	...	...	...	...	60	13	2	794	69	W 57	36	W 54	61	...	...	34	62	...	10-32					
		U	188	53	283	14	72	79	16	66	...	...	...	...	70	86	2	811	35	128	20	185	56	...	...	309	14	8-84	...					
		T	35	39	292	07	38	53	14	51	...	...	...	...	35	69	1	825	86	163	88	292	08	...	...	423	81	...	1-29	...				
		Bisanda Buzurg.	A	185	33	142	46	27	13	...	...	319	91	3	45	93	26	35	1	619	46	18	...	...	10343	00	2054	83	70-10	48-28	...			
			B	185	49	153	19	37	34	...	...	22	95	...	...	33	36	1	769	54	132	64	W 313	29	719	00	...	...	...	0-87	...			
			C	235	58	159	08	29	34	...	...	27	42	...	...	10	45	...	742	12	122	19	254	83	698	74	...	...	...	1-78	...			
			D	166	04	215	06	34	62	...	...	28	32	...	...	...	...	...	713	80	142	10	264	29	748	47	...	...	...	0-93	...			
			E	172	16	201	09	34	08	...	...	31	78	1	...	...	...	...	682	01	154	40	296	50	942	57	...	...	...	1-74	...			
			F	183	00	193	25	28	29	...	...	27	52	...	...	6	56	...	654	49	160	96	315	36	867	87	...	...	...	...	0-05	...		
			G	268	07	196	25	20	87	...	...	20	02	...	...	5	90	...	634	47	166	86	327	82	656	30	...	...	...	...	3-14	...		
			H	88	55	284	32	31	87	8	00	...	...	...	...	30	85	1	642	47	197	70	364	56	...	...	291	65	7-39	...	...			
			I	192	03	193	27	32	02	...	...	31	14	1	...	...	7	45	...	611	32	205	15	402	85	1254	88	...	...	...	...	3-77	...	
			J	182	10	205	30	25	66	...	...	23	16	...	...	11	05	...	588	16	216	20	421	35	975	85	...	...	...	0-39	...	...		
			K	193	48	207	40	25	96	...	...	22	99	...	...	12	05	...	565	17	228	25	444	45	1021	79	...	...	...	...	1-22	...		
		L	179	55	221	28	30	47	...	...	22	83	...	...	20	18	...	542	34	248	43	476	68	1088	26	...	...	...	0-43	...	...			
		M	216	55	221	23	10	94	...	...	8	21	...	...	7	23	...	534	13	255	66	504	09	413	86	...	...	...	...	0-81	...			
		N	226	43	258	18	25	30	...	...	5	13	...	...	24	77	1	529	00	280	42	536	08	275	01	...	...	...	0-46	...	...			
		O	101	13	305	01	29	22	16	77	...	...	...	...	23	93	...	545	77	304	35	584	77	...	...	980	66	0-38	...	...	...			
		P	107	38	226	14	21	07	...	...	14	57	...	...	15	22	...	531	20	319	57	623	92	909	05	...	...	...	0-21	...	...			
		Q	244	48	153	52	13	72	...	...	12	32	...	...	6	04	...	518	88	313	53	633	10	779	98	...	...	...	...	1-67	...			
		R	97	36	218	40	11	99	...	...	9	36	...	...	...	...	...	509	52	321	02	634	55	593	94	...	...	...	0-30	...	...			
38° + 38	34' 24" 05 06 39 30																										12886	77	1272	31	15-05	10-66		

* These are written in red.

Has- tam.	Adh- rauri	Kha- naura	Mut- ahra	Attara Buzurg				Nag- wara.	Basrehi	Sun- arya Kusal	Pindkhar	S T U V W X Y Z A B C D E F G H I J K L M N O P Q R S T U V W X Y Z W	222	28	136	16	17	85	...	...	...	12	90	12	34	...	...	N496	62	W303	68	W629	70	812	31	...	...	...	0.74																																																																																																																																																																																																																																																																																																																																																																										
				Nag- wara.	Basrehi	Sun- arya Kusal	Pindkhar																																	222	28	136	16	17	85	...	...	...	12	90	12	34	...	...	N496	62	W303	68	W629	70	812	31	...	...	...	0.74																																																																																																																																																																																																																																																																																																																																															
156	17	173	44	31	18	...	...	...	...	...	...	156	17	173	44	31	18	...	...	...	...	...	31	17	1	0	69	...	...	...	465	44	307	99	616	67	1922	78	...	...	0.45	...																																																																																																																																																																																																																																																																																																																																																																							
164	28	155	01	26	80	...	...	...	...	...	...	164	28	155	01	26	80	...	...	...	...	...	24	29	...	11	32	...	...	...	441	15	296	67	604	66	1468	72	...	...	...	0.57	...																																																																																																																																																																																																																																																																																																																																																																						
296	22	139	29	16	74	...	...	...	...	...	...	296	22	139	29	16	74	...	...	...	...	...	12	73	...	10	88	...	...	...	428	42	285	79	582	46	741	47	...	...	...	2.74	...																																																																																																																																																																																																																																																																																																																																																																						
66	54	255	51	15	42	...	...	...	...	...	...	66	54	255	51	15	42	...	...	...	...	...	3	77	...	...	...	14	95	...	...	424	65	300	74	586	53	221	12	...	...	...	2.05	...																																																																																																																																																																																																																																																																																																																																																																					
217	55	143	45	11	42	...	...	...	...	...	...	217	55	143	45	11	42	...	...	...	...	...	9	09	...	6	91	...	...	...	415	56	293	83	594	57	540	46	...	...	...	1.27	...																																																																																																																																																																																																																																																																																																																																																																						
131	03	180	40	18	02	...	...	...	...	...	...	131	03	180	40	18	02	...	...	...	...	...	18	02	...	...	...	0	21	...	...	397	54	294	04	587	87	1059	31	...	...	...	0.59	...																																																																																																																																																																																																																																																																																																																																																																					
230	14	131	43	25	05	...	...	...	...	...	...	230	14	131	43	25	05	...	...	...	...	...	16	67	...	18	70	...	...	...	380	87	275	34	569	38	949	16	...	...	...	0.87	...																																																																																																																																																																																																																																																																																																																																																																						
189	43	181	57	51	28	...	...	...	...	...	...	189	43	181	57	51	28	...	...	...	...	...	51	25	1	...	...	1	74	...	...	329	61	277	08	552	42	2831	70	...	...	...	3.89	...																																																																																																																																																																																																																																																																																																																																																																					
109	20	191	40	36	50	...	...	...	...	...	...	109	20	191	40	36	50	...	...	...	...	...	35	75	1	...	...	7	38	...	...	293	85	284	46	561	54	2008	07	...	...	...	4.81	...																																																																																																																																																																																																																																																																																																																																																																					
73	22	121	00	11	00	...	...	...	...	...	...	73	22	121	00	11	00	...	...	...	...	...	5	67	...	9	43	...	...	...	283	18	275	03	559	49	317	23	...	...	...	1.12	...																																																																																																																																																																																																																																																																																																																																																																						
279	58	14	22	11	96	...	...	...	...	...	...	279	58	14	22	11	96	...	...	...	...	...	12	46	...	27	55	1	...	...	299	77	272	06	547	09	...	...	634	08	...	0.07	...																																																																																																																																																																																																																																																																																																																																																																						
169	58	114	20	30	24	...	...	...	...	...	...	169	58	114	20	30	24	...	...	...	...	...	11	77	...	46	19	1	...	...	287	31	244	50	516	56	643	63	...	...	...	0.43	...																																																																																																																																																																																																																																																																																																																																																																						
158	56	104	18	47	67	...	...	...	...	...	...	158	56	104	18	47	67	...	...	...	...	...	11	77	...	46	19	1	...	...	275	54	193	30	442	80	521	18	...	...	...	4.91	...																																																																																																																																																																																																																																																																																																																																																																						
185	22	83	13	21	09	...	...	...	...	...	...	185	22	83	13	21	09	...	...	...	...	...	...	...	...	...	...	...	...	...	278	02	177	45	375	75	...	...	93	19	...	0.07	...																																																																																																																																																																																																																																																																																																																																																																						
179	37	88	12	16	82	...	...	...	...	...	...	179	37	88	12	16	82	...	...	...	...	...	...	...	...	...	...	...	...	...	278	73	148	72	326	17	...	...	23	16	...	0.51	...																																																																																																																																																																																																																																																																																																																																																																						
286	57	88	12	16	82	...	...	...	...	...	...	286	57	88	12	16	82	...	...	...	...	...	...	...	...	...	...	...	...	...	279	26	131	91	280	63	...	...	14	87	...	0.19	...																																																																																																																																																																																																																																																																																																																																																																						
89	02	195	09	29	45	...	...	...	...	...	...	89	02	195	09	29	45	...	...	...	...	...	28	43	...	...	...	7	70	...	...	250	83	139	61	271	52	771	93	...	...	...	1.29	...																																																																																																																																																																																																																																																																																																																																																																					
259	04	104	11	34	06	...	...	...	...	...	...	259	04	104	11	34	06	...	...	...	...	...	8	35	...	33	02	1	...	...	242	48	106	58	246	19	205	57	...	...	...	1.26	...																																																																																																																																																																																																																																																																																																																																																																						
188	33	183	15	32	56	...	...	...	...	...	...	188	33	183	15	32	56	...	...	...	...	...	32	51	1	...	...	1	85	...	...	209	96	108	43	215	01	699	21	...	...	...	0.09	...																																																																																																																																																																																																																																																																																																																																																																					
167	01	191	48	24	98	...	...	...	...	...	...	167	01	191	48	24	98	...	...	...	...	...	24	45	...	...	...	5	11	...	...	185	51	113	51	221	97	512	72	...	...	...	0.22	...																																																																																																																																																																																																																																																																																																																																																																					
115	35	178	49	16	61	...	...	...	...	...	...	115	35	178	49	16	61	...	...	...	...	...	16	61	...	0	34	...	...	...	168	90	113	20	226	74	376	62	...	...	...	0.33	...																																																																																																																																																																																																																																																																																																																																																																						
197	40	114	24	26	65	...	...	...	...	...	...	197	40	114	24	26	65	...	...	...	...	...	11	01	...	24	27	1	...	...	157	89	88	92	202	12	222	53	...	...	...	1.55	...																																																																																																																																																																																																																																																																																																																																																																						
248	20	132	04	10	49	...	...	...	...	...	...	248	20	132	04	10	49	...	...	...	...	...	7	03	...	7	79	...	...	...	150	86	81	13	170	05	119	55	...	...	...	1.46	...																																																																																																																																																																																																																																																																																																																																																																						
177	05	200	24	33	40	...	...	...	...	...	...	177	05	200	24	33	40	...	...	...	...	...	31	31	1	...	...	11	61	...	...	119	54	92	77	173	90	544	65	...	...	...	0.72	...																																																																																																																																																																																																																																																																																																																																																																					
101	38	197	29	37	43	...	...	...	...	...	...	101	38	197	29	37	43	...	...	...	...	...	37	75	1	...	...	11	26	...	...	83	78	104	03	196	80	703	76	...	...	...	0.62	...																																																																																																																																																																																																																																																																																																																																																																					
247	50	109	07	18	11	...	...	...	...	...	...	247	50	109	07	18	11	...	...	...	...	...	8	81	...	15	82	...	...	...	74	97	88	21	192	24	169	36	...	...	...	1.90	...																																																																																																																																																																																																																																																																																																																																																																						
195	58	186	56	23	21	...	...	...	...	...	...	195	58	186	56	23	21	...	...	...	...	...	23	04	...	...	...	2	80	...	...	51	93	91	01	179	22	412	92	...	...	...	1.15	...																																																																																																																																																																																																																																																																																																																																																																					
160	05	202	54	12	88	...	...	...	...	...	...	160	05	202	54	12	88	...	...	...	...	...	11	86	...	...	...	5	01	...	...	40	07	96	02	187	03	221	82	...	...	...	0.11	...																																																																																																																																																																																																																																																																																																																																																																					
105	06	182	59	15	60	...	...	...	...	...	...	105	06	182	59	15	60	...	...	...	...	...	15	58	...	...	...	0	81	...	...	24	49	96	83	192	85	300	46	...	...	...	1.41	...																																																																																																																																																																																																																																																																																																																																																																					
172	58	108	05	24	97	...	...	...	...	...	...	172	58	108	05	24	97	...	...	...	...	...	7	75	...	23	74	1	...	...	16	74	73	08	169	91	131	63	...	...	...	0.17	...																																																																																																																																																																																																																																																																																																																																																																						
157	03	101	03	38	01	...	...	...	...	...	...	157	03	101	03	38	01	...	...	...	...	...	...	...	...	...	...	...	...	...	9	45	35	76	108	84	79	34	...	...	...	5.30	...																																																																																																																																																																																																																																																																																																																																																																						
210	59	173	14	23	97	...	...	...	...	...	...	210	59	173	14	23	97	...	...	...	...	...	4	91	...	...	...	...	...	...	14	36	12	29	48	05	...	...	23	59	...	2.50	...																																																																																																																																																																																																																																																																																																																																																																						
259	51	189	01	14	62	...	...	...	...	...	...	259	51	189	01	14	62	...	...	...	...	...	4	80	...	13	81	...	...	...	9	56	1	52	13	81	...	6	63	...	...	...	0.18	...																																																																																																																																																																																																																																																																																																																																																																					
96	50	...	...	...	...	...	...	...	...	...	...	96	50	...	...	...	...	...	...	...	...	...	9	56	...	...	...	1	52	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0.08	...																																																																																																																																																																																																																																																																																																																																																																				
X	5.4*	105	50	...	...	20	22	...	...	...	...	X	5.4*	105	50	...	...	...	...	...	...	...	529	68	+	392	92	+	71	93	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...

ABSTRACT.

W K T R W	...	652	70	9	100	69	+	556	80	+	127	91	5	0	00	000	...	...	...	50190	55	3699	03	65.17	76.91
...	...	319	91	3	45	93	...	26	35	1	619	46	18	N551	91	E429	...	...	...	10343	00	2054	83	70.10	48.28
...	...	24	77	...	341	08	3	47	73	...	204	89	2	N825	86	W163	...	...	...	12886	77	1272	31	15.05	10.66
...	...	20	22	...	529	68	6	392	92	8	98	...	...	N509	52	321	0	...	...	19547	37	788	89	21.66	23.96
...	...	1017	60	12	1017	38	10	1023	80	19	1024	24	25	...	...	...	...	...	...	92967	69	7815	06	171.98	159.81
...	...	+	22	...	...	...	...	...	...	+	44	...	...	...	...	...	...	...	...	85152	63	...	...	12.17	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	42576	31	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	+12	17	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	42588	48	...	...	...	...

* This is written in red.

VILLAGE-CIRCUIT TRAVERSE TABLE.

Page

Mauza Orha

No.

District

530

A MANUAL OF SURVEYING FOR STUDENTS.

[CHAP. V.]

Names of Adjoining Villages.		Station Letters.	DISTANCES ON THE																		Sums of each successive pair of Co-ordinates.		PRODUCTS.				Offsets.									
			Observed			Angles.			Bearings.			Distances.			MERIDIAN.			PERPENDICULAR.					MERIDIAN.		PERPENDICULAR.				North N. E., or S. W.	South N. W., or S. E.						
			°	'	Cours.	°	'	Cours.	Chs.	Lks.	North.		South.		East.		West.		of each Station from the first in the Series.																	
											Chs.	Lks.	Cours.	Chs.	Lks.	Cours.	Chs.	Lks.	Cours.	Chs.	Lks.	Cours.	Chs.	Lks.	Chs.	Lks.	Chs.	Lks.	Acres.	Dec.	Acres.	Dec.	+	-		
Turrah	M. C. No. 6.	X	48	28	...	152	00	21	01	...	...	18	55	...	9	86	...	...	S	18	55	E	9	86	E	9	86	...	...	18	29	...	6-58			
		Y	25	32	...	120	28	29	85	...	...	15	14	...	23	73	1	...	33	69	35	58	45	44	...	...	68	80	9-53	...						
		A	50	34	...	66	00	22	57	9	18	...	...	...	20	62	1	...	24	51	56	19	91	77	84	24	...	...	0-87	...						
		B	49	57	...	36	34	46	15	37	07	...	...	...	27	49	1	...	N	12	56	83	67	139	86	518	46	...	...	5-74	...					
		C	19	05	...	106	31	25	58	...	...	7	27	...	24	52	1	...	5	29	103	18	191	85	...	...	139	47	...	1-14						
		D	51	33	...	145	36	18	33	...	...	15	12	...	10	36	...	...	S	9	83	118	54	226	72	...	...	342	40	...	2-08					
		E	74	10	...	117	09	7	64	...	...	3	40	...	6	86	...	...	13	62	125	34	243	88	...	...	85	11	...	0-47						
		F	86	23	...	11	18	19	02	18	65	...	...	...	3	73	...	...	N	5	33	129	07	254	41	471	47	...	...	...	0-71					
		G	94	46	...	17	41	54	40	51	83	1	...	...	16	52	...	...	57	15	145	59	274	66	1423	29	...	...	...	0-96						
		a	93	46	...	292	27	24	83	9	48	...	...	...	...	...	22	95	...	66	63	122	64	268	23	254	28	...	...	...	1-03					
Hadha Manji	M. C. No. 6.	b	62	46	...	306	13	31	65	18	70	...	...	...	...	...	25	53	1	85	33	97	10	219	74	410	91	...	...	...	0-18	...				
		c	04	40	...	289	00	20	79	6	77	...	...	...	...	...	19	66	...	92	10	77	44	174	54	118	16	...	...	...	0-54	...				
		d	96	47	...	213	40	40	42	...	...	33	64	1	...	...	22	41	...	58	45	55	03	132	47	...	...	445	76	...	2-54	...				
		e	78	01	...	230	27	38	19	...	...	24	32	...	...	...	29	45	2	34	13	25	56	80	59	...	...	195	99	0-65	...					
		f	45	21	...	228	28	29	32	...	...	19	44	...	...	...	21	95	...	14	63	3	61	29	17	...	...	56	71	0-21	...					
		X	38	11	...	193	49	15	13	...	...	14	69	...	...	...	3	61	...	0	00	0	00	3	61	...	...	5	30	0-72	...					
		Sum																...	...	3283	81	1358	23	17-09	16-26											
		Difference																...	...	1925	58	...	...	1-43	...											
		Half																...	...	962	79	...	...	...	...											
		Area of offsets																...	...	+1	43	...	...	...	...											
Statute Area																...	...	964	22	...	...	...	...													
Nagwara	M. C. No. 6.	8	2	20	00	0	152	00	200	33	151	68	1	151	66	1	145	63	4	145	56	3														

* These are written in red.

578. Example of a Field-book of Village-Circuit.—The form on p. 325 shows the method of entering the angular and linear measurements of a village-circuit as kept by the Survey of India Department.

574. Traverse Computation Form for a Village-Circuit.—Subjoined (p. 330) is the computation form of an actual survey showing the system of entering the angular and linear measurements with the process of calculations as generally adopted by the Survey of India Department. The figures for stations X to F are taken from the traverse table of Main-circuit No. 6.

575. Limit of Error.—The greatest permissible error in the angular work is 1 minute in every 5 angles for main and minor circuits, and 1 minute in every 3 angles for village traverses and sub-traverses. In the linear measurement the traverse distances of latitudes and departures should prove within $\frac{1}{2}$ link in every 10 chains for main-circuits, and of village-circuits within 1 link. If greater corrections than these be required a re-survey becomes necessary.

576. Triangulation when Necessary.—It may sometimes happen that traverse plots can not be supplied to the detail surveyors in time owing to the Traverse camp not having been able to complete the necessary calculations for the *traverse set-up* after taking the angular and linear measurements or owing to their inability to enter upon their work for some cause or other, and that the field survey precedes the traverse survey, working on the triangulation system. In the first case, either the field surveyor will rechain the traverse lines and divide the village under survey in such a way that the traverse stations will form the apices of the Δ s into which the ground will be split up for the detail survey, or after triangulating he will pick up all the traverse stations of the village by sight-vane measurements and rechain the traverse lines or partly by each of these methods. In the second case the field surveyor will fix the traverse lines in the same manner as the traverse surveyor would have done, and then triangulate the interior to suit the requirements of the detail survey. The stations thus formed either on the boundary of the village or in the interior should be marked by stout pegs 3' or 4' in length and covering their feet with earth, a foot or so high, and nicking a V-shaped trench round them. By doing so it would be easy for the traverse surveyor to find them out and to pick them up during the traverse survey. As soon as the survey of a village is completed by the field surveyor in this way, a trace of the sheet belonging to it should be transmitted to the officer in charge of the Traverse camp through the officer in charge of the Cadastral camp.

All the station marks fixed on the boundary to be taken up by the traverse surveyor should be shown on the trace by the mark Δ , and to be connected with each other by firm blue lines, the other triangulation station in the interior are to be marked by $\odot$ and connected by dotted lines to enable the traverse surveyors to pick them up easily and to connect the field work properly.

Where the triangulation system has been adopted for detached pieces of work situated away from the main-circuits, they may be connected with the main work by means of open traverses starting from stations on the main-circuit and ending on to stations of the detached survey.

577. The Following Points should be observed in making Village Traverses:—(a) The object of the traverse survey is to prepare skeleton maps of villages for the purpose of field survey. The surveyor should follow as closely as possible the real boundaries of villages in making the traverse survey of boundaries which he will find demarcated. If they are not demarcated the surveyor who is deputed to fix survey stations on the boundaries, previous to traverse survey, or to make traverse survey of the boundary, is to mark stations on the boundaries, according to possession acting on information obtained from the zamindar or his local agents, and having regard to the boundary according to Revenue Survey. (b) He will mark stations on the actual boundaries, and not within in a cultivated field, and join these by right lines which when plotted will form the skeleton map of the village. (c) These lines should be so run that offsets from them to the bends in the boundaries will not, as a rule, exceed $2\frac{1}{2}$ chains. (d) The traverse lines should not exceed 40 chains in length. (e) All large village sites should have sub-traverse lines running around and through them at distances not exceeding 40 chains apart, and as near as possible in a north and south or east and west directions. The stations should be made only on the real boundaries. (f) Jheels, jungles, plantations, and blocks of cultivations should be encircled by sub-traverse lines. Sub-traverses must run: (1) along edges of jungle blocks, at a distance of about 1 to $1\frac{1}{2}$ chains from the commencement of the jungle; (2) along the bases of hills or ranges of hills. It is not sufficient to erect traverse stations on the crests or sloping sides of such ranges. (3) Where jungle blocks adjoin hills no separate sub-traverses are necessary. (4) Through, round or near isolated patches of cultivation, situated in jungle or other difficult tracts. (5) All large rivers and unfordable streams of over 3 chains in width require stations on both banks. The traverse lines should be laid out as near as practicable to the banks of rivers within 1 or 2 chains of them. When a range of hills forms the boundary between two villages, the line should be run along the ridge which separates the villages; another line should be run along the foot of the hills bordering on cultivation. (g) In large villages if the *tolas* form compact areas their boundaries are to be demarcated and traversed for the convenience of the cadastral survey and record writing. (h) While traversing, any isolated objects of importance, such as G. T. Stations, temples and large trees, etc., should be connected by intersections from two or more stations or sub-traverses and plotted on the field sheets.

578. Methods of Traverse Survey.—When the survey is non-professional the boundary or traverse survey may be made either

with the plane table or with the prismatic compass. Stations may also be fixed on the boundary by intersections with the alidade from the ends of a base-line carefully measured within the village and from subsidiary points similarly fixed therefrom (q. v., p. 296, Fig. 275). The plane table is, however, the best instrument for the work of the boundary survey of small areas. Work should commence, if possible, from a tri-junction point on the north-west of the village or from some point on the boundary marked by an wooden peg, and should proceed in a south-west direction round the village so as to keep it on the left-hand. Two other points, either on the actual boundary, or as close to it as possible, will be marked by flags on either side of the starting point, and at a convenient distance not exceeding 20 to 30 chains from it. The surveyor then adjusts the plane table, having an imperial paper with one inch co-ordinate squares ruled in faint blue lines pasted on it, over the starting point to the magnetic north by the trough compass so as to have the ruled lines parallel with the north line. He now selects a point towards the north-west corner of the ruled sheet in such a position as to admit of the skeleton plot of the village coming within the sheet and aligns the alidade on the back (north-east) flag, and draws a ray along it. The alidade will then be moved by him to intersect the fore-flag (i.e., the south-west flag) and another ray will be drawn. This ray will give the angle at the starting point. He then measures the distances from this point to the back and fore flags, and thus fixes the positions of the stations, previously marked on the ground, on the paper. The plane table is now moved on to the fore-station and the back-flag to the next station ahead. The plane table is fixed at the new station by aligning the alidade on the ray to the first station, and then moved round till it is aligned with a flag placed at the next station. The plane table is then clamped and the ray drawn and distance measured as before. The work thus proceeds round the village. After fixing the magnetic north at the first station it is not absolutely necessary to use the magnetic needle again, as the measured line and back-flag will give the proper direction, but it is desirable to do so at each station as a check. In preparing the skeleton plot offsets to the actual boundary of the village need not be taken but the traverse lines should keep as near as possible to the boundary. No points should be more than 20 or 30 chains apart, and care should be taken to mark all the points, whence observations are taken, with such marks as can be readily found when the cadastral survey is being made.

579. Checking of Traverse Survey.—When the surveyor on returning to the starting point finds that he is unable to close up his work correctly he should go over the work again but in the opposite direction beginning from the station of origin and following the lines round until the error is eliminated. When there is a closing error it should be always shown on the plot (q. v., p. 236).

580. Shifting the Paper.—It may happen that the whole of a village can not be plotted in one sheet. In that case the surveyor will plot on one sheet so far as he can. He will then measure a straight

line across the village, being careful to keep actually to the straight line on the ground, which is drawn on the paper, with the use of flags and the alidade, and making marks as he goes along on the ground, and on boundaries of fields to show the cadastral surveyor where the village is divided on the plot. Such divisions should if possible be made along one of the ruled lines of the sheet. If it is not possible to clear a straight line on the ground across the village, the traverse stations may be placed zigzag on or near the margin of the sheets.

584. Section Papers.—Imperial sized papers will be supplied, divided into squares on the 16-inch-to-a-mile scale containing $2\frac{1}{2}$ acres to the square inch. This paper should be stretched over the plane table and pasted on its edges with a strip of cloth. The paper, on drying, will then stretch smoothly over the surface.

582. Surveying the Boundary with the Prismatic Compass.—This method (q.v., p. 187, Art. 334) should seldom be used, as it is very liable to error it being almost impossible to read the angles with sufficient accuracy or to plot them on the paper.

583. Plotting of Traverse Stations.—The 2nd index plots will give the surveyor definite idea as to the size of the paper to be taken for plotting the stations on the 16" sheets of a village so that ample room may be left for the delineation of details. The symbols used in denoting positions of the different kinds of stations in the field-book should be used on the 16" plot. These symbols, together with the station letters and numbers and the traverse lines, should be entered on the sheets in light blue, while the heading of the sheets, names of villages and references should be in black before the sheets are transmitted from the Traverse to the Cadastral camp.

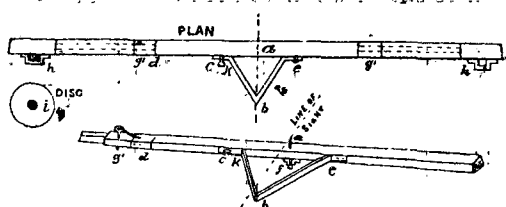
584. Bar-Subtense Survey.*—To meet the difficulty of carrying on accurate chain measurements over hilly and broken ground, and in taking traverse lines across rivers, an easy method has been introduced of determining the distances in such localities by computation, after measuring the angle subtended by a bar of known length placed at right angles to the ray whose length is required. At the meeting of the British Association at Cardiff, 1891, the late Colonel H. C. B. Tanner, I. S. C., read a paper on Bar-Subtense Survey, from which the following is extracted:—

The Bar-Subtense* method has none of the drawbacks attending the use of the chain or of micrometer instruments; it is more accurate than either, and is effected by means of an ordinary theodolite, together with bars of varying lengths, according to the nature of the work to be performed. The system is readily acquired by surveyors, after a week's instruction, and in their hands, over the roughest possible mountain tracts, is capable of furnishing horizontal measurements up to a maximum of some two miles with an error of about three feet per mile, and up to a distance of three miles with a somewhat greater error; and an adaptation of the process is capable of yielding reconnaissance traverses and approximate trigonometrical work far more accurately and expeditiously than can be looked for by any other means, unless a regular trigonometrical survey be resorted to.

* By permission of the Royal Geographical Society from "Hints to Travellers."

The theodolite used should be six-inch or larger; it should be simple in construction, and furnished with one vertical and one horizontal wire. The bars may be of varying lengths. In the Himalayas the 20-foot bar was in general use, but ten and two-foot bars were found convenient for some purposes. A 20-foot bar with 12-inch circular discs (Figs. 290-291) is capable of furnishing under favourable conditions of light and atmosphere and by a skilled observer, a $\frac{1}{3}$ -mile distance with an error of six feet. A ten-foot bar with eight-inch discs will give good results up to a mile and a half, and a two-foot Gunter's scale blackened at the ends with two-inch paper discs pasted on two feet apart, and properly mounted, will give distances up to 20 chains.

585. Description of Subtense Bar.—The bar is generally made 10' long (or 20' if for greater distances), and has at each end a diamond-shaped board, about $1\frac{1}{2}$ feet along one diagonal, and 1 foot along the other, facing the observer. The longer diagonals are vertical and so adjusted as to be exactly 10' apart. Fig. 290 represents a rod devised by Lt. Col. St. G. C. Gore, R. E., formerly Surveyor-General of India. The bar is made of hard wood in 3 sections. The central section is square in cross-section, $1\frac{3}{4}$ " $\times$ $1\frac{3}{4}$ "



Figs. 290-291.—Subtense Bar.

with iron sockets 6" long, g, g , at each end into which the outer portions of the bar fit, being pinned into place by the pins g' (Fig. 291) which is a perspective view of the central portion. The outer ends of the bar carry iron sockets, h, h , which have the recesses in them accurately machined out. Into these socket the discs, i , fit by means of carefully fitted hooks on their backs. The discs are of wood 10" in diameter, painted white with a black ring. Black cloth covers are also carried to fit tightly over the discs, in case of working with a light background. In the centre of the bar is a brass socket plate, by means of which the bar can be attached to a tripod. The sighting arrangement consists of a light iron frame, hinged at e, b , and k . The pin of the hinge, b , carries a point on the top, and a similar metal point is fixed at a in the centre of the bar. The end of the frame, e , is screwed to the bar, and the other end is fixed by a thumb-screw, c , in such a position that the line joining b to a is at right angles to the line joining the discs. For travelling, the thumb-screw, c , is unscrewed and the frame is closed up against the bar, in which position the thumbscrew screws into the hole, d , in a metal plate affixed to the bar. The bar is fixed in position by an assistant, looking along the sights, a, b , and laying them on to the theodolite.

Another form of bar and more easily made though not calculated to give such accurate results is the following:—The two boards at the ends are 1 foot square, painted white, with a black

cross on each. They are fastened on to a bamboo in such a manner that the centres of the crosses shall be a known distance apart.

586. Method of Observing with the Subtense Bar.—The *modus operandi* of a traverse surveyor must now be explained in detail. The forward signalman sets up the horizontal bar over the station mark, and then, by means of the folding sight-vane, directs the bar at right angles to the observer, who then intersects and records the reading of the back signal. Then, leaving the lower clamp fast, he releases the upper plate and intersects the right-hand disc of bar, the reading of which he records. Now release lower clamp (leaving upper clamp fast) and intersect left-hand disc. Again release upper plate and intersect right-hand disc, and for a second time the left-hand disc with lower plate, and so on, continuing the repetition until, say, 10 times, and then read and record the right-hand disc. In this operation the graduated limb of the theodolite will have moved over an arc 10 times greater than that subtended by the bar. Now repeat again, ten or twenty times, and record readings of right-hand disc, and then, having taken a vertical angle to bar, and leaving lower plate fast, intersect, and record the reading of back signal with upper tangent-screw, and such a record here shown will have been obtained :—

Signals observed.		Reading of A Vernier.	Differences.	Subtended angles.		Error of 20ft. bar—0.2 of an inch.
Back station...	A	126 14 20				
Right-hand disc.	B	206 26 30				
" " "	B ₂	209 48 30	3 22 00	d	20 12	
" " "	B ₃	213 10 15	3 21 45	d ₂	20 10.5	
" " "	B ₄	216 32 5	3 21 50	d ₃	20 11	
Back station ...	A ₂	136 19 55		M 20 11.2 = x		
(30 repetitions)...	...	10 5 35	From Table, chains	...	...	... 51 60*
		10 5 35	Correction†	...	...	... — 4
Subtended angle	x	20 11.2	Corrected distance	...	...	... 51 56

$$\text{Traverse angle :—B—A } (= B_4 - A_2) \quad 80^\circ \quad 12' \quad 10''$$

$$\frac{1}{2}x \quad - \quad 10 \quad 5$$

$$\left. \begin{array}{l} \text{Angle at station 1, between back station and} \\ \text{centre of bar at No. 2} \end{array} \right\} \quad 80 \quad 2 \quad 5$$

* For actual use the distances have been tabulated between 2 and 180 chains.

$$\begin{array}{rcl} \dagger \text{ Bar} = 20 \text{ feet} = 30.31 \text{ kls.} & \log & 1.48144 \\ 20' 11.9'' & \text{cosec.} & 2.23122 \end{array}$$

$$51.60 \log \quad 3.71266$$

A 10-foot bar with an error of 0·2 of an inch would give—

Chains	25·80
Correction	- 08

Corrected distance, ch. ... 25·72

A 2-foot bar with an error of 0·02 of an inch would give—

Chains	5·16
Correction	- 1

Corrected distance, ch. ... 5·15

I wish to draw attention to the complete system of checks on the observations furnished by the above record. In the first place, there are two values of the azimuthal or traverse angle $B-A$ and B_4-A_2 , both of which should nearly correspond, and show only trifling differences. The subtended angle, or x , which is D divided by the number of repetitions, should correspond very closely with d_1 , d_2 , d_3 , and, as a check on the arithmetic, it should agree exactly with the mean of d_1 , d_2 , d_3 . These values are taken out during the progress of the observations, and should one of them show even a small discrepancy, the work must be condemned and done *de novo*. Again, A_3-A_1 and B_4-B must agree very closely. The checks are such that, by examining his record, the observer can make certain before proceeding to his next station that he has obtained the correct distance. Up to a mile he can detect any error made by the signalman in placing the bar at right angles, for it is only when exactly set that the black lozenge at the end of the sight-vane of the bar appears to him in the middle of the white patch on the bar itself. The signalman soon learns to place the bar sufficiently near the horizontal for practical purposes. An error of 2° of dislevelment, which would seldom occur in practice, would only produce an error of about 3" in a mile. The manner in which this method may be made applicable to other classes of survey is shown in Col. Tanner's paper, published in "Proceedings of the Royal Geographical Society," Nov. 1891.

586. Computation and Distance.—Having obtained the subtended angle, the distance corresponding may be either taken from a table previously prepared, or computed by a table of nat. cotangents. The true distance in feet is obtained from the equation $d = \frac{1}{2} l \cot. \frac{s}{2}$, when l is the length of the bar in feet, and s is the subtended angle; but the formula $d = l \cot. s$ is quite accurate enough and more convenient. The approximate formula $d = \frac{l}{s \sin. 1''}$ is practically accurate, as $\frac{l}{\sin. 1''}$ may be taken as a constant, and then division by s in *seconds* is all that is necessary to give the distance without the aid of logarithms. Thus if $l = 10 \frac{1''}{\sin. 1''} = 2,062,649$, and the subtended angle is $3'$, then $d = \frac{2062649}{3 \times 60} = 11,459$ feet. Printed table of distances corresponding to angles measured on subtense bars of 10 and 20 feet can be obtained from the Surveyor General's Office, Calcutta.

587. Means of Aligning the Bar.—The principle of the *bar-*

subtense method requires that the bar should be held at right angles to the line of sight of the theodolite, and this is sometimes done by a small level and by means of sights made much after the fashion of gunsights. A short iron bar, about 1' long and

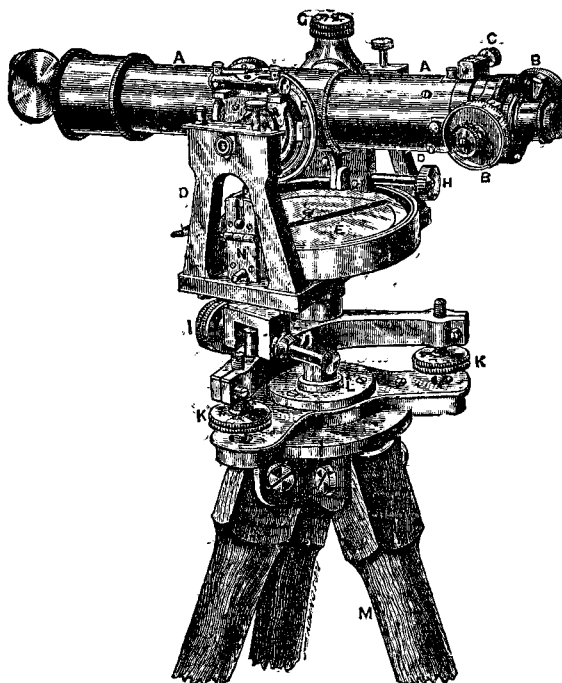


Fig. 292. —Tacheometer.

about $\frac{1}{10}$ inch thick carrying a V-sight at one end, and a bead at the other, is firmly screwed to the middle of the bar carrying the diamond-shaped boards, and so adjusted once for all that its line of sight is at right angles to the bar. The bar is capable of adjustment on an ordinary plane table stand and is revolved until the sight-line of the iron bar intersects the observer's theodolite at the other end of the ray. The assistant then levels it and holds it in this position until he receives a preconcerted signal

of some kind from the observer to indicate that the observation is completed. The bar-man, should be well-drilled in his work, as there is no check on him, unless the ray be short enough to enable the observer to see the sights himself, when looking through the telescope; this is generally possible up to 900 or 1200 feet.

588. A Tacheometer* is an instrument for measuring small angles. It consists of a telescope *A*, (Fig. 292) fitted with a pair of micrometers, *B, B*, which are used for measuring either vertical or horizontal angles, as they can be turned through an angle of 90° , and fixed in that position by the screw *C*. The telescope is mounted on standards, *DD*, over a prismatic compass, *E*, and is furnished with a small circle, *F*, for taking vertical angles, which can be read to minutes. *G* is the screw by which it is clamped in altitude; *H* is the vertical slow-motion screw. The instrument is fitted with a screw (not shown in the plate) for clamping it horizontally; and *I* is the horizontal slow-motion screw. The bearing of any object is read through the prism, *N*. There are 3 levelling screws, *K*, which fit into a tribrach *L*, that screws on a tripod, *M*. The instrument is

levelled by means of the screws K , and a level attached to one of the standards (not shown in the plate).

There is a disc of glass visible in the field of view, divided in such a manner that each division equals one revolution of the micrometer head, and each micrometer is divided into 100 parts. These divisions are both vertical and horizontal, to suit the corresponding positions in which the micrometers are used. The measurement of the distances by means of the tacheometer is based on the solution of a $\triangle$. In Fig. 293, suppose the instrument to be at A ,

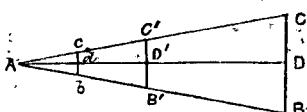


Fig. 293.—Use of the Tacheometer.

and a staff of known length to be represented by BC ; then if the $\angle BAC$ is measured, and the length of the staff, BC , is known, the distance AD can be easily computed. In order, however, to measure the $\angle BAC$, the value of the micrometer divisions must be determined in the following manner:—Set the telescope to *solar focus*, and carefully measure the distance, AD , from the instrument to a staff of known length; measure the $\angle BAC$ subtended by the staff with each micrometer, carefully noting the number of divisions and decimals of a division used with each. Divide the length of the rod by the distance, AD , between the instrument and the rod, and multiply this by the cosecant of $1'' = 206265$, and the result will be the value of the $\angle BAC$ in *seconds* as measured by that micrometer. Now divide BAC in *seconds* by the number of micrometer divisions used in taking it, and the result will be the value of each division of the micrometer in seconds and decimals of a second. As the value of the divisions will not be exactly the same in both micrometers their values must be separately determined. *It should be borne in mind that the values of the micrometer divisions must be determined at solar focus and the instrument used subsequently at solar focus, otherwise wrong values will be given for the micrometer divisions.*

Example:—Number of divisions used (Right micrometer), 1157.1; length of rod, 12 feet; distance between rod and instrument, 983.2 feet.

Log 12 = 1.079181	1157.1 2517.460 (2.17	} Value of each division.
Long distance. 983.2 = 2.992642	23142	
	20326	
	11571	
	87550	
Cosecant of $1'' = 206265$ Log = 5.314425	80997	
The whole $\angle = 2517.46 = \text{Log } 3.400964$		

The same process would have to be gone through to find the value of a division of the left micrometer. In combination with this instrument a rod of known length is generally used. For the description of the rod (see pp. 335-336).

Any theodolite can be used as a tacheometer, by having hairs in the diaphragm (as s , s , in fig. 47; page 52) fixed at such a distance apart as to read one foot on a staff when it is 100' distant from the instrument; 2' when the staff is 200' distant, and so on, and a theo-

solite fitted in this way will always give a proportion of 1 to 100 between the readings on the *graduated* staff and the distance. As the power of the telescope is usually small, the figures and marks on the graduated staff can only be read at a comparatively short distance. The following precautions must be taken, or no accurate results can be obtained. The fixed hairs must be adjusted to read in the proportion of 1 to 100 or, what is the same thing, the staff must be marked to read one foot, when it is 100' distant from a certain point. It is the determination of where this point is that is absolutely necessary, and the place from which to measure the distance is arrived at in the following manner:—Mark the ground immediately under the centre of the instrument by dropping the plummet from the centre of the tripod, in the usual manner, and from this measure a distance, in the direction the telescope points, equal to the focal length of the object-glass, added to the distance from the object-glass to the vertical centre of the instrument. Thus, if the focal length of the object-glass was 12", and the distance of the object-glass from the vertical centre of the instrument was 7", then the position of the point from which to commence the measurement of the 100' would be 19" from the place where the plummet let fall from the centre of the tripod touched the ground. The telescope must always be set to *solar focus*, otherwise no accurate results can be obtained. To all distance measured in this manner, a constant, equal to the focal length of the object-glass + the distance of the object-glass from the vertical centre of the instrument, must be added, otherwise there will be an increasing error in each distance that is measured.

589. Surveying with the Tacheometer.—The method of surveying with such a tacheometer as that shown (fig. 292), is, as regards fixing positions of distant objects, the same as with the prismatic compass. This instrument has, however, this advantage over the prismatic compass, that distant objects are seen much more distinctly through the telescope, and the bearings can therefore be more accurately taken than when the ordinary sight-vanes of the prismatic compass are used. In addition to which, the compass is larger than the prismatic compass. The principal advantage of the tacheometer however, will be found when it is employed for fixing positions within comparatively short distances. This is done by sending an assistant to the spot it is desired to fix with a staff (Figs. 290-291), and with the micrometers, measuring the angle it subtends when held (either horizontally or perpendicularly) at right angles to the line of sight; at the same time taking the compass reading through the prism. With the angle measured by the micrometers, if a ten-foot staff has been used, knowing the value of the micrometer divisions, the distance of the object can be at once obtained from Table III (see part II of this work). With the distance so found, and the bearing which has been taken, the position of the object can be at once laid down on the survey by setting out the bearing from the point of observation, and then measuring the distance, taken from the scale of the map.

* By permission of the Royal Geographical Society, London, from "Hints to Travellers."

With any other length of staff than 10 feet, the above table can not be used without calculation, and the distance of the object will have to be computed. It is usual, when observing the angle subtended by the staff, to measure half of it with each micrometer, the sum of which measures will, of course, be the whole angle subtended. The distance from the staff is computed in the following manner:—Multiply the total number of divisions used in each micrometer by the value of a single division of that micrometer, add the results together, and this will be the value of the angle in *seconds*. Divide the length of the staff, in feet, by the angle in seconds and multiply the result by the cosecant of $1'' = 206265$. This will give the distance between the instrument and the staff, in feet.

Example:—Length of staff, 12 feet; divisions used, Left Micrometer 581·9, value of each division, 2·31; Right Micrometer, 575·2, value of each division, 2·04.

Left Micrometer	Right Micrometer	Calculation:—
581·9	575·2	Log. 12 feet = 1 079181
2·31	2·04	Log. 2517·6 = 3 400986
5819	23008	
17457	11504	Cosec. $1'' =$ 3 678195
11638	1173 408	206265 Log. = 5 314425
1344·189	1344·189	
	The angle	Log. distance
	in seconds = 2517·597	in feet, 983 2 = 2 992620

The rod, though convenient, is not, however, absolutely necessary, as distances can be measured by this class of tacheometer without it, by making an assistant set up two staves at a carefully-measured distance from one another, and at right angles to the line of sight. The angle subtended by these staves is measured with the micrometers, and the distance computed in the manner already shown.

590. A theodolite with fixed hairs, (pp. 52 and 55) may often be used for measuring distances approximately when it is impossible to read the markings on a graduated staff, in the following manner:—An assistant should be sent to the object, the distance of which is required, and directed to place a staff on the ground. The surveyor must then cover the staff with one of the fixed hairs in the instrument, after which the assistant must move, very slowly, in a line at right angles to the line of sight, until he is covered by the second fixed hair, when he might be stopped by some pre-arranged signal, and place another staff there. He must then carefully measure the distance between these two staves, which distance multiplied by the ratio between the value of the hairs, which is generally 1 in 100, will be the distance of a point, midway between the two staves, set up by the assistant, and the observer. Thus, if the measured distance between the staves was 10 yards, the distance from the instrument would be $10 \times 100 = 1000$ yards. In Route Surveying of a rapid nature the traverse may be executed with these theodolites. The traverse may be computed with the aid of tables of latitude and departure and then plotted.

591: Method of Triangulating Across Water or any Similar Obstacle.—In traversing across rivers and creeks, it often

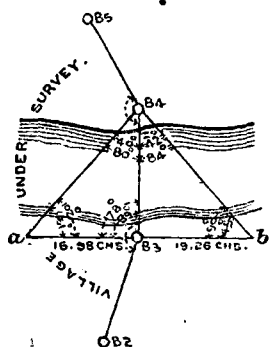


Fig. 294.—Triangulating Across Water.

be taken (q. v., Instructions, p. 225, Art. 406) as a check against any error in reading or observing; similarly he will observe the $\angle B_4 B_3 b$, then moving his theodolite to stations a and b respectively, will observe the $\angle B_4 a B_3$ and $\angle B_3 b B_4$, and also their doubles. He has now all the necessary data for computing the distance, $B_3 B_4$, for which he will obtain two values, one from each of the Δ s. The $\angle s a B_4 B_3$, $b B_4 B_3$ may also be observed as checks.

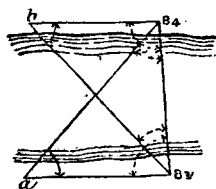


Fig. 295.—Triangu-
lating Across Water.

In $\Delta a B_3 B_4$.—

$$\text{Log base } a B_3 = 16.98 \text{ chains}$$

L. cosec. $B4$ ($\angle$ opposite base) $40^{\circ}00'00''$

L. sin. α ($\angle$ opposite required side) $51^{\circ}00'00''$

Log ($B_2 B_4$) = 20.53 chains)

In ΔbB_1B_2 —

Log base $bB_2 = 19.26$ chains

L. cosec. B_4 ($\angle$ opposite base) $42^{\circ}00'00''$

L. sin. b ($\angle$ opposite required side) $45^{\circ}00'00''$

Log ($B_1 B_2$) = 20.35 chains)

The mean of the results $\left(\frac{20.53 + 20.35}{2} = 20.44 \text{ chains} \right)$ may be taken as the width of the river or length of B_3B_4 .

The surveyor should draw a rough diagram in his field-book, entering each observed angle and measured distance in its proper

place on the diagram in ink. The calculations are not to be performed by the surveyor in the field but would be done in the office.

592. Method of Numbering Stations.—Every main-circuit surveyor and every sub or village-circuit surveyor is given a distinctive letter as *A*, *B*, *C*, etc., by which his stations on the ground can be recognised. Thus surveyor *A* would number all his stations *A*₁, *A*₂, *A*₃, and so on. The same numbers should also be booked in his field-book. When the stations of another surveyor are met with, his numbers should always be used, no new numbers being necessary. The main and sub-circuit stations should be expressed by *capital letters*, as *A*; and village-circuit stations by *small letters*, as *a*. To avoid confusion the same numbering as adopted by the surveyors should be used in the Traverse Computation Form, and the 16" and 24" plots.

593. Specimen Traverses of Main-Circuits.—Fig. 296 shows the sketch of a traverse survey of main-circuits. *A*₁ is the starting-point of the main-circuit *A*₁, *A*₂, *A*₃,...*A*₂₇...*A*₁. The interior blocks are sub-circuits, while traverse lines have been run

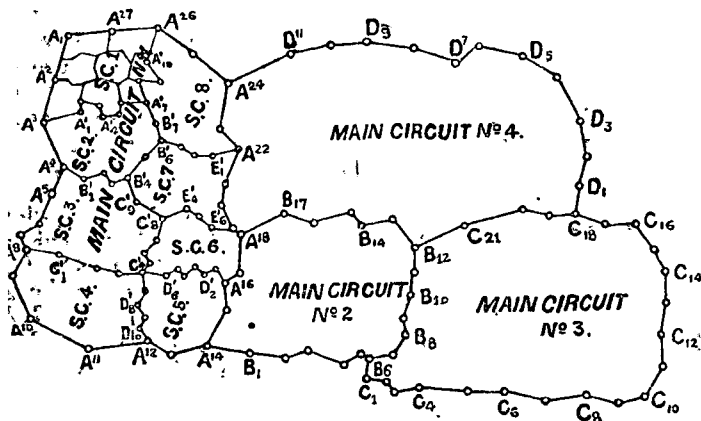


Fig. 296.—Specimen Traverses of Main-Circuit.

through sub-circuits to enclose village-circuits as shown within sub-circuit No. 1. The Main-circuit is surveyed by surveyor *A*; sub-circuit No. 1 has been surveyed by *A*₁, No. 2 by *B*₁, No. 3 by *C*₁, Nos. 4 and 5 by *D*₁, No. 7 by *E*₁. The figure explains the system of numbering the stations and the direction the traverses have to be run. A sub or village-circuit surveyor should on no account re-measure the lines or re-observe the angles of a common boundary already measured by another sub-circuit or village or main-circuit surveyor except at starting or closing points of his traverse, and this should be always done with *single* angular and linear measurements only.

594. Elevations to be ascertained.—Heights of plateaux, ridges in ranges of hills, principal peaks and ranges, debouchments

of rivers, from hills, junctions of rivers, and inundation levels, should be determined by surveyors by observing vertical angles to them.

The most correct system of determining a series of heights is by regular spirit-levelling, next to which is triangulation which should be adopted when the ground is suitable for triangulation.

By the aid of a diagram called the Height Indicator available at the Surveyor General's office, Calcutta, differences of heights corresponding to various distances and subtended angles may be ascertained with tolerable accuracy.

595. Tangent Scale Clinometer, Survey of India Pattern*—Fig. 297 shows the instrument for taking elevations

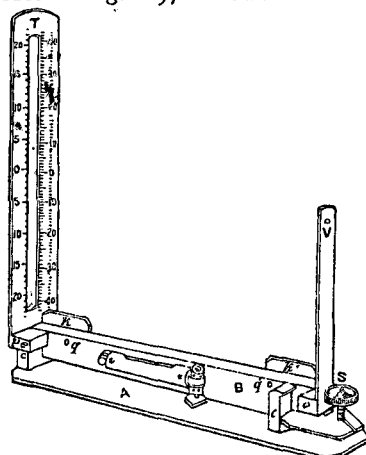


Fig. 297.—Clinometer, Survey of India Pattern.

and depressions from 0 to 45° . *A* is the base-plate, with 3 ivory buttons projecting below for the instrument to rest on and be moved over the paper of the plane table without soiling it. *B* is a bar $8''$ long, which carries the tangent-and-slope vane, *T*, about $7''$ high, and the sight-vane, *V*, about $3\frac{1}{2}''$, and which is levelled by milled-head screw, *S*; it is attached at one end to the base-plate by the pin *p* on which it works, and it is secured from shake at the other end by two clips, one of which *c* is shown. The

eye-hole in the sight-vane is at the same height above the upper surface of the levelling bar as the zero line of the tangent-and-slope-scale vane. Two struts, *k*, *k'*, turning upon pins, *q*, *q'*, as shown, are attached to maintain the perpendicularity of the two vanes. The tangent-and-slope-scales are divided upwards for elevations and downwards for depressions, degrees being shown on the left and tangents on the right with a central vertical slit between them to an arc of $8''$.

596. Adjustment of Clinometer.—A level is attached to the levelling bar and is provided with adjusting-screws; it can be adjusted by comparison with vertical angles taken with a theodolite; *i.e.*, when the line of sight from the eye-hole to any object indicates the angle shown by a theodolite reading of the same object, the bubble should be in the centre of the level. The levels should be adjusted before being sent out, and the adjusting-screws of the levels should not be touched. Before observing, the struts, *k*, *k'*,

* By permission of the Surveyor-General of India from "Descriptive Catalogue of Instruments."

should invariably be turned up to avoid error due to want of perpendicularity, and care should be taken to turn them back again before attempting to shut up the instrument, otherwise the vanes are liable to be bent.

Another Method of Adjustment.—On topographical work with fixed stations and heights it is well to set up the instrument on a plane table over a known station with known height. To this known height is added the difference of height between the mark *in situ*, and the height of the eye-hole, when the height of the eye-hole is obtained. From the plot on the table one or more other points are selected with given heights, and the horizontal length is known, so also the correction for curvature and refraction. Since the difference of height between A and B = nat. tangent of the angle between A and B $\times$ horizontal distance between A and B $\pm$ curvature and refraction (in this case since the object at B is bisected and the height of A deduced from B, curvature and refraction is minus), thus the nat. tangent of the angle is known. With the milled-head S make the instrument read this nat. tangent, and if the bubble is not in the centre of its run, then bring it to the centre by the bubble-nuts.

Ex: Heights of station A = 1560; distance A to B = 10,000 ft.

" " B = 1805; " A " C = 8,000 "

" " C = 1540; Height of eye-hole = 4 ft.

$\therefore$ Height of instrument at A = 1564. By the computation the nat-tangent of B = $+\frac{1805-1564}{10000} = .034$; of C = $-.003$; and if the instrument does not give these values, the bubble is not in the centre, raise or lower eye-hole to obtain these values and bring the bubble to the centre.

597. The Use and Care of the Tangent-Scale-Clinometer.—Trigonometrical heights should be supplemented by observations being made at plane table stations during a topographical survey with the tangent-scale-clinometer which will give a number of differential heights with fair accuracy. Objects should not be observed with the clinometer at a greater distance than 2 miles. When employing it at distances exceeding 2 miles, it becomes necessary to correct the resulting heights for curvature and refraction. In using it is placed on the plane table, and usually the right edge of the central vertical slit, that for natural tangents, is brought on the object so that when the observer looks through the eye-hole the object is seen to be bisected at or near a certain division of the scale. The bubble is now brought to the centre and a reading is taken, say it is .03. This gives the natural tangent of the angle which multiplied by the horizontal distance taken off the plotted map on a scale and when corrected for curvature and refraction, will give the difference of height between the instrument and the object over distances up to 5 miles, mean readings to within 5 feet is available. The card board scale used in taking off distances has also

the curvature and refraction correction ruled at intervals (co-efficient taken generally as '07).

In using the clinometer should be lifted by its base, *never* by its vanes. The vanes should be perpendicular and as a check they should be 8" apart. A clinometer should have a fairly long sensitive bubble, and an eye-hole just large enough to admit of no parallax. With a large eye-hole the object cannot be properly located. In the example above the value '034 is read as '03 line and an approximation of '004 above it. With practice an observer will get very nearer to the true reading but the eye-hole can be raised by the aid of a micrometer screw which renders such an approximation more exact.

The Table IV in Part II of this work will give tangents of angles for each 30" from 30° to 19°, and is intended to aid in the calculation of heights on the spot by measuring the distance from the maps. *as at*

(2) Traverse Section : Recess Season.

598. - The Work to be executed during the recess season comprises the traverse computations remaining over from the field season, azimuth computations, computation of rectangular co-ordinates, latitudes and longitudes, completion of traverse tables, plotting of balance of 16" sheets, preparing a 2" plot of the traverse, writing up tables of co-ordinates and plotting the graticule and station points on the 2" sheets.

599. Addition of Traverse Columns.—In the main or sub-circuit Traverse Tables; the columns of the distances on the meridian and perpendicular are not to be added up page by page, but separate totals shall be made at the tri-junctions of each district and main-circuit azimuth stations as also at the stations with which the G. T. S. Stations have been connected; and an abstract of these totals, with a grand total, is to be shown at the end of the Table (See sub-circuit-table pp. 327-29).

600. Azimuths as Observed and as Corrected for Convergency to be recorded in Traverse Tables.—Booking of the following should be made in red ink on Main-Circuit Traverse Tables alongside each station where an azimuth has been observed—
(1) The observed azimuth. (2) The corrections for convergency.
(3) The reduced bearing or inclination to the initial meridian.

601. Point of Origin.—If a trigonometrical survey has preceded the traverse and if a large tract of country is to be surveyed, the middle points of each geographical degree should be adopted as points of origin, but ordinarily the origin, to be adopted for the survey of a district or group of districts should be centrally situated. The point should be the intersection of some convenient geographical lines of latitude and longitude running through the district, districts and not a Trigonometrical Survey Station. If no Trigonometrical Survey has preceded the Traverse Survey, the origin should be chosen in a prominent position with a view to its being adopted as a T. S. Station, and should be permanently marked by the com-

struction of a circular pillar over it. For future identification of this station, the true azimuths of a sufficient number of permanent surrounding objects should be determined.

602. Initial Meridian.—The meridian of the *Station of Origin* will be adopted as the initial meridian of the Survey; and all azimuth observations made at other stations must be reduced to it, by allowing for convergency, which being recorded in the *Traverse Tables* as elements for correction of the angular work.

603. Convergence of Meridians: Cheek Azimuths.—While the method employed in the Revenue Survey fall within the sphere of Plane Surveying, still to the extent of allowing for the convergence of meridians, it takes into account the curvature of the earth. As the meridians converge to the poles the bearing of a very long straight line is not the same throughout its length. If a traverse is carried over a large tract of country extending over several miles in length, it becomes imperative to check the accuracy of the angular work by azimuthal observations to stars or the sun, for the determination of the meridian at intervals of 25 to 30 stations along main-circuits and sub-circuits, every main or sub-circuit tri-junction station being selected, if possible, as an azimuth station. The meridian thus determined is the true azimuth at station of observation and convergency corrections (angles of inclination of the meridians of the stations of observation to the meridian of the station of origin) must be applied to the observed meridians at other stations to make them parallel to the meridian of origin, the object of applying the convergency correction to the observed bearing being simply to maintain the parallelism of meridians required by the system of rectangular co-ordinates.

The Observed Bearing is that which is given by astronomical observations. It expresses the direction of a line correctly and is adopted as the working bearing in traverse calculations.

The Reduced Bearing is the observed azimuth reduced to the meridian of the Origin of Survey, by the application of the correction for convergency, for the true bearings (observed azimuths) at other points east or west of the Origin of Survey cannot be introduced into traverse computations as they are observed at converged meridians. The observed bearings at other stations must therefore be divested of convergency to form reduced azimuths which are really true inclinations of the traverse lines to the meridian of origin.

The Deduced Bearing is obtained by deduction from reduced bearing by the continual addition of the observed angles recorded in the *Traverse Tables*.

If the observed angles of the traverse were correct, the deduced and reduced bearings would be exactly the same, but if the difference on comparison of *Deduced* with *Reduced* Azimuths be not greater than the allowed percentage of error of one minute in 5 angles, corrections are to be applied by studying the angles susceptible of being changed by the rejection or addition of the fewest number of minutes so that the deduction through the traverse tables may work into the *Reduced Azimuth*. Notice the difference between this *Reduced*

Azimuth and that of *Reduced bearing* in Col. 5 of Traverse Computation Form, p. 232.

Convergency Correction is to be *added to or subtracted from* the observed meridian as the station of observation is *west or East* of the meridian of origin.

604. Theory of the Convergence of Meridians.—In Fig. 298, let S_1N_1 and S_2N_2 show two meridians whose difference in

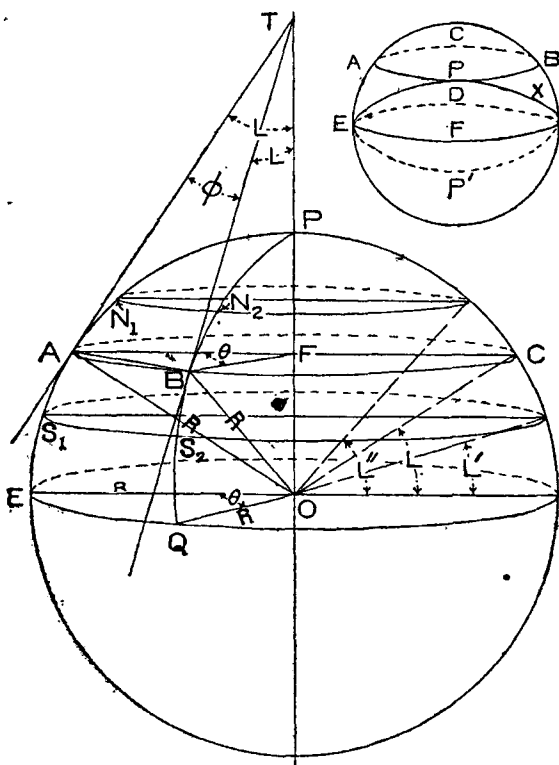
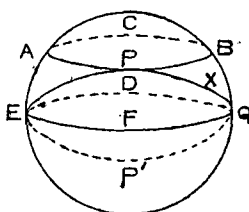


Fig. 298.—Theory of the Convergence of Meridians.

Fig. 299.



longitude in seconds between the extremities, A and B, of any line on the same parallel of latitude, L, as ABC, is θ , the length of each being equal to L. The earth is taken as a spheroid with centre O, and equator EQ. Also let AT and BT be the tangents of those meridians at A and B meeting in the earth's axis produced. The object of this article is, (I) the deduction of the value of the $\angle \phi$ between the two meridians, which is the convergency of the meridian of B; and,

(II) the determination of the difference in length of two east and west lines, S_1S_2 and N_1N_2 , lying between two meridians of equal length L. Let $R = OE = OQ = OA = OB$ represent the mean radius of the earth.

$$\begin{array}{l|l} \text{I (a). } \phi = 2 \sin^{-1} \frac{AB}{2AT} \dots (1) & AB = 2 AF \sin \frac{\theta}{2} \dots (3) \\ AT = R \cdot \cot. L \dots (2) & AF = R \cos. L \dots (4) \end{array}$$

Substituting (4) in (3), $AB = 2R \cos. L \sin. \frac{\theta}{2}$... (5)

Substituting (5) and (2) in (1), $\phi = 2 \sin^{-1} \frac{2R \cos. L \sin. \frac{\theta}{2}}{2R \cot L} =$
 $= 2 \sin^{-1} \sin. L \sin. \frac{\theta}{2}$... (6)

In cadastral surveying in the cases which require an application of equation (6) the values of ϕ and θ are small, hence, the arcs may be replaced by their sines, and we may write, $\sin. \phi = \phi \sin. 1''$; and $\sin. \theta = \theta \sin. 1''$ when ϕ and θ are in seconds. With these substitutions equation (6) becomes $\phi = \theta \sin. L$... (7)

When the points between which the convergency is required are in different latitudes, then $\phi = \frac{\theta \sin. \frac{1}{2} (L + L^1)}{\cos. \frac{1}{2} (L - L^1)}$. To apply equation

(7) L can be found from any map and θ taken from Tables. With these data ϕ can be found at once and the bearing corrected accordingly. For direct calculation the following method may be adopted:—
 I (b).—The angle of convergency is measured by the arc, EQ , on the equator divided by the earth's radius, or by the arc, AB , divided by its radius on the parallel of L . If the cone, formed by the tangents AT , BT , etc., be developed, it will be a sector whose radius $= AT = BT = CT$; and whose arc will equal the $\odot$ ce of the circle, ABC . Hence, Arc $AB =$ Circular measure of $\angle ATB \times AT$

$$= \text{circular measure of convergency} \times R \cot. L.$$

But for distances not exceeding 8 or 10 miles, the departure, D , of B from A may, with great approximation, be taken for the arc AB . Hence $D =$ circular measure of convergency $\times R \cot. L$.

$$\therefore \text{Circular Measure of } \phi = \frac{D}{R} \tan. L.$$

Let $N_m =$ number of minutes in angle of convergency of B 's meridian. Hence circular measure of $\phi = N_m \sin. 1'$.

$$\therefore N_m \sin. 1' = \frac{D}{R} \tan. L, \text{ or } N_m = D \tan. L \times \frac{\text{cosec. } 1'}{R}$$

$$= D \tan. L \times K \left(\text{since } \frac{\text{cosec. } 1'}{R} \text{ is constant} \right) \dots (9)$$

$$\therefore \log N = \log D + \log \tan. L + \log K.$$

If D is in miles, and R = Mean Radius of the Earth (Clarke's spheroid, 1866) is calculated as under :—

$$\text{Equatorial semi-axis in miles} = 3963.3 = \log 3.5980536$$

$$\text{Polar semi-axis in miles} = 3949.8 = \log 3.5965788$$

$$\text{Mean radius in miles} = 3958.78 = \log 3.5975620$$

$$\text{and } \log K = \log \frac{\text{cosec. } 1'}{3958.78} = \log \text{cosec } 1' - \log 3958.78 \\ = 3.536271 - 3.59756 = \bar{1}.93871.$$

$$\text{If } D \text{ is in feet, } \log K = \log \frac{\text{cosec. } 1'}{(3958.78 \times 5280)} = \bar{1}.93871 - 3.72263 \\ = \bar{4}.21608.$$

$$\text{If } D \text{ is in } 100' \text{ feet chains, } \log K = \log \frac{\text{cosec. } 1'}{(3958.78 \times 52.80)} \\ = \bar{1}.93871 - 1.72263 = \bar{2}.21608.$$

$$\text{If } D \text{ is in } 66 \text{ feet chains, } \log K = \log \frac{\text{cosec. } 1'}{(3958.78 \times 80)} \\ = \bar{1}.93871 - 1.90309 = 2.03562.$$

(II). Let L' be the latitude of S_1 and S_2 , and L'' that of N_1 and N_2 . The arc $S_1S_2 = R \cos. L' \times \theta$; and the arc $N_1N_2 = R \cos. L'' \times \theta$.

$$\text{And } S_1S_2 - N_1N_2 = c = R\theta (\cos. L' - \cos. L'') \\ = R\theta \left[-2 \sin. \frac{1}{2} (L' + L'') \sin. \frac{1}{2} (L' - L'') \right] \\ = -2 R \sin. \frac{1}{2} (L' + L'') \times \theta \sin. \frac{1}{2} (L' - L'') \quad \dots (10)$$

If A and B are respectively midway between S_1 and N_1 , and S_2 and N_2 , their latitude, L , is the mean latitude of the lines, S_1N_1 and S_2N_2 and equals $\frac{1}{2} (L' + L'')$. The extreme length of north and south lines that may be run without lateral correction is 24 miles, and within this limit, the expression

$$-2 R \sin. \frac{1}{2} (L' + L'') = 2 R \sin. \frac{1}{2} (L'' - L')$$

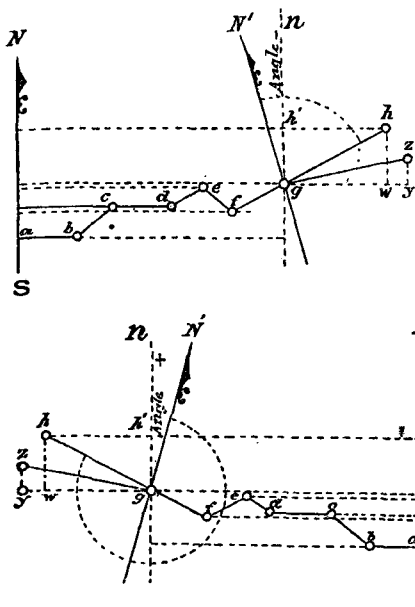
of equation (10) is practically equal to l .

$$\therefore c = l \theta \sin. L \quad \dots \quad \dots \quad \dots \quad \dots (11)$$

From equation (7), $\theta \sin. L = \phi = \sin. \phi$, for small values of ϕ occurring in practice. $\therefore c = l \sin. \phi \quad \dots \quad \dots \quad \dots (12)$

or the difference in length between two parallels included between two meridians is equal to the common length of the meridians multiplied into the sine of their mean angle of convergence. Equation (12) is used in checking the work of running north lines.

605. Illustration of the Rule for applying Correction for Convergency of Meridian. *—Let a (Figs. 300, 301) be the



Figs. 300-301.—Convergency of Meridian.

Draw gn parallel to aN , and the inclination of the meridian gN' to that of aN is measured by the angle $N'gn$. To obtain the true position of the point, h , by co-ordinates measured from the point a , we must ascertain the value of the departure $h^1h = gw$, and of the perpendicular $h^1g = hw$; these values added to the co-ordinates already obtained by traverse for g , will give those of h , and the distances so obtained, being plotted from the meridian aN will give the point, h , correctly on the map. Now, to obtain these co-ordinates we require to know the angle h^1gh , which is equal to the angle N^1gh , less the angle of convergency, N^1gn , which measures the inclination of the two meridians aN and gN' .

Let us, however, suppose that in lieu of using the above corrected angle, or *reduced* azimuth, which, if the angular work had been good, would be nearly the same as that *deduced* by traverse, we had used the *observed* azimuth of the lines gh , i.e., the angle N^1gh , and see what the result will be. At g lay off the angle $ngz =$ the *observed* azimuth N^1gh , and make $gz =$ the measured distance gh ; draw zy perpendicular to gw produced. If now we add the quantities zy ,

starting point of a survey where the azimuth of origin has been laid down, and to which it is necessary to refer all other points connected with the survey operations, in order to preserve their relative positions by the application of the traverse system of rectangular co-ordinates, drawn respectively parallel and perpendicular to the meridian of origin. Suppose the survey to proceed along the lines, $ab-fg$, either eastward or westward, to the station g , where owing to the distance traversed, and the number of angles observed between it and the first station, it becomes necessary to check the work by re-observation of azimuth; suppose the azimuth of the forward line, gh , to have been observed and to be represented by the angle N^1gh .

* By permission of the Surveyor General of India from "Hand-book of Professional Instructions" for the Revenue Branch, Survey of India Department.

gy, respectively, to the corresponding co-ordinates of the point *g*, already deduced by traverse, we find that by thus using the *observed* azimuth N^1gh , instead of the *reduced* azimuth $ng\hat{h}$, the point *h* is brought down to *z*, or very much too far to the South. The error thus shown to be engendered through the use of *observed* azimuths instead of *reduced ones*, is not the only one that may arise; for it is clear that the value of the angle N^1gn , which measures the convergency of the two meridians, if brought into the traverse computation, must affect some one or more of the bearings of the lines between the point, *g*, and the starting-point *a*, and will in like manner if neglected, throw all stations affected thereby too much to the south.

This may be practically shown by computing out a traverse with *observed* and also with *reduced* azimuths. A portion of one of the main-circuit traverses of the Punjab Survey (p. 353) has therefore been selected for the purpose, and a comparison of the results will show at a glance what serious discrepancies may be caused by the use of *observed* instead of *reduced* azimuths.

Note—In the first diagram (Fig. 300) the traverse lines lie to the East of the Meridian of origin and the $\angle N^1gn$ is subtractive from the $\angle N^1gh$, to obtain the true angle for Traverse computation, $ng\hat{h}$. Similarly, if the Traverse lines run to the West of the Meridian of origin the $\angle N^1gn$, becomes additive as shown in figures below.

606. To Find in Minutes the Convergency of two Meridians in any Latitude, and at any distance apart in Feet, Miles or Chains.

Example.—To find the convergency of two points 10 miles apart in latitude $17^{\circ}40'$.

For departure in Feet		For departure in 100' Chains.	
To the Const. log.	... 4.21608	To the Const. log.	... 2.21608
Add log tan. lat. $17^{\circ}40'$	... 9.50311	Add log. tan. lat. $17^{\circ}40'$	... 9.50311
And log. departure in 52800 feet	... 4.72263	And log. departure in 100' Chains = 52.80 ch.	... 1.72263
The sum is log.		The sum is log.	
Convergency = $2.766 = 2'46'' = 0.44182$		Convergency = $2.766 = 2'46'' = 0.44182$	
For departure in Miles.		For departure in 66' Chains.	
Constant log.	... 1.93871	Constant log.	... 2.03562
L. tan. lat. $17^{\circ}40'$	... 9.50311	L. tan. lat. $17^{\circ}40'$	... 9.50311
log. 10 (miles)	... 1.00000	log. 800 (chains).	... 2.90309
Log Convy. = 2.766 or $2'46'' = 0.44182$		Log Convy. 2.766 or $2'46'' = 0.44182$	

Table V in Part III of this work shows the convergency for $\frac{1}{2}$ mile or 1 chain of departure in latitudes 0° to 60° and given in seconds; if the departure is more than one mile or 1 chain, multiply it by the corresponding convergency given in the table, for that required as the convergency is proportional to the departure. If the latitude contains part of a degree, the convergency for it can be found by simple proportion. The length of departure is scaled off from the standard map. Thus, in the above example:—

Extract from M. C. 24 of the Sind Saugor Doab, Punjab Survey.

[illegible]

Note... By using Observed Azimuths, the point M is thrown N. 1.93 & W. 0.04 out of its true position. The meridian of origin of the Survey to which all observations of azimuth had to be reduced was 1481.07 chains *west* of Station X, and 230.02 chains *East* of Station M.

From Table V, Convergency in Latitude $17^{\circ}=15''\ 92$ for one mile

" " " " " " $18^{\circ}=16.91$ " " "

Difference $0''.99$

Also $40' = 0.66$, then $0.66 \times 0.99 = 0.65$;

$$\therefore \text{Convergency required} = 10 \times (15^{\circ} 92' + 0^{\circ} 65') = 165^{\circ} 7' = 2^{\circ} 46'.$$

607. **Observation for Azimuth.**—Two different stars should be observed in determining the azimuth of a line. They should

be as near the prime vertical as possible, and not at a high altitude. The method is to observe simultaneously the altitude of a star east or west of the observer and the horizontal angle between the star and a referring mark, when the azimuth of the star having been computed by means of its altitude, its north polar distance taken from the Nautical Almanac and the latitude of the place, the azimuth of the mark is obtained by the application of the angle between the star and mark. There are several large stars which may be observed early of an evening or in the morning when there is just sufficient day-light to observe an unilluminated referring mark and also to see the cross-wires of the theodolite without illumination. In such cases the angles may be taken as easily as if angles to terrestrial objects were observed. Not less than two complete sets of observations on each face of the theodolite are to be taken for each determination of an azimuth, and in order to eliminate the instrumental errors, an equal number of each face are to be used in getting the mean which is to be taken as the true azimuth. It is desirable in Northern India to choose stars of north declination only but not exceeding the co-latitude of the station, also to observe the star at an altitude between 10° and 25° . The Sun may be observed for azimuth when it will be found inconvenient to observe the stars at night; in this case afternoon observations should be made, as there will be less refraction then. Examples will be given in the Chapter on Practical Astronomy of this work.

608. Finding the Variation of the Compass.—Whenever an azimuth is observed the variation of the needle shall be ascertained. The vernier being set to zero, the telescope should be directed to the magnetic north by releasing the lower plate and turning the telescope until the compass-needle attached to it coincides with zero on the compass-box. The lower plate should then be clamped and the vernier-plate released and the cross-wires directed on to a mark placed on the true north line. The angle now indicated by the vernier is the variation of needle.

609. Redemarcation and Resurvey of Boundary.—The theory of convergency finds its practical application in an important operation which the surveyor very often finds himself called upon to perform, namely, the redemarcation of a boundary. This is a task which it is simply impossible to do correctly without ascertaining the convergency at any available point of departure of the resurvey.

The relaying of a boundary as its name implies is the relocating on the ground of the boundary of any tract previously surveyed in exactly the same position in which they are indicated in the previous survey. Now the boundary of a tract is set up by offsets taken from 'traverse lines' run round about the tract; and the data necessary for the resurvey at a subsequent period of the identical boundary are : 1. The point of origin of the old survey on the plan or on the traverse "set up." 2. One at least of the traverse stations of the original survey on the ground—as a point of departure. 3. A copy of the old plan on as large a scale as available shewing all the traverse stations and lines, the required boundary, pillars,

posts and land marks on the said boundary, rivers, streams and roads traversing the boundary, and temples, buildings and all other conspicuous objects near it. 4. The traverse set-up of angles and bearings, distances and co-ordinates, clearly recording the origin of survey. 5. The field-book from which the offsets from the several points of the traverse lines to the corners and angles of the boundary as well as all important objects standing thereon may be found.

It often happens that the field-book is missing. But if the old traverse lines can be retraced, the offsets to the boundary as well as all important objects thereon can be measured off by means of the scale and dividers from the old map and relaid, if not accurately, to the nearest degree of approximation as the old survey might not have been correctly plotted.

610. How to Resurvey.—The theodolite is the instrument best suited for such work and it is also advisable, before commencing work to test the length of the chain by a standard chain or bar.

Suppose it is required to trace out a boundary, such as, A B C D, (Fig. 302) whereof all the traverse stations except one, such as A,

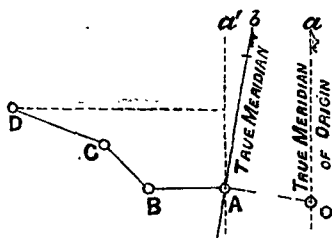


Fig. 302. — Re-Survey of Boundary.

have been destroyed, where A is the point of departure of the subsequent survey say 10 miles to the West of O, the point of origin of the old survey and situated at North latitude $20^{\circ} 29' 40''$. The surveyor ought to proceed step by step as follows :—1st. He should be satisfied as to the identity of the point, A, that is to say, he should take care to see that there is no doubt that the station,

A, selected by him on the ground is the station, A, noted in the map. 2nd. He should ascertain by the usual process (see Chapter on Practical Astronomy) the true meridian at the station A and set up a referring mark on the meridian line.

3rd. He should next set up his theodolite over station, A, and look up the traverse 'set up' and find the deduced bearing, as booked therein, of the traverse line, AB, which is, say, $269^{\circ} 06' 00''$. Here it is that the calculation for convergency comes in :—For the deduced bearing of the line AB noted in the old 'set up' is the bearing with reference to the true North line at the point of origin, O, of the old survey. But the station, A, is 10 miles West of O, the point of origin. If the terrestrial meridians were actually parallel it would not matter but they converge towards the poles and the meridian at A is inclined at an angle to the meridian at O :—Suppose Oa is the true North line at O and Ab the true North line at A, and Aa' is the line parallel to Oa at A. The line Ab makes with the line Aa' the angle $a'Ab$, the amount of convergency at point A. Now the deduced bearing booked in the traverse 'set up' is the angle $a'AB$, i.e., the angle which AB makes with reference to the true North line at O. But the true bearing as actually observed at A is the angle bAB , i.e., the $\angle a'AB$

— $\angle a'Ab$ or $\angle a'AB$ minus the amount of convergency. The surveyor should therefore deduct from the deduced bearing the amount of convergency to be found in the table of convergency, when the point of departure of the new survey is West of the point of origin, and add the same when the point of departure is East of the point of origin for obvious reasons. In the present case point A is 10 miles West of the point of origin, and from the table of convergency it is to be found that at $20^{\circ} 29' 04''$, the given latitude of the place, the convergency is $3' 15''$. The deduced bearing as pointed out above is $269^{\circ} 06' 00''$. The true bearing, therefore, will be $269^{\circ} 06' 00'' - 3' 15''$ or $269^{\circ} 02' 45''$.

4th. He should next set the theodolite to zero and direct the cross-wires on to the referring mark fixed on the true meridian line Ab , and clamp the lower plate. This done the upper plate should be released, the angle $269^{\circ} 02' 45''$ set off, and the plate clamped again.

5th. He should next send a flagman in the direction pointed to by the cross-wires of the telescope approximately at the required distance, and direct him to move to the right or left until the foot of the flag is bisected by the cross-wires.

6th. He should then find out the distance of the line AB as shewn in the traverse set-up or scaled off from the map and chain that length on the ground along the line indicated by the flag, measuring offsets on both sides of the line, to the principal bends and corners of the boundary, as also to all other notable features on or about. When the required length has been measured on the ground he should drive a peg and see through the telescope whether the cross-wires bisect it as an additional check. The traverse station will thus have been fixed.

7th. The next step for him to take is to remove the theodolite to station B , set the vernier to zero, direct the cross-wires to A and clamp the lower plate, after which the upper plate is to be released and made to read the interior angle as booked in the traverse 'set-up' between AB and BC —in this case, angle $130^{\circ} 11' 00''$. The line pointed out by the telescope is now to be ranged out as above, the distance measured, offsets taken and the peg driven in at C .

8th. The instrument should again be shifted on to C and the above operation repeated, and so on, till the traverse is completed.

Cases may occur when it may be necessary to determine the position of an isolated traverse station without referring to any of the intermediate stations, or the connecting traverse lines between them which are not required to be traced out. For instance, we may be required to locate the station D (see Fig. 302) without referring to the intervening stations B and C .

We have simply to look up the traverse 'set-up' and note the co-ordinates as given therein of the stations A and D , and find out the difference between these co-ordinates thus:

D from origin	...	N. 192° 96 chs.	W. 1846° 00 chs.
A " "	...	N. 32° 29 "	W. 1415° 64 "

The distance of D from A —N. 160° 87 chs. W. 430° 36 chs.

Now find out the true North line at *A*, as in the preceding example, and after setting the vernier to zero direct the telescope on to a referring mark fixed up in the North line, and make the vernier read the angle $3' 15''$ west, the angle of convergency at the station *A*. Range out the line which will be the meridian of origin at *A*. Now measure a length of 160.87 chs. North on this line and drive a peg at the point. Set the theodolite over this peg and make the vernier read 270° West. Range out the line indicated by the cross-wires and in it take off a length = 430.36 chs. west. The position of *D* is now fixed.

611. Connection with the G. T. Survey.—There should be perfect linking up of the Revenue Survey with the Great Trigonometrical Survey. The operation results in the following advantages:—1. The most severe test to which a Chain Traverse Survey can be subjected is the comparison of its results with those of a Trigonometrical Survey. 2. The errors which are almost unavoidable in chain measurements can by this means be eliminated and the traverse data rendered suitable for exact projection on Geographical maps. 3. The union of the two Surveys affords the means of referring the Revenue Survey maps to the true Geographical Parallels and Meridians of latitude and longitude.

612. To Locate a True Parallel of Latitude, or a True East and West Lines.—A *parallel* is the intersection with the surface of the earth of a plane perpendicular to the axis of the earth. Parallels are small circles and are strictly parallel to each other all the way around the earth. Meridians on the other hand are ellipses and converge towards the north or south pole. At any point on the earth's surface the horizontal angle, less than 90° , which any line makes with the meridian is the *true bearing* of the line. The same line may, if prolonged sufficiently have more than one true bearing. Supposing that a true east and west line (one which is at every point at right angles to a meridian passing through that point, hence a constantly curving line) is laid off at some point on the earth's surface as at *P*, (Fig. 299, p. 348). If this line should be prolonged, it would not coincide with the parallel *APBC*, but would describe a great circle *PEP'Q*, *EQDF* being the equator, and, while running due east and west at the point *P*, at another point, as *X*, its bearing would be very different. Hence a *true east and west line* cannot be prolonged indefinitely by means of fore-sights and back-sights. A practical method of locating a true parallel is to set up the transit at a point, as *A* (Fig. 303), and that it is desired to locate a point, as *B*, on the same parallel with *A*, but *x* miles further west. If a right angle is turned off from the meridian, *AP* at *A*, and the line thus determined prolonged, the great circle *GAE* will be run out. Let *EP* be a meridian midway between *AP* and *BP*. Then, in the right spherical triangle *PAD*, $\cos. AP =$

$$\cot. PAD \times \cot. APD. \therefore PAD = \cot.^{-1} (\cos. AP / \cot. APD) \dots (1)$$

$$= \cot.^{-1} \left(\sin. L / \cot. \frac{\theta}{2} \right) \dots (2)$$

Also $DAE = 90^\circ - PAD$, $\therefore DAE = \tan^{-1} \tan \frac{\theta}{2} \sin L$... (3)

In practice DAE and θ are both small, hence from equation (7) p. 349, Art. 604, $DAE = \frac{1}{2} \theta \sin L = \frac{1}{2} ATB$ (4)

If, therefore, we turn off from the meridian at A an angle equal to 90° minus one half the angle of convergence of the meridians at A and B, the line thus determined, if prolonged, will pass through the point B. Intermediate points will lie not on the parallel, ACB, but on the great circle ADB. Off-sets, however, can be calculated and laid off, thus locating points on the parallel as ADB is being run. In the U. S. Government Surveys the points, A, B, are located 6 miles apart—the length of a township line. At B a new meridian is established and the work repeated. Points are located on the parallel at every half a mile. The Manual of Surveying Instructions for the Survey of the Public Lands of the

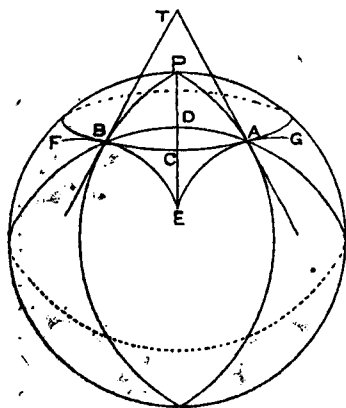


Fig. 303.—To Locate a True East and West Line.

U. S. A. recommends the *secant method*, which consists in running instead of the line ADB, an intermediate or *secant line* which passes south of A and B and intersects the parallel at two points one mile west of A and one mile east of B. Tables are given containing the azimuths of the secant (*i. e.*, angles made with the meridian), and offsets to the parallel at each half mile.

Ex : Locate a parallel of latitude, in latitude $22^\circ 33' 25''$ N. of Calcutta, and 1° apart in longitude.

Method 1.—From eqn. (2), $\cot. PAD = \sin L / \cot \frac{\theta}{2}$; ... (5)

$\therefore L \cot. PAD = L \sin L + L \tan \frac{\theta}{2} = 9.583880 + 7.940858 = 7.524738$;

$\therefore PAD = 89^\circ 48' 30''$;

i. e., a line AB laid down making an $\angle 89^\circ 48' 30''$ with the true north at A will pass through B.

Method 2.—Length of 1° of the parallel at latitude $23^\circ = 63.706$ statute miles, and $22^\circ = 64.166$ statute miles (Smithsonian Table 37, p. 167).

$\therefore$ At latitude $22^\circ 33' 25''$ length of $1^\circ = 63.91$ miles.

$\therefore$ log. convergence in Minutes =

Constant log. = 1.93871

L. tan. $22^\circ 33' 25'' = 9.61844$

log. 63.91 = 1.80557

log. $23'.05 = 23' 3'' = 1.36272$

If we now turn off from the meridian at A an angle of $90^\circ - 11' 31'' = 89^\circ 48' 29''$ the parallel is located. The difference is due to logs being taken to 5 places only.

In order to find the point D, take PD as the co-latitude of the middle point D, and then $\cos. PD = \cos. PA / \cos. AD$.

$$\therefore \text{Log } \cos. PD = L \cos. 67^\circ 26' 35'' - L \cos. \frac{1}{2} \\ = 9.583881 - 9.999983 = 9.583898;$$

$\therefore PD = 67^\circ 26' 32''$. $\therefore$ Latitude of D = $22^\circ 33' 28''$. Hence D is $3''$ N. of the parallel $22^\circ 33' 25''$. At latitudes 22° and 23° , $10''$ of latitude interval subtends 1009.10 and 1009.22 feet respectively. $\therefore 3''$ subtend about 302.75 feet (Smithsonian Tables 17, p. 78) or 302.75 ft. will be the offset south at a point 31.95 miles = 168722.4 ft. from either A or B along the parallel of latitude. One or more offsets may be computed in the same way and the rest put in by interpolation of values for similar triangles.

This method is recommended for those called upon to lay down new townships, canal colonies, etc.

613. Reduction of Co-ordinates of Stations from Rectangular to Spherical.—The latitude and longitude of a revenue-Survey Station can also be determined when its rectangular co-ordinates are given and have been corrected for errors of measurement and made consistent with the corresponding co-ordinates of the T. S. Stations in districts under survey. The latitude and longitude of the origin must of course be known, and the given rectangular co-ordinates must be perpendicular and parallel to the meridian of origin. The example 1 given on p. 360 illustrates the method of computation.

614. To Convert the Latitudes and Longitudes of the Trigonometrical Survey Stations into Rectangular Co-ordinates.—The operations of the Revenue surveyors are based on rectangular co-ordinates, and when Trigonometrical Survey Stations fall within range of their operations it is necessary to convert the latitudes and longitudes of the T. S. Stations into rectangular co-ordinates to obtain data for correcting the errors of their operations by simple proportion. These rectangular co-ordinates are very useful for the projection of small-scale maps. The example 2, given on p. 361 show how computations are made.

615. Co-ordinates transferred from One District Origin to Another.—The Origin of Survey (Fig. 304) for the District of Midnapur is the intersection of Lat. $22^\circ 30' 00''$ and Long.

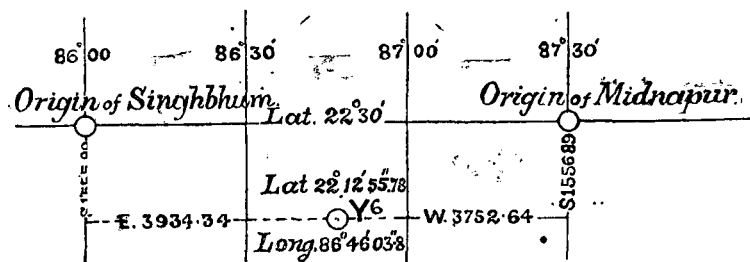


Fig. 304.—Co-ordinates transferred from one origin to another (Spherical).

$87^\circ 30' 00''$, and that of Singhbhum Lat. $22^\circ 30' 00''$ and Long. $86^\circ 00' 00''$. The co-ordinate (rectangular) value of P^0 (Fig. 304) as

derived from the origin of Midnapur has been fixed to be S. 1556'89 and W. 3752'64. In example 1, Col. 4, the Rectangular Value is converted into spherical, and found to be Lat. $22^{\circ} 22' 55''$ 78 and Long $86^{\circ} 46' 03''$ 82. In example 2, Col. 4, the Spherical Value of Y° is converted into Rectangular Value referred to the origin of Singhbhum, and found to be S. 1555'99 and E. 3934'34. The latitude of both the origins being the same, the southing is practically the same, the difference noticed is due to the latitude lines being spherical and the longitudes different.

Example 1.—Reduction of the Co-ordinates of Stations from Rectangular to Spherical.

1	2	3	4
Latitude of Origin (Ao)	32° 00' 00"	32° 00' 00"	22° 30' 00"
Longitude of Origin Lo	78° 00' 00"	78 00 00	87° 30' 00"
Name of Station (S)			Y6. M. C. 10 Midnapur.
Y in chains	S 3692.12	S 3030.48	S 1556 89
Log	3 567 2758	3 481 5112	3 192 2579
" 66	1' 819 5439	1' 819 5439	1' 819 5439
Sum=log Y in feet	5 986 8117	5 301 0551	5 011 8018
Y in feet			
$\frac{100}{100} = Y$ in arc approximately =	0 40 36 80	0 33 20 12	0 17 07 55
$\pm \frac{1}{2} Ya$	0 20 18 40	— 16 40 06	— 8 33 77
Ao	32 00 00 00	32 00 00 00	22 30 00 00
Algebraical sum	31 39 41 60	32 43 19 94	22 31 26 23
A. C. of log from Table XXVI* for			
Alg. Sum	3 995 5025	3 995 4984	3 996 0690
Log Y in feet	5 386 8197	5 301 0551	5 011 8018
Sum=log Y in seconds	3 382 3222	3 296 5535	3 007 8708
Y in arc	0 40 11 69	0 32 59 49	— 10 58 29
Ao+Y in arc=A'a	31 19 48 31	31 27 0 51	22 13 01 71
X in chains	W 958.47	W. 1547.24	W 3752.64
Log	2 981 6	3 18	3 574 3969
" 66	1' 819 5	1' 819 5	1 819 5439
A. C. of log from Table XXVII* for A'a	2 061 9	2 062 4	3 027 0993
Sum	2 563 0	3 071 5	3 420 9801
Twice Sum	5 726 0	6 143 0	6 842 0
Log from Table XXVIII* for Aa	6 034 2	6 035 1	7 931 0
Sum=log Arc-versine	1 760 2	0 178 1	0 773 1
Arc-versine	0 0 0 58	0 1 507	0 5 98
A'a—Arc-versine=Lat. of (S)	31 19 47 73	31 26 59 03	22 12 55 78
Log X in chains	2 981 5804	3 189 5590	3 574 3369
" 66	1' 819 5439	1' 819 5439	1' 819 5439
A. C. of log from Table XXVII* for Lat. of (S)	2 061 8594	2 062 4110	3 027 0943
Sum=log X in seconds	2 862 8537	3 071 5139	3 420 9751
X in arc	0 12 9 43	— 19 39	— 43 56 18
Lo	78 0 0	78 0 0	87 30 00 00
Algebraical sum = Long. of (S)	77 47 50 57	77 40 21	86 49 03 82

* X in arc and Y in arc have the same signs as they have in chains.

* Of the Auxiliary Tables 3rd Edition, 1887. or Tables VII, VIII, IX in Part III of this work.

616. Co-ordinates of a Point from One Origin to Another.—The computation is made as in ex: 3, p.362 (Fig. 305).

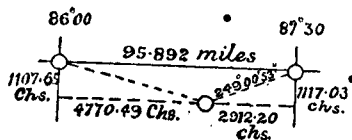


Fig. 305.—Co-ordinates from one origin to another plane.

617. Computation of Direct Distance between Any Two Stations of a Traverse and Mutual Azimuth from Rectangular Co-ordinates.—If it is required to find the direct distance

Example 2.—Computation of Rectangular Co-ordinates of Trigonometrical and other Stations of which the Latitude and Longitude are known.

From	Latitude	22° 30' 00"	22° 30' 00"	22° 30' 00"
Origin	Longitude	90° 30' 00"	90° 30' 00"	86° 00' 00"
	1	2	3	4
Number of sheet				
Name of Station (S)	Bhatra.	Gangapur.	Y6 M. C. 10 Midnapur.	
Long. of Origin (O)	90 30 0	90 30 0	86 00 00	
" (S)	90 10 26.66	90 29 55.81	86 46 03.82 P	
Difference	19 33.34	0 4.19	+ 46 08.82	
" in seconds	11.7834	419	2768.82	
Log	3 069 4202	0 622 2140	3 441 5093	
Log from Tab. XXVII* for Lat. of (S)	1 970 3973	1 970 4660	1 972 9057	
A. C. of log 66	2 180 4561	2 180 4561	2 180 4561	
Sum=log X	3 220 2746	0 773 1361	3 594 8715	
†X (Easting or Westing) in chains ..	W. 1660.63	5.9811	E 3934.34	
Twice log diff. of long. (4 places) ..	6 188 8	1 244 4	0 883 0	
Log from Tab. XXVIII* for Lat. of (S)	7 943 1	7 942 8	7 931 1	
Sum=log Arc-versine	0 081 9	5 187 2	0 814 1	
Arc-versine in seconds	1 2075	00000015	6.1152	
Lat of (O)	22 30 0	22 30 0	22 30 00.00	
Lat of (S)	23 0 52.12	22 59 34.77	22 12 55.78 P	
Difference (d)†	180 52.11	+ 29 34.77	- 17 04.22	
Arc-versine in seconds	+ 121	+ +	+ 6.52	
Algebraical sum=Y in arc	+ 30 5090	+ 29 34.77	- 16 57.70	
Y in seconds	+ 185 093	+ 1774.77	1017.70	
‡ Y in arc	+ 15 2545	+ 14 47.88	- 8 28.35	
Lat of (O)	22 30 00.00	22 30 00.00	22 30 0 0.90	
Algebraical sum	22 45 25.45	22 15 12.62	22 21 31.15	
Log from Tab. XXVII* for Alg su	2 003 9249	2 003 9254	2 003 9310	
Log of Y in seconds	3 267 3990	3 249 1421	3 007 6198	
A. C. of log 66	2 180 4561	2 180 4561	2 180 4561	
Sum=log Y	3 451 7710	3 433 5236	3 192 0060	
§ Y Northing or Southing in chains	N 2833.73	N 2713.63	S. 1555.99	

†X is $\frac{\text{east}}{\text{west}}$ of Origin according as the Longitude of the Station is $\frac{\text{greater}}{\text{less}}$ than that of the Origin.

* Of the Auxiliary Tables, Third Edition, 1887 or Tables VII, VIII, IX in Part III of this work.

§ Y is $\frac{\text{north}}{\text{south}}$ of the Origin according as Y in arc is $\pm$

† (d) is $\pm$ according as the Latitude of the Station is $\frac{\text{greater}}{\text{less}}$ than that of the Origin.

P—See example I, Col. 4.

between any two stations of a traverse with the bearing of such line the computation will be as in example 4, given below.

Example 3.—Computation of Co-ordinates of a point from one Origin to another (Fig. 305).

Village...
Trijunction.. From Origin 22° 30' 00" Lat., 87° 30' 00 Long.

I Old Co-ordinates	{	Mern (Y) 1117.08 South, Chains Log	..	..	3 048 0648
		A. C. of "	..	..	4 951 9352
		Perpr. (X) 2012.20 West, Chains "	..	..	8 464 2212
		Sum = Log tan. α	..	..	0 416 1564
		Log Sec. α	..	..	0 445 9617
		Log Y	..	..	3 048 0648
Log Direct Distance (3119.08 chs.) from Origin*..					3 494 0265
Bearing from Origin (a) 249 00 53					
Convergency correction 95.892 miles }					— 34 2
from one Origin to another					
Deduced Bearing (A) 248 26 31					
References—New origin Lat. 22° 30' 00", Long. 86° 00' 00"					
II	Sum of (1) and (2) = Log. y 1146.09	..	..	..	3 059 2174
	Log. Cos. Bearing A (1)	..	..	..	9 565 1909
	Log. Direct Distance* (2)	..	..	..	3 494 0265
	Log Sin. Bearing A (3)	..	..	..	9 968 5043
	Sum of (2) and (3) = log x 2900.89	..	..	..	3 402 6308
III	Deduced new Co-ordinates of point				
	Corrected Co-ordinates of x and y	..	..	1146.09 S	2900.89 W
	Computed Values of difference of Origin	..	..	38.44 N	7671.38 W
	Values in terms of new Origin	..	..	1107.65	4770.49 E

Note.—In example 1, Cols. 2 and 3, the rectangular co-ordinates of certain stations are converted to spherical values, similarly in example 2, Cols. 2 and 3 the Spherical values are converted into rectangular ones of certain stations. The student is requested to draw the index plans for these cases.

Example 4.—Computation of Direct Distance between any Two Stations of a Traverse and Mutual Azimuth from Rectangular Co-ordinates.

No.....	From.....A.....								
	To.....N.....								
Meridian Co-ordinate (γ°)	N. 5°75'	Chains Log	..	..	..	..	0	759	6678
		A. C. of	..	..	..	..	9	240	3322
Perpendicular Co-ord. (X)	E. 3°51'	Chains	..	..	..	..	0	545	3071
Bearing or $\alpha = 31^\circ 24' 00''$;	Sum =	Log tan. a	..	..	..	..	9	785	6393
		Log sec. a	..	..	..	..	10	068	7706
		Log Y	..	..	..	..	0	759	6678
Sum=Direct Distance by Traverso in Log Chains	6°74'		..	..	..	..	0	828	4384

Example 5.—Computation for Comparison of Direct Distance by Traverse, with that by Triangulation.

No.....	From...A.....				
	To.....N.....				
Meridian Co-ordinate (Y)	N. 5.75	Chains Long	..	..	0 750 6678
	A. C. of	"	..	..	9 240 3322
Perpendicular Co-ord. (X) E. 8.51		Chains	..	..	0 545 3071
Bearing or $\alpha = 81^{\circ} 24' 00''$;	Sum =	Log tan. α	..	..	9 785 6393
		Log sec. α	..	..	10 068 7706
		Log Y	..	..	0 759 6678
		" 66	..	..	1 819 5439
Sum = Direct Distance by Traverse in Log Feet	444.62		..	..	2 647 9823
Direct Distance by Triangulation	446.89		..	..	2 650 2006
	Difference of Logs	..	..	..	0 002 2183
Error of Traverse per mile in feet*	..	..	..	..	2.7

* The error per mile in feet is obtained by dividing the differences of logs of Direct Distances by .0000 823.

618. Computation for Comparison of Direct Distance by Traverse with that by Triangulation.—If it is necessary to compare the direct distance by traverse with that by triangulation, the computation will be as in ex : 5, above.

619. Cadastral Section : Field Season : Detailed. Field Measurements.—The following will give an outline of the system of survey which is adopted for all field-to-field surveys. It is called the **offset system**, and differs from the **planetable system** in not requiring the use of the alidade. It is the best System of Survey being that of direct lines between fixed points and offsets. This system is capable of rendering best results with a minimum of chaining ; it doubles the daily average outturn compared with any other system.

620. Survey of the Mauza (Village) Boundary.—The traverse and sub-traverse stations or *Chandas* are plotted on the Section papers (Art. 539) and the cadastral surveyor (amin) is supplied with this sheet on which the traverse and sub-traverse lines are shown with the connecting stations. The cadastral amin thus gets a skeleton map of the village on which to work in the detailed survey. In addition he is supplied with a rough sketch of the traverse plots called the *Khaka*. The first duty of a field-surveyor on entering a village is to make a reconnaissance and then to find the theodolite stations. He will note on the *Khaka* (rough copy of the skeleton plot) the different descriptions of the survey marks embedded, and will show them by the prescribed conventional signs. He will distinguish between stones, cylinders and bamboo pegs.

621. Defective Demarcations.—All defective demarcations and all theodolite stations not found should be noted and reported by the field-surveyor, through his Inspector to the Camp Officer. When such stations are absolutely necessary their position must be carefully relaid by intersection and measurements from adjacent stations. In other cases they should be dispensed with and the work should proceed,

623. Traverse Lines to be remeasured at Field Survey :
Survey of the Boundary.—The traverse of the village or area in question having been completed, computed, and plotted on the 16-inch scale on a sheet of drawing paper ruled into 1-inch squares (a side of which is, therefore, equivalent to 5 chains) the cadastral surveyor will proceed to map in the details commencing with the boundary first. During field survey the total distances of the traverse lines must be re-chained and compared with the plot. In case of disagreement a resurvey must be made, and if any error is found it should be reported to the Inspector. To survey the boundary the surveyor must choose a convenient station on his board and go to the corresponding theodolite station on the ground, sending a man with a flag to the next station on the boundary in the direction he proposes to work. He must now chain along this traverse line, taking offsets with the optical square to every bend in the boundary, and noting in his field-book the distances along the traverse lines from which he takes the offsets, and also the length of each offset. A specimen of a Cadastral Survey field-book is shown on p. 175 by Fig. 184. A field-book is generally prepared in form, Fig. 179, p. 167. The space, A N, represent the traverse sub-line or partial

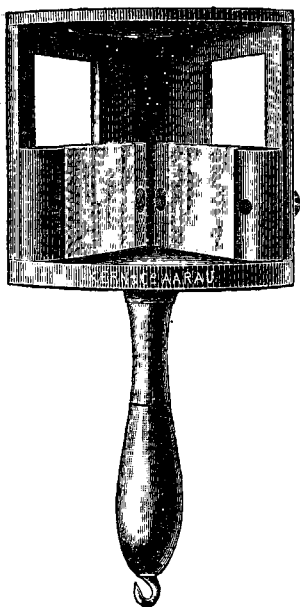


Fig. 306. —Kern's Optical Square.

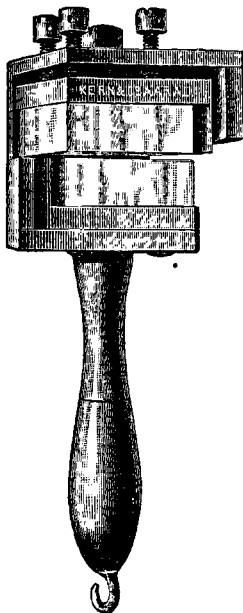


Fig. 308. —Line Ranger or Prismatic Square.

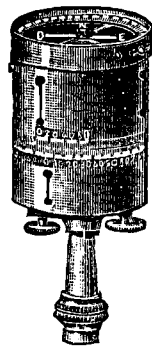


Fig. 307.
Kern's Cross-staff.

lines, along which the Amin is moving. The points, x_1 , x , p , r are the *Katans*. Their distances from the starting point are noted, as in the Figs. 178, 180, in which $\perp$ dotted lines are the offset distances.

Field corners, bends, or other points are shown on the right and left of the vertical lines according to their position on the ground with their offset measurements. The survey can be plotted from the field-book on to the sheet and a trace may be made. Fig. 306 represents an optical-square. Fig. 307 represents a cross-staff, and Fig. 308 a line-ranger. The offsets are plotted direct, now a days, on to the plane-table sheet. In measuring offsets of greater length than one chain the Gunter's chain is used. In case of offsets, one chain or less, a straight bamboo pole 20 links long will suffice. Each link is marked by a circle cut on the pole. The 5th, 10th and 15th links are marked by a specially broad circle.

623. Testing the Optical-Square.—To test its correctness

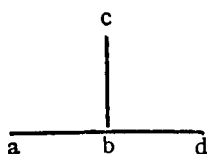


Fig. 309.—Testing Optical Square.

the surveyor will first fix an well-defined object, *c* (Fig. 310), on a level ground, and from a given point, *b*, lay out by the optical-square the lines, *ba* and *bd*, in opposite directions from the point, *b*. If staffs planted at *a*, *b*, *d* appear to be in one line, the adjustment is perfect. If not, measure the distance that one of the staffs *a* or *d* is out of the straight line, and adjust the mirror, by means of the adjusting-screws for that purpose, to coincide with a staff placed half way between. Repeat until quite correct.

624. Subsidiary Stations and Tie-lines.—Every offset of over $1\frac{1}{2}$ chains in length should be checked by a tie-line, and offsets of over 2 chains by two tie-lines. But whenever the village boundary runs far from the traverse lines, auxiliary stations should be established in the same way as long offsets are tied, and, on the lines running to and between these, the boundary offsets should be taken (*q. v.* Fig. 178, p. 166, and Fig. 184, p. 175).

625. Plotting of Disputed Boundary.—If in the course of the village boundary-survey, the surveyor comes upon any disputed place he will at once inform the Inspector. It is the duty of the Inspector to prepare the preliminary record of the boundary dispute. When the two Mauzas (villages) lie within the Halka of the same Inspector that Inspector is responsible for preparing the record. When the two villages lie within different Halkas under the same Head Inspector, the Head Inspector will decide which Inspector is to prepare the record. When the two villages lie within the jurisdictions of different Head Inspectors they will arrange mutually which Inspector is to prepare the record. Preliminary records of boundary disputes will be submitted to the Settlement Officer through the Survey Office. Inspectors will on no account indicate their own opinion on the merits of the dispute. Unless both sides claim present possession no dispute should be instituted.

Boundary disputes will not be entered in the Dispute Lists of the villages concerned. Inspectors will be provided with printed forms which they will use in reporting boundary disputes. The printed forms show in parallel columns the Khasras of the disputed area

according to the rival claimants. The entry in column 2 of the *Khasra* should be very brief and should show only the immediate claimant to possession and not his superior landlord. Along with this printed form the Inspector will attach a trace of the disputed area according to each village. The plots in the traces will be numbered. If the plotting according to the two villages coincides the traces will be exact duplicates of one another. If the plotting differs the traces will of course be dissimilar except in regard to the outside boundary. Each *Khasra* in the printed form will follow the numbering of the trace of that village. *A Khasra is the record in which the circumstances of each field are entered on the spot field by field in their order.*

After the Settlement Officer or his Assistant has decided the boundary dispute these traces will be received back by the Inspector. Along with each trace will be an abstract in the vernacular of the finding of the Court. This abstract is called a "Reference Slip." The Reference Slip and the trace will be filed with the papers of each *Mauza*. The Inspector who prepared the preliminary record is responsible for seeing that the maps of each *Mauza* concerned are corrected in accordance with the finding. If a boundary be not demarcated, but can be pointed out by both parties, it should be surveyed. When the Northern and Eastern boundaries of a *Mauza* are surveyed, a field-book of the same should be kept. Field-books need not be kept when a stream or a road forms the boundary.

626. When a Stream not more than 3 chains in width forms the boundary, it will be surveyed complete from one high bank to the other by the amins of both *mauzas*. When a stream more than 3 chains in width forms the boundary, it should be surveyed only in the *mauza* in which both banks have been plotted on the traverse sheets. When a stream is so wide that the two banks cannot be shown on one sheet it should not be plotted along with the *mauza* on either bank. Each surveyor should only plot the bank on his side. Separate arrangements will be made for mapping large streams of this nature. When the distance between the low water mark and the high bank of a stream is more than 20 links, the low water mark should be shown as well as the high bank.

627. When a large river forming the boundary between two districts is under survey, the channel, which the largest boats follow in the dry season, is to be noted in the map, as it indicates the deep stream which is the legal district boundary. If there is a disputed boundary and two lines are marked on the ground, both are to be surveyed twice, *i.e.*, once in each village touching on the disputed lands. The fields and other items in the disputed portion will only be surveyed once, *i.e.*, they will be surveyed with the village lying to the west or north of the disputed portion. When the field sheets are received in the Camp office, a tracing of the disputed portion will be made, and on it the boundary, according to the old Revenue Survey Sheets, enlarged as correctly as possible will be entered in red. The tracing with a copy of the *Khasra* will be transmitted to the Settlement officer for disposal. Until the dispute is disposed of, the disputed portion will be entered on the field-sheet in blue.

628. Common Boundaries.—When the outer boundaries touch on villages which are not included in the area under survey, they will be surveyed twice, the measurements being once booked in a field-book, and once plotted direct on the sheet. When the villages on either side are all in the area under survey, the boundaries of each village will be surveyed independently by each of the field surveyors concerned on his own village sheets, and plotted on each of the field sheets, and will then be compared in office. Boundary lines should not be inked in, unless they agree perfectly with those of adjoining villages. When there is any discrepancy the portions affected should be resurveyed by the surveyors who did the original surveys, or the inspector will personally resurvey the doubtful portion and correct it, if possible, before the record writing commences.

629. Damage to Crops not to be done.—The surveyor must see that no damage is done to the standing crops during the survey. When chaining through crops, such as paddy, pan, tobacco and sugarcane, the chain must be carefully lifted so that no injury be done to the crops. High crops should be bent over slightly, and not broken or cut, to permit the chain passing through. The amin should not cut down any tree.

630. Objects to be fixed by Offsets.—Large trees and all permanent objects must be offsetted to and booked, and it should be clearly stated which corner or other point of the object has been taken up. Every boundary pillar should be recorded and its material whether of mud, stone or masonry, should be stated against each.

631. Marks to identify Fixed Points on the Ground.—The surveyors should also note in addition the distances along the chain line at which field boundaries cut the same, marking the place on such boundaries by a small V-shaped cut, called a *Katan*, with the *Kodali* to assist him to find the spot again. The corners and bends of fields offsetted to are marked by a small $\odot$, a few inches in diameter.

632. Establishment of Stations for Quadrilaterals.—While chaining the traverse lines round the boundary, the surveyor should establish intermediate stations for his quadrilaterals. These stations should be from 10 to 14 chains apart, on any of the traverse lines, as may best appear to suit the work, but they should not be made in a cultivated field where they are liable to be destroyed.

633. Preparation and Survey of Quadrilaterals.—Having completed the traverse of and offsetting to the mauza boundary, the cadastral surveyor should plot the distances from his field-book on to the 16" plot and draw in the boundary. The surveyor will now proceed to divide the village or area, as the case may be, into quadrilaterals with sides of 10 to 14 chains, each averaging some 15 to 25 acres. To do this he will run straight lines on aligned flags from and to theodolite stations, or intermediate quadrilateral stations already fixed by him on the traverse lines. It is advisable to run the first quadrilateral lines across the narrowest breadth of the village so as to divide it into blocks with the least

possible amount of error in his measurements. Whilst running these lines new quadrilateral stations should be made on them at intervals of 10 to 14 chains apart, as may appear most convenient for the after work; and from and on these stations other lines will be run and closed until gradually the entire village or area is cut up into small blocks whose shapes should be as nearly rectangular as possible (Fig. 310).

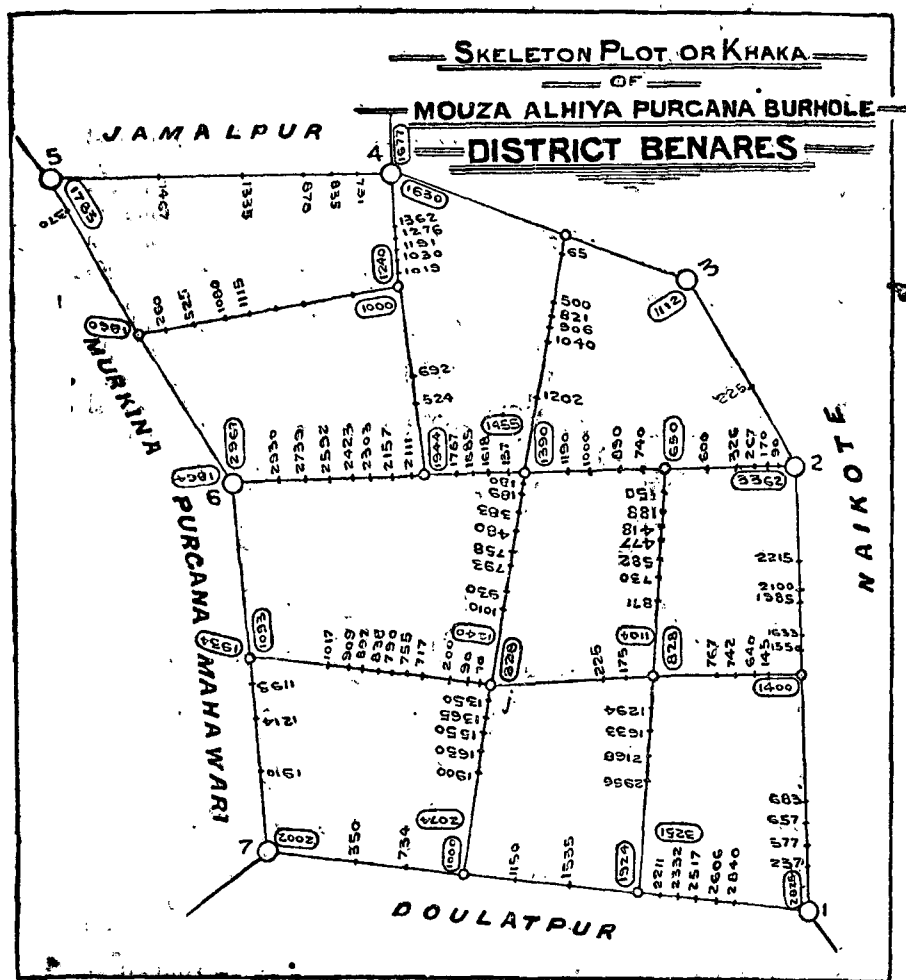


Fig. 310.—Example of a Khaka.

634. Survey of Quadrilaterals to be made with Special Care.—As the quadrilateral lines and stations form the backbone

of the work on which the accuracy of the rest of the plotting depends, special care should be taken in measuring accurately the sides of the quadrilaterals and the sub-lines that the lines are chained straight from station to station. Flags should be placed at either end of the line and the leader should be kept in a line covering the foreflag. The line-ranger will facilitate the work. The surveyor will compare the measured distances with those on the plot, will draw the lines straight, and distribute small differences correctly.

635. How Quadrilateral Stations are marked.—All the quadrilateral stations should be on *ails* and not in the middle of fields. To indicate the spots they should generally be marked by well-defined circles of 3' or 4' in diameter cut on the ground and also noted on the *Khaka*. Where however owing to the narrowness of the *ail*, the hardness of the soil, or any other reason, it is not practicable to mark such a circle on the *ail* a bamboo should be erected to mark the station.

636. An Example of a Khaka (Fig. 310): Rough Plot of Quadrilaterals on the Khaka.—The surveyor, whilst determining his quadrilateral stations, will make a rough plot of the quadrilaterals on his Skeleton plot (*Khaka*). Along each line he will note the distances between the quadrilateral stations, and also that of every field boundary crossed by his chain. These distances should be recorded in one series (not on any scale) from starting to closing stations. For easy reference in subsequent work the chain cuttings on each field boundary should be marked on the ground by a small V-shaped cut, called a *Katan*.

637. Plotting of Quadrilaterals on the Map.—When this cutting up of the village, and recording of all measurements on the *Khaka* have been completed, the surveyor will proceed to plot the same on his 16" sheet. The length of each line as measured on the ground must be compared with its length on the 16" sheet. If the total of the measurements on each line agrees with its length on the 16" sheet, the intermediate distances recorded along the line may be plotted, otherwise, the error, if only slight, must be distributed over the whole line; should the error be large, *i.e.*, more than $\frac{1}{2}$ link in each chain the line must be remeasured. The *amin* will not be permitted to begin the detailed survey (except in case of boundary disputes) until he has cut up a sheet into quadrilaterals completely. If, however, a part of a sheet is divided off completely by a subtraverse line, the detailed survey of this portion may be done before the quadrilaterals of the remaining portions are completed.

638. Distribution of Small Errors.—Small errors, such as, one link in 2 chains will be distributed as under:—

- (1) In a line of 43·30 chain, an error of 20 links* is found. As the total length is nearer to 40 than 50, lay off along the line 3·30 chains backwards from the closing station and divide the rest of the line into 4 equal parts, each of which will be 10 chains. The error is distributed along the line of 40 chains. All subsequent measurements for offsets or *Katans* should be taken from the nearest nominal 10 chains distance.

(2) In a line of 58·72 chains an error of 25 links is found: prolong the line 1·28 chains (60—58·72) beyond the closing station, and divide this total distance into 6 equal parts, and proceed as in (1), each length of 10 chains will then have received its proper proportion of error, and should be divided into halves if the difference is at all appreciable.

(3) If the error is considerable, but is allowed to pass, it is advisable to still further divide the nominal 10 chain lengths into halves, so that each 5 chain length will receive its due proportion of error.

(4) The same procedure should be adopted in plotting all the boundary and quadrilateral lines. Even when there is no discrepancy, the lines should be divided into 10 chain-lengths, so as to avoid stretching the compasses beyond 10 chains in subsequent plotting.

(5) If the above directions are carefully carried out, the surveyor will find he has his village out up into a number of small quadrilaterals, the corners of which are in correct positions, and all errors due to irregularities and stretching of the paper as far as possible eliminated.

639. Deep Water How avoided.—In laying out the quadrilateral lines, the surveyor should avoid, if possible, all deep *bils* (lakes) and tanks. If this cannot be done, a rope may sometimes be carried round and stretched across, and if it is not possible to measure the distance with a rope, the distance may be obtained with the optical-square by Prob. XV, p. 139, Art. 241. No man is ever to be sent with a chain into water beyond his depth.

640. Triangulation to be adopted when Jungle intervenes.—Triangulation with the chain may be adopted in lieu of cutting up the area into quadrilaterals when owing to obstructions from dense jungle, etc., the method of cutting up the village into quadrilaterals is impracticable. Each new point, however, must be fixed by at least 3 chain measurements as in the case of point, D, in Fig. 178 (p. 166) fixed from 3 fixed points, as n, m, o. All triangulation lines should be shown on the khaka with their measurements. The practice of running single lines with the sight-vane and building up *detailed Survey* on them is strictly prohibited. In a very close village it is only by the use of the alidade or a prismatic compass that the details within the village can be plotted.

641. Field Cuttings to be plotted on 16" Sheet.—When the quadrilaterals are completed, the *Katans* or chain cuttings on each field-boundary should next be carefully plotted on the sides of the quadrilaterals between each 10 chains; these points together with the quadrilateral stations give a certain number of fixed stations all over the area of the survey, and on the plotted sheet, from and to which, the *sub-lines* for the subsequent measurements of the fields can be started or closed (Fig. 311, q. v., specimen of a Final Map when plotted, p. 371).

642. No Booking of Measurements for Plotting of Fields except on Quadrilateral Sides.—Except along the quadrilateral lines on the Khaka no measurements are recorded. During the field survey all work is plotted on the map at once, as it is too expensive to keep records of minor measurements.

643. Survey and Internal Plotting of Fields.—The zaminders or their agents and the cultivators must be present to point out their plots,

and to state the names of occupants to the surveyor, when he begins surveying the fields, so that he may not survey *Kacha merhs*, *kiaries*, or *ails*, temporarily thrown up for irrigation or cultivation purposes only, unless especially ordered to do so. In rice tracts, however, plots which are traditionally known as fields have often numerous little subdivisions, of which the limits are permanent *merhs* (*ails*) rarely, if at all, distinguishable from the outer *merhs* of the tenant's entire block. In these cases, every *merh* should be carefully surveyed and shown on the map in firm pencil lines. But there is no necessity for giving each sub-division a separate number, nor any occasion for working out the areas separately. In inking up the map outer

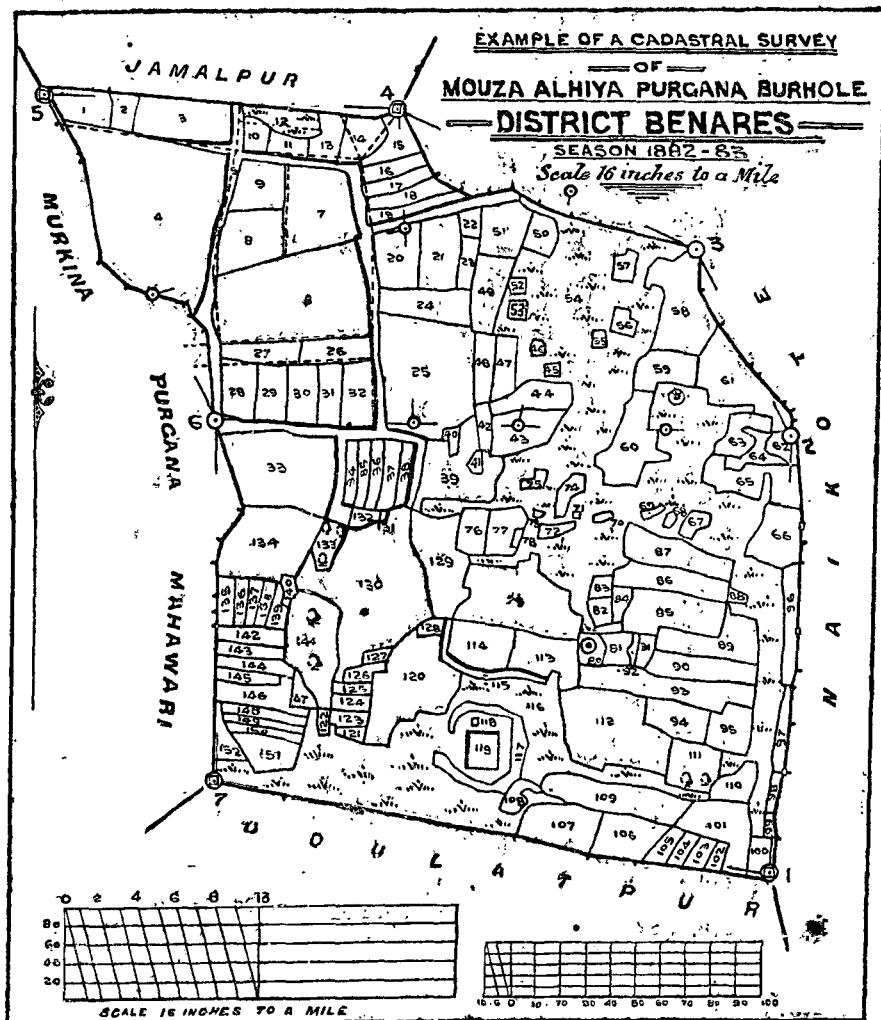


Fig. 311.—Example of a Cadastral Survey Village Map.

boundaries of plots should be inked in firm lines, and the interior sub-divisions in dotted lines. Numbers are given plot by plot. It is therefore, necessary to understand clearly what is meant by a plot. A plot (*dag*) is a piece of land possessed by one person or set of joint persons, round which one periphery can be drawn, which is held under one title, and consists of one kind of land, with one capability in the matter of crops. This is the *Unit of Survey*.

644. Sub-lines.—The survey of fields should, if possible, commence from the north-west corner of the north-western quadrilateral. The surveyor will now proceed to cut up the quadrilaterals by sub-lines from the *katans* which must conform to the general run of the field boundaries, in order that the offsets may be as short as possible to the corners of the fields. These sub-lines must start from and close on "*fixed points*," i.e., quadrilateral stations or chain cuttings of field boundaries on quadrilateral or traverse boundary lines. Points taken up by offsets are not to be used as the starting and closing points of sub-lines. The full distance of the sub-lines as measured by the chain must always be compared with the distance on the map. By running sub-lines where necessary from fixed point to fixed point, every field in the quadrilateral is worked down to and plotted. Whenever the chain intersects a field boundary the point of intersection will be marked on the ground, and will be at once marked on the sheet according to scale. An offset will similarly be taken to any corner of a field which lies within *one chain* of the line or sub-line. Each field corner when fixed by an offset from a sub-line should be marked on the ground by a small $\odot$ of a few inches in diameter. The surveyor will plot the field corners, boundary bends, etc., by taking offsets from the sub-lines, and proceed in this way till the survey of the village is completed. Sub-lines will be run judiciously so as to secure as far as possible that the offsets may not be greater than one chain.

To take an offset a man will stand at the corner or bend with the offset-staff. The surveyor will move with the optical-square in hand (see pp. 26-27) along the chain till he is at right angles to the corner, and the distance from this point to the corner will be measured, and marked on the map according to scale. When the four corners of a field with the bends of the field boundary have thus been plotted on the map, it will remain to connect those points on the sheet and the field has been plotted.

Sub-lines to be run along Crooked Field Boundaries.

—A special sub-line should be run near a field boundary which is not straight so that the curves and bends can be correctly taken up by offsets.

Sub-lines to be run at Right Angles to Long Field Boundaries.—In the case of long and narrow straight fields sub-lines must be run at about right angles across them, and at 4 or 5 chains apart in order to check the positions of the side in order that all important bends may be shown on the sheet. Long offsets entail useless chaining and the surveyor should learn to dispose his lines so that no superfluous chain may be measured.

645. Use of the Alidade in Fixing Stations.—In a rough

or jungle district where correct chaining is difficult, it will be often found necessary to use the alidade (Fig. 36, p. 42, F. F.) to establish

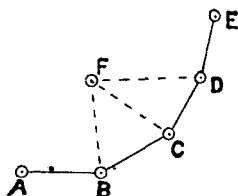


Fig. 312.—Intersected Point from Traverse Lines.

new stations. Suppose it is required to fix on the sheet the point F, (Fig. 312), the points A, B, C, D, E, being traverse stations. Plant flags at F, A, C, and the plane-table at B. Orient the plane-table by the aid of the traverse lines, B C and B A, so that the sight-rule placed against a pin at B will intersect the flags at A and C, when this is secured, place the alidade against the pin at B, and align it along the direction B F, moving it about until it intersects F, and draw a pencil ray along the alidade in the direction, B F. Then go on to C, and repeat the same process, showing the direction of C F also on the sheet. The point at which the two lines cut is the required point, F. To check the accuracy, bring the table to D, and set it as before. The alidade aligned along D F on the sheet should intersect the flag at F, if this flag has been fixed correctly. Another test is to run a line from F to any known point on the traverse lines. Its length measured on the ground should agree with its length on the sheet.

646. Missing Traverse Stations are also located on the Ground by means of the Sight-vane.—The plane-table is set as before in the same way, and the sight-vane is aligned along the traverse line, at the end of which is the traverse station to be located on the ground. A man is set forward with a flag in the direction of the missing station. He is made to move about until the sight-vane intersects his flag. The distance of the missing station from the point of observation, as shown by scale, is then measured in the direction of the flag, and the required point is thus fixed on the ground. Its correctness should be tested by an intersection from an adjoining traverse station (q.v., Art. 443, p. 246).

647. Conspicuous Objects to be plotted from Actual Survey.—Mile-posts, telegraph-posts, bench-marks, and all conspicuous objects, such as, temples, tombs, mounds, wells, isolated trees, etc., are not to be mapped by guess: they must be measured to, and plotted from actual survey, and entered on the sheet in their true positions by the proper conventional signs. As many conspicuous trees as there is room for will be plotted on the sheet. All valuable timber and fruit trees affecting the assessment are to be plotted in the same manner; when their numbers are too large to admit of their being shown on the plan they are to be carefully counted and recorded.

648. Survey of Streams within Cultivation.—If the bed of a stream running through a field is cultivated by the holder of the field, its limit will be surveyed and shown by dotted lines. If it

is, cultivated by another tenant the stream will have firm boundaries and a separate number.

649. Survey of Khal.—If the bed of a khal has silted up and been completely cultivated it will not be treated differently from other cultivated land. If however any part of the bed remain uncultivated that part should be distinctly plotted.

650. Shifting Cultivation in the bed of a Tank.—Small cultivated area in a tank, or lake, or along the margin of a stream for which the tenants have no occupancy rights need not be surveyed field by field. The limits of different properties must of course be shown.

651. Fields which can not be traced.—When the field boundaries cannot be traced on the ground owing to their being annually destroyed by floods or otherwise, it may be necessary to get the tenants and landlords to demarcate them before survey. Any piece of land, in which there is separate possession depending on imaginary lines, should be plotted according to those imaginary lines, if the parties concerned point them out. Otherwise the whole piece of land should be shown as one plot. Imaginary lines plotted should be shown by dotted lines if they run through water: otherwise by firm lines.

652. Field Boundaries not existing on the ground.—If it is required to show on the map divisions that do not exist on the ground, either in waste, or in lakes, or in cultivation, they should be marked by firm lines when they represent limits of properties, and by dotted lines when they merely mark off new fallow from old fallow, or irrigated, from dry cultivation, and such like.

653. Fields of the same number to be bracketted.—When two or more plots separated by firm lines, such as the lands on each side of a road, or the several enclosures in a village site, are given one number, they are to be bracketted thus—on the map.

654. Minor Drainages to be shown on Cadastral Maps.—Dry runs, *i.e.*, the general directions of the natural drainages which are invisible during the dry season should be shown by dotted lines if they can be ascertained, as such informations may be valuable to canal officers.

655. Survey of Ravines.—The main arteries of the ravines should be carefully surveyed, as well as the openings of all minor branches (say of 10 chains length) leading from them, the cultivation of the latter being shown dotted, and a note added to the map explanatory of the dots. Cultivation within ravines must be carefully surveyed.

656. Survey of Rivers and Streams with High Banks.—In tracts subject to alluvion and diluvion all existing high banks of rivers and streams should be carefully shown by short hachures. No change in the boundary between the alluvial and the permanent mahal in a village shall be made except by order of the Settlement Officer.

657. Survey of Creeks.—All creeks should be surveyed by

plane table traverses with slight jungle clearing, if necessary, in case the surveyor cannot proceed along its bed at low water; all main bends are to be accurately fixed by offsets.

658. Survey of Hills.—All hills falling under the survey should be fixed, either by plane table triangulation (see p. 296, Fig. 275) or by short traverses run to connect them from one known point to another on the plane table. The tops and feet of the hills, and their spurs should first be fixed on the table. Then measurements should be carried up the slopes of the hills, wherever required, and the contour lines drawn in for 10 feet vertical intervals, and minor details sketched in. If it is found impracticable to fix the contour lines by the method of stepping, *i.e.*, by short horizontal chain measurements, aided by a 5' rod, a clinometer for measuring the angle of inclination may be used and the sloping distance measured, from these data the horl. distances for 10' vertical intervals can be deduced.

659. Survey of Bench-marks.—Where the country under survey was levelled previously, all existing bench-marks should be found out and their positions carefully fixed on the maps.

660. In the Survey of Fields the Following Points to be attended to.—(1) A field map of every village should indicate the boundary of every field separately held. (2) The unit of field survey is an area of which the ownership is separate and the occupant is separate. (3) Each parcel or block of land within the same exterior boundaries in the occupation of one person, or of several persons holding jointly, under one title, provided it is all of one soil, should be mapped as a separate survey number. (4) Uncultivated plot should be numbered separately from a contiguous cultivated plot, and when required contiguous plots at different levels should be separately numbered.

661. Survey and Plotting of Village Sites, Roads, Railways, etc.—The surveyor is to survey every plot shown to him, as said before, as a separate field. When one field is sub-divided by *aïls*, for irrigation or cultivation, such *aïls* need not be separately measured. Every field boundary existing on the ground should be shown on the map by firm lines, and those not visible on ground, as in rice fields under water by dotted lines. The divisions of estates properly partitioned among the proprietors in a village may, if required, be shown by colors, and if not demarcated on the ground, will not be shown on the map otherwise than by broken boundary lines in blue. This should be done only in case of compact blocks. Every village site should be surveyed in detail. All houses, gardens, homesteads, roads, wells, etc., within it are to be surveyed and numbered omitting nothing except patches of waste, etc., under 5 links in breadth which cannot be clearly shown on the 16" scale. If the plots in any part of a village cannot be shown on the body of the map by the scale in use they should be surveyed on a larger scale, either 32" or 64" to a mile, and delineated on the margin of the map, a reference letter put on the blank space. Garden plots and homesteads in village sites should be surveyed and numbered separately from houses. Houses with masonry plinths should be

accurately plotted in their true positions ; but all other houses or huts should be shown merely by the conventional signs in their approximate position. When a masonry plinth which has been plotted under this rule is identical in possession with the adjacent homestead land it should be connected with it by the sign ω . At K̄hanapuri (Record-of-rights) only one number will be given to the plinth and the adjacent homestead land. Houses (not temporary huts) with their compounds in fields outside village site should be separately mapped, but unless the compounds are cultivated, one number marks both the house and compound. The sight-rule is to be used when absolutely necessary in plotting homestead lands and dense jungle. Claims may be made to obliterate existing field boundaries in order to rearrange them, but this cannot be permitted. It must be distinctly understood that the survey is to follow present possession. If the parties insist, the matter will be reported to Settlement Officer for orders. Roads and permanent footpaths (halats) through villages are to be surveyed and correctly plotted throughout, and separately numbered whether they run through cultivated or wastelands. A note is to be made on the map of the places the roads come from and lead to. All numbers and names of places, cut on bench-marks, and mile-stones, are to be shown on the margin of the map. All topographical features which the scale will admit of showing must be shown on the map. Large village sites are to be surveyed only by skilful surveyors. During the survey of railway and canal embankments no hole is to be dug ; red flags are never to be planted on a railway line ; all masonry pillars near the line, level crossings, etc., are to be carefully fixed. The line, with its banks and fences is to be mapped as shown in the list of conventional signs. All items (such as jungles, gardens) other than cultivated fields are shown by conventional signs on the map. Names of villages, rivers, creeks, adjoining villages, should be written in pencil. When the survey of fields is in progress the surveyor should give notice to the occupants of the lands in that part of the village in which survey is in progress but should not cause their attendance until their lands are under survey.

662. Numbering of Plots.—The numbering, of the plots in the new map, and the writing up of the *khasra*, shall go on together ; the surveyor should ordinarily insert a new number for each plot on the map just before he makes the entries for that plot in his *khasra*. Every plot on the map whether cultivated or uncultivated, must have a separate number. The numbering of fields in the new map should usually begin at the North-west corner of the village (see Fig. 311) and end at the South-east corner. But if there is a disputed boundary in the village, the numbering should be arranged so as to end at the disputed portion in order that any change rendered necessary when the dispute is settled may be more conveniently made. If on completing the village it is found that the number of any plot has been omitted, the plot will receive the number next in succession to the last number already recorded, and, as indicative of its position, the number of an adjoining field will be placed ab

the field number in fractional form thus : $\frac{64}{1345}$, 1345 being the last number already recorded, and the plot omitted is near field, 64. If there was, besides another fractional number to be entered, as $\frac{70}{1347}$, its denominator would be 1347; so on, for others. Should it be found during the revision that the number of such additional entries exceeds 10 per cent. of the number of plots in the village, the map and Khasra should be renumbered.

663. Check or Partial Survey by Inspectors.—During the progress of the boundary survey the Inspector must check the boundary field-books of every village in his circle on the ground, and where errors are found, he must order the surveyor to make new measurements, and a new field-book of the portion of the boundary affected. When the boundary follows a stream or lies in broken ground, the survey will be made direct on the field-sheet and not in the field-book. When the boundaries have been surveyed and plotted on the traverse plots, the Inspector will then make a careful comparison of boundaries of adjoining villages, inking up in blue all those that agree and adjusting all differences at once, either by ordering the surveyors concerned to resurvey the portions of their boundaries affected, or by personally making new measurements. While working these comparisons the Inspector should on no account detain his surveyors, but should direct them to proceed with the preparations for the interior survey of the North-west villages. When the Inspector has completed the comparison of the boundary surveys, he should send tracing of the boundaries with the field-books and traverse plots into Camp Office, retaining the plot of the village on the survey of which the surveyor is engaged. A final comparison of boundaries will be made in office before the boundaries are passed and inked up. If in the course of the detail survey the surveyor discovers any errors in the boundaries as plotted from the field-book he must correct the mistake in pencil and report to the Inspector who will then proceed to the spot, re-measure and make a corrected field-book of the wrong work, enter the corrected boundary on the sheet in blue, and send a report with the new field-book to office.

As soon as all the surveyors of an inspector's charge have started work in their respective circles, the Inspector fixing some village as centre, must constantly move about examining the work. He should test portions of the boundary especially where intricate or winding; check the plotting of all quadrilateral stations by the measurements on the Khaka of each village in his area, and affix his signature in ink on the khaka to show that he has done so; see that sub-lines follow the general run of the fields, that there is no superfluous chaining, and that the offsets are carefully measured and plotted, and not too long; from time to time run check lines across sections of the work as it is in progress; reject and rub out all bad work, correcting small errors under 10 links; see that the conventional signs are entered on the plans where needed; see that the banks of streams, especially near alluvial lands, are marked by hachures; and in short, satisfy himself that all the instructions are strictly

adhered to. An active Inspector should discover bad work before any large area has been surveyed. Each Inspector should be supplied with a test chain, and should make a permanent test line on the ground in each village. He should test the surveyors' chains whenever he visits them, and should send his own chain to office to be tested at intervals of 15 days throughout the field season. As soon as the survey of a village is completed, the Inspector must run his final check lines from station to station across the map as plotted, and note if the field boundaries intersected and the corners offsetted to, agree with those shown in the field surveyor's plotted map. He should also check the measurement of at least one quadrilateral in every sheet, and make such other examination as he thinks necessary. If he considers the work correct, he will make an entry to that effect on the back of the sheet, noting the date of entry. If he has to reject the work, he will note the area and number of days spent in Re-survey, and send the report in his fortnightly return. Before sending a map into office the Inspector will ink up his check lines in blue, as well as all conventional signs of conspicuous objects, such as temples, wells, and masonry buildings, other than cultivated fields, and check the reference lists as to names of streams, villages, hamlets, fairs, markets, thanas, tehsils, temples, schools, ferries, railway stations, etc. These reference lists are to be made by the surveyors during survey and entered on the map.

664. Check or Partial Survey by Survey Officer.—On receipt of plans in office the survey officer will arrange for their final check by independent field-book partals, or by re-survey of certain quadrilaterals, the original maps being in the meantime kept in office under lock and key. The Survey Officer should arrange for himself, and the assistant Survey Officer to check as much as possible of the work on the ground. Sheets thus checked may require little, or no independent partial, and may frequently be passed by the officer at the time of his partial. The final check lines will be marked on the *khasas* and those only sent to partallers. The partaller will register the check lines in field-books, and will send the field-books with plots to office to be compared by the Survey Officer personally. These plots should show entire fields through which the lines run, and all other items up to 2 chains on either side. It will frequently be found useful for the partallers to enter the details of crops and irrigation of fields in their field-books when partalling, to be used afterwards as a check on the *khasra* entries. Bad work, *i.e.*, work vitiated by wrong plotting of quadrilateral, should be entirely rejected and resurveyed on fresh plots, the originals being kept under lock and key in the mean time. When errors are confined within a quadrilateral that quadrilateral only need be rejected and re-surveyed but more check lines should be run in other parts of the survey. Independent final check lines should often follow or cross those marked on the plans as having been run by Inspectors or partallers during the progress of the work in order to test the latter's work as well as that of the surveyor.

665. The Amount of independent partial. *i.e.*, a partial

which is measured along a line indicated by a responsible officer on a sketch after the completed map has been received in office, will be 40 chains per every block of 320 acres. All partials are to be compared before the field is quitted, and are not to be left to be done during the recess-season.

666. Amount of Discrepancy allowed.—When on comparing the partial with the original survey plans, discrepancies of more than 10 links are found, which cannot be reconciled and corrected by reference to the original field-book entries, remeasurement of the discrepant items shall at once be made. There are two distinct sources of error arising : (1) from careless plotting of quadrilaterals, (2) from careless plotting of fields, the partialler should distinguish between these two kinds of error in classing work. The first is by far the more serious.

667. The Officer in charge of the Survey should test a certain number of villages himself, to enable him to form an opinion as to the general quality of the work.

668. Partial Line Field-book.—All partial or test lines should be booked in field-books, and be projected, for easy comparison with original survey, on tracing cloth or paper.

669. Classes of work.—Detailed field-work is divided into three classes : 1st class, which indicates agreement like a trace ; 2nd class, which is work generally out 10 or 15 links ; 3rd class, which is work that is rejected.

670. Signatures of Officers.—The persons who have surveyed or partialled the work, or inked up the work, or numbered the fields, or passed boundaries, etc., should put in their signatures, with dates on the back of the sheets. Such entries should be made legibly by the persons concerned.

671. Clean Maps a Necessity.—Maps should be kept clean and free from stains. A sheet mounted on the plane table when not in use should be covered with a sheet of paper over which a piece of water proof cloth of the required size should be tied.

672. Completion of the Map.—The surveyor will outline every item except the mauza boundary in ink before sending in the map for final partial and before Khanapuri, if possible. A fine pen will be used and not a drawing pen. Surveyors are not to ink up their work till it is passed by the Inspector. Surveyors and Inspectors are strictly prohibited from making any erasure in a map.

Surveyors must be careful that the fields sheets do not become greasy. Any grease on a sheet will spoil the sheet for reproduction, and necessitate the preparation and comparison of an intermediate trace. After Khanapuri, field numbers are to be inked upon the middle of each field. The numbers should read horizontally, the north being considered as the top of the paper. All fractional numbers should be noted at the end of the note sheet. No coloring or flat washing will be put on the map.

673. The Recess Season : Sections of the Cadastral Survey Office.—The principal divisions of a cadastral survey office consist of : (1) *Drafting section* : comprising boundary plotters,

draftsmen to ink up the maps, tracers, colorists and examiners.

(2) *Computing section* : area estimators and examiners.

674. Drafting Section : Inking Original Sheets.—The draftsman in the cadastral survey office should first ink in the field numbers which must be done serially from left to right from the North-west corner of the village and end at the south-east corner, and all conventional signs with the aid of *khassras*, and compare and ink in the boundaries. No shading is to be given to roads, tanks, canals or streams, thus the only thick line on the map will be the boundary, except the shading of houses in sites. Solitary trees should never be entered on the map unless their position has been carefully fixed by the surveyor in the field by actual measurement.

All the permanent theodolite stations should be shown by $\square$ for tri-junction platforms, $\triangle$ for stones, and $\odot$ for clay cylinders. This must be done from the *khaka*. The corresponding station letters are to be entered in black or red and the chain lines must be drawn in faint blue. After the original plan has been inked up it must be made over to an examiner, who will go over every number serially examining the map, while a muharrir (clerk), reads out the entries in the Khasra—field, grove, fallow, well, etc., etc.

The examiner will be responsible that all conventional signs have been properly entered, and no plot left unnumbered. He will keep a list of errors and omissions to be corrected hereafter. Subsequently, the title, giving the name and number of the village, with the pargana and district names, must be entered, also the scale of the map, and the year of survey ; also all surrounding village names, and names of village sites, roads, streams, rivers, etc. All these will be in the vernacular. On the upper margin will be given in English the names of the village, pargana, and district. A scale of chains will be inked up at the foot of the map.

675. Tracings of the Map.—One or two tracings are made of the original plan in the vernacular for the Civil or Settlement Officer. The numbering on the tracings will be given independently and will afford a check on the work.

(2). **The Computing Section : Computation of Areas ;—**

Areas of every plot of ground given in the maps must be computed in acres and decimals in duplicates by two separate estimators. The mean areas will be calculated, compared and passed by a third estimator. The areas of fields as taken out by the two estimators must agree within the following limits :—Difference $\frac{1}{4}$ or $-$ of

- 01 may pass in all cases ;
- 02 in fields under ·20 of an acre to be redone ;
- 02 in fields from ·20 of an acre upwards may pass ;
- 03 in fields under 2·00 acres to be redone ;
- 03 in fields of 2·00 acres and upwards may pass ;
- 04 to be re-done except in the case of very large blocks.

Areas that on comparison have to be re-done under the above rule will be left in pencil and their numbers noted on a slip for the third working which will, when finished, be compared by the examiner. All the fields now passed will be inked in. If the differ-

ences are still in excess, a fourth or fifth working will be made, till the fields are corrected and entered : In the case of large blocks the calculations may be passed if the two workings agree within 1 per cent. The total of each column of the area-form must then be found, and an abstract of the column-totals will be made to give the grand total of the village by summation of fields. The total will be compared with the U. T. area $\pm$ the offsets of each traverse line; and also with the area obtained by the planimeter.

The offsets must be written in duplicate; they must then be compared and entered on the sheet in pencil, and on an index map in ink. They will be sent to the officer in charge of the Traverse Party for entrance in his Traverse volumes.

The areas by summation of fields, by U. T. area $\pm$ offsets and by the planimeter, should all agree with each other within 1 per cent.

676. Instruments used for Finding Areas:—For the smaller blocks, the acre-comb or the talc-square is used for computing the areas, and for the larger ones the planimeter.

NOTE.—Students who wish to go more into the details of the subject are referred to the "Handbook of Professional Instructions for the Revenue Branch, Survey of India Department".

CHAPTER VI.

DRAWING MATERIALS AND INSTRUMENTS.

677a. The Drawing papers should be of good quality and new, but not too highly glazed, and should have as smooth a surface as possible. There are two classes of drawing papers: machine-made and hand-made, of the former, there are many qualities, from the common *cartridge* to the *extra superfine drawing*, either in sheets or continuous rolls, any width up to 62". For fine ink-drawing, the hard smooth surface quality, such as, Hollingworth's or Joynson's, are to be recommended. These same papers are to be had with "not" or medium-rough surface for general work. Hand-made, for all better classes of work, and in cases where drawings are to be handled much, is the kind to be preferred, and can be supplied in sheets only. The hand-made paper most generally used is Whatman's, either with *smooth*, "*not*" or *rough surface*. This paper is in large demand, but the quality now supplied is not equal to the same available some years ago. Sunder's, comparatively a new-make, seems to be much appreciated. It has all the characteristics of the hand-made, erases well, and is little cheaper than the other. Whatman's ("*not*" hot-pressed) medium thickness is a quality usually used for finished drawings. They bear wetting and stretching without damaging, and when so treated receive shading and coloring well. Whatman's *hot-pressed* paper and Bristol boards are useful for exhibition pen-and-ink drawings. For ordinary working drawings and details, cheaper cartridge paper, or cartoon paper, of which

there are many varieties in continuous rolls of 10 and 50 yards, 30," 54," and 60" in width, is to be preferred. For delicate small-scale line drawing, the thick blue paper imperial size, such as is used for ledgers, etc., answers well. The following are sizes in general use, although the exact dimensions vary slightly with different makers. *Those marked with a** are the sizes almost universally used:—

Demy—Hot pressed and Not, perfect...	20" × 15 $\frac{1}{2}$ "	
*Medium— " " "	22" × 17 $\frac{1}{2}$ "	
Royal—Hot pressed, Not and Rough ...	24" × 19 $\frac{1}{2}$ "	
Super Royal ...	27" × 19"	Emperor ... 72" × 48"
*Imperial, perfect and Retree	30 $\frac{1}{2}$ " × 22"	Atlas ... 34" × 26"
*Double Elephant	40" × 26 $\frac{3}{4}$ "	Columbia ... 35" × 23 $\frac{1}{2}$ "
*Antiquarian ...	52 $\frac{1}{2}$ " × 30 $\frac{1}{2}$ "	Elephant ... 28" × 23"

677b. Mounted Drawing papers for drawing have now come into much more general use than formerly, and are to be had in nearly all qualities, thicknesses, or surfaces, either machine or hand-made, mounted on cotton, white or brown holland, or linen, in any size of sheet or continuous, almost in any widths of (26", 30", 39", 52" 60") in rolls of 25 yards. In the case of drawings that are likely to be much handled or where a deal of work is to be put on the sheet, as in colliery, or estate plans, mounted papers thoroughly seasoned to prevent shrinkage may be considered as essential.

677c. Mounting Paper on Drawing Board.—Before the commencement of a drawing, the paper is to be stretched perfectly flat on the drawing board, hand-made paper being used, with drawing pins, or with paste in case of an important drawing that is to be colored on. The proper face of the paper upon which to draw is that surface which being held to the light, the water-mark of the maker's name can be easily read. Lay the sheet flat on the board, damp it with a wet sponge dipped in clean water all over the surface to within $\frac{3}{4}$ " of the edges all round, reverse and damp other side, observe that no pools of water are left, leave for a short time until the gloss entirely disappears, and redamp on both sides, place sheet in proper position on board, take a straight-edge, turn up the dry edge on one side of the paper against this, apply a brush dipped in a hot solution of strong glue, or a strong paste made of flour and ground resin, between the turned-up edge and the board, and rule down the turned-up edge on the board with a cloth or a paper-knife, repeat the operation on the other side, then treat the two edges in the same way; if not firmly fixed all round, rub well down with a folder, allow it to dry in a horizontal position, and in 2 or 3 hours the sheet will be found perfectly flat. The more expeditiously the gluing down is performed the better, because if there should be any part dry before the gluing is accomplished, it will have a tendency to unduly strain the paper, and cause wrinkles and unevenness on the surface. When properly strained, it should be perfectly flat and tight on the board; further, it should be quite dry and hard before being used, and as a safe precaution it is better to leave it for a day or so after being strained. When the drawing is finished the paper,

is cut with a cutting gauge, or a pen-knife drawn along an iron straight-edge. The edge of a T-square should not be used for cutting against. The glued margin left on the board should be thoroughly softened with cold water, and carefully removed with a folder without damage to the surface of the board; otherwise with a lumpy surface at the edges, it is almost impossible to work the T-square with any degree of accuracy.

677d. Caution as to the Drying of Paper.—The wetting of the paper is for the purpose of expanding it; and the edges, being fixed to the board in its enlarged state, act as stretchers upon the paper, while it contracts in drying, which it should be allowed to do gradually in the shade. All creases or undulations by this means disappear from the surface which forms a smooth plane to receive the drawing.

677e. Mounting Paper upon Canvas or Linen.—If plans are required on canvas, the paper should be mounted and dried before plan drawing is started, in order that the contraction in drying may not alter the dimensions. To mount the paper, a piece of linen, larger than the plan, is put on a sloping flat table. The edges, $1\frac{1}{2}$ " wide, are glued and pressed on the table, the linen being at the same time pulled tight. Two persons are required for manipulating larger sheets. The back of the paper is then pasted and it is necessary to let every sheet lie for a few minutes after the paste is put on it; for as the paste soaks in, the paper will stretch, and will lie more evenly on the canvas, whereas if it be laid on before the paste has moistened the paper, it will stretch afterwards, and rise in blisters when laid upon the canvas. When the paste has soaked in, it is as well therefore to go again over the paper with the paste-brush containing very little paste; this is done to just moisten the whole surface, and to remove any superfluous paste. The paper is then lifted up, and placed with the pasted side on the linen, and pressed from the centre to the edges. The rubbing-down may be effected with the hand or with a soft cloth; in either case a sheet of paper is interposed. The mounted paper should be allowed to dry in a dry room before it is cut off.

The paste must be sufficiently liquid without any lumps. To prepare it, a quantity of good starch is dissolved in as small a quantity of cold water as possible, and to this solution boiling water is poured with continual stirring of the paste thus formed. A little alum, if mixed, will enhance the adhesive properties of the paste and the drawing will not be so stiff as if only paste is used.

677f. Varnishing Plans.—The plans may be varnished by applying several coats of hot isinglass size or pale glue, allowing each to dry before applying the next, and finishing with a coat of Canada balsam diluted with oil of turpentine.

677g. Restoration of Damaged Drawing Paper.—Take a wash composed of one drachm of Isinglass steeped in 2 ozs. of water for 12 hours. Then simmer it for 15 to 20 minutes over a fire. When nearly ready, add common alum in powder 20 grains, strain through linen for use, apply it to the back of the paper when the paper is on drawing board and damp, and work it on, with a flat brush; when dry, wash the paper over with water, which will indicate whether a second wash of the above is necessary. When the paper is thoroughly recovered, wash it well with plain water, and a flat brush, to take off any superfluous Isinglass, absorbing the superfluous water with a clean linen rag. When thus prepared the colors can be thrown in, as safely as on good paper. A paper will be considered damaged when it will soak water like a blotting paper.

677h. Preservation of Drawing Paper.—Drawing paper should be wrapped up in flannel and kept closed up in a round tin-case to prevent its being affected by the dampness of the atmosphere.

677i. Joining Sheets of Paper.—In joining two sheets of paper together, by over-lapping, it is necessary, in order to make a neat joint to feather-edge each sheet; this is done by carefully cutting with a knife, half-way through the paper near the edges, and on the sides, which are to over-lap each other, then strip off a feather-edged slip from each, which being done dexterously, the edges will form a very neat and efficient joint when put together. About one-third of an inch overlap is enough.

678. Tracing Paper is a preparation of tissue paper. Most of that sold tears so readily as to be of comparatively little use, except for tracings to be enclosed in letters for mailing. Some are quite stout and strong, and good for line drawings and may be folded and stowed away conveniently but if likely to be of frequent use they should be mounted on cloth or on paper and cloth. They shrivel badly under broad washes of color, even when stretched forming little puddles which make them difficult to produce a uniform tint. Size, 40" × 30", and also in rolls of 20 yards, in widths of 30" and 40".

Tracing Paper, prepared from double-crown tissue paper by lightly and evenly sponging over one surface with a mixture of one part of raw linseed-oil or nut-oil and 5 parts of turpentine (5 gills of turpentine, and one of oil, will go over about $1\frac{3}{4}$ quires of 24 sheets), thus rendered transparent and qualified to take ink-lines on, and tinting, without spreading, is when placed over a drawing already executed the drawing becomes distinctly visible from above and may be copied or traced by draftsmen easily. Another method of preparation is to dissolve $\frac{1}{2}$ lb of white rosin (*ral*) in a saucepan on a char-coal fire and pour in 1 bottle of turpentine, stir with a palette knife till the rosin mixes with the turpentine. When it cools, coat the tissue paper with a camel-hair brush. Turpentine should not be added to the rosin while on the fire, as the ingredients might explode.

678a. Tracing Cloth, usually called tracing muslin, and sometimes vellum cloth, is altogether preferable to tracing paper on account of its durability and great strength. Widths, 30", 36", 43", 48", 54", length 24 yards in a roll. It is a similar preparation of linen as the tracing paper. In tracing, Indian ink is to be used. Neither the cloth nor the paper takes colors as kindly as the drawing paper. In coloring drawing on tracing cloth, or paper, color should be put on the side opposite to that on which ink lines are drawn.

679. Sectional Papers and Cloths (Fig. 203).—Paper and cloth carefully ruled into small squares so that the divisions answer for a scale for the drawing is exceedingly useful for sketching out plans. They are ruled in 20, 12, 10, or 8 divisions to the inch. The sizes of the papers divided in fine gray lines are :—Royal;

24" $\times$ 19"; Medium, 22" $\times$ 17"; and Fools-cap, 16" $\times$ 12"; also in continuous rolls of 10 and 50 yds by 26". Mounted sectional papers can be had in rolls of 10 yds. by 26"; sectional tracing cloth and papers in rolls of 10 yds by 26". Metric scales are also used.

680a. Indian or Chinese Ink.—A good ink should be black, perfectly water-proof, working smoothly and free from grits which are fatal to clear lines. It dries quickly, does not corrode the pen. The usual size sticks sold run from 4 to 80 pieces to the pound (Fig. 313).



Fig. 313.—Stick Indian (properly speaking, chinese) Ink.

680b. To rub up Indian-ink from the Stick.—Put a few drops of water on the palette and keep rubbing lightly with a circular motion gradually keep adding a drop of water, and rub until the required consistency is obtained. Ink rubbed from a stick is to be preferred, specially if it is to be diluted to obtain gradations in brush work. To test whether the liquid ink thus prepared is of proper consistency and required degree of blackness blow over the ink on a white saucer, or draw a line on paper and smear it. In the first case, if it leaves the saucer at the place blown over, or if by either method the ink exhibits a brownish tint instead of a black, then more rubbing is required, and the rubbing must be continued until the ink stick to the saucer at the place after it is blown over again or the ink lines drawn show a jet black tint. The stick Indian ink, after rubbing, should be dried immediately, or it will work gritty when next used and will run if colored upon. Keep the ink covered up, and, if possible renew it by rubbing up fresh every day. To make the ink quite **fast** put in a drop of a not very strong solution of bichromate of potash into the ink in order to color over the ink lines. Too much of the solution will turn it brown.



Fig. 314.—Liquid Indian Ink.

680c. Liquid Indian Inks are now made by different makers (Fig.

314), some are good, others indifferent, and some few really bad.

Good ink of this class should be dense black, and permanent, flowing freely, thoroughly water-proof, and that will not coagulate. Chin-Chin, Streuber's, and Higgins' are good. A good liquid ink is most essential, being more economical than the stick, and it saves much time, and as there is no rubbing up it is always ready for use, Pint or quart bottles are the cheapest, and from these the smaller bottles in use can be filled from time to time as required. The bottles should always be kept corked when the ink is not being used, otherwise it rapidly thickens. Water-proof inks are largely made, but for general work are inferior to the ordinary ink, as they tend to dry rapidly and clog the pen. *Inferior inks* which run when the drawings are colored, must not be used. If the ink run, the whole drawing must be washed with clean water, which will remove all free ink. The water must be carefully removed by means of a sponge and a piece of blotting paper.

681a. Hard and Moist Water Colors.—The best, made by Winsor Newton, are to be had, either moist, in pans and half-pans



Fig. 315.—Whole Cake Color.



Fig. 317.—Moist Color Whole Pan.



Fig. 316.—Moist Half Tube Color.



Fig. 318.—Liquid Color.

(Fig. 317), tubes and half-tubes (Fig. 316), or hard in sticks or cakes and half-cakes (Fig. 315), and in every variety of color and tint. Hexagonal stick colors have come largely into use, they are cheaper in proportion than the ordinary cakes, and are convenient to handle for rubbing down, and are preferable for technical drawing, but moist colors in pans are more readily mixed, and are rapidly displacing sticks and cakes of which the most soluble, and brilliant are the best.

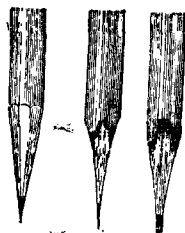
681b. The Liquid Colors (Fig. 318) in nearly all tints are becoming popular, and certainly are a great convenience when compared with the preparing process each time the color is to be used. The advantages claimed being that the same tint is always secured and which cannot be relied on if rubbed up each time as required. *Liquid water-proof colored ink* works agreeably with

the brush or pen, always remains fluid and ready for immediate use. The colors can be thinned to any degree with water and they are much appreciated. Streuber's and Higgins' colored drawing inks are good.

681c. Mixing Colors.—Colors are generally mixed but this method is not to be recommended, as it is not easy to obtain the desired tint, and then to match it again. When mixing colors sufficient quantity must be prepared for the work in hand. The whole of the mixed tint should be stirred with a brush every time before use. Some of the colors are obtained by mixing other colors. Payne's gray: indigo, Indian ink, and crimson lake; Hooker's green (for glass in section or grass plots): Prussian blue and gamboge; Burnt-umber. Sepia and burnt-sienna; Indian yellow: gamboge and pale burnt-sienna.

682a. Lead Pencils.—Those marked HB are used for notes and sketches; HH, HHH pencils are used for drawing lines intended to be inked in. F for drawing in the result with a firmer line.

682b. Pencil Sharpening.—A pen knife and a smooth flat tapering file should be used for the purpose, or a piece of fine glass-paper glued to a strip of wood is a ready means of restoring the fineness. For ordinary use a round fine point should be made (Fig. 319), but for drawing lines against straight-edges, with hard H, HH, pencils, a chisel point (Figs. 320 & 321) is preferable, as it will not blunt the edge so much as it would the round point. Pencils in compasses should always be chisel-pointed.



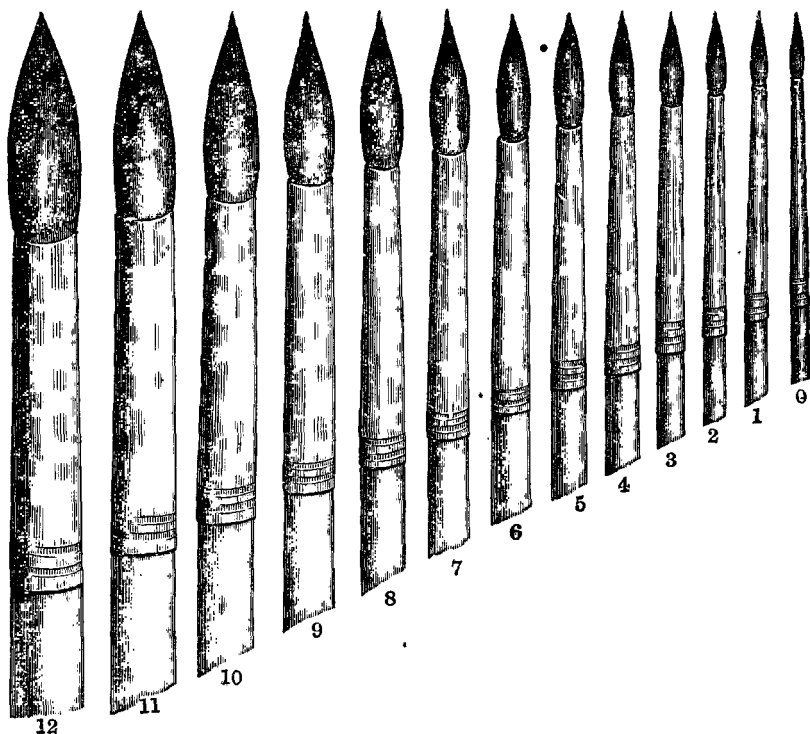
Figs. 319, 320, 321.
—Pencil Sharpening.

683. Erasers, for either ink or pencil, or combined, are to be had in every conceivable shape. Vulcanised India-rubber is found best for erasing pencil marks and cleaning off drawings. *Ink-eraser* is made of vulcanised-rubber and powdered glass and used for erasing or lowering the tone of color of hard ink lines or removing dirty marks or stains. Small sandpaper blocks are frequently used for erasing and pointing pencils. A pen or erasing knife is also used for erasing ink lines. A better means is to use a piece of glass paper (Oakeys No. 1) folded several times until it presents a round edge; this leaves the surface of the paper in much better order to color upon than it is left from knife erasure. A kind of size known as erasure fluid may be applied with a brush to prevent color running upon the erased part.

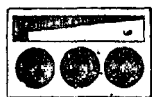
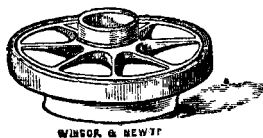
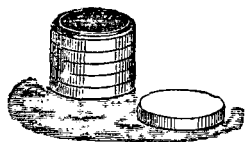
684. Brushes.—Best sable or camel-hair brushes of different degrees of fineness having good sharp points are required for laying on the color on the drawing. Red sable is the best. A good brush, if rinsed in water should immediately form a well-defined conical point and be springy (Figs. 322-334).

685. China Palettes, Saucers.—These are required in assorted sizes for rubbing colors on (Figs. 335-339).

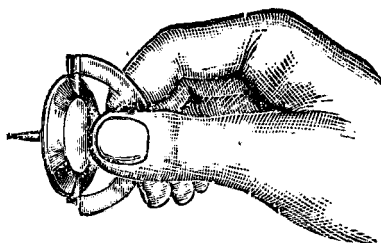
686. Drawing Pins are stout steel-pins with circular brass or electrum heads of $\frac{1}{2}$ " to $\frac{3}{4}$ " diameter with thick milled-edges used for securing the paper on a board at the 4 corners (Figs. 340-343).



Figs. 322-334.—Colour Brushes of Different Degrees of Fineness.



Figs. 335-339.—China Palettes and Saucers.



Figs. 340-343.—Drawing Pins and Lifter.

687. **Mapping Pens**,—Gillot's mapping pens and lithographic crow-quills are the best for printing purposes.

688a. **Drawing Boards**.—The qualifications requisite in a first class drawing board are that it must be made from best selected wood, yellow pine, sequoia, teak preferably, thoroughly well-seasoned, and effective means must be taken that it will remain true under varying conditions of temperature. To allow the expansion and contraction of the fibres to take place without materially affecting the surface or squareness of the board, the board is made up of narrow pieces of selected planks, glued together with the heart side of each piece to the surface, with mahogany or teak battens screwed to the back side. The screws pass through the battens in oblong slots bushed with brass, which fit closely under the heads, and yet permit the screws sufficient play to move freely when drawn by the contraction of the board. To give the battens power, to resist the tendency of the surface to warp, a series of grooves are sunk in half the thickness of the board over its back. These grooves take the transverse strength out of the wood and allow it to be controlled by the battens without impairing the longitudinal strength of the wood. To obviate the unpleasantness to work against the end grains at the working edges with the T-square, a slip of ebony is glued in, which is afterwards sawn apart about every inch to admit of its contraction with the body of the board. (a'. Fig. 344 shows a Drawing board fitted with hardwood battens, slotted and fitted with metal plates and screws, saw-cuts along back, and having ebony-edge. (b) Fig. 345 indicates a drawing board fitted with battens on back by means

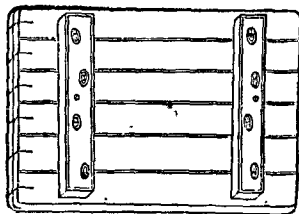


Fig. 344.—Improved Drawing Board.

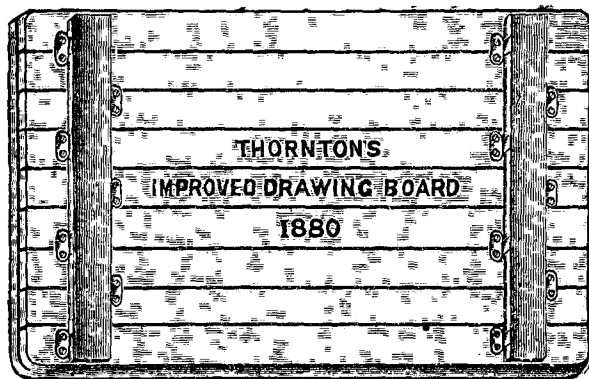


Fig. 345.—Improved Drawing Board.

of brass clips which work in grooves on each side of the battens.

Ebony working edge projects about $\frac{1}{4}$ " beyond the board's edge, thus giving a solid ebony for the T-square to work on. Saw-cuts along back, and grooved battens allow any shrinkage or expansion in the wood. This construction is better than the one shown in Fig. 344 above.

688b. Triplex Drawing Boards are made of three layers of well-seasoned wood, the centre one being placed transverse to the other two, firmly glued together under great pressure. By this method the liability to warp and twist is reduced to a minimum. Fig. 346 shows a triplex board with hardwood battens slotted to allow for expansion. Sizes: 15" x 11", 20" x 15 $\frac{1}{2}$ ", 23" x 16", 28" x 19", 31" x 23".

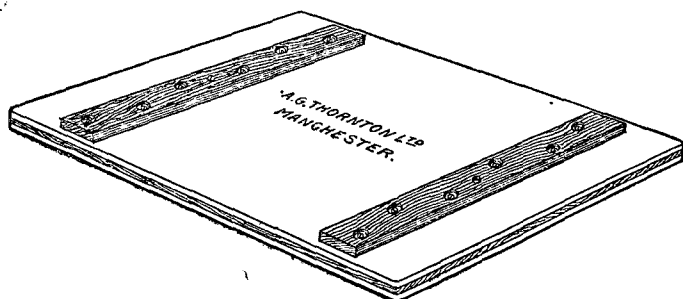


Fig. 346.—Triplex Drawing Board.

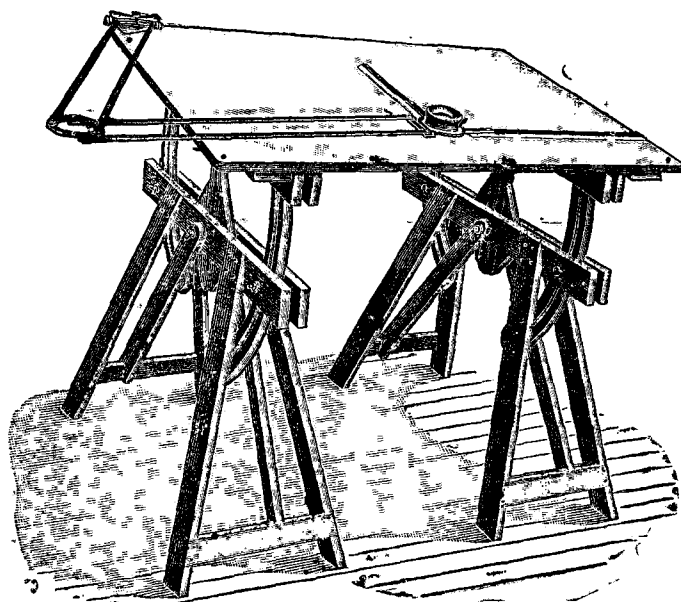


Fig. 347.—Universal Drafting Machine.

688c. John Davis & Son's Universal Drafting Machine.—It (Fig. 347) consists of a graduated square having an accurate

parallel motion about the drawing board. Both blades of the square are graduated for drawing and measuring lines at the same time. A straight-edge is provided for inking. It may also be used for long lines. The parallel motion is obtained by means of two pivoted parallelograms which constitute an arm joining the square to the board. This arm allows the blades to lie flat on the drawing. It is hinged at the connection to the board, and may be easily lifted out of the way of the paper. It may be changed from one board to another.

Adjustable Trestle.—In Fig. 347, below the board, is shown a trestle which may be raised from 2' 3" to 3' 4", and may be canted to any desired angle; it is provided with slotted carriers and thumb-screws.

688d. Paper Stretching Drawing Board.—The board (Fig. 348) is grooved all round about one inch from edge. Strong, galvanised rods are made to fit into these grooves, and filed out at the corners, with screws at each end, which go through and are fixed on the back of the board with wing-nuts, so that the rods are flush with the surface of the board, when the paper is stretched, thus allowing the T or Set-square to be used.

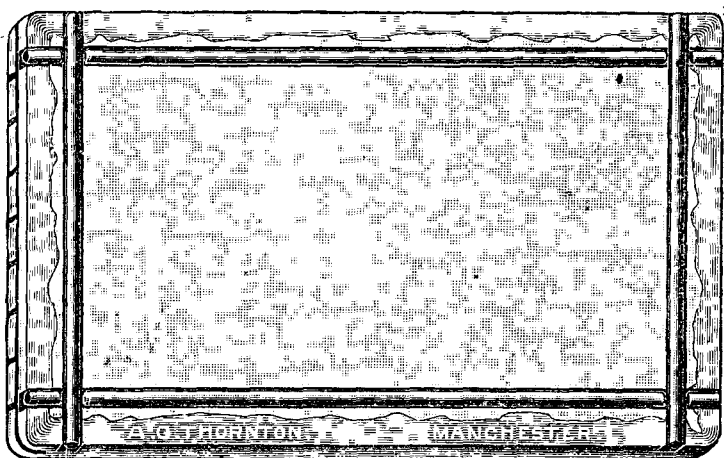


Fig. 348.—Paper Stretching Drawing Board.

Directions for Use.—Take rods out of grooves, thoroughly wet sheet, place it in the centre of the board, so that the rods will grip the edges all round, press the rods in the grooves, fix wing-nuts on back; allow sheet to dry, and it will then be found stretched perfectly flat.

688e. Stanley's Drafting Machine.—It consists of a mahogany frame which carries the whole of the apparatus, and any board of the size may be slipped into position or removed instantly (Fig. 349). A machined steel bar running upon rollers is so mounted that it may be moved over the surface of, and is always parallel with

the top and bottom of the board. Upon this, a carriage is moving easily along the steel bar; this carriage carries the square which carries the two scales, set at right angles to one another, so that by the combined movements of carriage and bar, the square can be moved to any position on the drawing, and the scales always retain their original angle. The two scales serve also as drawing edges

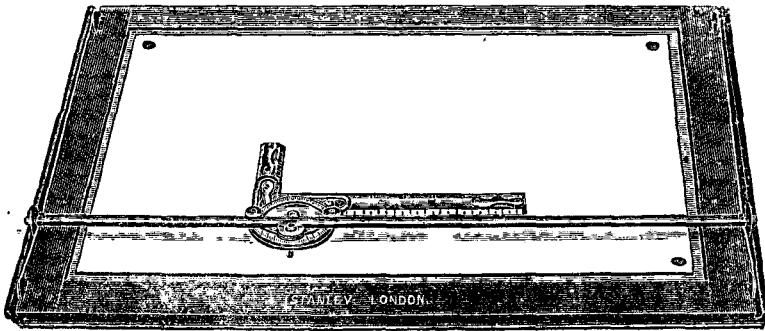


Fig. 349.—Drafting Machine.

for pencil drawings, and a straight may be substituted for inking in. Lines at any required angle may be drawn, as the Scale Holder may be revolved. It reads to a divided arc, and stops are provided for instantly locking at 30° , 45° , 60° and 90° , and a clamp for other positions, therefore all the straight line work of a drawing can be put in with this machine.

It is made in two sizes.—To take $31\frac{1}{2} \times 23$ Drawing Board (Imperial)
 „ 42×25 „ „ (Double Elephant).

688f. Adjustable Drawing Board Frame (Fig. 350) is in-

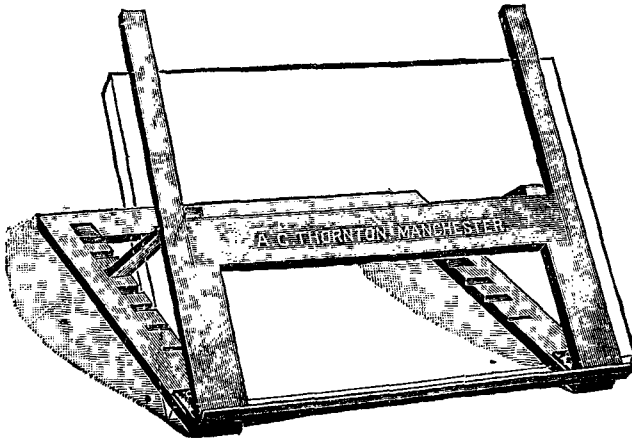


Fig. 350.—Adjustable Drawing Board Frame.

tended for use on a table or trestle, and has the advantages of being fitted with a notched rack arrangement so that the Drawing Board can be tilted to

convenient angle—the illustration clearly indicates the idea—in this form very portable, and no doubt by many will be considered a more useful addition to the ordinary Drawing office equipment.

689. T-Square.—It is a straight-edge or blade, *a* (Fig. 351), usually made of mahogany, ebony, rose or pear wood, fitted at one end with a stock, *b*, at right angles. The stock being so formed as to fit and slide against an edge of the drawing board, the straight-edge slides over the surface, and presents an edge at right angles to the edge of the board by which parallel lines may be drawn upon the paper. T-squares are made in lengths of 18" to 54"; the edge should be $\frac{3}{8}$ " in section, and the stock 2" $\times$ $\frac{5}{8}$ ". Each drawing board should have a T-square an inch shorter than its length.

Counter-sinking the blade flush with the stock or head is now abandoned, it is now found useful to have the blade raised on the head, admitting of the squares working close to the stock to prevent drawing the better the blade is to taper at the end as shown in Fig. 351. Some T-squares are made in metal which is objectionable, as with handling, it rusts and spoils the drawing surface. Ebony is also objectionable unless thoroughly seasoned, the sap of the wood, which comes off, soils the paper. Thoroughly seasoned pear wood is the best for T-squares, but mahogany seasons the quickest. Mahogany blades are made with one or transparent celluloid edge polished and keyed to prevent attachment at the end.

689a. Shifting Head T Square.—One-half of the stock, *b*, is sometimes made loose to turn upon a brass pin to any angle with the straight-edge, *a* (Fig. 352), and to be tightened by a screw and nut. The moving stock is used for drawing parallel lines obliquely to the edges of the board.

T-Squares should not at all be varnished like other wood instruments employed in drawing. The edges should be slightly rounded to allow the hand to play freely but not to be chamfered.

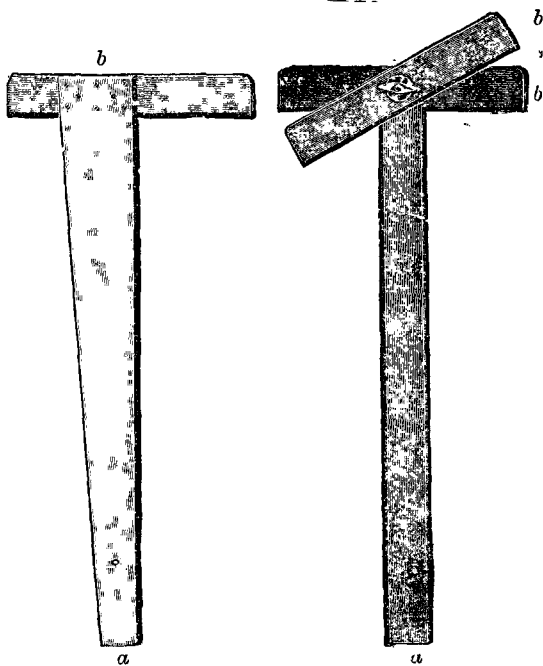


Fig. 351.—T-Square by Watts & Son.

Fig. 352.—Shifting Head T-Square.

689b. Roller Attachment for T-Squares.—The contrivance consists of a small roller, (Fig. 353), in hemispherical outer casing fixed close up to the stock of the square; the roller projects slightly below the under surface of the blade, and bears on the drawing board or on the paper placed thereon. The outer casing while carrying the roller, also shields it from the hand. By this attachment friction is reduced and the drawing is maintained in a clean condition. When the board is placed on a slope the Square with roller fixed does not run down as might be expected nor does the roller project sufficient to allow of set-squares getting under the blades.

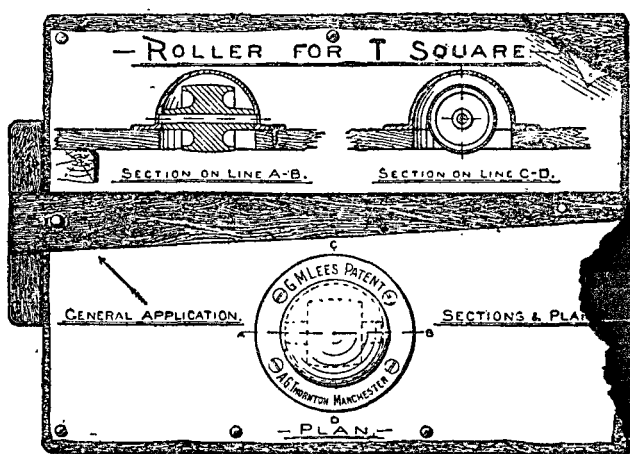
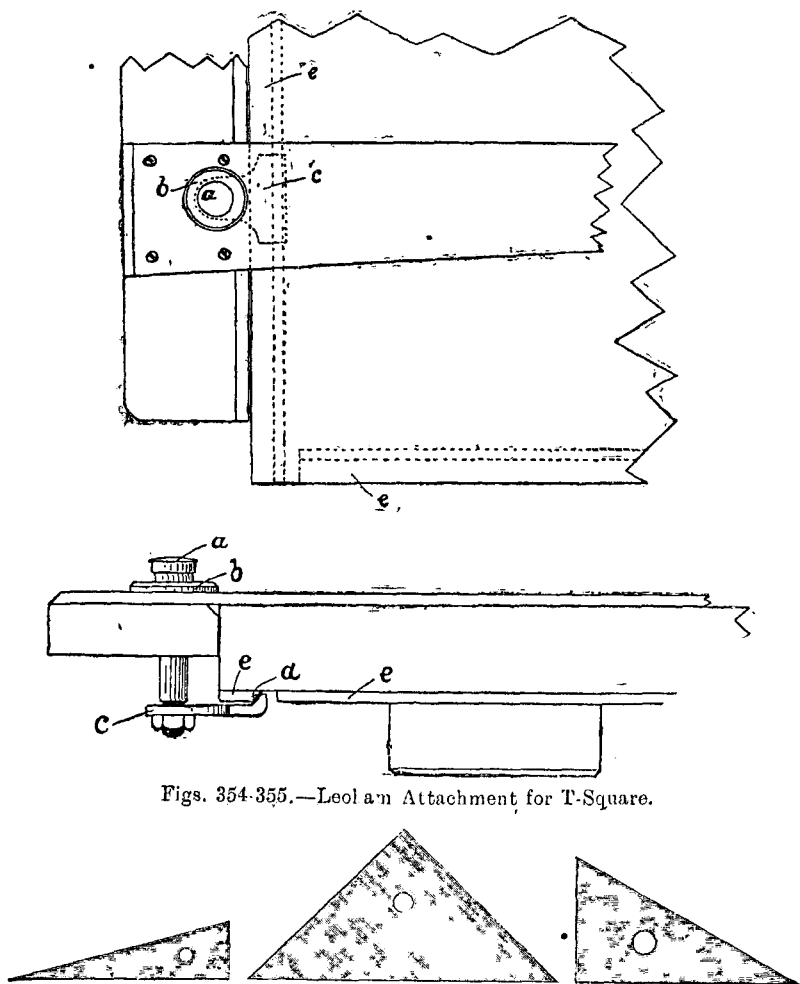


Fig. 353.—Roller Attachment for T-Square.

689c. The Leoham Attachment.—This is a device to keep the stock of the T-Square firm and true to the edge of the drawing board during drawing, particularly when both hands are required, in working set squares. By a slight pressure of the finger, it is free to move over the board as the ordinary T-Square. Referring to illustrations: Fig. 354 is a plan of part of a T-Square and drawing board fitted to this invention; Fig. 355 is an elevation thereof. *a* is a metal piece moving in the sleeve, *b*, on the lower part of *a* is a clamping shoe with a bevelled portion, *d*, which bears against the corresponding bevelled surface of the strip, *e*. In the push piece, *a*, is a well-tempered spring that draws the clamping shoe, *c*, upwards, which, it bears against the bevelled surface, *e*, draws the stock of the T-square to the edge of the drawing board, and thereby keeps the T-square true and firm.

To work the T-square place the left-hand on the stock as in the ordinary way of moving same, the device is so placed that the thumb as it rests on the top of the stock can also easily rest on the push piece, *a*. Then a slight pressure of the thumb instantly enables the T-square to be moved with the utmost nicety; releasing the push piece, *a*, clamps the T-square instantly. A full depression of the push piece, *a*, and the T-square may be removed from the drawing board.

690. **Set-Squares:**—These (Figs. 356-358) are small Δ r pieces of pearwood, transparent xylonite, celluloid, or vulcanite, having one of their $\angle$ s a right angle, the rest either 45° and 45° , 60° and 30° or $67\frac{1}{2}^\circ$ and $22\frac{1}{2}^\circ$. They are available in sizes of from 4" to 16" in sets containing 2 of 45° of different sizes, 2 of 60° , and one of $67\frac{1}{2}^\circ$ in a card-board box. These instruments are most useful in conjunction with a T-square and a Drawing Board for drawing lines $\parallel$ or $\perp$ to others; or two set-squares may be used for the same purpose.



Figs. 354-355.—Leola'n Attachment for T-Square.

Figs. 356-358.—Set-Squares.

690a. **Uses.**— a_1de and abc are a pair of set-squares. The edge a_1e of the one has been set against the edge ab of the other, making $a_1d \perp ab$. If the set-square a_1de be moved, and set as $a_2d_1e_1$; and a line be drawn

along a_2d_1 , it will be $\parallel a_1d$. The set-square can also be used along the edge of a T-square or on any st. edge as of the Marquois scale. The figure shows also different positions of the squares (Fig. 359.).

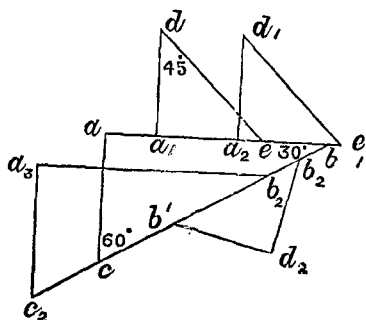
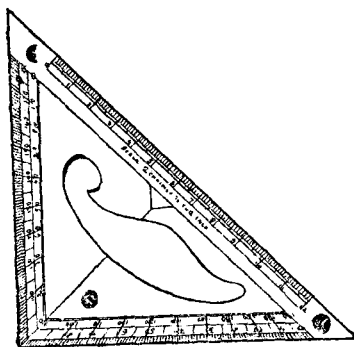


Fig. 359.—Uses of Set-Squares.



Figs. 360.—Protractor Set-Square.*

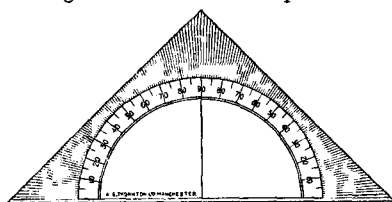


Fig. 361.—Protractor Set Square.

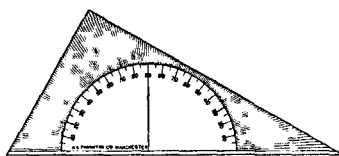


Fig. 362.—Protractor Set-Square.

690b. Protractor Set-Squares.—Fig. 360 is a 6 in. set-square, with combination of scales, protractor, curve, and 2-chain scale, made of celluloid, brass or vulcanite. Figs. 361-362 are 45° and 60° protractor Set-squares of celluloid.

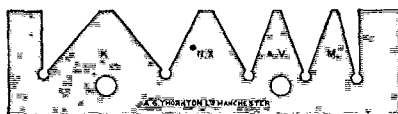
690c. Lettering Set-Squares (Figs. 364-366) are available in sets of 3, either of transparent celluloid or of vulcanite. Fig. 363 illustrates another idea (q.v., p. 397) which adds to the usefulness of a transparent set-square, so that a drawing or tracing can be lettered without drawing lines. Simply keep this set-square tight on the T-Square, hold the pencil at right angles to the board and work inside the cuts shown, for size of letter required. Fig. 367 shows the American pattern made of vulcanite or of transparent celluloid.

Fig. 367.

Fig. 364.

Fig. 365.

Fig. 366



Figs. 364-367.—Lettering Set-Squares.

690d. Framed Mahogany Set-Squares (Fig. 368) with solid ebony edges retain their angles correctly from the fact of the

grain of the wood running longitudinally on all sides. The center, being open, makes them lighter, and obscuring less of the surface of the drawing than the solid form.

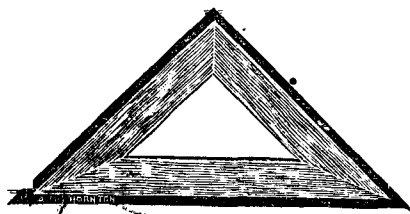
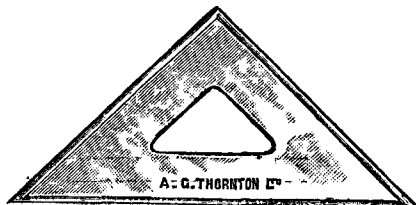
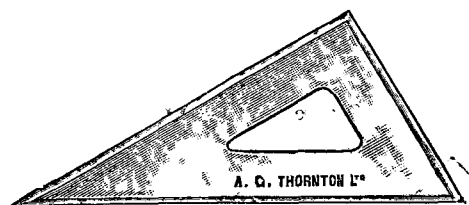


Fig. 368. — Framed Set-Square.

the drawing beneath the set-squares, when in use, is evident. The centre being cut out is an advantage making them lighter.

690e. Thick Celluloid Set Squares with Cut-out Centres and with Bevelled Edges on one or Both Sides. (Figs. 369, 370), $\frac{1}{10}$ " to $\frac{1}{8}$ " thick, are made. The great value of seeing



Figs. 369-370. — Set-Squares with Cut-out Centers and Bevelled Edges.

690f. Transparent Cornering Set-square (Fig. 371) will be found a great advantage with the useful additions shown, apart from being used in the ordinary way. Size: 6 ins. side, 45° angles.

To use it, place the square angle, 90° , of the set square on the corner of the tracing; then run a pencil point into and along one of the cuts according to the width of the border required, repeating this operation at each corner of the tracing. The figures on the Square show the width of border required, in inches, which the various cuts will give.



Fig. 371. — Cornering Set-Square.

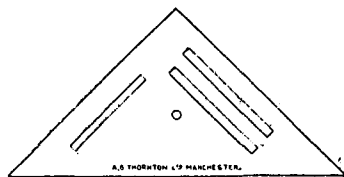


Fig. 363. — Lettering Set-Square.

690g. Universal Adjustable Celluloid Set-square (Fig. 372).—Its chief features are the white celluloid (divided) arc, which is read with a transparent cursor in the same way as a slide rule, thus enabling the square to be set to any angle; and the jointless edges of the square. Size of arm is 7" to 12".

690h. Universal Set-square (Fig. 373) for hatching, ruling calligraphing, etc., is used by keeping the square, A, against the T-square, B, and by putting vertically in one of the conical holes the point of a hard pencil, and drawing the line by keeping the square

against the guide. For circles use the first hole of the scale as centre.

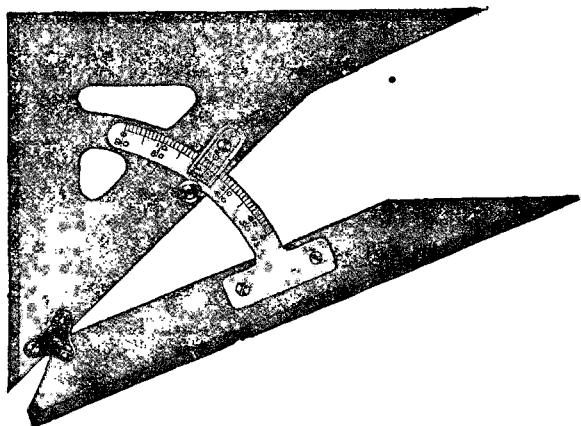


Fig. 372.—Universal Set-Square.

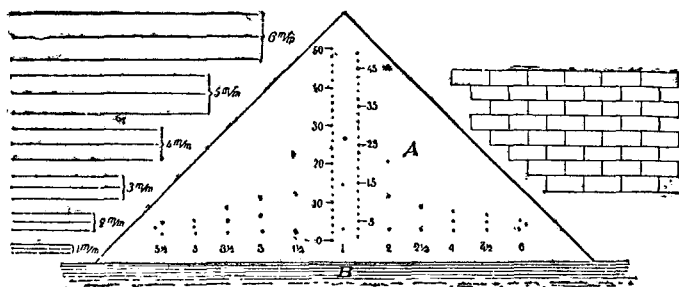


Fig. 373.—Universal Set-Square.

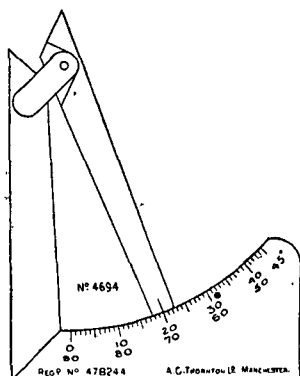


Fig. 374.—Clinograph.

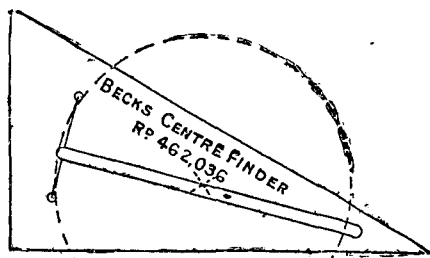


Fig. 375.—Centre Finder.

690i. The Clinograph or Duplex Angle is a set-square with a swivelling blade moved and retained in any position by friction; and is used in conjunction with a T-square for drawing lines $\parallel$, or $\perp$, or symmetrically inclined to any given line or direction. It is

made of pearwood, or mahogany polished, or mahogany with ebony-edge or from boxwood brass-mounted, divided on both sides to 45° (Fig. 374). This is used for drawing equal angles in opposite directions.

690j. Beck's Centre Finder (Fig. 375) is a transparent celluloid (ordinary 8 in., $60^\circ \times 30^\circ$) set-square, and also gives 75° and 15° . By placing the two small circles tangentially upon an arc of a circle, a line drawn along one edge of the slot will pass through the centre, and by repeating the operation from a different position the intersection of the two lines will give the point required.

691. Parallel Rules or Bars (Fig. 376).—These are constructed in lengths of from 6" to 2'. The common ebony, boxwood, or ivory parallel rule is made of two straight-edges which are joined together on their upper surface by two parallel and equal electrum or brass bars which always maintain their parallel position, the bars moving freely on their respective centres and greatly facilitates drawing of parallel lines.

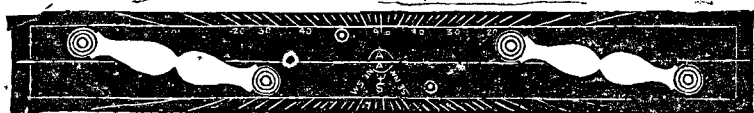


Fig. 376.—Watts & Son's Capt. Field's Protractor Parallel Rules.

To draw a line parallel to a given straight line.—Set the edge of the uppermost rule to the given line, press the edge of the lower rule tight to the paper with one hand, and with the other move the upper rule, till its edge coincides with the given point; and a line drawn along the edge through the point is the line required.

691a. Section Parallels (Fig. 377).—It can be adjusted to the greatest nicety, and any number of lines can be drawn at equal distances. The sectioning arrangement can be removed by releasing the screw holding the slotted plate, when it can be used as an ordinary parallel rule. It is made of ebony with brass mounts. Size : 6 to 24 ins.



Fig. 377.—Section Parallels.

691b. Rolling Parallels or Rulers (Fig. 378) made in lengths, of 6" to 24", is a useful instrument for drawing $\parallel$ lines. It may be used instead of a T-square. It is a rectangular parallelogram of ebony, brass-mounted, or of gun-metal, or of electrum, supported by two brass wheels, which are connected together by a long axis, the wheels being exactly of the same size; and their rolling surface $\parallel$ the axis. When they are rolled backwards and forwards, the axis and

the rule will move in a direction || themselves. Ebony rulers have white edges divided in $\frac{1}{2}$ " $\frac{1}{4}$ " $\frac{1}{8}$ " $\frac{1}{16}$ " to the foot.



Fig. 378.—Parallel Ruler.

691c. Section Liner.—Equally spacing rulers are very useful (Fig. 379). By means of an adjusting screw in the centre of spring lever the distance required between each line is first set, then simply pressing upon the setscrew on the top of the movable folder,

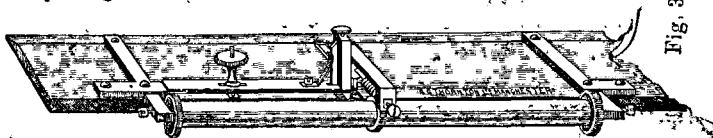


Fig. 379.—Section Liner.—Size : 10" with Ebony or Electrum straight-edge.

In rolling these rules, one hand only must be used; and the fingers should be placed nearly on the middle of the rule so that one end may not have a tendency to move faster than the other. The wheels only should touch the paper when the rule is moving, and the surface of the paper should be smooth and flat. This rule is sometimes made with slips of ivory laid on the edges and divided into inches and tenths, as shown in Fig. 378.

692. Drawing Instruments.—A good electrum instrument box contains:—(1) a 6" sector-jointed compasses with movable lengthening bar, plain steel points, pen and pencil legs, (2) a 5" sector-jointed hair-divider, (3) a $3\frac{3}{4}$ " bow pen, (4) a $3\frac{3}{4}$ " bow pencil, (5) a 6" jointed and a $4\frac{1}{2}$ " plain ivory-handle drawing pens, (6) one 6" ivory protractor, (7) one 6" ivory parallel rule, (8) one pricker, (9) one eraser knife key and file, (10) 3-3" spring bows, (11) 4 drawing pins, (12) one 6" proportional compasses, (13) one 6" ivory sector, (14) some curves, (15) a pair of small set-squares, (16) Ivory Lead-box, etc.

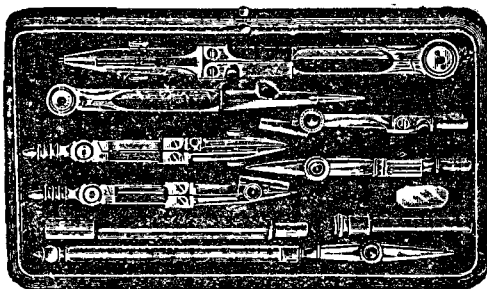
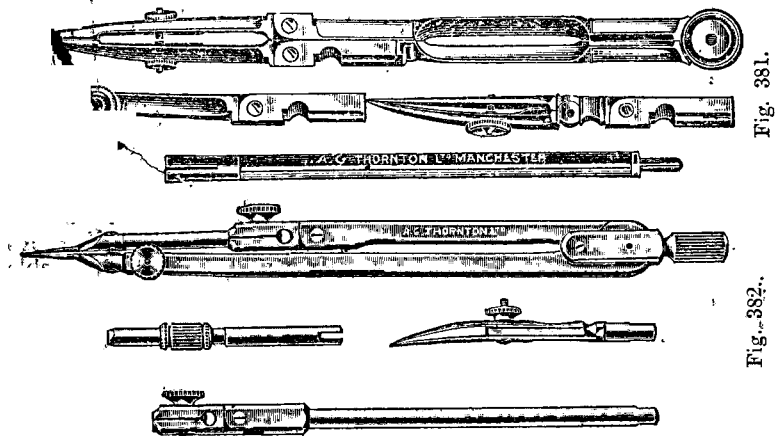


Fig. 380.—A Small Case of Drawing Instruments.

Of these 1, 2, 3, 4, 5, 6 and 7 are enough for a surveyor's use in office. In the field Nos. 2, 4, 6 and 7 are only necessary. Cheap cases are also sold of either brass, or electrum but they do not contain all the above instruments. A small serviceable case of drawing instruments, containing: (1) a pair of compasses with

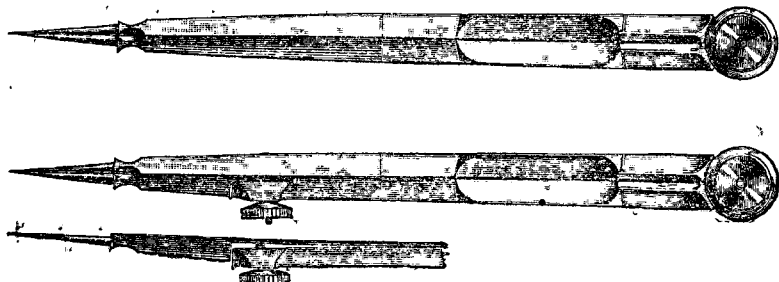
movable legs, and a lengthening bar, (2) a 6" rectangular ivory protractor, (3) a 6" parallel ruler, is absolutely necessary for field use. Fig. 380 shows a small case of Drawing Instruments.

692a. Drawing Compasses (Figs. 381-382) are instruments for describing circles. These have one or both of the legs removable. When the movable point is taken away, a socket is left, into which a point to carry ink or pencil may be placed. Fig. 381 shows half sets of compasses, $6\frac{1}{2}$ or $4\frac{1}{2}$ ", with double steel joints and with combo needle points, and Fig. 382 shows Continental pattern of half sets, $6\frac{1}{4}$ ". Both with pen, pencil and dividing points, opening bar and key.



Figs. 381-382.—Drawing Compasses, Lengthening bar, Pen, Pencil Points, etc.

692b. Dividers are compasses terminating in two points and employed to divide spaces into equal parts, to set off or transfer distances from a plan to a Scale, or *vice versa*, and to copy drawings (Figs. 383-384). *Hair-Dividers* (Fig. 384) differ from plain dividers (Fig. 383) in having one of the points movable for a short distance

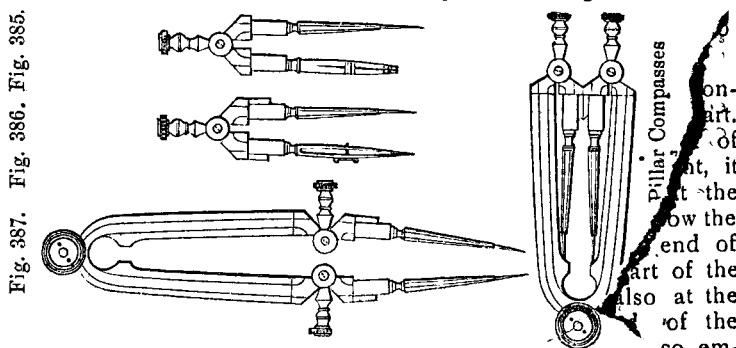


Figs. 383-384.—Dividers, Plain and Hair.

by a screw adjustment, which enables the draftsman to adjust the point with greater delicacy than could be attained by moving the

head joint only. The points of dividers should be sharp and fine and when closed should fall over each other forming as it were one point.

692c. Pocket Dividers.—Two other forms of pocket dividers and compasses in common use, termed **Pillar compasses** and



Figs. 385-388.—Pillar Compasses.

dividers (Figs. 385-388), and **Napier Compasses and dividers** (Figs. 389-390) are constructed with knuckle joints which enable the points to fold in to render the compasses portable as shown in Figs. 388, 390.

Fig. 389.

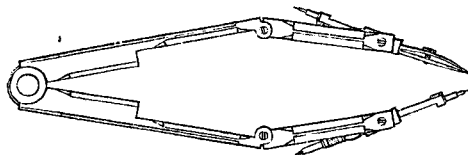
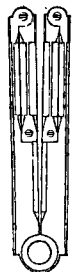


Fig. 390.—Napier Compasses folded.



Figs. 389-390.—Napier Compasses.

692d. Universal Compasses (Fig. 391) forms the divider ruling pen, pen and pencil compasses and is a portable instrument.

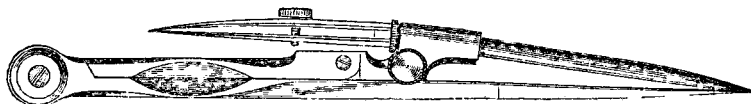
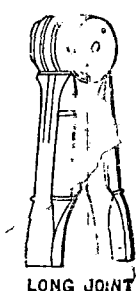


Fig. 391.—Universal Compasses.

692e. Joints of Compasses or Dividers.—The compasses or dividers consist of two pointed branches or legs usually jointed at the top by a rivet on which they move. Compasses admit of many refinements and vary in construction in many ways. The first consideration is a perfect top joint, in making which, there are 4 distinct methods, one is called the *long-joint* (Fig. 392) and the others: the *sector-joint*, (Fig. 393), the *disc-joint*, (Fig. 394), the *pivot-joint*

(Fig. 395, 396). In long-joints, the head extends some distance down the body, hence, with the closing of the compasses a larger amount of surface comes into action, producing much greater friction and stiffness when the instrument is nearly closed than when wide open. It is used for common instruments for it can be easily made. The sector-joint is now made to all good compasses. In this joint the working surfaces are circular in form, equally distributed round the centre; hence the compasses move, with equal pressure, whether nearly closed or wide open. It is not necessary to screw the centres



LONG JOINT



SECTOR JOINT

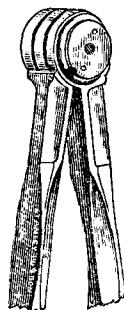
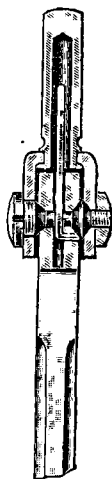


Fig. 394.—Disc Joint.



Fig. 395-396.—Pivot Joint.



of sector-jointed compasses tightly, as the surfaces are not required to come in perfect contract—for this reason, that after the sector-joint is made, the workman lubricates between the joint with hot beeswax, which aids in producing that deadness in movement so much esteemed in the sector-joint. If the joint be made true, the wax will never leave it. Care should be taken that no oil gets into the joint as it speedily dissolves the wax and spoils the joint.

The Disc-joint (Fig. 394) combines some of the qualities of the long and sector joints. This is constructed like the long joint, but it is not extended beyond the true joint. Instead of steel plates soldered into the sides of the compasses, 2 circular steel plates are inserted between the parts.

The Pivot Joint (Fig. 395-396) entirely suppresses any lost motion or injurious friction, and the stability of the compasses head is increased.

The top portion of the compasses is made of brass or electrum and the points made of steel. Each pair of compasses is provided with a key of which the two points fit into the holes on the face of the top joint, and by winding the joint it can be loosened or tightened at pleasure.

692f. Points of Compasses.—The plain points of compasses are either of pointed steel or are constructed to hold needles, which form the points. The plain points are sometimes left triangular to the end, but they are more generally rounded or needled. Points

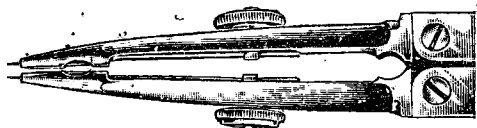
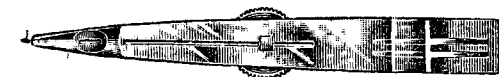
to carry needles are described and illustrated below. The plain points of double-jointed bows are often constructed to carry needles, and except being smaller resemble in every respect the points of compasses.

Screw-off Needle Points.—Fig. 397 shows the legs of a 6 in. compasses from the knee joints, full size, with reliable method of firmly securing the needle points.

Bolt-and-Nut Needle Points.—Figs. 398-399 show the contrivance for extricating a broken needle from the solid drilled part.



Fig. 397.—Screw-off Needle Points.



Figs. 398, 399.—Bolt-and-Nut Needle Points.



Figs. 400, 401.—Combo Needle Points.

compasses from the knee joints. The points are made of steel of special quality and temper. The saw-cut, as shown in the back view, is made so that it forms a spring. The solid part of the point, where the needle passes through, is grooved to take any ordinary sized needle, and grips the needle at the extreme point only, so that if the needle breaks it can be taken out without difficulty and a new one inserted. After adjustment to the setting required, the needle is secured by a bolt and nut.

692g. To Take a Distance with the Hair-Dividors open them as nearly as you can to the required distance. Set the fixed leg on the

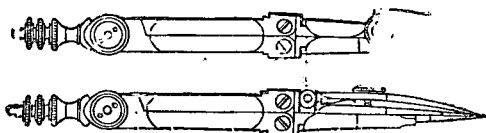
In the usual form of Bolt-and-Nut point, it often occurs that the needle snaps below the bolt, at the top end of the drilled part of the point, and also at the extreme end of the point, and is so embedded that the instrument-maker's help has to be resorted to to have the broken needle drilled out. In this arrangement a part of the underside of the drilled part is filed out (exposing the needle) sufficiently to admit of a penknife being used in the aperture, to press the broken needle upwards towards the head of the compass and, so release it.

Combo Needle Points.—Figs. 400-401 show, full-size, the points of a 6 in. com-

point from which the distance is to be taken, and make the extremity of the other leg coincide accurately with the end of the required distance, by turning the screw.

In using the compasses it should be held at the top with one leg between the thumb and the 2nd finger, the other leg between the third and fourth fingers, and with the thumb against both the legs close to the joint. In drawing concentric circles care should be taken to avoid enlarging the centre hole.

692h. Bow Pencil and Bow Pen (Figs. 402-403).—To describe small circles or arcs, a bow pencil with pointed pencil legs, and a bow pen with pointed pen legs are used. They are used in the same way as the compasses. They are either single-jointed, or double-jointed which latter are the best. They have head-joints and points like the compasses but they have handles above the head-joint, and the points are not made changeable in the way of points of compasses.



Figs. 402, 403.—Bow Pencil and Bow Pen.

692i. Beam Compasses (Fig. 404) has a beam made of ebony or well-seasoned wood, from 2' to 4' long, to which are adapted two sockets, one of which is fixed, and carries a steel point, and the other sliding which carries the pencil point. The sliding socket can be made to slide up and down and can be clamped to any part of it by a milled-headed screw. Along the beam is engraved a scale of inches. These compasses are very useful when the radius of the circle or arc required is too large for ordinary compasses to be used, and for setting off lines, and measuring long lengths required in laying down co-ordinate or graticule lines for the projection of large maps.

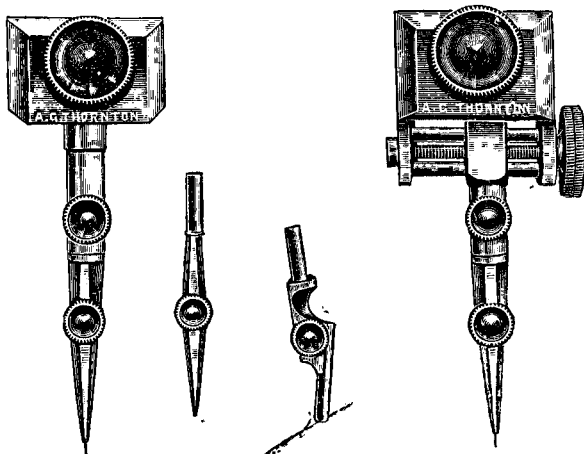


Fig. 404.—Beam Compasses.

692j. Beam Compasses (Fig. 405) will be found the lightest, and quickest in action. The heads are formed by two runners attached by a flexible strap which forms a bow and has sufficient spring to cause the runners to grip the beam tightly, but if compressed slightly, by pressure of the thumb and finger instantly releases the runners and allows the head to be easily moved and

adjusted to any desired position. They are supplied with the pair of heads in electrum, two steel points, one electrum pencil point and one pen point and 18" mahogany beam.

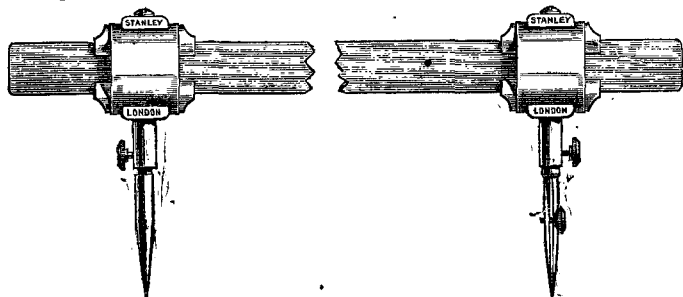


Fig. 405.—Beam Compasses.

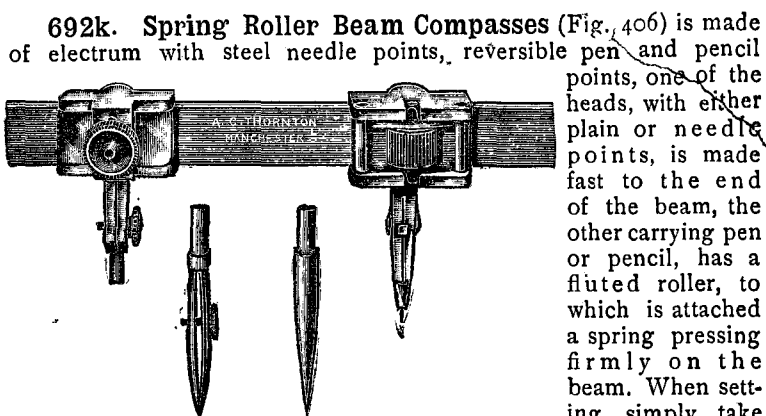
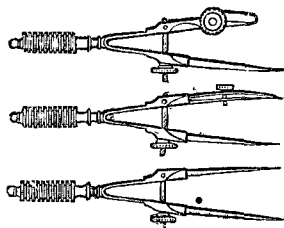


Fig. 106.—Spring Roller Beam Compasses.

(Fig. 406) is made of electrum with steel needle points, reversible pen and pencil points, one of the heads, with either plain or needle points, is made fast to the end of the beam, the other carrying pen or pencil, has a fluted roller, to which is attached a spring pressing firmly on the beam. When setting, simply take end of beam in

left-hand, and with the right run the rolling beam-head along beam to required distance, and adjust with thumb.

6921. Ordinary Spring-bows.—(one with two steel-pointed legs (Dividers), another having a steel-point and pencil legs (Bow pencil), and a third with a steel-point and a pen (Bow-pen) are used for drawing small arcs and $\odot$ s of less than 1" radius (Figs. 407-409).



Figs. 407-409.—Ordinary Spring Bows.

Rapid Adjustment for Spring Bows.—Figs. 410-411 show a device which admits of the two points being set approximately to the distance required almost instantly; the exact adjustment of the points to the required setting is obtained by the nut in the usual way. In comparison with the old system the wear on the bolt and nuts is practically nil.

Instructions for use.—Press A' against the face of the bow, this releases the spring, and the two points can be pressed together

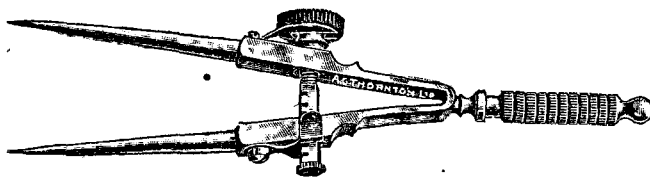


Fig. 410.

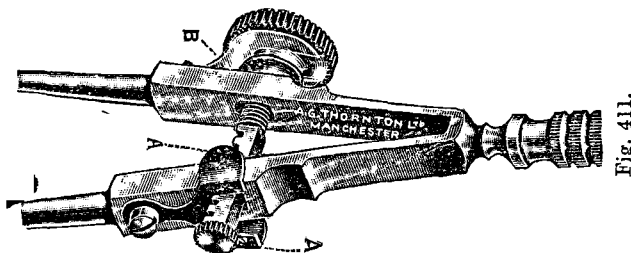


Fig. 411.

Figs. 410-411.—Rapid Adjustment for Spring Bows.

to the approximate distance. Pressing A, against the other face, causes the teeth to become engaged and fixes to the position required. B keeps the bolt and nut in its proper position when pressing or releasing the spring.

692m. Parallel Spring Bow Pen (Fig. 412) is so made that the point of the pen is always parallel with the needle point, and that it will draw circles of any diameter within its range, holding the instrument in a perfectly vertical position.

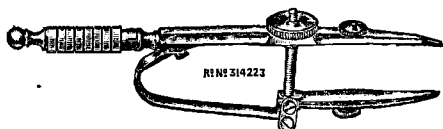


Fig. 412.—Parallel Spring Bow.

692n. Rotating Compass.—The peculiarity of this instrument (Fig. 413) consists in the centre point remaining stationary, whilst the pen or pencil, resting by its own weight on the paper, is guided round gently. This is of the type, known as **pump bows**, is far better than the ordinary one.

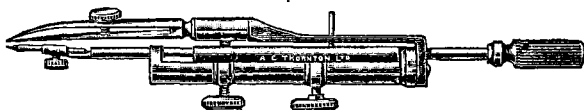


Fig. 413.—Rotating Compass.

692o. Triangular Compasses (Fig. 414) are convenient instruments to assist in the production of copies of the same dimensions as the original. They are constructed by jointing a third leg to the centre pin of the head-joint of the ordinary pair of compasses. They are usually made 5 ins. long in electrum.

Use it, its 3 points are set to coincide, at first, with any three points on the original, and then transfer them to the copy. Then two points of the compass are set upon two points of the drawing already copied, and the third leg, B, made to coincide with a new point of the drawing;

then by putting the two first points on the corresponding points on the copy, the third point of the compasses will transfer the new point to the copy. They are also useful to test the accuracy of the copies.

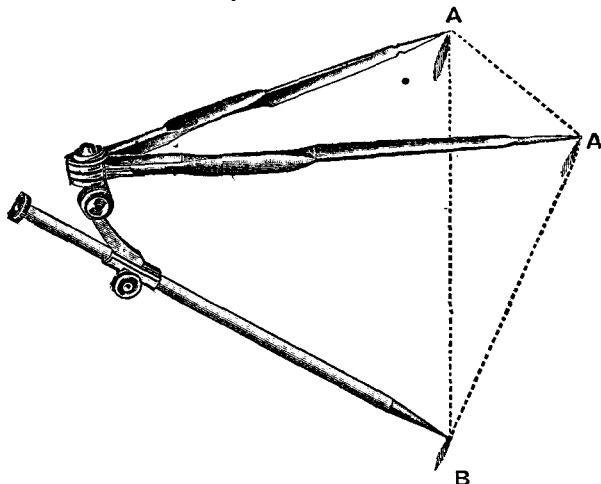


Fig. 414.—Triangular Compasses.

692p. **Triangular Beam Compasses.**—This (Fig. 415) is similar to the plain beam compasses with a kind of knuckle joint in the middle of the beam. The centre of the joint and the points of the beam-heads remain vertical and are of equal lengths. These instruments may also, by unscrewing the point in the joint, be used as ordinary beam compasses, for measuring distances, etc.

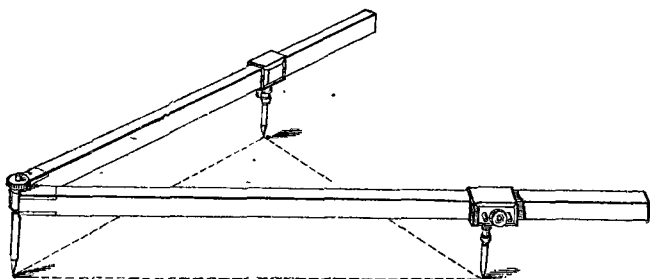


Fig. 415.—Triangular Beam Compasses.

692q. **Wholes and Halves, or Bisecting Compasses.**—Fig. 416 shows the instrument of which the longer legs are twice the

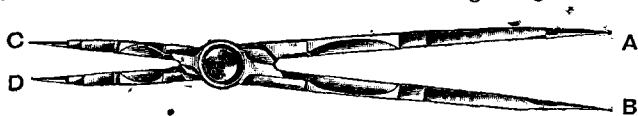


Fig. 416.—Wholes and Halves.

length of the shorter ones, and therefore when the former are opened to any given length, the shorter ones will be opened to half that length.

Use.—Drawings may be copied so that all lines in them may be reduced to one-half, or enlarged to double their length, by means of these compasses, but they are superseded by the proportional compasses for this purpose. This instrument is chiefly employed, in fact, for dividing lines into equal bisections. To effect this the points A, B, of the longer legs are used.

equal to the length of the line to be divided, then the distance between the points, *C*, *D*, of the shorter legs set off from either end of the line, will bisect it. The distance of the points *A*, *B*, being now made equal to one of the divisions just set off, the distance between *C* and *D* will divide the distance into 4 equal parts, and by repeating the process, it will be divided into 8 equal parts, then into 16, and so on.

692r. Proportional Compasses (Fig. 417) consist of two equal and similarly shaped flat brass or electrum limbs, *AE* and *DB*,

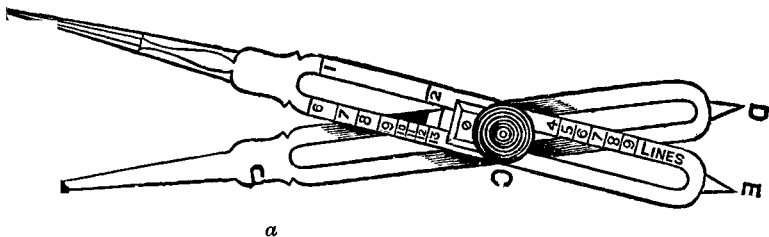


Fig. 417.—Proportional Compasses when Open for Use.

being a double pair of compasses with steel points, *A*, *B*, *E*, *D*, joined by turning the milled-head screw, *C*. This screw passes through the two slides which move in the longitudinal slots with a line or index marked on either face. On one face of the compasses, are marked divisions for lines and lineal ratios, and on the other, for plans and solids on either side of the slots. When shut up, the limbs appear as one (Fig. 418), and a stud, *a*, fixed on one

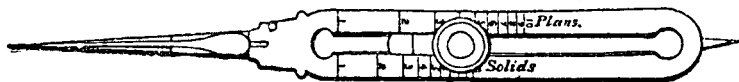


Fig. 418.—Proportional Compasses when Closed.

fits into a notch made in the other. The adjustment of the instrument for use is to be made, when the two limbs exactly coincide, as it is exactly in this position, that the slides can be moved up and down the groove. This instrument is used for enlarging and reducing drawings in any given proportion, and the extraction of cube and square roots, etc.

692s. Bar Proportional Compasses (Fig. 419) are made with an adjusting screw. This enables the slides to be set exactly to the division, 9" compasses will be found more useful than 6" ones.

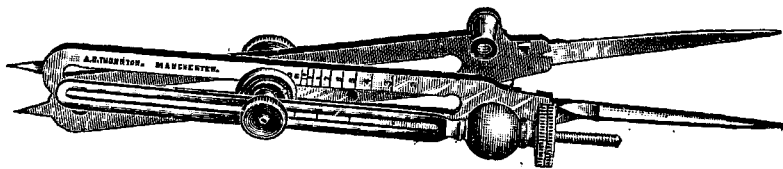


Fig. 419.—Bar Proportional Compasses.

692t. Proportional Compasses with Hooked Points (Fig. 420).—One drawback to the usual form of proportional is that the points can not be set. If the points are altered in any way in sharpening, or otherwise, the proportions engraved on the instrument are completely spoiled. To overcome this serious defect the instrument is made with hooked points which can be sharpened to fine points without interfering with the proportions engraved.

Lines.—When the index on the slide is set to the division marked 1 on the Line of Lines, and clamped by the mill-headed screw, *C*, equal lengths will be obtained from both the extremities. When the index is set to 2 on the Line of Lines, any distance set off by *D* and *E* will be half the distance measured by *A* and *B*, and so on, for other divisions up to 10 (Fig. 417).

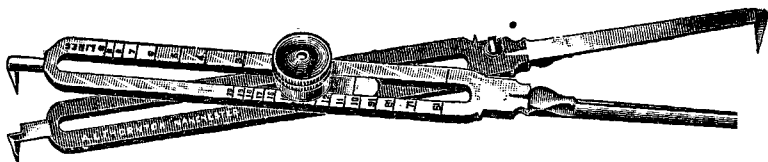


Fig. 420.—Proportional with Hooked Points.

Lineal Ratios.—The divisions are marked 1, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$. When, for instance, the index on the slide is set to the division, $\frac{2}{3}$, the length measured by the points, *D* and *E*, will be $\frac{2}{3}$ of that measured by the points *A* and *B*, similarly for other ratios.

The Scale of Circles.—In some compasses, as in the figure, lineal ratios, divisions of circles are marked. This scale is used to divide the circumference of a circle into any number of equal parts up to 20. The index being put to the number of divisions required, when the instrument is closed, to whatever radius the longer points are opened, the shorter points will divide the circumference of the circle struck by it into that number of equal parts, according to the setting. This scale is rarely used; and for accurate drawing none of these scales should be depended upon.

The Scale of Plans.—The scale is used for reducing or enlarging the area of a plan in a given proportion; e.g., if the index on the slide be set to 4, then the area of the square struck with the longer points, *A* and *B*, will be 4 times the square struck with the shorter points; any other figure follows the same rule.

The Scale of Solids.—It is used for reducing or enlarging the contents of a solid in a given proportion in the same way as the scale of plans. To reduce, longer points are to be used for measuring the original solids, and the shorter points will set off the required dimensions. To do this, set the index on the slide to any number on the scale of solids, then the distance between the points, *A* and *B*, will give the side of a solid of so many times the content as the number indicates; and the distance between the points, *D* and *E*, will give the reduced side of the solid, and *vice versa*.

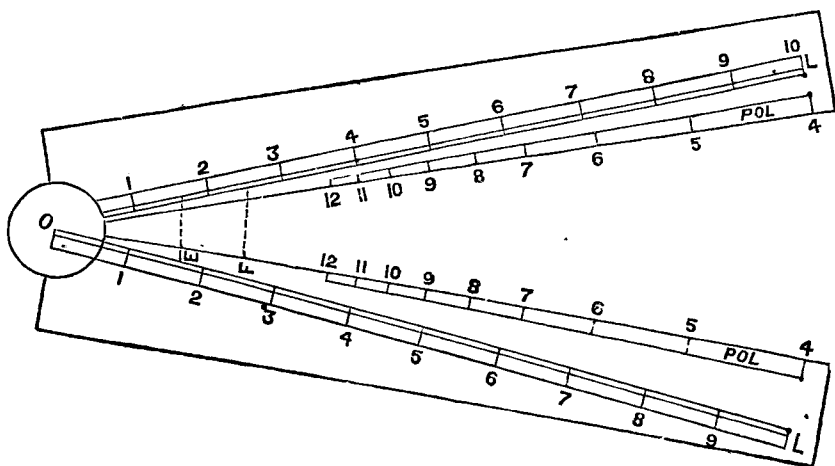


Fig. 421.—Sector.

693. The Sector (Fig. 421) is a foot-rule 12" long, and about $\frac{3}{4}$ " broad, joined in the center, so as to allow of its being folded together in the direction of its depth. It is made of ivory or box-wood. The instrument, derives its name from Euc. III., def. 10, and is used by measuring the arc between the graduations on the legs. Upon one face are : (1) *the line of lines*, marked L, (2) *lines of chords*, marked C, (3) *lines of secants*, marked S, (4) *lines of polygons*, marked POL, (5) *a scale of 12"* in which each inch is divided into equal parts; upon the other are (6) *the lines of sines*, marked S, (7) *lines of tangents*, marked T, (8) *the lines of logarithmic numbers*, sines and tangents, marked N, S, T respectively, (9) *a scale of foot* divided into equal parts, each of which is also sub-divided into 10 equal parts called the *decimal-scale* and placed on the edge of the instrument. Wooden sectors should be used in this country as the ivory ones get warped, and become useless. The most important scales are "*the line of lines*," "*the lines of polygons*" and "*the lines of chords*."

The Principle of the Use of the Line of Lines :—Let AB , AC be equal sectoral lines (Fig. 422). Set off upon AB any number of equal parts 1, 2, 3, 4, 5, 6, 7, etc., and from these points, draw lines $\parallel BC$, cutting AC in a, b, c, d , etc. Now the distances, $1a, 2b, 3c$, etc., have, to each other, the same ratio as the numbers 1, 2, 3, etc., e.g., the distance $2b$ has the same ratio to $5c$, as the number 2 has to the number 5, or $\frac{2}{5}$. If the equal divisions be continued to B, C , making 10 at these points, and if BC measure 2.5"; then $1a$ will be $\frac{1}{10}$, $2b$ will be $\frac{2}{10}$ or $\frac{1}{5}$ of a line 2.5" long. In this way we can find out any required part of a given line.

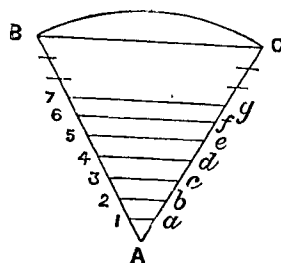


Fig. 422.—Use of the Sector.

Examples :—(1) To find $\frac{2}{5}$ of a line 3" long, apply the given line, i. e., make it a transverse distance (as the lines $1a, 2b$, etc.) between the numbers 9 and 9 on the scale marked L; then the transverse distance between 5 and 5 will be the length required.

(2) To divide a line 3.4" into 9 equal parts.—Apply the line as before, then the transverse distance between 1 and 1 will be the required length.

(3) To Bisect a Line 3" long.—Open the sector until the transverse distance from 10 to 10 on the scale marked L, equals the given line 3" long; then the distance from 5 to 5 will be half the line.

Note.—(a) Measure with the dividers from the inside line, where the dots are marked. (b) The transverse distances at 8 and 4, or 6 and 3, or 4 and 2, may also be used for the bisection.

(4) To Divide a Straight Line into 8 Equal Parts.—Open the sector until the transverse distance from 8 to 8 on the scale, L, equals the given line; then the transverse distance from 1 to 1 will be $\frac{1}{8}$ th of the given line.

(5) To Find the Fourth Proportional to 3 Given Lines (Fig. 423).—From the center O on L (Fig. 421), set off the lateral distance $OF = A$; open the sector until the transverse distance at F equals B . Then, if OE be set off equal to C , the transverse distance at E will be the fourth proportional.



Fig. 423.—Use of the Sector.

(6) To Divide a Line $A'B'$ into Eleven Equal Parts (Fig. 424).—

Since the divisions on the line of lines only extend to 10, we must take a sub-multiple of the number of parts into which the line is to be divided, and make the given line a transverse distance between the numbers corresponding to this sub-multiple. Make, therefore, $A'B'$ a transverse distance between 5.5 and 5.5, which is a sub-multiple of 11; then take the transverse distance between 5 and 5, and set it off from A' to a . Take aB' the remainder, and step it along $A'B'$, and the line will be divided into eleven equal parts.

Fig. 424.—Divide a Line into 11 equal parts.

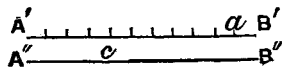


Fig. 425.—Determine $9/27$ of a Line.

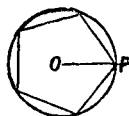


Fig. 426.—Inscribe a Regular Pentagon.

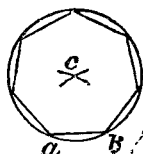


Fig. 427.—To Construct a Regular Heptagon.

(7) To Determine $\frac{9}{27}$ of a Line, $A'B''$ (Fig. 425).—As the denominator of the fraction exceeds 10, the distance can not be taken from the primary divisions, and we must, therefore, have recourse to the decimal notation. Multiply the numerator and denominator of the fraction by 3. We thus get $\frac{3}{9}$. Take the given line $A'B''$ in the dividers, and make it a transverse distance between 8.1 and 8.1; then the transverse distance between 2.7 and 2.7, set off from A' to c , will contain the number of parts required. The problem may be verified thus: $\frac{9}{27} = \frac{1}{3}$; therefore, the number of parts required will measure $\frac{1}{3}$ of $A'B''$. Now $A'c$ is $\frac{1}{3}$ of $A'B''$.

Note.—The divisions on the line of lines agree with the decimal notation, each primary division being divided into 10 sub-divisions, and according to the value attached to each of these sub-divisions, so 10 may stand for 100, 1000 etc., and, thus, a line may be divided into 50 or 500 parts, as well as into 5 parts.

Line of Polygons.—This scale, marked *POL*, is used for dividing the $\odot$ ce of a $\odot$ into any number of equal parts from 4 to 12, and by joining which furnishes means of inscribing regular polygons in a $\odot$.

Exs. (1). To Inscribe a Regular Pentagon in a Given Circle (Fig 426).—Let PO be the radius of the circle. Open the sector until the transverse distance from 6 to 6 on the scale marked, *POL*, equals PO . Then the transverse distance from 5 to 5 will equal the side of the pentagon.

Note.—(1) Always make the radius of the circle equal the distance from 6 to 6. (2) If a heptagon were required, then the distance from 7 to 7 would equal the side. If a square were required, then the distance from 4 to 4 would equal the side.

(2) To Construct a Regular Heptagon on a Given Line (Fig. 427).—Let ab be the given line. Open the sector until the transverse distance from 7 to 7 equals ab . With a and b as centres, and the distance from 6 to 6 as radius, describe arcs intersecting at c . With centre c and the same radius describe a circle. Set off the distance ab round it.

Line of Chords.—This scale is similar to the one, marked *CHO*, on the protractor, and is used for the same purpose; but the double scales on the sector are more useful than the single scale on the protractor; as on the sector, the radius with which the arc is described may be of any length, between the transverse distance of 60 to 60 when the legs are closed, and the transverse distance of 60 to 60 when the legs are opened as far as the instrument will admit of; but by the scale on the protractor, the arc described must always be of the same radius.

Examples:—(1) To lay down an $\angle$ less than 60° ; say 42° .—With the point a of the line ab as centre, and with any transverse distance between 60 and 60 as radius describe the arc bc (Fig. 428). Now take the transverse distance of the given degrees 42, and lay off this distance on the arc from b to c . Join ac ; the $\angle bac$ is the angle required.

(2) To lay down an $\angle$ greater than 60° ; say 142° .—Draw the arc bcd (Fig. 428) with radius equal to the transverse distance of 60 to 60 as before. As the scale is up to 60° , divide the given number of degrees 142° into 4 equal parts to bring the subdivisions below 60° . Then take the transverse distance of the subdivision, and step it along the arc bcd 4 times as from b to e , e to f , f to g , g to d . Join ad ; bad is the required $\angle$ of 142° .

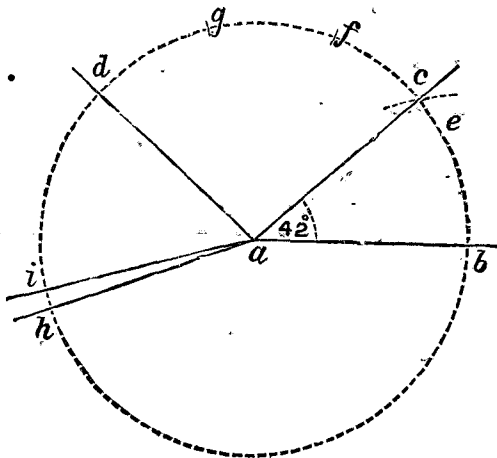


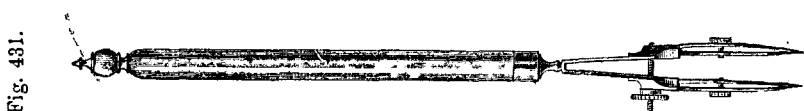
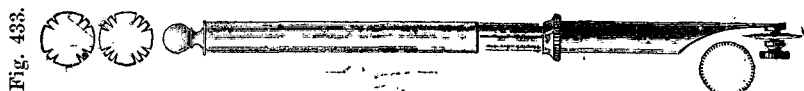
Fig. 428.—To lay down Angles with the Sector.

694. **Drawing or Mathematical pen** (Fig. 429).—It consists of two curved blades of brass or electrum ending in steel nibs fixed to an ivory handle. One of the blades is hinged to facilitate cleaning. The blades with a spring between them are clamped with a mill-headed screw to allow the distance between them to be varied slightly and thus altering the thickness of the line as required.

To use it, ink is inserted with the corner of a piece of paper between the nibs, and the pen is tested on a piece of stiff paper. When it is found that the pen is marking a line of even thickness, it is then held firmly against the rule, slightly inlined in the direction of the line to be drawn, taking care to make both nibs touch the paper. In inking over pencil lines, work should be done from the top towards the bottom. When a drawing is to be colored, light lines are to be drawn and then washed with sponge and water, otherwise the lines are likely to "run". The thick black lines to give expression to the drawing should be drawn at the very last. If the pen gets clogged, as it frequently does from taking up the lead of the pencil lines, it should be cleaned by drawing between the nibs a slip of paper which removes the obstruction. After the pens have been used they should be wiped to prevent rust and the blade-points kept perfectly clean. Two pens should be used, one for drawing thick lines and the other for drawing fine ones. Above all things, the paper should be kept clean. It should not be touched by the hands more than possible, as the hands make the paper greasy, and whenever a paper has acquired this defect, clean sharp lines on it are impossible. The nibs of a pen should be equal in length and width, and the points should be set to give as fine a point as possible. A hone is all that is needed to set the points properly.

694a. **Fine Drawing Pen** (Fig. 430) is made out of a piece of steel wire, the entire working part being in one piece. The nibs are not jointed but clamped with a milled-head screw. For fine-line drawing, it is preferred, as the nibs are each equally firm when in contact with the paper. The fine pen is used much for plotting surveys.

694b. Road Pen (Fig. 431) consists of two drawing pens adjustable to any required width by means of an adjusting screw; and fitted to a handle, and is used for drawing parallel lines, as for roads, canals, railway lines upon surveys, joists, etc.



WINSOR & NEWTON LTD.

Fig. 429 —Drawing or Mathematical Pen. Fig. 430 —Fine Drawing Pen.
Fig. 431—Road Pen. Fig. 432. —Dotting Pen. Fig. 433.—Dotting Pen.

694c. The Road Pencil is similar to the road pen except that it has two spring sockets fitted to carry pencils in place of the pen.

694d. The Dotting or Wheel Pen (Fig. 432) is similar to the drawing pen with lift-up nib. At the extremity of one of the nibs a pin is fixed to form the axle for the wheel or rowel shown in the illustration. In this pen a set of wheels is kept within the top of the ivory handle. Fig. 433 shows another variety with loose wheels. The wheel pen is used for drawing dotted lines of different patterns. The trouble is that a really good dotting-pen does not exist. The form (Fig. 433) in which dotting wheel receives its ink from an open reservoir is the best. The pen should be used with great care. It works better if it is not held quite straight with the line to be produced. The Indian ink should be made thicker than for the ordinary drawing pen. Previous to drawing any line, it is advisable to run the pen over a piece of waste paper, to see if it is properly charged with ink and working neatly.

694e. Bordering Pen (Fig. 434) differs from the ordinary drawing pen by having an extra inner nib, contains double the quantity of ink to that of the ordinary form, and will produce a line of double the thickness. The adjustment to width of line is made by two nuts which adjust from each side of the pen. The use of

this instrument is to border drawings, and color narrow lines frequently used upon drawings, such as, shadow lines, roof timbers, etc.

694f. Curve Pen

(Fig. 435).—The curved form prevents the interference of the hand in the observation of the nibs, thus enabling the pen to be held upright; it also possesses the advantage that it will follow any straight or curved edge with a steady motion.

694g Tracer (Fig. 436) is a tapering point of hard steel or agate, fixed to an ivory handle, and is used for copying drawings by pressing firmly with the point over the lines to be reproduced. The drawing is placed over the sheet of paper which is to receive the copy, with a carbon paper between them. It is also used for etching on metals. A needle-point pricker is shown by (Fig. 437).

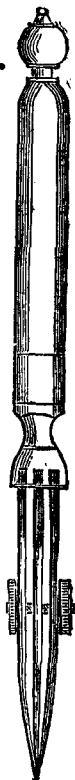


Fig. 434.—Bordering Pen.



Fig. 435.—Curve Pen.

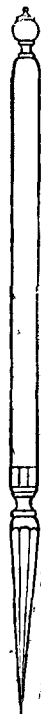


Fig. 436.—Tracer.



Fig. 437.—Needle-point Pricker.

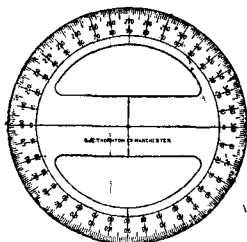


Fig. 438.—Circular Protractor.

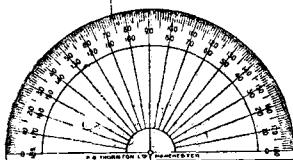


Fig. 440.—Semi-Circular Protractor.

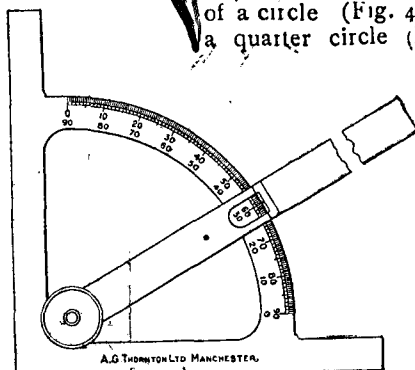


Fig. 439.—Quarter-Circle Protractor.

695. The Protractor consists primarily of a circle (Fig. 438), a quarter circle (Fig. 439), or a semi-circle (Fig. 440), the

circumferences of which are divided into 360, 90, or 180 equal parts respectively, each part subtending at the centre an angle of 1° . A brass circular protractor (Fig. 200) is graduated into degrees and

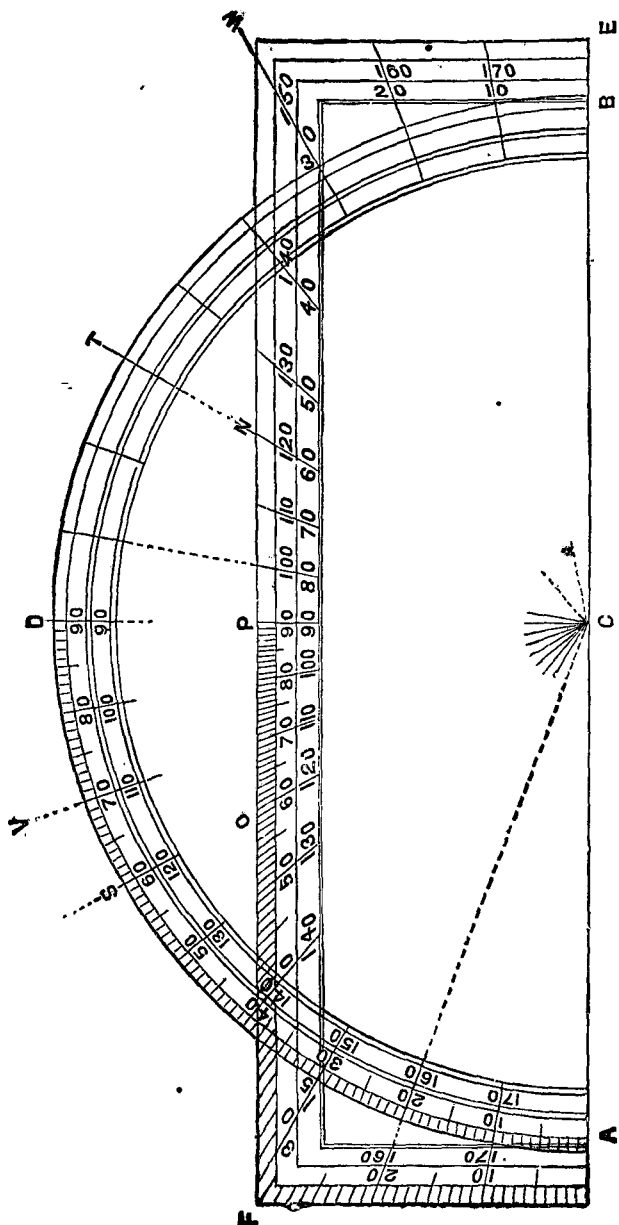


Fig. 441.—The Principle of Construction of a Semi-circular and a Full-size Rectangular Protractor.

$\frac{1}{2}$ degrees from 0 to 360° on the inside and outside rows, or in each quadrant from 0 to 90° on the inside. The circular protractor made

of talc is also a very serviceable instrument for field-work. The protractor is generally a rectangular piece of ivory, or of box-wood, $6'' \times 1\frac{3}{4}''$, (Fig. 441). Along three of its borders in front, the angles are marked in degrees by lines radiating from the centre of the fourth side, and in two rows, the outside from 0 to 180° , and the inside from 180° to 0° . Various other scales, however, are engraved on both sides of the instrument. The ivory protractor is generally used for setting

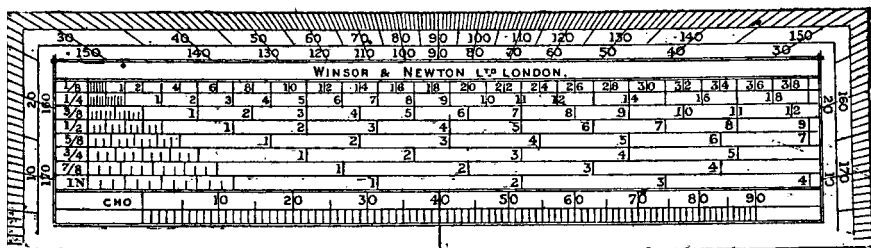


Fig. 442.—Rectangular Draftsman's Pattern Protractor.

off on paper any given angle of any number of degrees, or to find out the number of degrees contained in a given angle. The semi-circular protractor made of brass, ealuminum, or celluloid, is also in use.

695a. The rectangular and the semi-circular protractors though different in shape are constructed on the same principle, and to demonstrate this the rectangular protractor is placed over the semi-circular one (Fig. 441). ABD is a semi-circular, and EF a rectangular protractor, and both are graduated in the same way. In both of them 180° have been divided into 180 equal parts. The numbers on divisions at O, P, N , on the rectangular protractor correspond respectively to S, D, T , on the semi-circular. The centre and the radii are the same in both.

695b: Use of the Protractor.—To set off any given angle, say 120° , at the point C on a given line AB , place the protractor so that its centre coincides with the point, C , and its edge with AB , (Fig. 441). Then 120° should be read off the graduations, and a dot marked with pencil against 120° , i.e., at T , or N , which when joined to C will give the $\angle ACT = \angle ACN = 120^\circ$.

When an angle is already plotted, its magnitude can readily be found with the protractor. Suppose, the $\angle ACV$ is on the plan and its magnitude is to be determined. To do this, place the centre of the protractor at the point C , its lower edge coinciding with the line AC , then read the number which corresponds to the line, CV . This will give the number of degrees contained in the angle, i.e., 70° . Similarly $\angle ACW = 150^\circ$; and $\angle ACD = 90^\circ$. Thus a perpendicular may also be drawn from any point on a given straight line. To do this, place the centre of the protractor at the given point, the lower edge coinciding with the straight line. Now mark a dot on the paper just beyond 90° . The line joining the dot with the centre will be the perpendicular to the given line.

The length of the circumference of a circle $= 2\pi r$ (r = radius), and varies as the length of the radius or the length of the protractor. Hence on the length of the circumference depends the number of graduations into which it can be divided. It will be shown hereafter that 80 divisions can be conveniently packed together on an inch visible to the naked eye, so that on the length of the circumference depend the clear graduations.

When plotting survey work, the position of the protractor will be on the right side of north line when the bearing is less than 180° and on the left when the bearing is greater than 180° (*q.v.*, Art. 344, p. 193).

695c The advantages, the semi circular protractor has over the rectangular, are that with a semi-circular protractor any angle is protracted at full length of the radius of the protractor from the centre, but this is not so with the rectangular (*q.v.*, $\angle$ s ACT and ACD). Again in plotting off any line CT, if there be an error equal even to a sharp pencil point at N nearer to C, the error in the direction of CT increases the more CT is produced, but if the error is made at T it will be much less.

Scales.—Rectangular protractors, as made by different makers, vary much in the number, description and arrangement of the scales marked upon them. Rectangular protractors are usually of two patterns: (i) the *draftsman's* pattern, and (ii) the *military* pattern. (Fig. 207, p. 208).

695d. The draftsman's pattern (Fig. 442) contains a variety of Scales marked on the back face, with Nos. 6, 5, 4, 5, 4, 3, 5, 3, representing the number of parts into which the inch is divided, and each major division is then divided into 12 equal parts, each representing 72th, 60th, 54th, 48th, 42th, and 36th of an inch. The use of these scales is found when we have to use for a drawing a scale such that one of these divisions represents a convenient unit of measurement, such as 1', 1 yard, 10', 10 yds., etc., etc.

Two diagonal scales are also engraved on the back of the rectangular protractor, one on the right-hand giving $\frac{1}{200}$, and the other on the left-hand $\frac{1}{100}$ of an inch, for $\frac{1}{2}$ " and 1" are divided into 10 equal parts. The principle of their construction will be explained in Chapter VIII.—Construction of Scales.

The scales marked in front: $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, and in in some (Fig. 442) are also simple proportional scales. The numbers $\frac{1}{8}$, $\frac{1}{4}$, etc., refer to the length of one division which is divided into 12 equal parts. The one marked in is one inch long, those marked $\frac{1}{8}$, $\frac{1}{4}$, etc., being $\frac{1}{8}$, $\frac{1}{4}$, etc., of an inch long respectively. The scales are useful for measurements involving feet and inches owing to the divisions being divided into 12 parts.

695e. The Scale, marked CHO, in front, is a Scale of Chords, and is constructed as under (Fig. 443):—Take a quadrant BAC, divide the arc BC into 9 equal parts, giving 10° divisions on the arc from B to C. With B as centre, and radii from B to these divisions describe arcs cutting BA in the points 10° , 20° , 30° , 40° , ..., 90° . These radii being the chords of the different arcs, BA is the Scale of Chords.

Each Scale will vary with the length of the radius. By Euc. IV. 15, the side of a hexagon is equal to the radius of the circumscribing circle, or the radius of the circle=chord of 60° . The radius of a circle may therefore be set off exactly six times round the circumference; hence, if an arc be

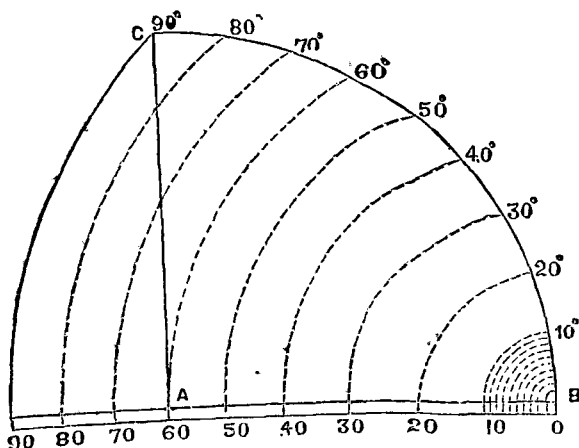


Fig. 443. — Scale of Chords.

described, and a portion cut off equal to the radius of the arc, an angle containing 60° will be obtained. With this principle, in mind, construction for angles of 60° , 30° , 120° , 15° , 45° , 75° , 105° , 135° , $22\frac{1}{2}^\circ$, $67\frac{1}{2}^\circ$, can be made by the aid of practical geometry without using a *protractor* or a *scale of chords*.

695f. To Lay with the Scale of Chords an Angle of 40° .—With centre A (Fig. 444) and radius equal to the distance from 0 to 60° on the scale, describe an arc BC, cutting AD in B, and with centre B and radius equal to the distance from 0 to 40° describe another arc cutting BC in C; join AC, then the angle BAC is 40° . This method of laying off angle is to be preferred to that laid off by the *protractor*, as it is more accurate, and the greater the radius the greater is the accuracy. When the angle is greater than 90° , it should be laid down in two operations.

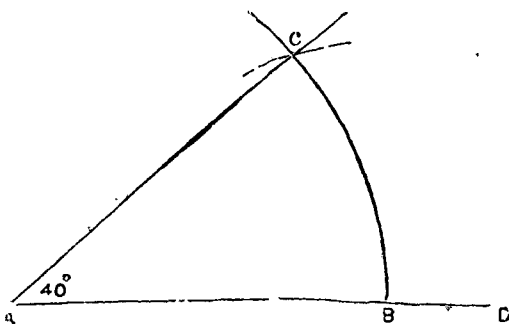


Fig. 444 — Use of the Scale of Chords.

695g. To measure the size of an angle, say 40° , by means of the scale of chords. Let BAC (Fig. 444) be the given angle. With radius 0 to 60° from the scale describe the arc BC. Take the chord BC with a pair of dividers and apply it to the scale which will give it to be 40° .

695h. The Military Ivory Protractor (Fig. 207, p. 208) is similar, as regards the protracting edges, to the last described. The distinguishing feature is a series of lines across it, ruled up from a scale divided along the base to which they are vertical, being parallels with the line which reads from the centre to 90° . It is useful in plotting readings taken with the prismatic compass (q.v., Art. 375, p. 208). On the back of this protractor is the scale of chords and two diagonal scales: one giving $\frac{1}{200}$ and the other $\frac{1}{400}$. It can also be used as a clinometer by forming a hole near the edge and suspending a small weight with a thread.



Fig. 445.—Eraser Knife Key and File.

696. Eraser Knife Key and File (Fig. 445) is sparingly used for erasing ink lines off a drawing when corrections are necessary.

696. Plotting Scales and Offset-pieces (Fig. 180, p. 168). Plotting scales are made of boxwood, or boxwood with celluloid edge, about, $\frac{3}{8}$ " thick, $1\frac{1}{4}$ " broad, and 6", 12", $12\frac{1}{2}$ ", 18", 24" and 30" long; and of ivory 6", 12", $12\frac{1}{2}$ " long. These scales are made in flat and oval sections, but the edges of flat ones are chamfered, and all are graduated along the length. The surveying scales are flat in section and the full length is divided into inches, and each of these parts again is subdivided into 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 parts for each kind of scale. The offset-pieces are 2", 3", 4", long $\times 1\frac{1}{4}$ " broad $\times \frac{1}{8}$ " thick. Each of them is graduated like the scale with which it is used. For all ordinary purposes, cheap cardboard scales are used; and if varnished, they will last a long time. Measurements should on no account be taken from a scale by means of dividers and then transferred to the paper. For accuracy and time-saving, the division on the scale to be marked should always be applied directly to the drawing with a pencil point.

To use a plotting scale, it is laid along a line already plotted, and the offset-piece is made to slide along the scale, so that its graduated edges are $\perp$ to the graduated edges of the scale. (q.v., Art 291, p. 167).

697a. Triangular Scales and Offset-Scales are made of boxwood, ivory, or eucurum. Six or twelve scales can be cut on these, and they lie quite flat like the flat section described above, but they are expensive. They are very easy to handle and to work.

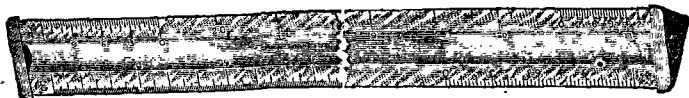


Fig. 446.—Watt's Triangular Metal Scale.

The liability of wood scales to crack, warp, chipping of edges, and their variation from standard measurement are well-known. Metal Scales are free from these defects. Fig. 446 shown is a metal scale made from brass tubing nickelled with a dull finish, weighing $3\frac{1}{2}$ ozs.

The ends are covered with hardened steel plates that slightly raise the scales from the paper, and prevent the graduations wearing.

697b. Foot-rules.—These are made of ivory or boxwood about $\frac{1}{8}$ " thick, 2' long (four fold), and $\frac{1}{2}$ " or $\frac{3}{4}$ " wide (Fig. 448).

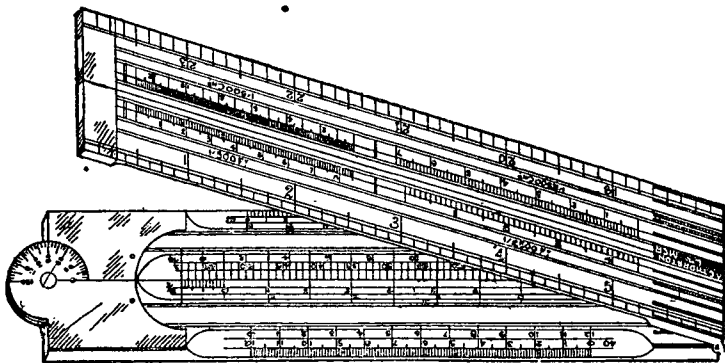


Fig. 448.—Foot-rules.

They are necessary for measuring short distances, and are graduated on one side into inches and $\frac{1}{16}$ th of an inch, and on the other into inches and $\frac{1}{8}$ th of an inch on one edge; $\frac{1}{2}$ " and $\frac{1}{4}$ " are shewn on the other edge.

698. Marquois' Scales.—Level Sections may be plotted with great rapidity by the aid of the Marquois' scales. These consist of a right-angled Δ (set-square) the hypotenuse of which is three times the length of the shorter side, and two rectangular rulers. Fig. 449

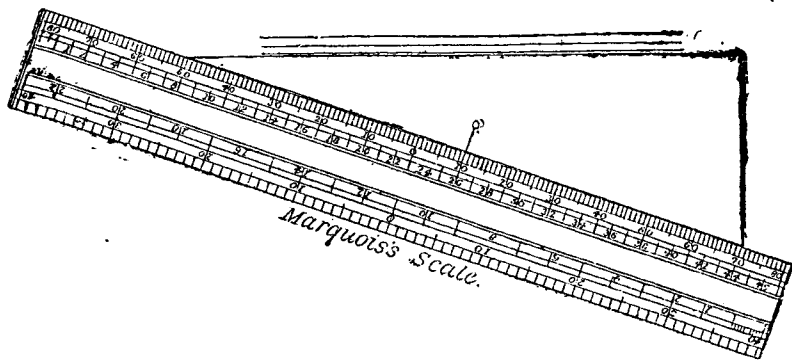


Fig. 449.—Marquois' Scales.

represents the Δ and one of the rules, as being used to draw a series of parallel lines. Either rule is one foot long, and has parallel to each of its edges two scales, one the *outer* or *artificial scale* placed close to the edge, and the other *inner* or *natural scale* immediately within this. The divisions on the artificial scale are three times the

size of those on the natural scale, so as to bear the same proportion to each other that the longer side of the Δ bears to the shorter. Each natural scale is a scale of equal parts with the primary divisions numbered from the left to right. The first primary left-hand division is sub-divided into 10 equal parts, and the number of these sub-divisions in an inch is marked underneath the scale and gives it its name. On one of the pair of Marquois' scales, as shown in the figure, there are, on one face, scales of 30 and 60, on the obverse, scales of 25 and 50, and on the other one face scales of 35 and 45, and on the reverse, scales of 20 and 40. In the artificial scales the zero is placed in the middle of the edge of the rule, and the divisions are numbered both ways from that point to the two ends of the rule, and are every one, sub-divided into 10 equal parts, each of which is, therefore, 3 times the size of the subdivision of the corresponding natural scale. The Δ has a short line drawn perpendicular to the hypotenuse near the middle to serve as a pointer, and the longer of the two sides has a bevelled edge.

To Draw a Line parallel to Another at a given Distance from it.—1. Having applied the given distance to one of the natural scales, which is found to measure it most conveniently, place the Δ with its sloped edge coincident with the given line, or rather at such a small distance from it, that the pen or pencil passes directly over it, when drawn along this edge.

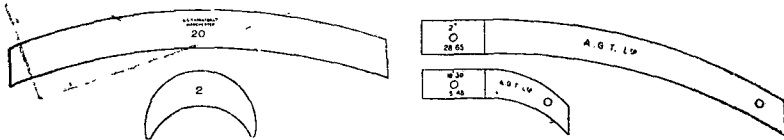
2. Set the rule closely against the hypotenuse, making the zero of the corresponding artificial scale coincide with the pointer upon the Δ .

3. Move the Δ along the rule, to the left or right, according as the required line is to be above or below the given line, until the pointer coincides with the division or sub-division corresponding to the number of divisions or subdivisions of the natural scale, which measures the given distance; and the line drawn along the sloped edge in its new position will be the line required.

To Draw a Line Perpendicular to a Given Line from a Given point in it.—1. Make the shortest side of the Δ coincide with the given line, and apply the rule closely against the hypotenuse. 2. Slide the Δ along the rule until a line drawn along the sloped edge passes through the given point; and the line so drawn will be the line required.

The Advantages of these Scales are: 1st., that the sight is greatly assisted by the divisions on the artificial scale being so much larger than those of the natural scale to which the drawing is constructed: 2nd., that any error on the setting of the index produces an error of but one-third the amount in the drawing.

§99. Railway Curves.—For drawing railway curves of large radii beyond the reach of ordinary compasses, thin pieces of pear-wood, transparent celluloid, or vulcanite, cut to required radius at the edges are used most conveniently.



Figs. 450-451.—Railway Curves.

Railway curves in sets of 25, 50, 75 and 100 in a case, $1\frac{1}{2}$ " to 50", $1\frac{1}{2}$ " to 120", $1\frac{1}{2}$ " to 150", $1\frac{1}{2}$ " to 240", radius respectively, $1\frac{1}{2}$ " to 2" wide, and from 3" to 18" long are sold; (Figs. 450-451).

699a. French Curves These are made of well-seasoned pear-wood, transparent celluloid, or velo veneer in about 42 shapes. Size : 4" to 36". The curves other than $\odot$ or ones, of variable curvature, called universal sweeps (Fig. 452-454) are very serviceable for drawing curves, where the use of the compasses is inadmissible.



Figs. 452-454.—French Curves.

699b. Graded Scale Curves (Fig. 455) are designed to act as substitutes for the great bulk of French and irregular curves. They form a series of four, similar in shape but of different sizes, graded to various scales. At one end the curve is very flat, at the other it is almost circular. Beginning at the flat end, the curve gets more and more convex, gradually and continuously to the circular end. Calling the scale of the largest "Unity," the scale for the series would be on the 5 in. curve, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$; on the 7 in. curve $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$; $10\frac{1}{4}$ in. curve $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{6}$; and on the $13\frac{1}{4}$ in. curve (largest) 1, $\frac{1}{2}$, $\frac{1}{4}$; and this without taking account of the fourth convex edge. They are made in pear-wood, celluloid and velo veneer. This latter is a new material similar to vulcanite, but with polished dark bronzed figured surface. It keeps true, will bend, yet does not break.



Fig. 455.—Graded Scale Curve,

699c. Brooks' Flexible Curve (Fig. 456) has a flexible steel strip about $\frac{1}{4}$ in. wide, on which are fixed stiff hinged links attached to tabs. The tabs are held down by the points of the fingers to the required curve, along which a pen or pencil line is drawn, simply bent to shape required, bit by bit, commencing at one end, and the links being self-clamping will remain in position. Do not press on the strip between the tabs. Size : 9", 12", 18", 24".

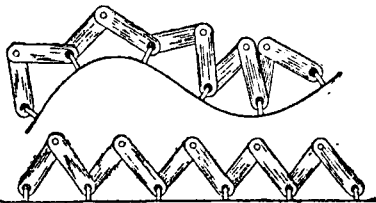


Fig. 456.—Brooks' Flexible Curve,

699d. Brooks' Adjustable Curve (Fig. 457) is made for drawing long curves and is made in sizes of 9", 12", 18", 24". It is a steel band curve which is held to position required by a series of sliding rods, movable upon a narrow straight rod.

699e. Beck's Radius Curve, (Fig. 458) made of transparent celluloid, is composed of curves of stated radii, from $\frac{1}{2}$ in. to $3\frac{3}{4}$ in.; the commencement and end of each being distinctly shown.

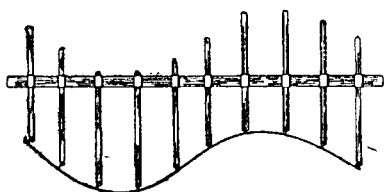


Fig. 457.—Brooks' Adjustable Curve.

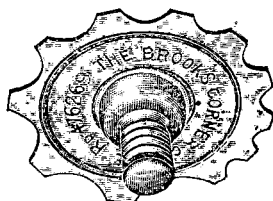
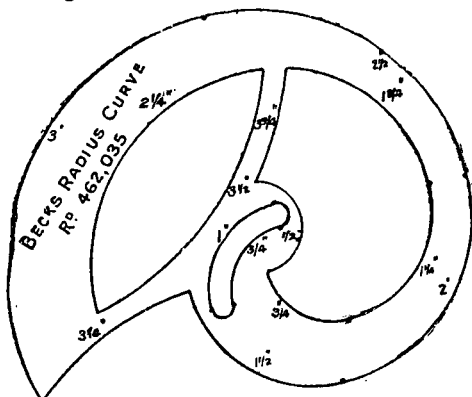
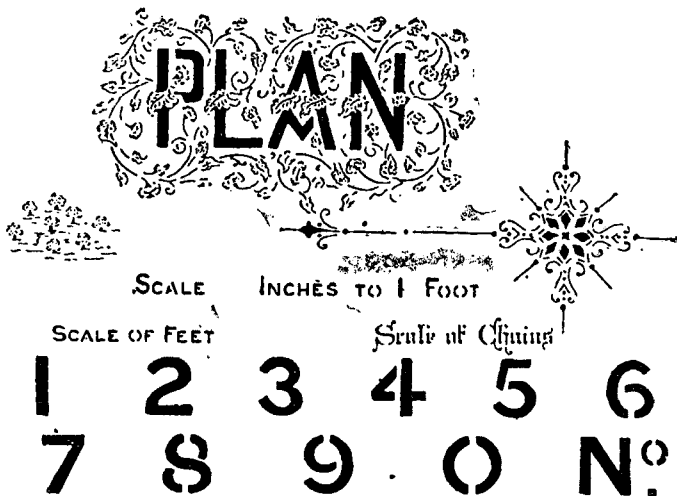


Fig. 460.—Corner Curve.



Scale Feet to 1 Inch
 Scale Feet to one Inch
 SCALE FEET TO ONE INCH

DRAWING N°
 LONGITUDINAL SECTION



Plan referred to

Horizontal Scale

Vertical Scale

Scale of an Inch to a Foot

SCALE

GROUND PLAN

REFERENCE

DESIGN

Plan

PLAN

PLAN

SECTION ON LINE

M A P

A B C D E F
 G H I J K L
 M N O P Q R
 S T U V W X
 Y Z &

SCALE INCH TO ONE FOOT SIDE ELEVATION

reversed and draw a line. If the two straight lines coincide the edge is true. It should be hung in a vertical position after use.

701. Stencil Letters and Words.—Letters used for the title of a drawing may be put in very expeditiously with the aid of stencil plates. The corners of borders and meridian lines may also be easily put down by means of these plates. These are thin copper or celluloid plates having the letters very neatly cut out from them ;



Fig. 468.—Corner Stencil Plate.

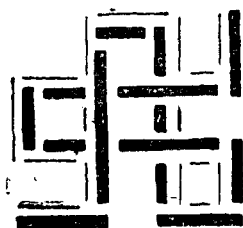


Fig. 469.—Corner Stencil Plate.

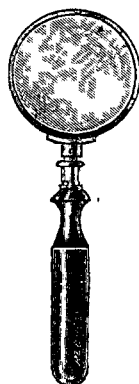


Fig. 471.—Magnifying Reader.

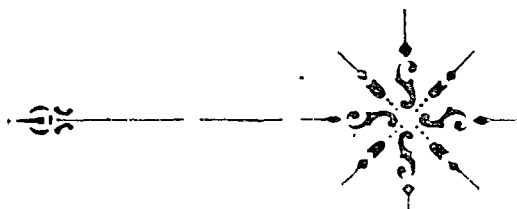


Fig. 470.—North Line Stencil Plate.

by laying them on the paper, and brushing the ink over them with a hard brush it is painted on the paper through the openings, forming a very elegant typography. Transparent celluloid plates overcome the difficulty of alignment and spacing which can be accurately adjusted as the letters stencilled can be seen through the plate, (Figs. 462-470).

701a. Magnifier or Magnifying Readers (Fig. 471) having powerful bi-convex lens will often be found useful.

CHAPTER VII.

PROBLEMS ON PRACTICAL GEOMETRY.

702. Geometrical Problems.—The following are a few of the useful problems in plan drawing, ground tracing and field-survey operations, which the surveyor should carefully bear in mind, so that he may be prepared to use a measuring tape, or a cord for the compasses when describing arcs in the field, and the former when setting off distances (Art. 21, p. 5 ; Arts. 227-251, pp. 135-154, q.v.)

Prob. I. *To bisect a given straight line, ab .*

From centers, a and b (Fig. 472), with any radius greater than one-half ab , describe arcs cutting each other in c and d , and draw the line, cd . This will bisect the given line at e , [Euc. I. 10].

Prob. II. *To divide a given line, ab , into any number of equal parts.*

(a) Draw any indefinite line, ac (Fig. 473), at any angle to ab ; on the latter set off as many of any equal parts as, ad , de , ef , fc , as those into which the line ab is to be divided; join bc , and draw fg , eh , di $\parallel$ to it. Then these will divide ab as desired.

(b) By *trials*.—When the number n into which the line is to be divided is small, open the dividers to what appears to be the n th part of the line, step this distance along the line from one end, and if it be found to exceed or fall short of the n th part, correct the opening by decreasing or increasing it by the n th part of the error, and repeat the trial till the exact n th part be ascertained. With a little attention, 2 or 3 trials are sufficient to divide the line into the required number of equal parts. This method is recommended for practice. In constructing scales, the major divisions should be found by *trial* method, but the sub-divisions should be found by the method (a).

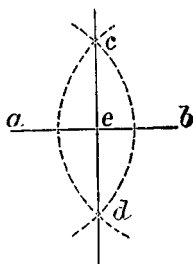


Fig. 472.

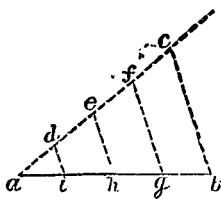


Fig. 473.

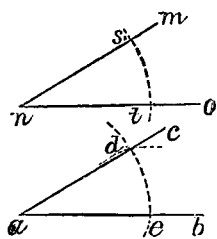


Fig. 474.

Prob. III. *To divide a given line, ab , into two parts which shall have a given proportion to each other (Fig. 473).*

This is done on the same principle as the last; thus let the proportion be as 1 to 3. First draw any line ac , and with any convenient opening of the dividers make $ad=1$ step, $dc=3$ steps, and join cb ; and draw di $\parallel$ to it. Then $ai:ib::1:3$.

Problems II and III may be used in drawing proportional scales for enlarging or reducing a drawing to a given proportion.

Prob. IV. *At any point a in a given line ab , to make an $\angle cab =$ a given $\angle mno$ (Fig. 474).*

From n and a , with any convenient radius, describe the arcs st and de ; take st with the dividers, and make $ed=st$; through a and d draw ac [Euc. III. 27]. Then $\angle cab = \angle mno$.

Prob. V. To bisect a given $\angle bac$ (Fig. 475).

From center a , with any radius, describe an arc de ; and with centers d and e , with any radius, describe arcs intersecting in f ; draw af , and it will bisect the $\angle bac$, [Euc. I. 9].

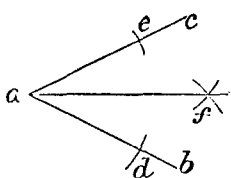


Fig. 475.

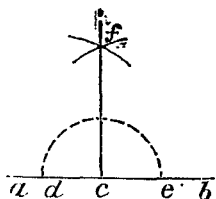


Fig. 476.

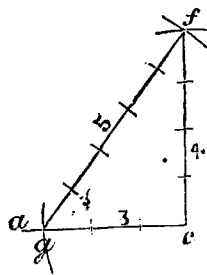


Fig. 477.

Prob. VI. At a given point c in a given line ab , to erect a $\perp$.

(a) From the point c (Fig. 476), with any radius, cut off any equal parts, cd and ce , on ab , from the centers d and e , with any one radius, describe arcs intersecting at f . Join cf which will be $\perp$ to ab , [Euc. I. 11.]

(b) Or from c , with any convenient scale, describe two short arcs g and f , either one of them with a radius of 3 units, and the other with a radius of 4 units. Then from g with a radius 5 of the same units, describe arc f cutting the arc from c in f . Join fc , which is the $\perp$ to ac (Fig. 477).

(c) To erect a $\perp$ if the point c is at one end of the line or very near it.

From any point, d (Fig. 478), above the line as center, through the given point, c , describe an arc cutting the given line at e . Through e and d draw diameter edf ; join cf which is the $\perp$ to ab .

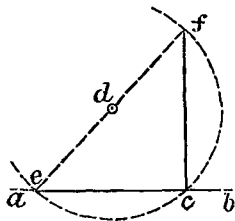


Fig. 478.

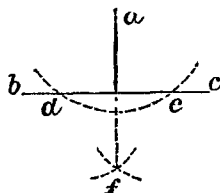


Fig. 479.

Prob. VII. (a) From a given point a to let fall a $\perp$ on a given line bc (Fig. 479).

From the point a as center, with any convenient radius describe an arc cutting the given line at two points, d and e ; and from centers d and e , with any radius, describe two arcs cutting at f ; draw af , which will be the $\perp$ to bc . [Euc. I. 12.]

(b) To erect a $\perp$ when the given point a is nearly opposite the end of the line.

From any point d in the given line bc (Fig. 480) as center, describe an arc through the given point a cutting bc in e ; from center, e , with radius ea , describe another arc cutting the former in f ; draw af , which will be the $\perp$ to bc .

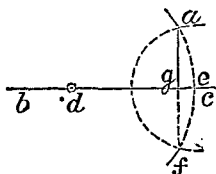


Fig. 480.

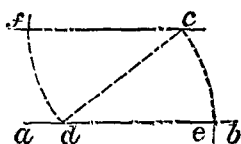


Fig. 481.

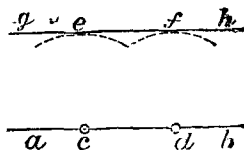


Fig. 482.

Prob. VIII. Through a given point c draw a line $\parallel$ to a given line, ab .

(a) Take any point d in ab (Fig. 481); from d and c , with radius cd , describe two arcs, ce and df . Make $df=ec$; through f draw ef , which will be parallel to ab [Euc. I., 31].

(b) When the parallel is to be drawn at a given distance from ab (Fig. 482). From any two points c and d in ab , with a radius equal to the given distance, describe arcs, e and f ; draw line gh tangent to those arcs; gh will be parallel to ab .

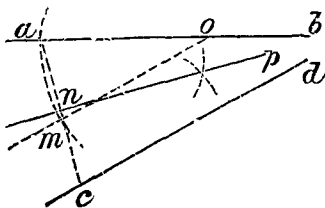


Fig. 483.

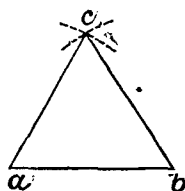


Fig. 484.

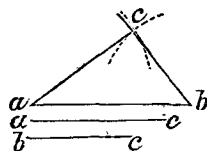


Fig. 485.

Prob. IX. To determine the direction of the line which would bisect the $\angle$ contained by two straight lines intersecting beyond the limits of the drawing (Fig. 483).

Let ab and cd be the lines. In ab take any point o ; draw om parallel to cd ; make $om=oa$, join am , and produce it to meet cd in c ; draw np bisecting ac at rt . $\angle s$; np if produced, would pass through the point of intersection of ab and cd , and bisect the $\angle$ between them.

Prob. X. To draw an equilateral Δ on a given line, ab . From centers a and b (Fig. 484), with ab as radius, describe arcs cutting in c . Draw ac , bc , and abc will be the equilateral Δ .

Prob. XI. To make a Δ with three sides, equal to ab , ac , bc (Fig. 485) respectively. With center, a , and radius, ac , describe an

∞ c ; with center, b , and radius, bc , describe another arc cutting the first in c , draw ac , bc , and abc will be the Δ^* required.

If ac and bc are equal, the Δ will be an isosceles Δ . If $ac = bc = ab$, it will be an equilateral Δ .

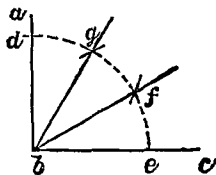


Fig. 486.

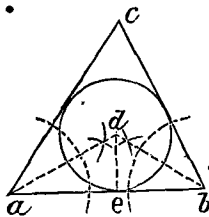


Fig. 487.

Prob. XII. To trisect a rt. $\angle abc$.

From b with any radius describe arc, de (Fig. 486), cutting ab and bc in d and e ; from d and e with the same radius, db or be , describe arcs f and g ; lines, bf and bg , joined, will trisect the rt. $\angle$.

Prob. XIII. To inscribe a $\odot$ in a Δabc .

Bisect the $\angle$ s at a and b with two lines ad and bd (Fig. 487). From the intersection d , which will be the center of the $\odot$, draw $\perp de$, and it will be the radius of the $\odot$ required.

EXAMPLES ON PRACTICAL GEOMETRY FOR PRACTICE.

1 (a). Make an $\angle$ of 70° , and bisect it; (b) describe a circle, with 1" radius, tangent to both lines containing the $\angle$; (c) draw two other circles tangents to the first, and also to the lines containing the $\angle$.

2. Draw a line 2' long $\perp$ to a given line 3' long, from a point $\frac{1}{2}$ " from one end by geometrical construction.

3. Draw a line 3'47" long. Let this represent a length of 5' 5", and from one end mark off a distance of 1'6".

4. Construct an equilateral Δ having a perimeter of 6'14", and a regular hexagon of which the center shall be one $\angle$ of the Δ , and a side one side of the Δ . What is its perimeter?

5. Construct an isosceles Δ equal to the sum of the four squares of which the sides are $\frac{1}{2}$, $\frac{5}{8}$, $\frac{7}{8}$, and $1\frac{1}{4}$ inches respectively. Having found a square equal to the sum of the four squares, the isosceles Δ will have a base twice the side of this square, while its altitude will be equal to the side of the square.

6. Make a Δ of which the sides are 3'7.25 and 2'15 inches respectively. Measure with protractor and note down values of each of the 3 $\angle$ s.

7. Divide a line 3'57" long into 9 equal parts.

8. Draw ab $4\frac{1}{2}$ " long. Through a draw ac $2\frac{1}{4}$ " long making an $\angle cab$ (by construction) 75° . At b draw (by construction) bd (on the opposite side of ab and ac) $\perp$ to ab and $1\frac{1}{8}$ " long. Join cd and divide it by construction into 5 equal parts.

9. Construct, on a given line 2' long as a base, an isosceles Δ having a vertical $\angle$ of 30° . What is its perimeter?

10. Construct a rectilineal figure similar to a given rectilineal figure but less in area.

11. To describe a circle, which shall pass through 3 given points not in the same straight line.

12. To find by construction a mean proportional between two lines 2'7" and 3'9" respectively.

13. To make a rectangle = a given Δ as in example 6.

14. To make an isosceles Δ = any other Δ .

15. To make a Δ equal in area to a regular pentagon of 5" sides.

16. To make a square = a given rectangle, 3'74" long $\times$ 2' wide.

17. To make a square = the sum of two or more squares.

18. To draw a square = any given rectilineal figure.

19. Set off with the protractor an $\angle$ of 57° , and bisect this $\angle$; again bisect the $\angle$ s so obtained.

20. Draw five concentric squares .25" apart, the smallest having a side of .75". Ink in these squares with lines that increase in strength from the smallest to the largest.

21. From a line 7'5" long, cut off $\frac{3}{16}$ of the length by using the sector.

22. Divide a straight line 6'7" long into 9 equal parts, and through the points of division draw $\parallel$ straight lines $\frac{1}{2}$ " apart, making them alternately dotted and continuous.

23. Draw two circles with radii of .92" and .64" touching each other externally, and about them, circumscribe a Δ whose $\angle$ s shall be 46° , 62° , and 72° .

24. Construct an equilateral Δ 3'75" high.

CHAPTER VIII.

CONSTRUCTION OF SCALES.

703. Construction of Scales.—It would be manifestly impracticable to draw a full-size drawing of an object on paper for its large size, as the drawing would be too large. The usual practice, therefore, is to make a drawing in such a reduced scale, that each line drawn upon the paper has a known and fixed ratio to the line which it represents on the object. The ratio of reduction is termed the *Representative Fraction*, e.g., in Fig. 487, let abc be a Δr field of which ab measures actually 300'. Now by applying a scale of equal parts to ab , it will be found to measure 1", i.e., a length of 300' is represented by a line 1" long, and the drawing is said to be made to a scale of 300' to 1"; and all other sides of the Δr field will be represented, therefore, by lines drawn in the same proportion.

Further, it is clear that the actual length of each of the lines of the Δr field is 3600 times the length of the line which represents it on the drawing, i.e., every line in the drawing is $\frac{1}{3600}$ th part of the corresponding line in the field. This fraction which represents the ratio of the drawing to the field is called the *Representative Fraction*.

Representative Fraction, or R. F., expresses the ratio which the measurements on the plan bear to the corresponding actual measurements of an object surveyed in the field. Both dimensions given must be brought to the same denomination. The numerator will always be 1. The R.F. should be entered on every plan.

Examples :—(1) The scale of an inch to a foot would be a scale of $\frac{1}{12}$, for 1" represents 12". (2) In a scale, 6" represents 25 yds, the R.F. is $\frac{1}{180}$, for 6" represents 25 yds. or 900"; $\therefore \frac{6}{900} = \frac{1}{180}$. (3) Again suppose 1" represents $\frac{1}{4}$ of a mile; then the R.F. = $\frac{1}{47520}$, as $\frac{1}{4}$ mile = 47520".

The student is recommended to make an attempt to answer the following questions without seeing the book:—(1) A drawing is made to the scale of 50 yds. to 1"; find the R. F., or relation between the drawing and the real object. Here $50 \text{ yds} = 50 \times 3 \times 12 = 1800'$; $\therefore$ the R. F. $= \frac{1}{1800}$, i.e., every line in the drawing is $\frac{1}{1800}$ th part of the corresponding line of the object.

(2) A drawing is made to the scale of 3 leagues to 1"; find the R. F. $3 \text{ leagues} = 3 \times 3 \times 1760 \times 12 \times 3 = 570,240'$; $\therefore$ the R. F. is $\frac{1}{570240}$.

(3) A drawing is made to a scale of $2' = 1''$. Give the R. F.

Here $2' = 12 \times 2 = 24'$; $\therefore$ R. F. $= \frac{1}{24}$.

(4) If the R. F. be $\frac{1}{18}$, then 1" on the scale would represent 48" or 4' on the ground.

(5) If R.F. be $\frac{1}{1013780}$, then 1" on the scale represents 16 miles on the ground.

The R. F. should be entered in a conspicuous place on every plan, to represent the ratio. Sometimes, instead of, or in addition to the R. F., a *graduated straight line* termed a *scale* is drawn on the plan so that dimensions can be measured from it at any time; before the commencement of a drawing a scale should be drawn at the bottom or at the lower left-hand corner or wherever convenient, the scale and the drawing being equally affected by the contraction and expansion of the paper. The true distance on ground between any two places, from a plan of the same, can be ascertained by taking off the length between the two places with a pair of dividers and then by reading the distance between the points of the dividers on the graduated scale drawn on the plan; or by means of a plotting scale.

When it is said that a plan is drawn *full-size*, it means that the full-size of the object is drawn on the plan. 'On a drawing is written thus; *scale $\frac{1}{2}$ full-size*. Here every inch on the drawing represents 8' on the object, i. e., $\frac{1}{2}$ th of the object is drawn on the plan. When it is said that the scale of a map is 8" to the mile, it means that a mile distance on the ground is represented by a length of 8" on the map. A scale of 40' to 1" or $1'' = 40'$ shows that 1" on the map represents 40' on the ground.

704. The Scales may be Classified into:—(1) Plain scales; (2) Diagonal scales; (3) Vernier scales; (4) Comparative scales; (5) Proportional scales.

705. Simple or Plain Scales show equal primary (major) and secondary (minor) divisions only. In all scales, the unit of length of the scale must bear the same ratio to the real unit of length, as shown by the representative fraction, as a line in the drawing bears to the line which it represents.

Ex. 1. To Construct a Scale of 4' to 1" to show show feet (Fig. 488). Rule two $\parallel$ lines in pencil 6" long, and say $\frac{1}{10}$ " apart. Mark off 6" on the lower $\parallel$ line, and divide it into 6 equal parts, then subdivide the left-hand part into 4 equal parts. The major divisions show 4' each, and the subdivisions 1' each. In dividing, the $\perp$ s through the divisions are drawn up to the top line, and alternate horizontal lines are drawn between the two $\parallel$ lines. After the scale is drawn, it is inked in. The line at the right-end of the left-hand major division is marked 0. The major divisions are numbered 4', 8', 12, etc., from 0 to the right, and the subdivisions 2, 4 to the left of 0; and the word denoting the unit of measurement (here it is feet) is written on the right after the last figure; unless this is done the scale means nothing. To take 15' from this scale, put one leg of the dividers on the major division marked 12, and the other leg on the subdivision marked 3, the distance between the points of the dividers represents 15'. The end left-hand subdivision may again be subdivided into 12 equal part to show inches.

Exs. 2 and 3. Similarly, if the scale be: (i) $2' = 1''$; or (ii) $20' = 1''$ (Figs. 489 and 490). In each case draw two $\parallel$ lines, say about $\frac{1}{10}$ " apart, mark off 6" on the lower line; divide these into 12 equal parts,

Fig. 488.—Plain Scale.

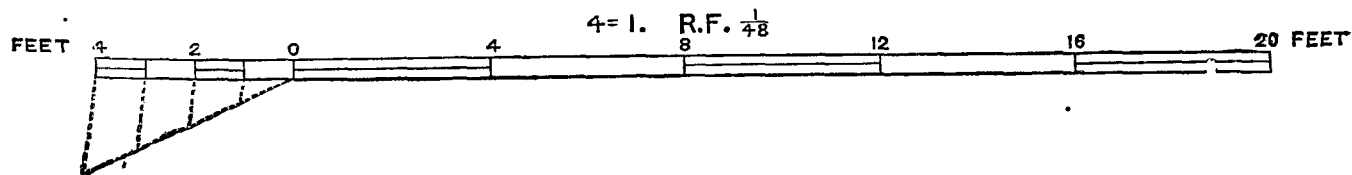


Fig. 489.—Plain Scale.

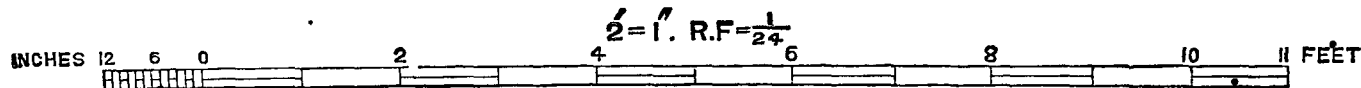


Fig. 490.—Plain Scale.

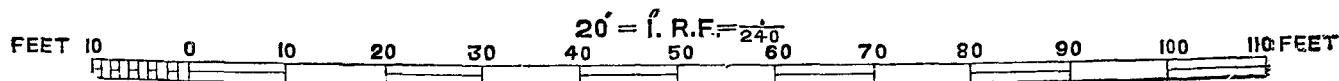


Fig. 491.—Plain Scale.

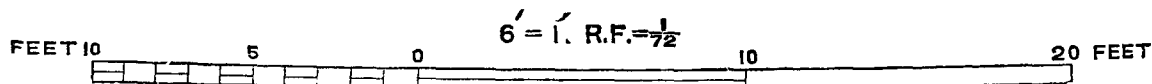
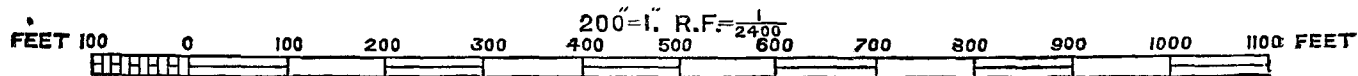


Fig. 492.—Plain Scale.



subdivide left-hand subdivision into 12 and 10 equal parts respectively. Each primary division will give 1' in (Fig. 489) and 10' in (Fig. 490), and the subdivision will show 1' in (Fig. 489), and 1' in scale (Fig. 490) respectively.

Ex. 4. To construct a scale of 6' = 1 (Fig. 491). Take 5" which will show 30', divide it into 3 equal parts, and the left-hand major division into 10 parts—each major division will show 10' and each subdivision 1'.

Ex. 5. To construct a scale of 200' = 1" (Fig. 492). R. F. = $\frac{1}{2000}$. Take 6", divide it into 12 equal parts, each major division will represent 100'. Divide the left-hand major division into 10 equal parts, each showing 10'.

Ex. 6. To construct a scale of 300' = 1" (R. F. = $\frac{1}{3000}$).

Divide the length of 6" into 18 equal parts, each showing 100', and the left-hand division into 10 equal parts, each showing 10'.

Ex. 7. To construct a scale of 400' = 1" (R. F. = $\frac{1}{4000}$).

Take 6", divide it into 24 equal parts, each showing 100', and the left-hand division into 10 parts, each showing 10'.

Ex. 8. To construct a scale of 16' = 1 mile or 330' = 1"; or 5 chains (Gunter's) to the inch. R. F. = $\frac{1}{33000}$ (Fig. 493). Suppose, 2000' will be required to be shown on the scale.

Then $330' : 2000' :: 1' : x'' = 6.06''$.

Divide 6.06" into 20 equal parts, each representing 100'. Subdivide the left-hand major division into 10 equal parts, each will represent 10'.

Ex. 9. To Construct a scale of 16" = 1 mile to show Bighas and Cottahs (Fig. 494).

Here 1 mile = 5280' = $80 \times \frac{1}{2} \times 44 = 44$ bighas (linear) or *rashis* of 120' = 80 haths. $\therefore$ 16" represent 44 bighas or 4' represent 11 bighas. Lay off a length of 4'; divide it into 11 equal parts, each division will represent a linear bigha. Sub-divide the left-end division into 10 equal parts, each showing 2 cottahs. If the left-end division is sub-divided into 4 parts, each will represent the measuring chain of 30' or 5 cottahs. This last is the Thakbust or Bigha-cottah scale.

706. Diagonal Scales.—Very minute divisions can not be obtained from plain scales, diagonal method is resorted to for obtaining very minute sub-divisions. It will be clear from the preceding examples of plain scales that if we wish to measure a fractional part, such as tenths, of a secondary division, we can not do it with any accuracy; but by means of a diagonal scale we can measure hundredths of primary divisions, hence representing a method in decimal notation. From plain scales we get two dimensions such as yards and feet; inches and $\frac{1}{10}$ of an inch, etc.; from diagonal scales we get three dimensions, as yards, feet, and inches. *To draw a diagonal scale the left-hand sub-division must be divided into the number of units of the 2nd dimension required, and the number of parallel lines drawn above the scale line must be the same as the number of units there are of the third dimension in a unit of the 2nd dimension.*

706a. The Diagonal Scale in the Draftsmans' Pattern Protractor is $5\frac{1}{2}$ " long divided into $\frac{1}{2}$ " except the left-end division as *ab* in Fig. 495, which is 1" long, and is divided into 10 equal parts for the scale to read to $\frac{1}{100}$ th of an inch as under:

10. To Draw a Diagonal Scale showing Inches, Tenths, and Hundredths of an Inch.

Rule 10 parallel equidistant lines above *ab*, and draw *a a'* and *b b'* $\perp$ to them (Fig. 495).

Fig. 493.—Plain Scale.

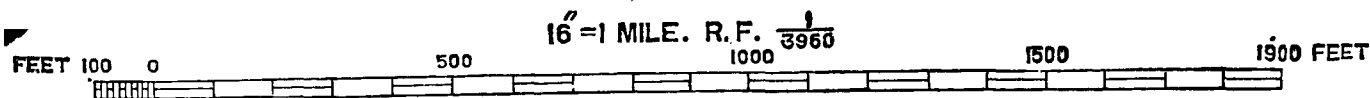


Fig. 494.—Plain Scale.

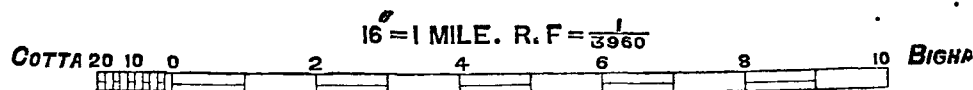


Fig. 495.—Diagonal Scale.

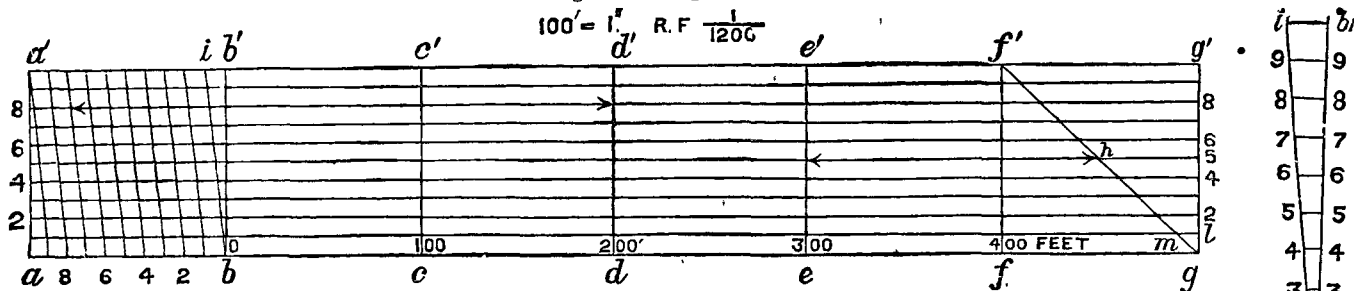


Fig. 497.—Diagonal Scale.

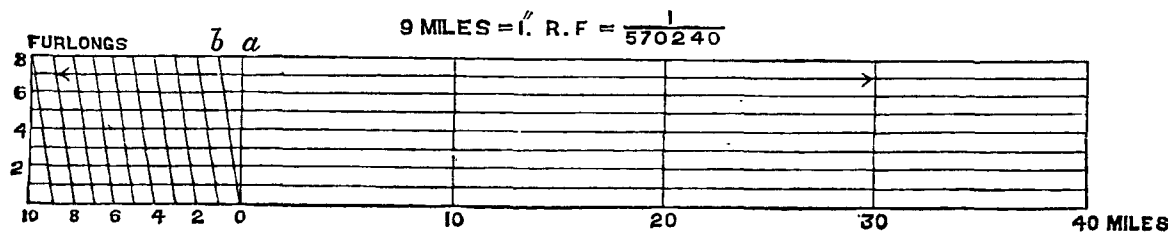
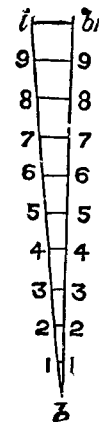


Fig. 496.



Divide ab and $a'b'$ each into 10 equal parts, mark the points of division on ab by the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Join the points of division of ab and $a'b'$ by diagonal straight lines; i.e., join b with the point, i , adjacent to b' , 1 with the next point in $b'a'$, 2 with the next point, and so on. Thus the scale is constructed. Each of the divisions of $ab = ab/10$; and the divisions on the $\parallel$ lines intercepted between bb' and bi are: $ab/100$, $2ab/100$, $3ab/100$, ..., $10ab/100$; i.e., $1/100$, $2/100$, $3/100$, ..., $10/100$ inch respectively as ab represents 1 inch. The principle may be explained as under:—Let bi and bb' (Fig. 496) be two straight lines meeting at b , and let bb' be divided into 10 equal parts (figure drawn to a larger scale). Join ib' , and through the points of division in bb' draw lines $\parallel ib'$; $\therefore$ the Δs ibb' and $b1i$ are similar, and $ib' : bb' :: 1-1 : 1-b$; or $ib'/1-1 = bb'/1-b$; but bb' is 10 times $1-b$; $\therefore$ ib' is also 10 times $1-1$, i.e., $1-1 = ib'/10$; but $ib' = ab/10$; $\therefore$ $1-1 = ab/100 = 1/100$ inch. Similarly $2-2 = 2ab/100 = 2/100$ inch, and so on. The length between arrows on parallel through 8 = $2.78''$.

706b. In the Diagonal Scale in Draftsman's Pattern on the right-side to read to twentieth and two-hundredths of an inch, the $\frac{1}{2}''$ is divided into 10 equal parts, i.e., each equal to $\frac{1}{20}$ of an inch. The diagonal scale is made like Fig. 495. The distances, 1-1, 2-2, etc., (Fig. 496) in the $\frac{1}{2}''$ -scale are $1/200$, $2/200$, inch and so on.

706c. The Diagonal Scales in Military Protractor in Instrument boxes are 2 in number. In one, the distance of the primary divisions is $\frac{1}{2}$ an inch, and in the other a quarter of an inch. One primary division at each end is divided into 10 secondary divisions, i.e., $\frac{1}{20}$ th and $\frac{1}{40}$ th of an inch. The diagonal divisions are made as in Fig. 495. The distances as 1-1, 2-2, etc., (Fig. 496) in the $\frac{1}{2}''$ -scale are $\frac{1}{200}''$, $\frac{2}{200}''$, and so on. The distances as 1-1, 2-2, etc., in the quarter-inch scale are $\frac{1}{400}''$, $\frac{2}{400}''$, etc., inch.

Ex. 11. To Construct a Diagonal Scale of $100' = 1'$ to show Feet (Fig. 495). Take a length $ag = 6'$. Divide it into 6 equal lengths, ab, bc, cd, de, ef, fg , each representing $100'$. Rule any straight line $a'g' \parallel ag$, and on it set off 6 lengths $a'b', b'c', c'd', d'e', e'f', f'g'$, each $= ab$, so that $aa', bb', cc', dd', ee', ff', gg'$ are $\perp$ s to ag . Now divide aa' into 10 equal parts; mark the points of division by the numbers 1, 2, 3, ..., 9, 10, and through these points rule straight lines $\parallel ag$. Divide ab and draw the diagonal scale as in Ex. 10. Each of the divisions of $ab = ab/10$, and the divisions on the $\parallel$ lines intercepted between bb' and bi are $ab/100$, $ab/200$, $ab/300$, etc., i.e., 1, 2, 3, ..., 10 feet respectively as ab shows 100 feet. To find the length of a straight line of $278'$, place one leg of a pair of dividers at the intersection of the line dd' and the 2nd $\parallel$ line from the top, and the other at the intersection of the diagonal line from 7 and the 2nd $\parallel$ line from the top i.e., through 8 (Fig. 495).

12. To Draw a Diagonal Scale showing Inches and Tenths.

In Fig. 495, fg is $1''$ long. gg' is $\perp fg$, and is divided into 10 equal parts, and $\parallel$ s are drawn from the points of division to ff' . Now, if the diag. fg is drawn, we get from this scale inches and tenths thus:—Suppose $1\frac{5}{10} = 1.5'$ be required. As the rectangle $f'g$ is divided into 10 equal parts by $\parallel$ s to fg , and the diag. fg is drawn, then a number of 10 similar Δ s is formed as explained before by Fig. 493 (Euc. VI. 2). Now, if $g5$ is $\frac{1}{2} gg'$, then $5h$ is $\frac{1}{2} fg$; so place the dividers on the arrow on vertical ee' , and open it until the point h is reached as to get $1.5'$. Here $lm = 0.1'$, and the other $\parallel$ lines intercepted between gg' and gf' above ml are 0.2 , 0.3 , 0.4 , 0.5 , 0.6 , 0.7 , 0.8 , 0.9 , and 1 inch respectively.

Ex. 13. To Construct a Diagonal Scale of $16' = 1$ mile, showing links to measure 3000 links. Here $16' = 5280'$ or $1' = 5$ Gunter's chains = 500 links. Then $500 : 3000 :: 1' : 6'$. Take $6'$, divide it into 6 equal parts; each will therefore represent 500 links. Draw the diag. scale as above, then each of the left-hand sub-divisions equals 50 links, and the divisions on the $\parallel$ lines intercepted between bb' and bi (Fig. 496) are $\frac{500}{100}$,

$\frac{2 \times 500}{100}$, $\frac{3 \times 500}{100}$, $\frac{4 \times 500}{100}$, $\frac{5 \times 500}{100}$, $\frac{10 \times 500}{100}$ links, i.e., 5, 10, 15, 20, 25, 30, 35, 40, 45, 50 links. Therefore in this scale the minutest division that can be read is 5 links. In Fig. 495, instead of reading 278', the length between the arrows will represent 1390 links. The scale is not separately drawn.

Ex. 14. To Construct a Diagonal Scale of 9 miles to the inch, showing Furlongs, and measuring 50 miles (Fig. 497).

$\therefore 9 \text{ mi.} : 50 \text{ mi.} :: 1' : 5'55''$. Take a length of $5'55''$. Divide it into 5 equal parts, each division showing 10 mi.; subdivide the left-hand major division into 10 equal parts, each showing 1 mile. Above this plain scale draw 8 equidistant $\parallel$ lines and the diagonal scale as shown. Then each of the divisions on the $\parallel$ lines between oa and ob is 1, 2, 3, 4, 5, 6, 7, 8 furlongs respectively. The length between the arrows shows 38 mi. 7 fur.

Ex. 15. To Construct a Scale of $8' = 1''$ diagonally to show yds., feet, and inches, and mark 4 yds. 2' ft. 2" ins. (Fig. 498). Here the R. F. $= \frac{1}{96}$. Take 3"; it will represent 24' or 8 yds., divide it into 8 equal parts, each will represent 1 yd. Divide the left-hand major division into

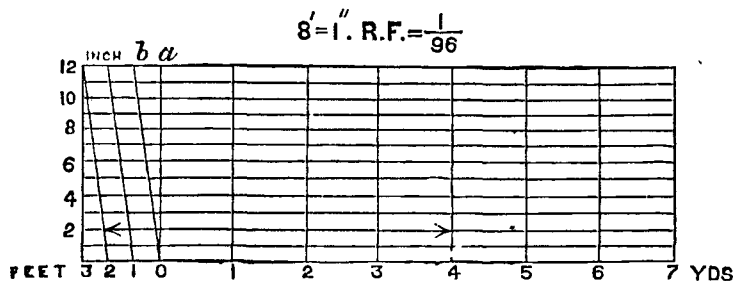


Fig. 498.—Diagonal Scale to show Yards, Feet and Inches.

3 equal parts, each showing a foot. Draw 12 equidistant $\parallel$ lines above this plain scale to form the diagonal scale for inches. Each subdivision as $ba = 1' = 12''$, and the subdivisions on the $\parallel$ lines intercepted between oa and ob are 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, and 1 inch respectively. The length between the arrows shows 4 yds. 2 ft. 2 inches.

Ex. 16. To Construct a Diagonal Scale of $1' = \frac{3}{4}''$ to measure feet, inches and tenths of an inch, and measuring 4' (Fig. 499).

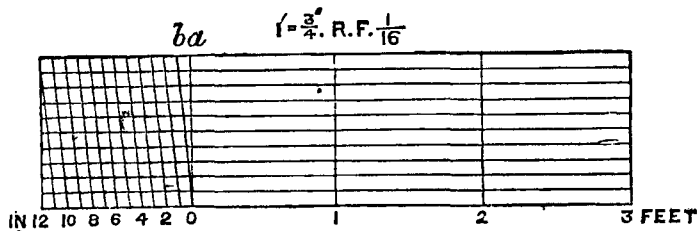


Fig. 499.—Diagonal Scale to show Feet, Inches and Tenths of an Inch.

Here $1' : \frac{3}{4}'' :: 4' : 3''$.

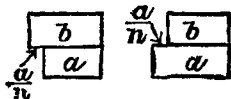
Take 3"; divide it into 4 equal parts, each showing 1'. Divide the left-hand major division into 12 equal parts, each will represent 1". Now rule 10 equidistant $\parallel$ line below this plain scale to form the diagonal scale for $\frac{1}{10}$ of an inch. Each sub-division $ba = 1''$, and the divisions on the horizontal $\parallel$ lines intercepted between oa and ob are $1'', \frac{9}{10}, \frac{8}{10}, \frac{7}{10}, \frac{6}{10}, \frac{5}{10}, \frac{4}{10}, \frac{3}{10}, \frac{2}{10}, \frac{1}{10}$.

If on the scale, feet, inches and eighths of an inch were wanted, then instead of 10 horizontal parallel lines 8 would have been necessary.

706d. The Scales for Latitudes and Longitudes.—In order to divide up the geographical maps into graticules of some integral order parts of degrees, diagonal scales of degrees, minutes and seconds are used for plotting data on such maps.

Ex. 17. Draw Scales for a Standard Map of India on the 1" = 1 mile scale which is 15' in longitude by 15' in latitude; the scale for plotting would be of 5' each. To draw the scales, take $\frac{1}{2}$ of the length in latitude and in longitude, divide each into 5 equal parts each equalling 1', and mark from the right-hand side : 4, 3, 2, 1, 0, 1. Subdivide the space 0-1 into 6 equal parts each equalling 10", and with 10 divisions perpendicularly, the lowest division as 1-1 in Fig. 496 will represent 1'. The scales will vary except at the equator.

707. Vernier Scales.—They consist of two plain scales (to read minute divisions like the diagonal scales), the primary and the secondary or vernier scale, the graduations of which are obtained from those of the primary scale, which is a plain scale divided into major and minor divisions. The vernier attached to a plain scale is usually fixed, unlike those in theodolite and other surveying instruments. The principle of the vernier is as under:—Let a represent the length of a minor division on the primary scale, and b represent the length of a graduated space on the vernier to be constructed; and the length of this graduated space is required. Now, if the number of divisions on the primary scale be n , and $n+1$ such parts be taken from it, and divided into n equal parts for the vernier, then each latter subdivision will be greater than the former



Figs. 500, 501.—Principle of the Vernier.

By the $\frac{1}{n}$ th part of a subdivision on the primary scale (Fig. 500). $\therefore (n+1)a = nb$, or $na + a = nb$; and $a + \frac{a}{n} = b$ or $b - a = \frac{a}{n}$. Similarly the difference between 2 divisions of the one and two of the other will be $\frac{2a}{n}$; between 3 of one and 3 of the other $\frac{3a}{n}$, and so on. If instead of $(n+1)$ divisions, $(n-1)$ divisions are taken from the primary scale and divided into n equal parts, then $(n-1)a = nb$, or $na - a = nb$, and $a - \frac{a}{n} = b$ (Fig. 501), i.e., each subdivision on the vernier is less than the primary subdivision by $\frac{1}{n}$ th part of a division on the latter. Hence to obtain a length, it is only necessary to make a minor division of one scale coincide with one in the other, *e. g.*, at the zero division in the following examples, and to read on the left for the length required.

Ex. 18. As an example of the foregoing conclusions, suppose a certain length to represent 8 ft., and it is necessary to take off inches from it. Divide the length into 8 equal parts, then each part will show one foot. It is necessary to take $n=12$ as $\frac{1}{12}$ of a foot = 1"; and therefore $(n+1)=13$, or $(n-1)=11$. Then take a length equal to 13 or 11 minor divisions on the primary scale, and divide this length into 12 equal parts for the vernier: each of these will be larger or smaller than the divisions on the primary scale by $\frac{1}{12}$ of the space representing 1', i.e., the vernier minor divisions will represent $\frac{13}{12}$ or $\frac{11}{12}$ of a foot, i.e., 1' 1" or 11"; and the difference between a division on the primary scale and a division on the vernier will represent 1.

Ex. 19. To Construct a Scale of $\frac{1}{48}$ to show Feet and Inches.

The plain scale will be $4' = 1''$; but the vernier is different, and each of the minor divisions of the primary scale equals one foot. In Figs. 502 and 503; 13 and 11 parts of the primary scales are taken and set off on the upper lines from zero of the scale on the left, and divided into 12 equal parts respectively. The space of each vernier division in the two scales is $1\frac{1}{12}'$ or $1\frac{1}{12}''$, i.e., $1'1''$ or $11''$ respectively, so that the several distances on the vernier from zero will give $1'1''$, $2'2''$, $3'3''$, and so on; or $11''$, $22''$, $33''$, and so on, respectively on the left of the scales. To compass off $11'7''$, hold one leg of the dividers on the 4th primary division, and another on the 7th division of the vernier (Fig. 502). On Fig. 503 hold one leg of the dividers on the 7th primary division, and another on the 5th division of the vernier. In one case seven divisions of the vernier give $7'7''$ which with 4 divisions on the primary scale equals $11'7''$; in the other 5 divisions on the vernier give $4'7''$ which with 7 divisions on the primary scale comes to $11'7''$.

Ex. 20. To Construct a Scale of $\frac{1}{48}$ to show Feet and Tenths.

The plain scale will be exactly like the one in Ex. 19 above, but the vernier is different, as tenths instead of twelfths are to be shown. Each minor division on the plain scale equals $1'$. Now set off on the upper line to the left of zero a distance equal to 11 or 9 minor divisions of the plain scale, and divide them into 10 equal parts for the vernier. Since 11 or 9 minor divisions are divided into 10 equal parts, each division on the vernier will represent $1'1''$ or $'9''$ of the minor divisions of the plain scale, and each of them will show $1'1''$ or $'9''$. Therefore the several lengths from zero of the scales to successive divisions on the vernier will show $1'1''$, $2'2''$, $3'3''$, $4'4''$, $5'5''$, $6'6''$, $7'7''$, $8'8''$, $9'9''$ and 11 feet, or $'9''$, $1'8''$, $2'7''$, $3'6''$, $4'5''$, $5'4''$, $6'3''$, $7'2''$, $8'1''$ and 9 feet respectively. The scales are not drawn but left to the student as exercises.

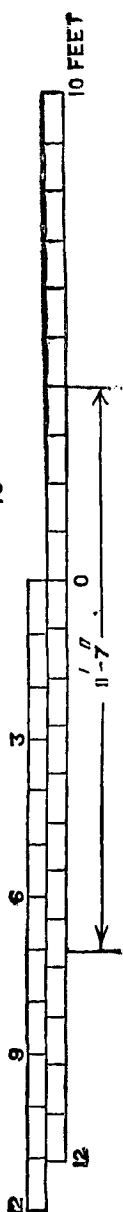
Ex. 21. Suppose, the vernier in the scale in Ex. 19, Fig. 504, is movable; and the zero of the vernier at m comes up to n (i.e., a little distance $'a''$ after the first primary division to the right of 0 on the primary scale), and is marked with an index m . Find out the length between 0 of the primary scale and the index. This evidently one foot + a fraction of a foot. If $a = \frac{1}{12}$ of the minor division of the primary scale or $1''$, the first division on the vernier will coincide with the first division on the primary scale directly under it. If $a = 2''$ or $3''$, the 2nd or 3rd division will coincide with a graduation on the primary scale, and so on. So a movable vernier can be read easily by mere coincidence of the divisions of the primary and vernier scales. In the figure 9th division of the vernier coincides with a division on the primary scale; hence $a = 9''$.

The vernier scales are used for the same purpose as the diagonal scales to meet the deficiency of the plain scales which can not be constructed to read to a very small unit. In Fig. 488 (Plain Scale of $4' = 1''$) it would not be easy to read with accuracy inch divisions, if one of the minor foot-divisions were divided into 12 equal parts to show inches, as the vertical lines making these divisions would be very near each other; whereas in the vernier Scales, (Figs. 502 and 503), inch divisions can be read with great exactitude.

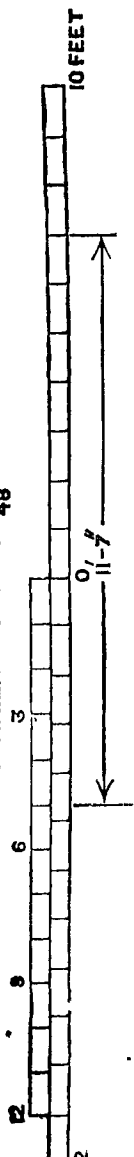
708. The Advantages and Disadvantages of the Diagonal and Vernier Scales are as under:—(1) Many $\parallel$ lines are to be drawn in drawing a diagonal scale, therefore making it too broad and unwieldy before it can be read to very great accuracy; whereas a vernier scale can be easily constructed to read to the required exactitude, making it thinner than the former. In the vernier only three $\parallel$ lines have to be drawn; whereas in the diagonal scale many more are to be drawn to complete it.

(2) On the diagonal scale lengths are taken off as easily and directly as on a plain scale; while on the vernier they are to be taken both on the plain scale as well as on the vernier after some

VERNIER - $4' = 1''$. R.F. = $\frac{1}{48}$



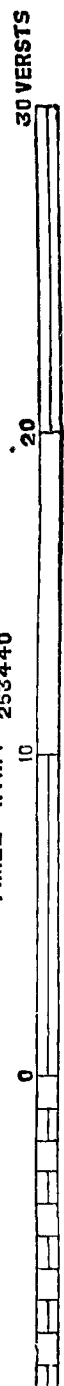
VERNIER $4' = 1''$. R.F. = $\frac{1}{48}$



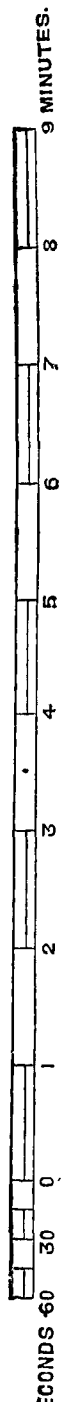
MOVABLE, VERNIER. $4' = 1''$



$4 \text{ MILE} = 1''$. R.F. = $\frac{1}{253440}$



$6' = 1''$. MILE. R.F. = $\frac{1}{10560}$



mental calculations ; the former part of the distance depends on where the vernier division falls, *i.e.*, how many divisions before the zero on the primary scale. In a movable vernier this disadvantage disappears.

709. The Comparative or Corresponding Scales:—One scale is said to be *comparative* to another when the distances measured by the one can be measured by the other ; *i.e.*, when the given scale of a plan reads in a certain measure and *it is desired to construct a scale for the plan reading in some other measure*, this new scale is a comparative scale, *e.g.*, (1) if we have a scale of miles to which a drawing is made, we can construct another scale by which the distances from place to place can be measured in furlongs. The scale of furlongs would then be comparative to the scale of miles. (2) If the scale on a French plan reads in leagues, and it is desired to take measurements off the plan in miles, a comparative scale must be constructed. In making one scale comparative to another, therefore, we must, of necessity, have two units of measure, as an inch and a foot, a foot and a yard, a mile and a chain, etc.

Ex. 22. Suppose it is required to make a Scale of Yards comparative to Ex. 1 of the Plain Scales. By proportion we have— $4' : 1'' :: 3' : x = \frac{3}{4}''$ *i.e.*, the number of inches to represent $30'$ (10 yds.) will be $(3 \times 10)/4 = 7.5''$. Make the scale $7.5''$ long, and divide it into 10 divisions to measure single yard.

In fact these are scales which have the same R. F. ; but which are graduated to read to different units, and used mainly to save calculations, to read direct on the scale and in the unit required, the information required from the map.

Ex. 23. To Construct a Scale of 4 miles to an inch or $\frac{1}{10560}$ to show 40 Russian Versts. One Verst = 1166.6 English yards (Fig. 505).
 $(4 \text{ mi.} = (1760 \times 4)/1166.6 \text{ versts}) : 40 \text{ versts} :: 1'' : x = 6.63''$

Take a line $6.63''$ long ; divide it into 4 equal parts, each will represent 10 versts, and the first primary division into 10 equal parts, each to show 1 verst.

Ex. 24. To Construct a Comparative Scale of $6'$ to the mile to read $\frac{1}{4}$ minute, adapted to the paces of a horse that trots 6 miles an hour (Fig. 506). R. F. = $6/(5280 \times 12) = \frac{1}{10560}$.

Then $10560'' : 6 \text{ mi.} :: 1'' : x'' = (6 \times 5280 \times 12)/10560 = 36''$.

But 6 miles expressed in the unit of time correspond to one hour.

$\therefore$ 60 minutes are represented by $36''$, or 10 minutes by $6''$.

Set off a line $6''$ long, divide it into 10 major divisions of one minute, and subdivide the first division into 4 subdivisions, each representing $\frac{1}{4}$ minute.

710. The Proportional Scales are used for enlarging or reducing a drawing to a given proportion.

Ex. 25. If the same distance on two plans on different scales be represented by ab and ac (Fig. 473), then all lines parallel to bc will cut off lengths from a in the ratio of $ab : ac$. Hence, taking any dimension on one plan in the dividers, or plotting scale, and applying it from a along its line, say ab , the length of the same dimension on the other plan will be found by moving the right leg of the dividers down the other line ac .

Ex. 26. Another form is shown by Fig. 507. Suppose we have to enlarge a drawing drawn on a scale of $400' = 1''$ to one on a scale of $150' = 1''$, *i.e.*, in the proportion of 8 : 3. Draw a line AB of convenient size to suit the measurements in the drawing, and produce it to C making $AB \frac{5}{8}$ of BC . Erect $\perp$

BD of any height; join AD and DC. Divide BD into any number of equal parts and draw lines parallel to AB. These are simply guide lines to enable measurements to be taken parallel to the base, *e.g.*, on placing a measurement from the original drawing on the scale, we find it occupies the position of *ab*; the distance between *b* and *c* will then give the length of the measurement to the enlarged scale, *i.e.*, in the proportion of 8 : 3. We go on in the same manner with every other measurement in the original. Should we wish to *reduce* a drawing in the same proportion, *viz.*, 3 : 8, the original measurements would be placed on the right-hand side of the scale, and the required proportion taken from the left-hand side.

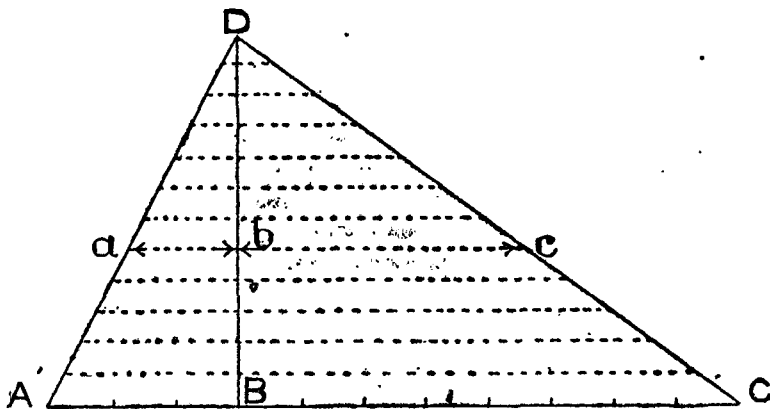


Fig. 507.—Proportional Scale.

711. Examples for Practice*.—Plain Scales.

Exs. 27—30. Draw the following Scales; give R. F. of (a), and the ordinary nomenclature of (b), (c) and (d).

Ex. 27. (a) 180 miles to the inch (b) $\frac{1}{55680}$. (c) $\frac{1}{108800}$. (d) $\frac{1}{10880}$.

Ans:—(a) R. F. = $\frac{1}{180 \times 5280 \times 12} = \frac{1}{1140480}$. Show 900 miles.

$\therefore 11404800'' : 1'' :: 900 \text{ mi} : 5''$.

Divide 5'' into 9 equal major-divisions, each representing 100 miles; subdivide the first major-division into 20 equal parts, each showing 5 miles.

Ex. 28. It is a scale of $1\frac{1}{2}$ miles to an inch. Suppose the scale is required to show 8 miles. Then $1\frac{1}{2} \text{ mi.} : 1'' :: 8 \text{ mi.} : x = 5.33''$.

Divide 5.33'' into 8 equal parts, each showing 1 mile. Subdivide the extreme left major division into 8 equal parts, each will represent 1 furlong.

Ex. 29. It is a scale of 80 miles to the inch. Suppose the scale to read 400 miles. Then $80 \text{ mi.} : 1'' :: 400 \text{ mi.} : x = 5''$. Divide 5'' into 4 equal parts, each representing 100 miles. Subdivide the first part into 20 minor divisions, each will represent 5 miles.

Ex. 30. It is a scale of $6'' = 1 \text{ mile}$. Show 5000'.

Then $5280 : 5000 :: 6'' : x = 5.7''$. Divide 5.7'' into 10 major divisions, each representing 500'; subdivide the extreme left major division into 20 equal minor divisions, each representing 25'.

Ex. 31. If the scale in Ex. 30 is made to represent yards, suppose 2000 yards, to be shown, then $1760 \text{ yds.} : 2000 :: 6'' : x = 6.82''$.

Divide 6.82'' into 20 equal parts, each equalling 100 yds.; subdivide the first part into 10 equal parts, each showing 10 yds.

* Students are to Draw the Scales as Exercises.

Ex. 32. Again, if the same is made to represent furlongs and chains of 66'. Divide 6' into 8 equal major divisions, each representing 1 fur.; and subdivide the end left-hand division into 10 equal parts, each showing 1 chain of 66'. R. F. = $\frac{1}{8000}$.

Ex. 33. To Draw a Scale of 9' = 1" to show 60', and give R. F.

as 9' : 1" :: 60 : $x = 6 \cdot 66'$. Here R. F. = $\frac{1}{1000}$.

Set off 6'66', and divide it into 6 equal parts, each representing 10'; subdivide the extreme left division into 10 equal parts, each showing 1'.

Ex. 34. To Draw a Scale of $\frac{1}{80}$ to show yards and feet, to measure 15'.

This is a scale of 5' = 1". Divide 3' into 5 equal major divisions, each representing one yard; subdivide the first into three parts each equalling 1'.

Ex. 35. To Draw a Scale of 875' = 10' to show 40'. Give R. F.

Here 40' would be represented by 3'5". Divide 3'5" into 4 equal parts giving lengths of 10'. Subdivide the left-hand fourth into 10 equal parts showing feet. R. F. = $\frac{1}{8750}$.

Ex. 36. To Construct a Scale of $\frac{1}{10}$ to read inches, and show 10'.

Here 19' : 10' :: 12' : $x = 6 \cdot 31'$. Divide 6'31" into 10 equal parts showing feet. Subdivide the first unit into 12 equal parts showing inches.

Ex. 37. To Construct a Scale of 2 miles to an inch, showing miles and furlongs. Take 5'5 to represent 11 miles. Divide it into 11 equal parts, each representing a mile; subdivide the first part into 8 equal parts to show furlongs.

Ex. 38. To Construct a Scale to show 70 yds. when 1" represents 9 yds. Give R. F. 7'77" will represent 70 yds. ∴ Divide 7'77" into 7 equal parts, each giving 10 yds. Subdivide the first primary division into 10 subdivisions, each showing a yd. Here R. F. = $\frac{1}{3112}$.

Ex. 39. The distance between two places is 4 miles, and is represented by 1'5". Construct a scale to show furlongs, and to measure 20 miles. Give R. F. Here 4 miles : 1'5" :: 20 miles : $x = 7 \cdot 5'$.

Divide 7'5" into 2 parts, each to show 10 miles. Subdivide the first primary division into 10 equal parts, each to read 1 mile. Divide the 10th subdivision on the left-hand into 8 equal parts for furlongs. R. F. = $7 \cdot 5 / (20 \times 1760 \times 36) = 1/168960$.

Ex. 40. To Construct a Scale of 50' = 1" showing 1'.

Divide 6' representing 300' into 30 equal parts, each showing 10', and the left-hand major division into 10 equal parts, each subdivision showing 1'.

Ex. 41. To Construct a Scale of 16' = 1" showing feet and measuring 110'. Then as 16' : 110' :: 1' : $x = 6 \cdot 87'$. Lay down 6'87". Divide it into 11 equal parts, each showing 10'; and subdivide the extreme left division into 10 equal parts, each showing 1'.

Ex. 42. Another way of constructing a scale of 4' = 1". Divide 5' to represent 20' into 2 equal parts, each showing 10'; subdivide the left-hand division into 10 equal parts, each showing 1' which may be divided into 12 parts for inches.

Ex. 43. To Construct a Scale of feet to measure 400', least dimension being 10', if 60' be represented by 75". Give R. F.

There will be 400' in 5'. Divide 5' into 4 equal parts, each showing 100'; and the first division into 10 equal parts each showing 10'. R. F. = $\frac{1}{8000}$.

Ex. 44. In a scale 5 furlongs are represented by $1\frac{1}{8}"$. Draw the scale and give R. F. R. F. = $\frac{1}{38400}$. Here 5'4" will represent 24 furlongs or 3 miles. Divide 5'4" into 3 equal parts, each showing a mile, and the left-hand third into 8 equal parts, each equalling a furlong.

Ex. 45. A line on a map is 7" long, and if the scale be $\frac{3}{4}"$ to a foot what length in feet and inches does it represent?

Ans. — as $\frac{3}{4}"$: 7" :: 1' : $x = 9' 4"$. To draw the scale, lay down 6' representing 8'; divide it into 8 equal parts, each showing a foot, and left-hand eighth into 12 equal parts, each representing 1'.

Ex. 46. If a distance of 15 miles on a map be represented by a line 2·27" long, mark off on your scale a distance of 37 miles.

Divide 6·05" representing 40 miles into 4 equal parts; subdivide the left-hand fourth into 10 equal parts, each representing one mile. To mark off 37 miles, place one leg of your dividers on the 30th mile point on the right of 0, and the other on the 7th mile point on the left of 0.

Ex. 47. To Construct a Scale of 1/79200 to show Miles and Furlongs. 79200" = $1\frac{1}{2}$ miles is represented by 1"; hence 4" represent 5 miles. Divide 4" into 5 equal parts to show miles; subdivide the left-hand fourth into 8 equal parts to show furlongs.

Ex. 48. The R. F. of a Survey is $\frac{1}{304}$. Draw a scale of yards. 204" or 17' or $1\frac{1}{3}$ yds. is represented by 1"; hence 30 yds. is represented by 5·29". Divide 5·29" into 3 equal parts, each representing 10 yards, subdivide the left-hand third into 10 equal parts, each showing a yard.

Ex. 49. To Construct a Scale of $\frac{1}{175840}$ to read chains of 100' each. If on a plan drawn to this scale two points are 2·5" apart, what is the distance in feet between the points?

Here 1" represents 15840' or 1320'. $\therefore$ 5" represent 6600'. Divide 5" into 11 equal parts, each equalling 600', and subdivide the part on the extreme left into 6 equal parts, each representing 100'. $\therefore$ 2½" represent 3300'.

Ex. 50. When a Survey is plotted to a Scale of 5 chains to an inch, how much larger is the actual ground than the plan?

5 Chains on ground = 1" on paper. $\therefore$ 25 sq. chains on ground = 1 sq. in. on paper. $\therefore$ $5 \times 66 \times 12 \times 5 \times 66 \times 12 = 15681600$ sq. ins. on the ground = 1 sq. in. on paper, i.e., the ground is 15681600 times larger than the plan.

Ex. 51. What is the Scale of a Plan on which a Slope of 30°, and 100' high, measures 2" in length? The horizontal length of slope shown on plan is $100 \times \cot. 30^\circ = 100 \sqrt{3} = 100 \times 1.732$. Hence 2" represent 173·2' or 1" represents 86·6'. $\therefore$ the scale of plan is 1" = 86·6'.

Ex. 52. A Survey of a Country 14 miles $\times$ 10 miles is to be plotted so as to fit on to an "imperial" sized paper. What scale would you adopt? The size of an imperial sheet of paper is 30" $\times$ 21". It is evident, if the scale be 2" = 1 mile, the paper will hold the plan.

DIAGONAL SCALES.

Ex. 53. To Construct a Scale of 5" = 1' showing Feet and Inches.

Divide 3" into 5 equal parts, each will represent a yard. For foot division, divide the first part into 3 equal parts, draw 12 equidistant || lines above this plain scale, and complete by drawing diagonals for inches.

Ex. 54. If the scale whose R. F. is $\frac{1}{27}$ be 2½" = 1', measure up to 10' and show inches. Divide 4·44" to represent 10' into 10 equal parts, each showing 1'; take 3 divisions representing a yard, and over this draw 12 equidistant || lines and diagonals all to form diagonals for inches.

Ex. 55. To Construct a Diagonal Scale of 8" = 1 mile to measure furlongs and diagonally paces of 60'.

Divide 6" into 6 equal parts, each will represent a furlong. Draw 11 equidistant || lines below or above this scale, and draw the diagonals in the first segment to read to paces of 60' or 1/11 of a furlong.

Ex. 56. Scale $\frac{3}{4}$ " to represent 1 chain. R. F. = $\frac{3}{4} \div 66' = \frac{1}{88}$.

The smallest distance to be taken off the scale is 5 links. Draw 5 parallel lines. Set off 3", divide it into 4 equal parts, through the points of division draw verticals. Divide the first space into 5 equal parts, giving distances of 20 links. Draw diagonals, thus getting distances of 20, 15, 10 and 5 links.

Ex. 57. To Construct a Diagonal Scale of 2" = 1' to read to $\frac{1}{100}$ of a foot.

Here 1' : 2" :: 2" : $x = 4'$. Divide 4" into 2 equal parts, each showing a foot. Subdivide the left-hand segment into 10 equal parts; each equals $\frac{1}{10}$ of a foot. Draw 10 parallel lines, erect 1s from the points of the primary division, draw diagonals through the points of division in the first segment to complete the scale. The smallest division will be $\frac{1}{100}$ or .01 of a foot.

Ex. 68. To Construct a Diagonal Scale showing Miles, Furlongs, and Chains, to show 6 miles. R. F.=1/63360.

The scale is 1 mile=1". Divide 6" into 6' equal parts each representing 1 mile. Subdivide the left-hand major division into 8 equal parts to show furlongs. Draw 10 equidistant parallel lines above this plain scale to form the diagonal scale for chains of 66'.

VERNIER SCALES.

Ex. 69. Construct a Movable Vernier to read Minutes to a surveying instrument of which the arc is graduated to $\frac{1}{2}^\circ$ (Fig. 508).

The subdivision on the graduated arc is 30 minutes. Take a length of 29 subdivisions and divide it into 30 for the vernier. When the instrument is to be read, read to nearest half degree, and add the number of minutes shown by the vernier between the index and the point of coincidence of the vernier and the primary scale on the limb.

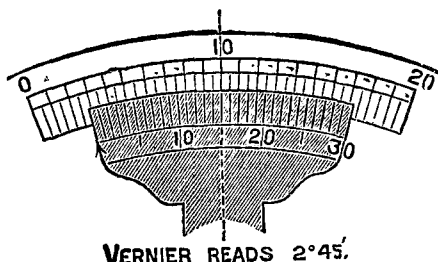


Fig. 508.—Movable Vernier to read to 30'.

Ex. 60. The Vernier of an Instrument whose arc reads to one degree is required to read to one minute. Draw the vernier.

The difference between the graduations of the vernier and the arc should be 1 minute or $\frac{1}{60}^\circ$. Therefore the vernier division should be $\frac{59}{60}$ th of the arc division. Take 59 divisions of the scale on the arc, and divide it into 60 divisions for the vernier.

Ex. 61. The Primary Scale of a Theodolite is divided into 20 minutes describe the construction of the vernier which reads to 30 seconds (Fig. 509). For description Ex: f, q.v., p. 81).

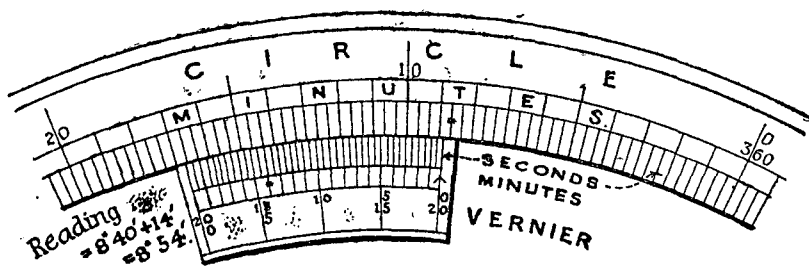


Fig. 509.—Movable Vernier reading to 30 seconds.

Ex. 62. Figs. 500 and 501 shows the Scale (enlarged) of a Standard Barometer divided into $\frac{1}{2}$ —10ths, or .05 of an inch. The vernier CD, moved by a rack and pinion, is made equal to 24 of such spaces and is divided into 25 equal parts, hence one space on the fixed Scale is $\frac{1}{25}$ th of .05 larger than one on the vernier, so that it shows a difference of .002 of an inch. The vernier reads from 0 (zero) upwards; D, therefore indicates the top of the mercury column. In Fig. 500 the zero of the vernier is between 29.65 and 29.70 on the scale. Glancing up the vernier and scale, the second line above 3 will be found in a direct line with one on the scale; this gives 0.03 and 0.001 to add to 29.65 so that the actual reading is 29.684. In Fig. 501, zero on the vernier is exactly in line with

29 inches and 5-10ths of the fixed scale; the reading, therefore, is 29.500 inch. The vernier line a falls short of a division of the scale by .002; b by .004, c by .006; d by .008; e by .010. If the vernier be adjusted to make a coincide with z on the scale, it will have moved through .002; and if 1 on vernier be moved to coincide with y on the scale, the space measured will be .010. Hence long lines marked 1, 2, 3, 4, 5 on vernier measures 100ths and the intermediate lines even 1000ths of an inch.

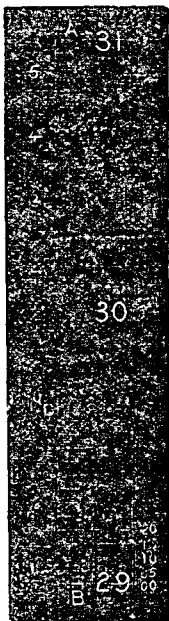


Fig. 510.—Vernier Reading to 100ths of an inch.

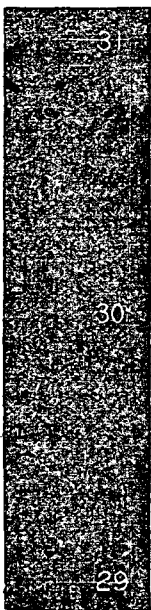


Fig. 511.—Vernier Reading to 1000ths of an inch.

Ex. 63. For Household Barometers the Scales are divided only into 10ths, and the Verniers are made to read to 100ths of an inch which is effected by making the vernier 9/10ths or 11/10ths of an inch long and dividing it into 10 equal parts.

Ex. 64. A Chain Line 2600' long is represented on the Plan by 10". What is the scale of the plan? Illustrate the use of the Vernier by drawing a comparative scale of sachines. A Russian Versta=500 Sagenes or Sachines=3500 English feet, or 1 Sachine=7'.

Here 10" represent 2600', or 1' represents 260".

$\therefore 5'38''$,, 1400' or 200 sachines.

Divide a line 5'38" long into 20 equal parts, each will show 10 sachines for the primary scale. For the vernier to read to sachines, take a length equal to 11 graduations on the primary scale, and divide it into 10 equal parts.

COMPARATIVE SCALES.

Ex. 65. On a Mesopotemian map a scale of Turkish berris (Berri =1827 yds.=3 Chinese Lis) is shown by measuring which by an English Scale 100 berris are found to coincide with 5 inches. Construct a Comparative Scale of Bengal Koshes to measure 50 Koshes.

A Kosh=2 English miles. Hence 50 Koshes=100 miles.

$\therefore (1827 \times 100)/1760 : 5 :: 100 : x = 4'81''$.

Divide 4'81" into 10 equal parts, and the left-end division into 10 equal parts each showing a mile= $\frac{1}{2}$ Kosh.

Ex. 66. One inch represents 13'.4". Draw a Corresponding Scale divided generally into 10' lengths with one such sub-divided into single feet. R. F.=1/13'.4"=1/160. The unit is 10'. If 160' be represented by 1", 10' will be represented by $\frac{1}{16}$ ". Set off spaces of $\frac{1}{16}$ " representing 10'; subdivide the left-hand division into 10 equal parts, each showing a foot.

Ex. 67. Draw a Scale of Kilometers comparative to Ex. 39. above. The kilometer=1093.61426 yds. The number of miles represented in the above scale is 20. Hence $20 \times 1760 : 7'5'' :: 20 \times 1093.61426 : x = 4'66''$.

Take 4'66"; divide it into 2 equal parts and the first primary division into 10 equal parts; each will represent a kilometer.

Here $R. F. = (7.5 \times 20 \times 1093.61426) / (20 \times 1760 \times 20 \times 1093.61426 \times 36) = 8960$.

Ex. 68. In a French plan, I find only a scale of decimeters 10 to 1'. Apply a comparative scale of feet. The decimeter = 0.327'. Show 10 decimeters = $10 \times .327' = 3.27'$. Then $3.27' : 1' :: 20' : x = 6.11'$.

Make the scale 6.11' long, divide it into 2 equal parts, and the first primary division into 10 equal parts, each showing a foot.

Ex. 69. In a Plan drawn to a Scale of Haths it is found that 1" represents 6 haths. Draw a comparative scale of feet to show 50 feet. Hath = 18". $R. F. = 1/108$.

$\therefore 108'' : 1'' :: 600'' : x = 5.55'$. Take 5.55' and divide it into 5 parts, and the left-end division into 10 parts each showing a foot.

Ex. 70. To Construct a scale of 6' to the mile required for military sketch in paces of 33' or 2.75' to show 2000 paces.

Here $6' = 5280'$ or $5280/2.75 = 1920$ paces. 2000 paces will be represented by 6.25'. Lay down 6.25', divide it into 20 equal parts, each representing 100 paces; sub-divide the first primary division into 10 equal parts, showing 10 paces each.

Ex. 71. The length of an ordinary pace is 30". Draw a scale of $\frac{1}{3000}$ to show 600 paces. Here 1' represents 3000" or $\frac{1}{3000} = 100$ paces. Lay down 6', it will represent 600 paces. If the first inch be divided into 10 equal parts each will represent 10 paces.

Ex. 72. To Construct a Scale to measure in c. ft. the rainfall on a map drawn to a scale of 1" = 200', the rainfall per hour being $\frac{1}{4}$ ".

The rainfall on 1 sq. ft. = $1 \times 1 \times 1 / (4 \times 12) = \frac{1}{48}$ c.ft.

$\therefore$ " " 48 " = 1 c.ft. Then 48 sq. ft. correspond to 1 c.ft. of rainfall. $\therefore \sqrt{48}$ sq. ft. or 6.9' correspond to $\sqrt{1}$ c.ft.

If the map were divided into squares of 6.9' side, the rainfall on each square would be 1 c. ft., and there would be as many c. ft. of rainfall as there are squares. Therefore to construct the scale;—as 200' is represented by 1".

$\therefore 6.9'$ or $\sqrt{1}$ c. ft. is represented by .034" = 1 subdivision.

or 10 $\sqrt{1}$ c. ft. " " " .34" = 1 primary division.

$\therefore 12 \times 10 \sqrt{1}$ c. ft. " " " 4.08" = 12 " divisions.

Suppose, one side of a rectangular field is "a" units or $a \sqrt{1}$ c.ft. and the other side $b \sqrt{1}$ c. ft. Then the rainfall on the area = ab c. ft.

Ex. 73. On a Russian Map a Scale of Verstas is shown by measuring which by English scale 150 verstas = 5". Construct a comparative scale of English miles to show 100 miles. Versta = 1166.6 yds. say.

$\therefore 150$ verstas = $1166.6 \times 150 / 1760 : 5 :: 100 : x = 5.28'$. Draw a line 5.28' long, divide it into 10 equal parts each part representing 10 miles; divide the left-end division into 10 equal parts each showing 1 mile.

Ex. 74. Construct a Scale of English Miles when the Distance between two places 20 leagues apart is represented on the map by 2.5". 1 French league = 4262.84 yds. The scale to show 110 miles.

20 leagues = $(4262.84 \times 20) / 1760 = 48.44$ miles.

$\therefore 48.44$ mi. : 110 mi. :: 2.5" : $x = 5.77''$.

The R. F. of both the French and English Scales will of course be the same.

Divide 5.77" into 11 major divisions, each representing 10 miles; subdivide the first major division into 10 equal parts to show miles.

Ex. 75. Draw a Comparative Scale of Hectometers and Decameters (one hectometer = 109.36 yds.) to a scale of 32" = 1 mile.

Here 1760 yds. are represented by 32".

$\therefore 109.36$ " " " " 1.98" = one major hectometer division.

Take 3 major divisions of a total length of 5'94"; divide it into 3 parts for hectometers, and sub-divide the first major division into 10 parts to show decimeters.

Ex. 76. Draw a Scale of Knots Comparative to a Scale of 10 miles=1". 1 Knot=1'15 miles.

Here 10 mi. : 1" :: 1'15 mi. : $x=0'115"$.

∴ 60 knots will be represented by 6'9".

Take 6'9", divide it into 6 equal parts, each showing 10 knots; and the first part into 10 equal parts, each to show 1 knot.

712. EXERCISES ON SCALES AND PROTRACTORS.—

1. What is meant by the scale of a plan?
2. What is meant by saying that (a) a plan is drawn full size, (b) a plan is drawn to a scale of 8" to the mile, and (c) 1"=40'?
3. What is meant by the representative fraction of a scale?
4. When a map is given to you, how would you find out the true distance between two objects on the ground?
5. How is the simple scale graduated and used?
6. What conditions should the scales fulfil to be of any use?
7. State the way in which plain scales are numbered. To what limit of accuracy can a plain scale be constructed to read?
8. What are diagonal and vernier scales? Explain their construction and uses. Why are they employed to meet the deficiency of plain scales? Compare the advantages and disadvantages of diagonal, vernier, and plain scales.
9. What is a comparative scale? And what is its use? Give some examples of the use of comparative scales.
10. Explain by an example how comparative scales are constructed.
11. What are the contrivances for measuring and setting off angles?
12. What should be the length of the edge of a semicircular protractor to read distinctly 10 minutes, taking 80 distinct divisions to the inch?
13. Explain the construction of a scale of chords, and construct the angles of 75° and 120° by its means. How will you measure the size of an angle by means of the scale of chords?
14. How many dimensions can you get from plain scales? And how many from diagonal and vernier scales?
15. Explain the construction of the rectangular and semicircular protractors, and explain why the graduations should depend on the length of the edge of the protractor. Show by a diagram how rectangular and semicircular protractors are graduated and numbered. State the advantages the semicircular has over the rectangular protractor.
16. Explain how you will measure the size of an $\angle$ by a protractor, and how you will construct an angle with it.
17. Explain the construction of a vernier to read small subdivisions of angles. Explain what you understand by a movable vernier, and show by diagram how you will read a certain length or angle.
18. What is a vernier? If a vernier be divided into n parts, prove that the difference between each of the divisions on the respective scales is equal to $1/n$ one of the divisions on the primary scale.
19. Show how you will divide a given straight line by means of a sector.
20. Lay down an $\angle$ of 125° by means of the sector.
21. Divide a straight line 6" long into 7 equal parts, and on one of the divisions construct a regular heptagon by the use of the sector.
22. To Construct a scale of 11 feet to an inch to measure single feet.
23. Explain clearly how vernier scales are constructed, and construct a scale of $\frac{1}{100}$ to show feet and inches.
24. To Construct a plain scale of mile showing furlongs $\frac{1}{10}$, natural size.
25. To Construct a diagonal scale of meters proportional to a scale of 1 yard=1 inch, reading decimeters and centimeters, and mark on it a length of 1 metre 2 dec. 3 cent. 1 meter=10 decimeters=100 centimeters=39'37 inches.

26. State the principle of the vernier. Construct a vernier, reading degrees and minutes and showing $3'9''$.

27. Construct a diagonal scale of 7 miles to one inch to show miles and furlongs and chains 66 feet. What is the R. F.?

28. Describe briefly but clearly the principle on which movable vernier scales are made. In an upright vernier the primary scale $3''$ long and divided into tenths of an inch. Draw this as well as the movable scale in its proper position when the reading is $1'67''$.

29. Construct a comparative diagonal scale to one of 1 mile = $1''$, of Knots to show decimals of a Knot to two places of decimals. 1 Knot = 6080'.

30. Construct a diagonal scale of $1\frac{1}{2}''$ to the mile to show miles and furlongs, and chains, and mark on it a length of 2 miles 3 furlongs and 2 chains. What is the R. F.?

31. Draw a diagonal scale of $1/96$ to read feet and inches.

32. Draw a rectangular figure with an area of 45 sq. ins. State to what scale it should be measured to give an area of 20 sq. ft.

33. A meter is 39.37079 ins. A Russia versta is 1066.78 meters; make a scale to show $1/100$ of a versta upon the India G. T. S. Maps whose scale is $1''=1$ mile.

34. Your proportional compasses falls, and a little bit is found to be broken off one of the points (say $1/32$ of an inch). How would you set the compasses right so as again to work accurately.

35. Construct a scale of $10' = 1$ mile showing feet, tens of feet, and hundreds of feet. Give R.F.

CHAPTER IX.—MAPPING.

713. Examination of the Drawing Paper.—The drawing paper should be examined to see which is the rightside and that no mouldy spots exist. If any such spots are detected the paper should be rejected as it is impossible to color over the discolored spots. (See Art. 677, pp. 381—384).

714. Space the Drawing will occupy to be estimated.—Before commencing to plot a survey a rough estimate should be made of the space which the drawing will occupy; for this purpose, it will be sufficient to take the extreme dimensions of the survey and to cut the paper accordingly. The starting point of the plot should be selected in such a position on the paper, that the entire plan may be drawn on it, with an equal width of blank spaces on the sides and bottom and nearly double of it on the top for printing the title and scale. It will be found a good plan to roughly plot the chain lines to a small scale to obtain the shape of the survey, this will facilitate the laying down of it on the sheet in proper position.

715. Placing the North Point.—The top of the sheet should always be towards the north, so that, the top, bottom, right and left of the sheet are respectively to N., S., E., and W., and a north line, representing the direction of the magnetic needle with the variation of the compass with date marked on it, should be drawn on either the East or West edge of the paper or through the first-point of the survey as may be convenient.

716. Scale of Survey to be First Drawn.—The use to be made of a map determines the scale to which it shall be drawn. If it is necessary to take off distances from the plan it should then be drawn to a large scale. $100' = 1''$ scale is convenient for engineering purposes. Before commencing to plot, the scale should always be drawn at the foot of the paper if the plot is likely to occupy more than a day so that if the paper shrinks the scale will shrink equally, and will still apply to the plotted survey. All measurements should be taken from this scale. For the scales in *general use* see p. 7, Art. 23.

717. Plotting the Survey.—The first and the most important stage of a drawing is the pencil work, called **plotting**; which consists in representing on paper, to a smaller scale the linear and angular measurements taken in the field, and noted in the field-book. The delineation is made according to scale adopted. The position and size of the objects should be truly marked on the plan by means of accurate scale measurements. Avoid drawing all unnecessary lines especially in pencil work. It is difficult to erase all pencil lines thoroughly, and, covered with ink line they may give a soiled appearance to the map. The pencil work is to be done with a moderately hard (Faber's HH) pencil. The pencil lines should be fine, and the drawing should be neat and clean, the eraser should not be much used. Care should be taken not to overshoot the corners where two lines intersect. The T-square with a swivelling stock is often used in plotting field-work, and in doing so the movable stock should be kept at, and moved along the left edge of the drawing board, and the straight-edge coinciding with the chain lines. The meridian lines of a chain and compass survey are often drawn in this way. Avoid pressing heavily with the points of the compasses on the paper to prevent perforations. The draftman's touch should be light and firm, obviating the necessity of using the rubber, thus keeping the paper clean and smooth.

Plotting Offsets.—See p. 167, Art. 291.

718. Inking in the Chain Lines.—The chain lines are next ruled with fine blue or red lines, if required to be shown, exhibiting the basis of the whole work. Generally they are shown on a separate plan to avoid confusion in the details. It is not a good plan to plot the survey from pencil chain lines, as any rubbing out causes redrawing bits of the chain line which may have been rubbed out. The details should be plotted after all the chain lines have been plotted, together with the tie-lines, and after the accuracy of the work has been tested by the lines *lying in* properly. When the chain lines are not required to be shown there is then no alternative but to plot from pencil chain lines.

719. Inking in the Details.—When the pencil work is finished the various boundaries and objects are drawn in light Indian ink with the lining or mapping pen according as the lines to be drawn are straight, such as those of buildings and roads, or curved, such as those of streams, winding roads, fences, the lining pen being

held upright with its broad edge parallel to the direction of the line to be drawn, and the mapping pen being used freely by the hand. The roads should be inked in plainly and evenly, and their sides parallel, except where the survey shows a deviation from the general width. In drawing houses care must be taken that the corners and angles exhibit a sharp, clear outline, which adds much to the appearance of the map. In order to form a neat junction between a straight line and a curved one, it will be found convenient, to first ink in the latter. In drawing circles, care must be taken not allow the point of the compasses to perforate the paper, the holes thus formed are liable to become filled up with color and cause an unsightly spot. All the ink lines should be drawn thin and even, and at one stroke of the pen. To ensure this, the ink and drawing pens used must be of the best quality. The ink should flow freely, and the pens will have to be cleaned often. As it is very difficult to draw very fine lines, do not attempt to draw them. Auxiliary lines, such as that of offsets, if required to be shown, should be dotted. These should be drawn by a good dotting pen. The pencil lines are then erased. After inking in the fine outlines of a drawing, the paper should be thoroughly cleaned with India-rubber used as sparingly as possible, and then with stale bread, and then it is necessary to wash the drawing with sponge and water to enable the paper to receive colour if that is necessary to be put on.

722. Coloring Drawing.—(a) Mixing Colors.—At first little water should be used while rubbing cakes or mixing moist colors; more being added afterwards according to the degree of shade required. It is well always to mix more than sufficient color at first to last for the whole of the wash required, as it is almost impossible to mix color again of exactly the same strength of tint; any sediment should then be allowed to settle and the clear solution poured into another saucer for use.

(b) Flat Washes.—To tint large surfaces a large camel-hair wash brush is used as the more color a brush contains the more equally will it deliver the required tint. In laying on a flatwash the drawing is to be always tilted so as to let the color float downwards. The coloring is commenced from the upper left-hand corner of the surface, making short strokes the width of the brush along the top edge of the space to be colored, immediately following with another row of similar strokes into the moist edge of the first row, and so on as far as required removing the last surplus color with a nearly dry brush. The theory of the above is that wet color may be perfectly united to a moist edge but it can not be united to a dry edge without showing the juncture. The brush, if used full of color, and the color carried down stroke by stroke, will also ensure a fine even wash. The brush should be replenished with color as soon as half its contents are thrown out except near the lower end of the drawing when only that quantity sufficient to cover the remaining surface is retained in the brush. Red lines should not be washed over as the color will run. Beginners should be cautious in not overstepping the edges of the part to be covered. In coloring

large surfaces it is a better plan to first damp the paper with a sponge and water, any superfluous moisture being removed by placing a blotting-paper on the drawing so moistened; or the color to be put on must be prepared thin. A darker tint can always be obtained by washing over it again when dry. Take care to preserve harmony in the drawing. Bright colors match with bold lines and printing, while light shades of color, fine lines and modest printing go together.

(c) **For Shading**, brushes, termed *softeners*, are generally used; these have a brush at each end of the handle, one being much larger than the other. To use the softener for shading is to fill the smaller brush with color, and to thoroughly moisten the larger one with water; the color is then laid upon the drawing with the smaller brush to represent the dark portion of the shade, and immediately after while the color is still moist the water-brush is drawn down the edge intended to be shaded off; this brush is then wiped upon a cloth or blotting paper, and drawn down the outer moist edge to remove the surplus water which will leave the shade perfectly soft. If dark shades are required this has to be repeated when the first is quite dry. In shading the water lines of rivers, lakes, etc., the coats should be very light at first, and repeated gradually until of sufficient strength for by these means *softness* and *roundness* are obtained while a thick heavy shade laid on all at once, never can produce a proper tone and looks extremely bad.

(d) **Shade Lines**.—Buildings are usually drawn in outline and hatched with lines, or colored with crimson lake. Shade lines should always be used for the outlines of buildings, etc., the light being taken as coming from the left-hand top corner of the sheet at an angle 45° . All objects in elevation such as buildings, banks, roads and trees must be shown by dark lines on the right-hand and lower sides, while all depressions, such as rivers, lakes and tanks, or other excavations are to be darkened on the left-hand and top sides.

(e) **To produce a finished drawing no portion of it should be erased** otherwise the color applied will be unequal in tone; thus, when highly finished drawings are required, it is well to draw an original and to copy it, as mistakes are almost certain to occur in delineation. When sufficient time cannot be given to draw and copy, a good plan is to take the surface off the paper with glass paper before commencing the drawing; the color will then flow equally over any erasure it may be necessary to make, although not in the perfect manner it will flow over a new clean surface recently damped. Where ink lines are drawn a little over the intended mark, and it is difficult to erase them without disfiguring other portions of the drawing, a little *chinese white* may be applied with a fine sable brush or a little touch of *opaque color* will erase false lines on a colored surface. It renders a small defect much less perceptible than by erasure.

(f) **Use of the Sponge when a Flatwash is Uneven or**

contains a cut shade, probably from allowing that portion to dry, or it is required to erase out lines, or washes of color, which are errors in drawing, apply boldly but lightly a small sponge soaked in water, and blot off the excess moisture with a piece of blotting paper. When the color to be erased is contiguous to other colors, which the sponge might also touch, it may be enclosed by a piece of thick paper with the part above the color to be washed cut out, and the sponge can then be applied to the exposed part without soiling the adjacent colors. The sponge should not be used over or near thick ink lines; the ink is sure to run. When a stain or a small dirty-mark in a flat wash of color has to be erased and fresh color to be afterwards applied, take care to keep the texture of the paper as intact as possible. The ink eraser should therefore be rubbed very lightly to produce a clean surface. The color is then to be stippled in by separate strokes of the brush, any excess of color to be taken off by a piece of blotting paper. Ink lines on tracing cloth can be removed by means of a moist brush and blotting paper. The spot touched upon will, however lose its glaze and any ink line drawn over it afterwards, will be liable to run, if the texture of the tracing cloth is damaged.

Care should be taken, in making an elaborate drawing to receive color, that the hand should at no time rest upon the surface of the paper, as it is found to leave a greasiness difficult to remove. A piece of blotting paper placed under the hand will prevent this. Should the colors from any cause work greasy, a little prepared ox-gall may be dissolved in the water before mixing color, and will cause them to work freely.

723. Conventional Signs for Delineating Various Objects on Plans.—Each material or object should be represented on plans uniformly by a color or sign. If any special signs or abbreviations are used, an index or table showing their meaning should be entered on the plan. Fig. 195 (Plate III.) show the revised Conventional Signs with the conventional colors in use by the Survey of India Department.

List of General Color Conventions in Use.

<i>Hills.</i> —Brown (Burnt Umber).	<i>Wet Cultivations.</i> —Blue.
<i>Sand Hills.</i> —Ditto.	<i>River and Streams.</i> —Cobalt blue if perennial, otherwise black.
<i>Natural Drainage.</i> —Black.	
<i>Tank or Jhils.</i> —Cobalt blue if perennial, otherwise black.	<i>Village</i> { Brick houses.—Vermilion.
<i>Natural Ravines and Dry Nalas.</i> —Black.	<i>Sites.</i> { Mud Do. —Brown also Vermilion.
<i>Canals.</i> —Blue.	<i>Roads metalled.</i> —Vermilion.
<i>Waste Lands.</i> —Green.	<i>„ unmetalled.</i> —Do. broken lines.
<i>Dry Cultivations.</i> —Yellow.	<i>Railways.</i> —Black.

In topography the following colors may also be used; *Hooker's Green* to represent grass. *Burnt Sienna* to represent cultivated ground. *Burnt Sienna and Hooker's Green* to represent uncultivated ground. *Indigo and Hooker's green* to represent Swamp. *Gamboge and Hooker's green* to represent trees. *Yellow Ochre* to represent roads and streets. *Indigo* to represent water. *Garmine* to represent buildings, bridges and masonry. *Sepia* to represent hills, shade lines and shadows.

724. Representation of Ground.—The slopes of ground are represented by contour lines in those cases in which the scale is large

enough to admit of the contours being shown. In small scale maps the old method of hill-shading by hachures is still adhered to.

The method of obtaining contours by means of a *level* is described after the Chapter on Levelling. When the contours are plotted on the plan, it is easy to *accent* the slopes of the ground by shading them in by means of a brush.

Another way of representing the slopes is by shading them in by means of a brush and a little Indian ink or sepia; in this case the light is considered as falling nearly vertical, but sufficiently oblique to allow of a decided light and shade to the slopes of the hills, trees, etc. The hills are shaded, not as they would really appear in nature, but on the conventional system of making the slopes darker in proportion to steepness; the summits of highest ranges being left white. This system is easily understood and is also easily excused.

725. Border.—A neat plain border of a thick black line looks best, but often ornamental designs are used at the corners. Outside the border there should be a considerable margin of paper never less than an inch and a half in width; a very narrow margin spoils the appearance of a drawing.

726. Lettering.—The most difficult part of the inking for a beginner is the lettering, which now follows, and for which samples are given. It is expected that every surveyor shall have learned to draw sufficiently well to ink his sheet in a clear and distinct manner, and letter it with some regard to neatness and graphic effect, as the appearance of an otherwise well-inked sheet is sometimes marred by careless or indifferent lettering.

SPECIMENS OF ALPHABETS AND NUMERALS:— ROMAN CAPITALS.—

A B C D E F G H I J K L M N O

P Q R S T U V W X Y Z

0 1 2 3 4 5 6 7 8 9

Roman Small.—

a b c d e f g h i j k l m n o p q

r s t u v w x y z

Upright Block—Capitals and Small.—

A B C D E F G H I J K L M N O P Q R S T

U V W X Y Z

a b c d e f g h i j k l m n o p q r s t u v w x y z

1 2 3 4 5 6 7 8 9 0

Stump Writing—Italics.

a b c d e f g h i j k l m n o p q r s t u v w x y z

a b c d e f g h i j k l m n o p q r s t u v w x y z

1 2 3 4 5 6 7 8 9 0

SPECIMENS OF BLOCK TYPE PRINTING:—

PLAN. LONGITUDINAL SECTION. HORIZONTAL. VERTICAL. END. DRAINAGE
IRRIGATION. ELEVATION. TRANSVERSE. OF. GROUND. FRONT. SIDE. CROSS.

DALHOUSIE SQUARE,

CALCUTTA.

WROUGHT-IRON BLACK AND GALVANIZED TUBES AND FITTINGS.

SPECIMEN OF TITLE OF ENGINEERING DRAWING:—

BHAGULPORE WATER WORKS.

PLAN OF THE CIVIL STATION OF BHAGULPORE
SHEWING MAIN PIPING.

Scale 6 Inches = 1 Mile.

SPECIMEN OF ITALIC PRINTING:—

Surveyed and Drawn by

Transits of 6¼ in. graduations or larger, the horizontal circles are graduated to 20 minutes, and Verniers graduated to read single minutes, 30 or 20 seconds, as desired. Unless otherwise ordered, the graduations will be numbered with one row of figures, from 0 to 180, in opposite directions, as shown on page 25.

For the smaller size Transits, the circles will be graduated to half degrees, and numbered the same; Verniers will read to minutes.

For Mining Transits, and those to be used for triangulating, it is sometimes preferred to have an additional row of figures, from 0 to 360, or both rows, from 0 to 360, in opposite directions.

SPECIMENS OF LETTERING FOR MAPS.

HEADING OF MAPS:—

BENGAL SURVEY.

DISTRICTS CUTTACK & PURI.

Seasons 1877 to 79 & 90 to 92.

Magnetic Variation 2°-0' East.

Scale 2 Inches to a Mile, or 1:31,680.

SHEET No. 195

NAMES OF VILLAGES:—

Agpara. Balbhadrapur. Matiakund. Sahaspur. Rahanlo.

Jamunadad. Tayla. Deuda. Safipur. Sithalo.

Ramchandrapur. Baligora. Pakhar. Chhotagaria. Fajilpur.

Kharisan. Nuagaonesherpur.

NAMES INDICATING ROADS:—

from Cuttack to Jajpur.

NAMES OF RIVERS, STREAMS, CANALS. DRAINAGE:—

KHARSUA N. GANGUTI N. Joanai N. PATAMUNDI CANAL.

Peta N. DRAINAGE HANSUA.

LETTERING BELOW SHEET:—

Surveyed under the direction of Colonels C. Strahan, R.E., and J. E. Sandeman, I.S.C., Deputy Surveyor Generals.
by W. H. Patterson, Esq., Deputy Superintendent, Lieutenant R. T. Crichton, S.C., and H. R. Littlewood, Esq.
Assistant Superintendents, assisted by Messrs. R. C. Ewing, S. O. Madras, G. Campbell, Extra-Asst Supdts.,
A. W. Smart, C. S. Kraal, E. F. Berkeley, G. Vander Beek, J. P. Barker, C. S. Gasper, & Babu Nilmani Chatterji,
Sub-Assistant Superintendents.

SYMBOLS and ABBREVIATIONS

Pargana Boundary.....	Distributary Canal.....
Village.....do.....	Lock.....
Unmetalled Road.....	Canal Bungalow.....
Cart-Track.....	Inspection.....do.....
Embankment.....	Road Cess.....do.....
Telegraph Line.....	Post Office.....
Limit of Cultivation.....	Trijunctions marked by stones.....
	Do.....do.....by clay cylinders

NOTE — Surveyed on Scales of 16 & 32 Inches = 1 Mile, reduced
by pentagraph to Scale 2 Inches = 1 Mile, and
reproduced by photography.

The Longitudes are referrible to the Greenwich Meridian,
taking that of Madras Observatory as 80°-17'-21" East.
They require a correction of = 2'-30" to make them
accord with the most recent value of that Observatory,
viz., 80°-14'-51" East.

Index to adjoining Sheets.

194 S.W.	194 S.E.	222 S.W.
195 N.W.	195 N.E.	223 N.W.
195 S.W.	195 S.E.	223 S.W.

REFERENCES

No.	Pargana.	No.	Pargana.
27.	Utikan.....	62.	Nahakhand.....
30.	Chhedra.....	63.	Sukani.....
31.	Tikan.....	64.	Paena.....
58.	Kusmandal.....	65.	Balubisi.....
59.	Asureshwar.....	85.	Romito.....

Several styles of letters are used for this purpose but as a rule, the Egyptian or Block letters are decidedly the best for all kinds of Headings of maps or plans. The lettering set-squares will be found useful for giving the correct slopes to the letters.

The letters should not be very large. On a map $2\frac{1}{2}$ feet square the largest letters should not be more than a quarter of an inch high. The title, which should name the tract surveyed, and scale of the map, should consist of these larger letters. In College work the kind of survey, as "Chain-Survey," "Compass-Survey," etc., and the class and party by whom surveyed, should be stated. For detail matter within the drawing, Capital and small Italic is used. It is usual to place the first letter of the name as close to and to the right of the site as possible.

Disposition of Letters and Figures, (Plates).—The location of the names upon the sheet should be such as not to cover or obliterate any detail or feature of the survey, and the letters should be put in neatly and gracefully and in point of size and form according to specimens furnished (Fig. 512-524). The title should finally follow, with such notes as may be necessary to explain any peculiarity of the sheet or survey. This title and lettering should, as far as practicable, be so placed, that when the sheet is held with the top (usually the north or east end of the map) from you it can easily be read; or in other words, as nearly parallel to the top or upper end of the sheet as the nature of the work will admit. All names, well-established and recognised in the neighbourhood, both general and local, should be collected during the survey, and their correct orthography ascertained. No illuminated or German text, Old English, or what is known as "fancy printing" should be indulged in, a strict adherence to simplicity being the greatest ornament.

Parallel lines at a distance apart equal to the height of the letters to be printed, should always be drawn for capital and small letters; and also a line to determine the height of these letters, which, *a*, *t*, *b*, etc., reach above the original lines. For large Block printing, vertical lines spaced at equal intervals may be used with advantage; and for inclined Capitals parallel lines at the required angle will secure uniformity. For large printing, draw in the outline of the letters, and then fill in with the pen, care being taken to fill up the corners so as to make them sharp and defined. The spacing of letters is best determined by the eye; but it is always advisable to sketch in the position of the letters of a heading, or title with a pencil before proceeding to put them in ink.

The Following plan may be adopted by the beginners both for letters and their spacing:—For ordinary block printing divide the height chosen for size of letters into 5 equal parts. The breadth of all letters will be $\frac{4}{5}$ th of the height, with the exception of I whose breadth will be $\frac{1}{5}$, of J whose breadth will be $\frac{3}{5}$, of E, F, L, P, R, whose breadth will be $\frac{1}{10}$, of C, G, M, O, Q, T, W, whose breadth will be $\frac{5}{8}$ of the height. The spaces between the letters may be taken $\frac{3}{10}$ or $\frac{2}{5}$ of the height, and between the words equal to the height. Then to print a heading, arrange as under, to determine the height of letters and the spacing between them:—

S	U	R	V	E	Y	A	N	D	S	E	C	T	L	O	N	
4	4	3½	4	3½	4	4	4	4	4	3½	5	5	1	5	4	= 62½
1½	1½	1½	1½	1½	1½	5	1½	1½	5	1½	1½	1½	1½	1½	1½	= 29½
																TOTAL = 92

To write the words "**Survey and Section**" take the length = 92 parts each part being $\frac{1}{5}$ of the height chosen. Then rule the boundary lines of each letter and afterwards the single space lines within each letter. If a more narrow and upright letters are to be printed, take spaces of $\frac{1}{6}$ or $\frac{1}{7}$ of the height instead of $\frac{1}{5}$.

Curved Printing.—The printing should be curved and placed parallel to their courses on winding rivers, canals, roads, railways, lakes, coast-lines, etc. Curved lines of printing are used for the names of districts, thanas, etc., and should run as nearly parallel to the general outline as possible, and when several districts, subdivisions, etc., are shown on the same plan, the name should be stretched across the map with the letters at equal intervals. The printing should be placed clear of the inked-up details. When the outlines of the roads and streams cross names, they will off course be broken at the places but this course should be as well-defined as possible.

727. Shrinkage of Plans and Maps.—One of the greatest sources of errors in plans, which gives endless trouble, is the alteration in the sizes of the drawing paper and cloth due chiefly to the variations in the degree of moisture contained in the paper. Even with the best mounted paper thoroughly seasoned and kept in a room maintained at a constant temperature, shrinkage occurs in a few years' use. If a considerable amount of coloring be done on a plan, or if it be frequently subjected to varying temperatures and degrees of humidity, much alteration in the size results. Tracing cloth usually expands, and is unsuitable when accurate scaling is necessary. Good tracing paper with reasonable care, varies little in size, and is the best to use for accurate tracings, and for copying plans; but tracing paper is not very durable, as it becomes brittle after a few years use.

The dull side of the tracing cloth is the better to ink on, but color should be applied to the back or glazed side, as it is much more easily done, and shows up distinctly. The greasy nature of cloth often renders it difficult to make the ink run freely; this is remedied by rubbing the cloth with pounce or powdered chalk before inking in. A little soap in the color also answers the same purpose.

CHAPTER X.—CALCULATION OF AREAS.

728. Measuring Areas.—There are three methods of finding out areas of ground from plans: (1) *The method of Δ s*, (2) *the method of ordinates*, (3) *the mechanical methods*:—

728a I. Method of Δ s.—(a) The ordinary formula for the area of a $\Delta = \frac{1}{2}$ (base $\times$ height) $= \frac{1}{2} b \times h$; ... (1)

The base (b) and $\perp$ (h) height of each Δ are scaled off from the plotted survey unless measured beforehand.

Ex. 1. Find the Area of a Δ of which the base = 23', height = 16'.

$\therefore$ Area $= \frac{1}{2} (23 \times 16) = 184$ Sq. ft.

(b) **Calculation of Area from Figures in the Field-Book.**—The area of a Δ is then calculated from the formula,—

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \quad \dots \quad (2)$$

where a, b, c are the sides of the Δ , and $s = \frac{1}{2}(a + b + c)$. Here a, b and c are the measured lengths obtained from the field-book of a chain survey or from a Trigonometric triangulation survey.

Ex. 2. The Sides of a Δ r Field are 175 yds., 200 yds., 250 yds. long respectively. Find the area. Here $s = \frac{1}{2}(175 + 200 + 250) = 312.5$;

$s-a = 137.5$; $s-b = 112.5$; $s-c = 62.5$.

$\therefore$ The area of the Δ r field $= \sqrt{312.5 \times 137.5 \times 112.5 \times 62.5} = 3 \text{ ac. } 2 \text{ ro. } 14.56 \text{ po.}$

(c) Calculation of Area when two Sides and the included angle of a Δ are given: Then Area = $\frac{1}{2}(ab \sin. C)$. (3) here a , b and C are given. For examples, see pp. 262, 265.

(d) Calculation of Area when One Side and the Adjacent Angles of a Δ are given, i.e., B, C , and a are given.

$$\text{Area} = a^2 \sin. B \sin. C / 2 \sin. (B + C) \quad \dots \quad (4)$$

728 b. Casting by Equalising.—It is advisable to reduce a polygon to a single triangle and then to find its area as in Prob. XXII, p. 147, Art. 248.

728 c. Casting by Triangles.—Every polygonal field can have its area calculated by dividing its plot into Δ s, computing their areas

Ex. 3.—Calculation of Area of a Chain Survey (Fig. 185; p. 176).

Triangle.	ΔABC	ΔACG	ΔGCD	ΔGDF	ΔFDE	Summation of Partial areas.
$a =$	630	1020	840	860	1000	Bengal : * 49 bi. 16 ka. 9 chks. 19.2 sq. ft. U. P. : 26 bi. 7 Bis. 2 Bisw. 8 kan. 2.1 sq. ft. Punjab : 49 Bi. 0 Kan. 17. Mar. 5.7 sq. ft. Madras : 12 Cawny 10 Grds. 2344.2 sq. ft. Bombay : 3 Ru. 2 bi. 6 pan. 0 Ka. 81.3 s. ft.
$b =$	540	840	470	1000	560	
$c =$	1020	380	860	340	480	
$2s$	2190	2240	2170	2200	2040	
$s =$	1095	1120	1085	1100	1020	
$s-a =$	465	100	245	240	20	
$s-b =$	555	280	715	100	460	
$s-c =$	75	740	225	760	540	
Log s	3.03941	3.04922	3.03543	3.04139	3.00860	
„ $s-a$	2.66745	2.00000	2.38917	2.38021	1.30103	
„ $s-b$	2.74429	2.44716	2.85431	2.00000	2.66276	{ 145581.0 132886.0 206797.0 141645.0 71185.2 717544.23 sq. ft.
„ $s-c$	1.87506	2.86923	2.35218	2.88081	2.73239	
	10.32621	10.36561	10.63109	10.30241	9.70478	
Log Ap.	5.163105	5.182805	5.315545	5.151201	4.852390	
Ap.	145581	152336	206797	141645	71185.2	

* Memorise :—14400 sq. ft. = 1 Bengal Bigha ; 720 sq. ft. = 1 Kata ; 45 sq. ft. = 1 Chhattak.

by either of the first two formulæ, and adding the results together; thus, to compute the area of the polygon, ABGDEFG (Fig. 185 on p. 176) we have all the sides and the diagonals, AC, CG, GD, DF. Then if the lengths of the 3 sides of the $\triangle ABC$ are denoted by a, b, c ; the area of the $\triangle$ can be found by equation (2), above p. 456. In the same way, the area of the remaining four triangles in Fig. 185 are calculated. The sum of the areas of the 5 triangles gives the area of the whole polygon as exemplified by the above *Systematic Tabulations of the Computations* whereby errors are much less liable to occur, and, when they do occur, can be much more readily detected and corrected. As a check on the computation, the lengths of the diagonals BG, GE, GD and DB may be measured. Five new $\triangle$ s. would thus be obtained. The total of these five areas should equal the result in the first computation.

Again in order to compute the area of the polygon ABCDEF (Fig. 186) the diagonals AC, CE and EA, and the perpendiculars from B, D and F on them are measured, and the areas calculated by equation (1) as under from figures in the Field-book on p. 177 :—

Ex. 4. Calculation of Area of a Chain Survey in Fig. 186, p. 177.

Area of $\triangle ABC = \frac{1}{2}(342 \times 80) = 13680$ sq. ft.
 „ „ $ACE = \frac{1}{2}(342 \times 180) = 30780$ „
 „ „ $CDE = \frac{1}{2}(236 \times 80) = 9440$ „
 „ „ $EFA = \frac{1}{2}(261 \times 100) = 13050$ „

Memorise :—
 Uria Man = 43560 sq. ft.
 „ Goont = 1742.4 sq. ft.
 „ Bishwa = 108.8 sq. ft.
 (See p. 3)

1 man 13 goonts 6 biswa 86 sq. ft. = 66950 sq. ft.

The part between the $\triangle$ s, and any irregular boundary is then calculated from the offsets, and added or deducted as the case may be. The calculation of this part of the area is as under :— Each portion included between two adjacent offsets, the chain line and the irregular boundary, is considered to be a trapezoid, and its area = $\frac{1}{2}$ sum of the offsets $\times$ distance between them on the chain line; the total being the sum of all these trapezoids. The offsets and distances between them on the chain lines are taken from the figures in the field-book. This method of calculation from figures in the field-book is very laborious and only adopted when great accuracy is required and seldom adopted by the surveyor.

728d. Computation of Acreage.—When the area of a survey is required in acres, divide the area into convenient $\triangle$ s, and multiply the base of each $\triangle$ in links by half the $\perp$ in links; add all the $\triangle$ r areas so obtained in square links; cut off 5 figures to the right, the remaining figures will be acres; multiply the 5 figures so cut off by 4, and again cut off 5 figures, and the remainder is in roods; multiply the 5 figures by 40, and again cut off for perches.

In the U. S. A.—A tract of land 1000 Varas square contains 177 acres. Hence to reduce square varas to acres, multiply by 177 and point off 6 places. 1 acre = 5645 sq. varas nearly.

Ex. 3. Find the Area of a Rectangular Field of which the Breadth is 350 links, and Length 465 links, the measurements having been

made with a Gunter's chain. Area = $350 \times 465 = 162750$ sq. links. Cutting off 5 figures from the righthand gives 1.62750 acres.

$$\left. \begin{array}{r} 2 \cdot 51000 \text{ roods.} \\ 40 \\ \hline 20 \cdot 40000 \text{ perches.} \end{array} \right\} \therefore \text{Area} = 1 \text{ acre } 2 \text{ roods } 20 \cdot 40 \text{ perches or poles.}$$

If the measurements are in chains and the area found in square chains, the acreage is found by simply moving the decimal point one place to the left. If it is then necessary to show in roods and poles the decimal fraction of an acre, simply multiply it by 4 and 40 respectively.

Ex. 4. Suppose the Area of a Survey is 513 34 sq. chains.

Then the area = 51 334 acres.

$$\left. \begin{array}{r} 1 \cdot 336 \text{ roods} \\ 40 \\ \hline 13 \cdot 440 \text{ poles} \end{array} \right\} \therefore \text{the area is } 51 \text{ acres } 1 \text{ rood } 13 \cdot 44 \text{ poles.}$$

728e. Giving and Taking.—The areas of figures with irregular outlines can be obtained with accuracy and despatch by dividing them into Δ s, preceded by the process, called “**equalising**,” which consists in drawing “give and take lines” through the curved boundaries, which shall enclose within them, as nearly as can be judged by the eye, as much space not belonging to the figure as they exclude space belonging to it. The reduction of the crooked sides to straight ones is often done by applying the straight-edge of a piece of talc, or of a set square, or a horse-hair or fine thread to the crooked line. The easiest way of using the horse-hair is to string a small bow of thin split bamboo with it, the bow keeping the hair always stretched. It can be easily applied with one hand, while with the other make dots by the side of it to draw the straight line by. The process is also called “**casting**.” In Fig. 525 the *give and take lines*, ab , bc , cd , de , ea , are drawn through the irregular boundary of the figure making the area of the simplified figure thus formed equal to that of the complicated one. The simplified figure can be divided into Δ s and their areas easily determined, to give the area of the whole figure by the aid of equation (1) p. 456.

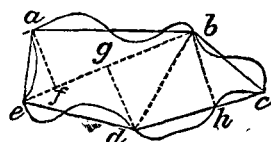


Fig. 525.—Example of Give and Take lines.

728f. Squared Paper.—Small irregular areas may be found from a drawing by laying upon it a transparent sectional paper or cloth ruled into small squares, each representing a given area, say an acre, a square mile or their parts, each; and by first counting the whole squares, and then adding the fractions of squares.

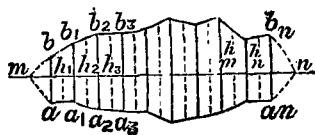


Fig. 526.—Irregular strip of Land.

728f. (2) Casting by Ordinates.—This method is applied to find approximately the area of a long irregular figure like $ab b_n a_n$, Fig. 526, such as a strip of land required for a road or railway. Between its ends ab , $a_n b_n$, space off *equal* distances

(the shorter they are the more accurate will be the result,) through which draw intermediate parallel lines, $a_1b_1, a_2b_2, a_3b_3, a_4b_4$, etc., across the breadth of the figure to make the spaces between them approximate to trapezoids. *Scale off these intermediate lines; add them together; and to the sum add half the sum of the two end breadths ab, a_nb_n . Multiply the entire sum by one of the equal spaces (d) between the parallel lines. The product will be the area.* This rule answers well, if either one or both ends terminate in points as at m and n . In the last of these cases both ab and a_nb_n will be included in the intermediate lines; and half the two ends will be nothing. The above reduced to a formula becomes:—

$$\text{Area} = (ab/2 + a_1b_1 + a_2b_2 + a_3b_3 + \&c., + a_nb_n/2) d \quad \dots \quad (5)$$

728g. Simpson's Rule.—A modification of the above trapezoidal formula founded on the assumption that the lateral boundaries of the figure consist of short parabolic arcs is as follows: the number of intervals should be a multiple of 2, *i.e.*, an even number of divisions of the line mn must be made with an odd number of ordinates. This is *Simpson's Rule* which gives—

$$\text{Area} = \{ab + a_nb_n + 2(a_2b_2 + a_4b_4 + \text{etc.}) + 4(a_1b_1 + a_3b_3 + \text{etc.})\}d/3.$$

= {first ordinate + last ordinate + 2 $\times$ sum of all other odd ordinates + 4 $\times$ sum of all even ordinates} $\times \frac{1}{3}$ of the common distance, d , between the two adjacent ordinates. $\dots \dots (6)$

728h. Weddle's Rule.—Another modification of the above is *Weddle's Cubic formula* which requires that the number of intervals should be a multiple of 6, and gives the best results.

$$\text{Area} = 3d/10 (5 \times \text{sum of even ordinates} + \text{middle ordinate} + \text{all odd ordinates}) \quad \dots \quad (7)$$

The following formulæ are more convenient than Simpson's rule, and (6) and (7) give generally, and (8) sometimes more accurate results. Σ stands for *sum of*.

728i. Franke's Rule* is:

$$\text{Area} = d \times \Sigma h + \frac{d}{7^2} (8ab + h_2 - 9h_1) + \frac{d}{7^2} (8a_nb_n + h_{n-1} - 9h_n) \quad \dots \quad (8)$$

728j. Poncelet's Rule* is:

$$\text{Area} = d \times \Sigma h + \frac{d}{12} (ab - h_1) + \frac{d}{12} (a_nb_n - h_n) \quad \dots \quad (9)$$

728k. Another Rule is— $\text{Area} = d \times \Sigma h \quad \dots \quad (10)$

The above 3 formulæ are given in the order of their accuracy beginning with the most accurate. The surface is divided into any number (n) of parallel strips having the same widths d and whose middle ordinates are represented by $h_1, h_2, h_3, \dots, h_{n-1}, h_n$, (Fig. 526), and is bounded by two ordinates $a b$ and a_nb_n and the irregular curves as per figure.

Ex. 5. The length of a line is 584 ft. and at equal distances along it, the following offsets were taken to an irregularly curved fence,

* "Pocket Companion by Carnegie Steel Co.," 1903. Pittsburg, Pa. U.S.A.

viz., 93', 84', 72', 63', 43', 54', 37', 29' and 23': Find the area included between the extreme offsets, the fence and the chain line by the Simpson's rule. Here the common distance comes to 73'.

$$\text{Area} = \frac{73}{3} \{ 93 + 23 + 2(72 + 43 + 37) + 4(84 + 68 + 54 + 29) \} = 3,3093.33 \text{ sq. ft.}$$

Ex. 6 Find the contents of an irregular area by means of seven equidistant ordinates whose lengths are 225, 231, 246, 273, 290, 306, 314 ft; the common distance between them is 65' (by Weddle's rule).

$$\text{Area} = \frac{3 \times 65}{10} \{ 5(231 + 273 + 306) + 273 + (225 + 246 + 290 + 314) \} = 105,261 \text{ sq. ft.}$$

The most accurate way to find the areas of all the pieces of lands included in a survey is to use the dimensions as given in the field-book alone, calculating the areas of the Δ s by equation 1 or 2, and the areas of the strips of land lying between the chain lines and the fences by equation 6 or 7 or 8 or 9.

728l. Area by means of Rectangular Co-ordinates.—The method as described on p. 241, Arts. 433, 434 is the standard for the calculation of the area of any closed traverse without necessitating the preparation of a plan.

728m. Mechanical Method (a).—Calculation of Area by the Talc or other Area Square.—The Talc-Square is an instrument for finding the contents of irregular figures of any shape. They are made thus:—First, large and small squares are correctly drawn on a piece of drawing paper or card board. This is kept as a pattern; and when a talc-square is required; a piece of talc is laid over this, and with a pointed iron, lines are scratched over across the talc. When all lines are thus scratched a mixture of lampblack and oil should be rubbed well into the marks. The edges of the talc, to prevent splitting should receive a narrow edging of paper pasted round.

The Principle and Construction of Talc-Square.—To construct a talc-square, take a piece of talc, or transparent horn paper, about 6" square, upon which describe as before a rectangle 5" square, and divide it into squares of acres on whatever scale the work is plotted on the map. The surveyors of the Revenue Department are supplied with talc-squares of which each smallest square is equal to $\frac{1}{16}$ of an acre. Now suppose, the scale of the map is 5 chains to the inch, or $16'' = 1$ mile. The length of the sides of the squares is obtained thus:—Construct a square of 5" sides on the talc, then divide each side into 5 equal primary divisions; and from which rule lines parallel to the sides of the square. There will thus be 25 smaller squares within the 5" square. As the scale is 5 chains to the inch, each small square, which is 1 sq. in. on the talc, will represent $5 \times 5 = 25$ sq. chains = $2\frac{1}{2}$ acres on the ground. Again, if the sides of each of these small squares are subdivided into 5 equal minor divisions, and lines drawn parallel to the sides of the squares; then each 1" square will contain 25 smaller squares, each of which will represent 1 sq. chain or $\frac{1}{16}$ of an acre, i.e., on a scale of $16'' = 1$ mile, the length of the side of a square representing $\frac{1}{16}$ of an acre will be $\frac{1}{5}$ of an inch on the talc; or in other words, if a square of $\frac{1}{5}''$ side be placed on a map

drawn to a scale of $16'' = 1$ mile, the portion of the map under the square will be equal to $\frac{1}{10}$ of an acre. Similarly, if the scale be $8'' = 1$ mile, the side of the square equal to $\frac{1}{10}$ of an acre will be $\frac{1}{10}$ of an inch on the talc. When the scale is very small, the smaller squares should be made to represent a larger area than $\frac{1}{10}$ of an acre; e.g., if the scale be $4'' = 1$ mile, the side of the square representing $\frac{1}{10}$ of an acre will be $\frac{1}{20}$ of an inch, which is too small to be of any practical use, but if the larger square on the talc be subdivided into smaller ones representing 1 acre each, the size of the smaller squares will be ten times larger. The side of the smaller square is thus obtained:—1 acre = 4840 sq. yds., hence on the ground the side of the square is $\sqrt{4840} = 69.57$ yds., and on the talc will be $69.57 \times 4/1760 = 0.15812''$ which can be easily drawn.

Ex. (1). To Construct an Acre-square on a Scale of $16'' = 1$ mile to measure 30 acres (Fig. 527). The side of a smaller square = $\frac{1}{10}$ on this

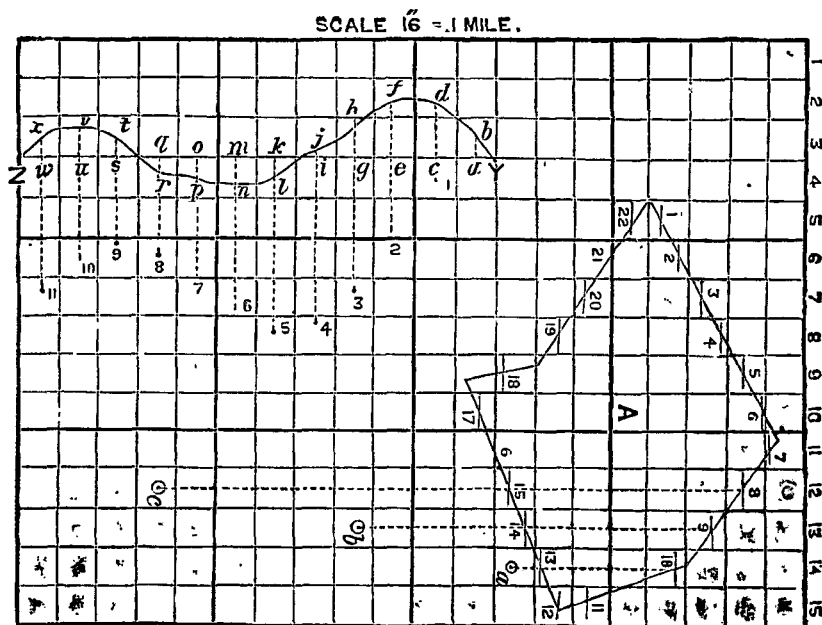


Fig. 527:—Talc-Square.

scale. $\therefore 30$ acres = $300 = 20 \times 15$ smaller squares of $\frac{1}{10}$ acre each. The length of 20 such squares will be $20 \times \frac{1}{10} = 4''$, and of 15 = 3". Now construct a rectangle with 4" and 3" sides; divide the sides into 20 and 15 equal parts. Join the corresponding points of divisions on the opposite sides. Then each of these squares will represent $\frac{1}{10}$ of an acre (Fig. 527).

Ex. (2). To Construct an Acre-square on a Scale of $8'' = 1$ mile to measure 50 acres. 50 acres = $50 \times 10 = 500$ small squares of $\frac{1}{10}$ acre each.

= 25×20
 $\therefore$ The length of 25 such squares = $25 \times \frac{1}{10}'' = 2\frac{1}{2}''$;
 and the breadth of 20 " " = $20 \times \frac{1}{10}'' = 2''$.

Construct a rectangle $2.5'' \times 2''$ on the talc, divide the length into 25, and the breadth into 20 equal parts. Join the corresponding points of divisions on the opposite sides.

Ex. (3). To Construct an Acre-square on a Scale of $4'' = 1$ mile to measure 1200 acres and one acre.

1200 acres = 1200 small squares of 1 acre each.

$= 40 \times 30$ small squares of 1 acre each

$\therefore$ The length of 40 such squares $= 40 \times 0.15812'' = 6.32''$

and „ breadth „ 30 „ „ $= 30 \times 0.15812'' = 4.74''$

Construct a rectangle $6.32''$ long by $4.74''$ broad on the talc, and divide the length and the breadth into 40 and 30 equal parts respectively; join the corresponding points of divisions on the opposite sides, thus forming 1200 smaller squares, each of which = 1 acre in area.

Similarly if the scale were $8'' = 1$ mile on the talc, the side will be $= 0.31622''$ instead of $0.15812''$ to represent the side of an acre.

Ex. (4). To Construct an Area-Square on a Scale of 1 inch = 2 chains to show $\frac{1}{100}$ th and $\frac{1}{10000}$ th of an acre and to measure 6 acres.

Construct a rectangle $5'' \times 3''$, divide the length into 10, and the breadth into 6 equal parts, and join the corresponding divisions on the opposite sides. Each small square will represent 1 sq. chain or $\frac{1}{100}$ of an acre. Subdivide the sides along the breadth into 5 equal parts, and the sides along the length into two equal parts. Then there will be 10 rectangles within the small squares and each of them will represent 1 cent, i.e., one hundredth part of an acre. The student is recommended to draw this square for himself on a piece of tracing cloth.

The Area square is made also of a piece of tracing cloth or tissue paper on which are neatly drawn or printed to the scale required, black ink squares each containing 10 cents, i.e., $\frac{1}{100}$ th of an acre, and red ink rectangles each containing 1 cent, i.e., $\frac{1}{10000}$ th part of acre. The Area square is laid over the plan, so that the boundary of the field, the area of which is to be found, is clearly visible, and so that the extreme East or West point of the field is at the corner of one of the 10 cent squares. The area should then be calculated from the North to South. The number of black and red squares contained within the field boundary can be counted in a few seconds, and the area thus found is very accurate. When counting 1 cent squares, half a square or more should be reckoned as one cent, and less than half a square rejected.

728n. To use the Talc-Square, more correctly, lay it over the plan the area of which is to be determined, and keep it fixed by a weight. Next begin to measure every column vertically containing any part of the field from one side to the other. If the horizontal line of the talc does not coincide with the boundary of the plan, nor parallel thereto, then equalise irregularities as far as the eye can judge by horizontal lines, as at the points 1, 2, 3, 4, ... 22 in figure A (Fig. 527) so that the space within the field, and that without may be equal to each other, i.e., when a portion of a field is overlapped by a portion of a square, only, the former has to be so divided into two parts that one of them lying outside may be equal to the other which lies within the square. The irregularities in cols. 5 to 15 have been equalised by the horizontal lines shown in the irregular figure A.

To measure the area of A, Fig. 527, hold a pair of dividers and place one of the legs on the point 11, and open out the other leg till it reaches the point 12. Then place the upper leg of the dividers at 13 without closing the legs; the lower leg will now be at a in a vertical line through 13. Fixing the lower leg at a , open the

dividers till the upper leg reaches 10. Again place the upper leg at 14 without closing legs, the lower leg will now be at *b*. Fixing it there open the dividers till it reaches 9. Again place the upper leg at 15 without closing the legs, the lower leg will be at *c*. Fixing it there open the dividers till it reaches 8. If the process is repeated, then the lower leg of the dividers goes beyond the talc-square; so it is necessary to read the distance between the two legs, *i.e.*, the distance of 8 to *c*, off a diagonal scale of $16'' = 1$ mile. When this distance has been read off and recorded, the process of measuring should recommence after closing the legs from Col. 11 to 5. As the side of each of the smaller squares represents one chain, the total length gives the number of chains and therefore the same number of smaller squares constructed on them.

To Find the Area of an Offset piece, YfpvZ, Fig. 527, place the talc-square over the chain line, YZ. The crooked line as seen through the talc represents the boundary line. To calculate the area of the offsets on YZ, commence at Y, and measure the distance *ab*, keeping this distance in the dividers, place the upper leg of the dividers at *c*, the lower leg being at 1 and open them *out* to *d*, place the upper leg again at *e*, the lower leg being at 2, and open them *out* to *f*, similarly from *g* to *h* and from *i* to *j*, here the offset going to the left of the line; place the upper leg of the dividers on the chain line at *k*, and draw them *in* towards *l*, then at *m* drawing them *in* to *n*; at *o* drawing them *in* to *p*, and at *q* to *r*, the offset again proceeding to the right of the line, the upper leg of the dividers must now be placed on *s*, opening them *out* to *t*, from *u* to *v*, *w* to *x*, then will the number of squares contained within the dividers, *i.e.*, *x* and 11 be the number of acres and parts of an acre contained in the offset on the line, YZ. In this case the net area of the offsets, is found to be 0.39 acre which is the remainder on the points of the dividers.

The talc-squares supplied to the Revenue Department have sides 2.4' long, divided into 12 equal parts, each measuring $\frac{1}{12}'$. Therefore the large square is divided into $12 \times 12 = 144$ small squares of $\frac{1}{12}'$ sides, and is made to a scale of $16'' = 1$ mile; so that the side of a small square represents one chain on the ground, and each square represents $\frac{1}{144}$ of an acre. The large square represents 14.4 acres. On the side of the larger square the small squares are divided into 5 equal parts, each such subdivision representing $\frac{1}{720} = 0.02$ acre; one-half of this equals $\frac{1}{1440}$ of an acre. This graduation therefore enables a calculator to take down acreage to two places of decimals.

To check the area of a figure obtained by means of the talc-square, turn the plan round so that the horizontal columns may become vertical and *vice versa*, now place the dividers in the horizontal columns, taking them now as verticals, and proceed as before till the area is obtained. If the result agrees with that obtained before the measurement is correct, otherwise not.

728o (3) Mechanical Method (b): by Area Combs.—The Area Comb or acre-comb (Fig. 528) is an open frame of thin electrum, or of stout card-board, perforated on two of its opposite sides with rows of holes at distances equal to the sides of the squares of the "Area square;" and having lines of fine silk stretched between the holes across the frame so as to give the instrument the appearance of a comb. The comb being placed over a map, the plot of which the

area is to be calculated, will be divided into a series of trapezoids by the parallel lines of silk; the lengths of each of these trapezoids can then be measured in succession, and the area will be obtained in the same manner by measuring on the "area scale" the length to which the compasses have been extended. It will be well to bear in mind that the system of computation by the "Comb" is exactly similar to that by "Square," and that the trapezoids obtained by the comb are composed of rows of imaginary squares. The connection between the compass measurement taken from the *comb* and the *area scale* will then be at once understood. The comb is more handy, and less liable to injury than the talc-square and is generally preferred for small areas and they measure $6'' \times 2''$ or $7'' \times 2\frac{1}{2}''$.

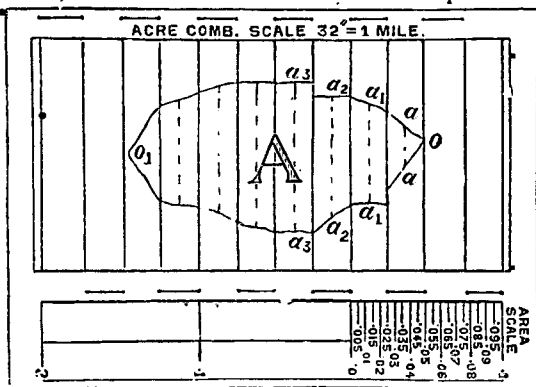


Fig. 528.—Acre-Comb.

Ex. If it is necessary to find the area of any figure A (Fig. 528) with the *Area Comb* on the $32' = 1$ mile scale, place the comb so that one of the silk lines fall upon *o* or any other point of the figure; then, if the distance, $a\alpha$, be measured with a pair of dividers, the area of the portion of the figure contained between the first two silk lines will be obtained by reading off the area from the scale corresponding to the distance measured; in like manner if $a_1\alpha_1$, $a_2\alpha_2$, etc., be measured, the whole area will be obtained by addition. The area of any portion, such as that near *o*, which does not reach the next silk line, must be taken by estimation. The area of field, A, in the example is 0.57 of an acre.

728p (3). **Mechanical Method (c): by Computing Scale or Computer.**—It is a very useful instrument for computing areas from plans on large scales. It consists of a scale of box-wood, Fig. 529, generally 20" long, along the centre is an undercut groove with a sliding indicator frame having a fine line across it, which forms the index to read off the contents on the plan. The scales, ordinarily

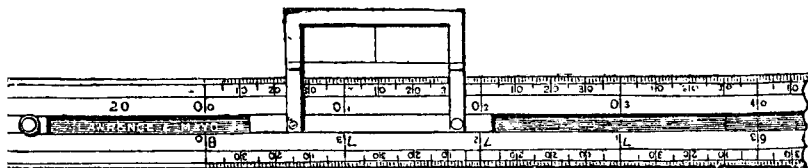


Fig. 529.—Computer.

any two, as two and three chains to the inch, are fully divided in acres, roods and perches at the edge, are figured from left to right,

and are read off by one observation to a line upon the edge of the frame. The computer is designed to measure only the areas of figure constructed to one or two scales only. To obviate this difficulty the **Universal Computing Scale** is designed. This computer (Fig. 530) is one in which the division is placed on a separate

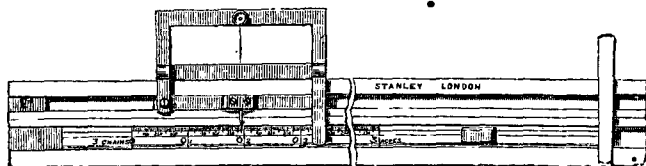


Fig. 530.—Universal Computer.

slip of box-wood. The slip slides into an undercut groove, and is read by an index. By this design one computing frame only is used for as many scales as may be required, such as, 1, 2, 3, 4, 5, 6 chains to the inch, 6 inch and 5 feet to the mile or any other scale.

Horn Paper.—To use the scale a sheet of transparent material is required, ruled entirely over in one direction, with lines which, for 1, 2, 3, 4 chains to the inch, may be placed one chain apart over the drawing. For closer scales, or scales of no even fraction of an inch the paper should be ruled to $\frac{1}{4}$ " spaces, and the division of the scale computed to read to this. Tracing paper or cloth may be used but it does not suit the purpose, being insufficiently solid. Horn paper is the best to use with the computer. It is made in the following way:—"Obtain a sheet of stout paper of the kind used for type printing; it should be of rather loose texture. This should be damped and fixed upon a drawing board. The distance of the lines, one chain apart, should be carefully pricked off along one edge of the paper, and lines ruled entirely over it from the punctures by a T-square or parallel rule. To render this transparent, it is necessary to pin it to an open frame and varnish it thoroughly from both sides several times with very good copal varnish. It will require to be kept a month or longer before it is fit to use." Transparent celluloid ruled with lines, $\frac{1}{4}$ in., $\frac{1}{2}$ in., or 1 in. apart, is also a suitable material for computing parallels.

To Compute Areas with the Computer the horn paper is first placed over the plot of which the area is required, and secured by leaden weights, that it may not slip. The plot as seen through the horn paper is thus made to appear as divided into slips of 1 chain width, such as *abcd*, *efgh*, etc., (Fig. 531). By adding up the lengths of the whole of these slips within the plot we get the area. The process is as under:—the computer, with its index at zero, is placed parallel with the parallel lines on the horn paper in such position, that the index will commence to read from the left-hand end of the first slip of land, as *ab*. While it is held in this position the index is then moved along to the position *cd*, the right-hand end of the slip to read the length of this slip on the computer. The instrument is then bodily moved, being careful not to shift the index, until the index is in the position *ef*, the left-hand end of the next slip of land, then by moving the index along

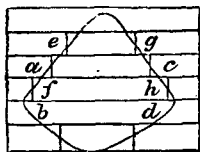


Fig. 531.—Finding Area by Computer.

the left-hand end of the next slip of land, then by moving the index along

the scale to *gh* the right-hand end of this slip, the second slip will be added to the first, and so on, until the slips are computed. If the plot contain more than, say, five acres, which will be the end of the scale of 2 or 3 chains to the inch, a pencil mark may be made upon the horn paper; a mark also should be made on a piece of paper that 5 acres have been computed. The index may now be brought back to zero upon the scale, and the computation continued from the mark on the horn paper until the index comes to another 5 acres, and so on. It is needless to add that *ab*, *cd*, *ef*, *gh*, etc., are equalising lines.

728q (3). Mechanical Method (d): by the Planimeter.—

Planimeters are instruments for measuring areas bounded by irregular curves directly from the drawing. They are made in many different forms, but in every case a *pointer*, generally called the *tracer*, is guided round the border of the area, causing motion in the mechanism which results in the rotation of a wheel; from the amount of this rotation the area of the curve is determined.

728r. Prof. Amsler's Fixed Scale Planimeter.—No. 1 of brass and No. 2 of Electrum (Fig. 532). It consists of two metal bars *CF* and *CE*, hinged at *C*. At *E* is the needle-point to be driven in the

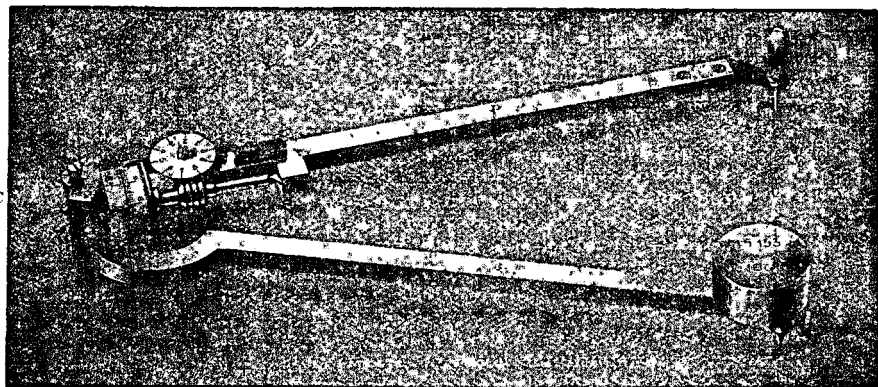


Fig. 532.—Amsler's Nos. 1 & 2 Fixed Scale Planimeter.

drawing board, and at *F* is the tracer. As this is moved round the border of the area, a wheel, *D*, mounted on *CF*, rolls on the paper, and the turning of this wheel measures to one scale only, the area reading in square inches or square centimeters, or acres, of actual measurement. It can be applied to the computation of land by a multiplier of the square of the number of units to the inch, as 4 for 2-chain scale, 9 for 3-chain, etc. The tracer-arm is of a standard length.

728s. Amsler's Proportional Planimeter.—(No. 3 of brass and No. 4 of Electrum).—In this (Fig. 533) the unit can be changed by altering the radius of the *tracer-arm* to any of the several scales engraved on the instrument.

728t. The Details of Construction of the Above Two Instruments.—The needle-point, *E*, forms the point of attachment

of the instrument, on the drawing, and the bar, *EC*, carrying it may be of any convenient length. The other end of this bar is

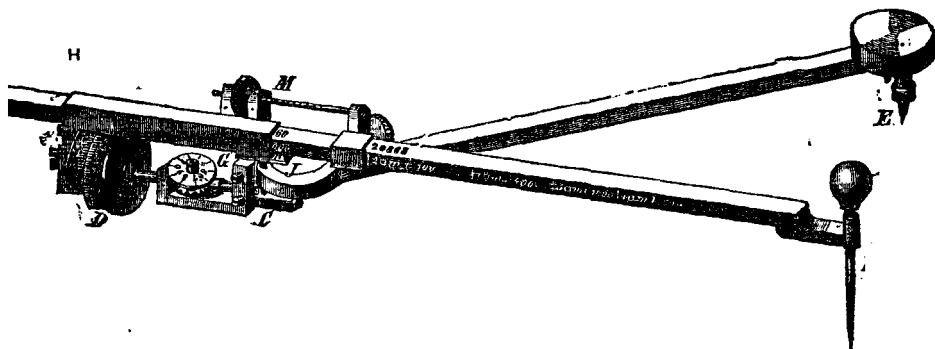


Fig. 533. — Amsler's Proportional Planimeters, Nos. 3 & 4.

cranked to obviate contact with part of the apparatus when in movement, (where a pair of vertical pivots are centered) and can move freely upon vertical pivot, *C*, horizontally. The axes of the pivots are carried either upon the *tracing-arm* as in Fig. 532, or upon a movable sliding fitting, *H*, upon it, as in Fig. 533. The tracer, *F*, at the end of this arm, is used to move round the border of the area to be measured. The measuring mechanism is mounted on the tracing-arm (Fig. 532); or on the sliding fitting or sleeve, *H* (Fig. 533). This consists of a roller *D* mounted on an axis parallel to the tracing-arm, *i.e.*, parallel to the imaginary line joining the tracing-point, *F*, and axis, *C*, of the needle or polar arm. In Figs. 532-533 the tracer, *F*, pivot *C* and the axis of the wheel, *D*, should be in the same vertical plane, and the instrument (Fig. 533) is so constructed, that *D* cannot pass between *E* and *C*. When in use the planimeter rests on the drawing at three

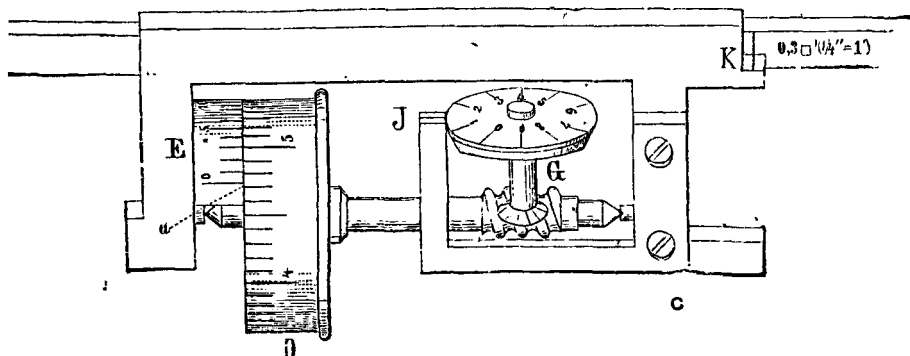


Fig. 534. — Enlarged Mechanism of Planimeter.

points : one on the circumference of the wheel, *D*, the others at *E* and *F*, *E* being fixed. The roller, *D*, carries a drum, graduated into 100 parts, reading into a vernier, (*E* in enlarged mechanism, Fig. 534),

which gives the reading of the drum's revolution to the $\frac{1}{1000}$ part of its circumference. On the axis of the roller an endless screw is cut, that works into a record disc, *G*, of 10 graduations which counts the number of complete revolutions of the roller. The proportional scales of the Nos. 3 and 4 planimeters are derived from the length of the vertical axis of the tracing-arm and the circumference of the roller. The lateral displacement of the roller multiplied by the length of the arm gives the area of the figure contained.

728u. Setting the Planimeter.—If of the *fixed* kind, the readings will be in inches. If of the *proportional* kind this has first to be set to the scale. For this one of the divisions upon the bar is taken to suit the nature of computation; thus if the area is required in inches, 10 sq. inches will be the most convenient. To this division the index-mark, *J*, on the bevelled edge of the slide, *H*, upon the bar is set, and the instrument is ready for use. There is a clamp and tangent-screw, *M* (Fig. 532), to get this to position exactly so that the tracer, *F*, may be set at different distances from *C*.

728v. Instructions for using Amsler's Planimeters.—No. 1 Planimeter (brass) and No. 2 Planimeter (Electrum) arranged for measuring areas in square inches, square centimeters, and acres.

Range of Instrument reading in square inches of actual measurement :

Needle point outside the figure to be traced.

Circle of 8" diameter—Square 7" sides—Rectangle 5" × 14" sides.

Needle point Inside the figure :

„ „ 18" diameter—Square 14" × 14" sides—Rectangle 10" × 18" sides.

Needle Point Outside the Diagram.—Put the instrument on the drawing surface, set the tracing point, *F*, at a mark on the outline of the area, press the needle point, *E*, slightly into the paper outside the curve, and read off the roller, *D*, and the counting wheel, *G*, taking the whole circumference of the recording roller as unit of reading (the roller need not be set to zero.)

The counter in Fig. 533, for example, would read 1.473, this figure being obtained thus : 1 from wheel, 47 from roller, 3 from vernier.

Then move the tracing point, *F*, round the area in the direction of the movement of the hands of a watch, till it reaches again the starting point. Now take another reading, and subtract the first from the second reading. The difference multiplied by .10 will then be the area of the curve in sq. ins.

Ex. 1. To measure the Area of a Square of 4 inches Side.

Result : Second reading 3.073

First reading — 1.473

$1.600 \times 10 = 16 \text{ sq. ins.}$

If the reading before starting had been 9.521, then the reading after circumscribing the same square as before would be 1.121. As the travel of the roller is in both cases the same, it is evident that the zero mark of the wheel has passed the fixed index mark and that, consequently, the second reading has now to be supplied with one more digit on the left before subtracting the first reading. Thus

Complete second reading 11.121

First reading — 9.521

$1.600 \times 10 = 16 \text{ sq. ins.}$

Needle Point Inside the Diagram.—Circumscribe the diagram with the tracing point clock-wise, watching at the same time the counter in order to see whether the total rotation of the roller is a forward or a back motion,

This preliminary rough operation being completed, proceed as before or explained, now following the curve carefully with the tracing point. If the total rotation of the roller has been a forward motion, subtract the first from the second reading, and add the difference to the figure engraved on the top of the small weight used for keeping the needle point in its place. The sum multiplied by 10 will then be the area of the curve in sq. ins.

Ex. 2. To Measure a Circle of 18 inches in Diameter. (You will see that the total rotation of the roller is forward).

Result : Second reading 9.884
First reading -0.868

9.016

Figure on weight + 16.431 (This figure is slightly different for different instruments).

$25.447 \times 10 = 254.47$ sq. ins.

If the total rotation of the roller is a backward motion, subtract the second from the first reading and subtract the difference from the figure on the weight.

Ex. 3 To Measure a Square of 11 inches Side. (You will see that the total rotation of the roller is backward.)

Result : Complete first reading 13.009 | Figure on weight 16.431
Second reading -8.678 | Difference of reading -4.331

4.331

$12.100 \times 10 = 121$ sq. ins.

Note : When the mark o on the roller is at the mark o of the vernier, a mark of the counting wheel should be opposite the fixed index mark. Any slight non-coincidence due to imperfection of worm wheel gear may readily be allowed for in taking readings. The area corresponding to a total revolution of the roller—10 sq. ins. in the above example—is engraved on the weight along with the figure before mentioned.

No. 3 Planimeter (brass), and No. 4 Planimeter (Electrum), arranged for measuring areas in several units, and in several English and French scales.

Range : *Needle-point Outside the Figure :*

Circle of 11" diam.—Square 10' × 10' sides, Rectangle 6' × 18' sides.

Needle-point Inside the Figure,

Circle of 24" diam.—Square 16' × 16' sides, Rectangle 12' × 20' sides.

Set the index-mark, J, on the bevelled edge of the slide to a convenient division on the bar. There is a clamp and tangent-screw to get this position exactly. The number of units of area per unit of reading and the nature of unit or scale are marked to the right of each division. The divisions on the bar are the following :

on back		10 ac. 1 : 2500		5 ac. $\frac{1}{4}$: 2500
		22.107		
on top	20.811 20.818 21.042	22.122		
on front	$100 \square \text{ cm.}$ $100 \square \frac{1}{8}'' = 1'$ $200 \square \frac{1}{4}'' = 1'$ $50 \square \frac{1}{2}'' = 1'$ $10 \square \text{ in.}$ $100 \square \frac{1}{4}'' = 1'$ $400 \square \frac{1}{8}'' = 1'$			

The figures on the top of the bar are slightly different for different instruments. The denotation $\square$ means square foot. $\square \text{ cm.}$ = square centimeter, $\square \text{ in.}$ = square inch, ac. = acres. If desired other divisions instead of these are put upon the bar. The slide being set on the bar, place the instrument upon the drawing and measure the diagram as with No. 1 planimeter.

Ex. 4. To Measure a Circle of 5 inches in Diameter on a $\frac{1}{4}$ " plan.

Set the index mark J, on the slide to the division 200 $\square$, $\frac{1}{4}'' = 1'$, press the needle point E, outside the circle into the paper, set the tracing point F, to any mark of the circle, read off the counter—the reading may be 1.322—follow the circle with the tracing point until you arrive at the starting point and read off again the counter. The reading will then become 2.893.

CHAP. X.] CALCULATION OF AREAS BY THE PLANIMETER. 471.

Result : Second reading ...	2.893	(The multiplier 200 is identical with the figure close to the right of the division used.)
First reading • ...	1.322	
	1.571	

$1.571 \times 200 = 314.2$ sq. ft.

For measuring small diagrams drawn on the $\frac{1}{4}$ " or $\frac{1}{8}$ " scale, the slide may conveniently be set to the last division on the bar marked 100 $\square \frac{1}{4} = 1$ " and 400 $\square \frac{1}{8} = 1$ ".

If the needle point, E, be placed *inside* the diagram to be measured, then one of the figures on the top of the bar must be used in the same way as the figure on the weight of No. 1 planimeter.

Ex. 5. To Measure a Square of 12 inches Side on a map (scale 1" = 2500').

Set the index-mark on the back of the slide to the division 10 ac. 1 ; 2500. Suppose that the counter reads 2.482 before starting the tracing point. In following the outline of the square you will see that the total rotation of the counter is backward and that the zero mark of the counting wheel passes once the fixed index.

Result : Complete first reading	12.482	(The multiplier 10 is identical with the denotation 10 ac. close to the right of the division used.)
Second reading	—4.723	
	7.759	
Figure on top of bar just over the division	22.107	
Difference of readings	7.759	

$14.348 \times 10 = 143.48$ acres.

No. 5. Pantograph Planimeter, Electrum, arranged for measuring very Large and very Small Areas.—For measuring large areas tracer, F, is used, for small areas the tracer *f*. It is constructed for different units and scales like the No. 4 planimeter.

Range : *Needle-point Outside the Figure :*

Circle 16" diam., Square 16" $\times$ 16" sides, Rectangle 12" $\times$ 28" sides.

Needle-point Inside the Figure :

Circle 40" diam., Square 24" $\times$ 24" sides, Rectangle 20" $\times$ 32" sides.

1. For measuring large areas; set the slide *B* to the proper division on the bar *A* (Fig. 535). For doing this, hold the bar, *A*, with the right-hand and shift the slide with the left whilst the parallelogram hangs down. Then place the instrument upon the drawing and proceed exactly as with No. 3 planimeter. The sliding point *f* is of no use now and may be taken off.

2. For measuring small areas, the smaller sliding bar, *a*, is to be adjusted so that the proper mark on the bar comes even with the end face of the tube in which the bar slides (the slide *B* need not be adjusted upon the bar *A*). Then let the point *f* travel round the diagram by moving the point, *F*, with the hand, but following the point, *f*, with the eye. This can be done with great ease and accuracy, the two points, *f* and *F*, describing nearly similar paths. The area will then be expressed in the unit shown at the mark on bar, *a*. Small areas can be measured in this way with about the same accuracy as large areas.

No. 6 Planimeter (German silver).—Same as No. 4, but with special arrangement for finding rapidly the mean height of indicator diagrams, which need not be described here (Fig. 536).

No. 7 Planimeter (Electrum).—This is the same as the No. 4; but of considerably large size for measuring very large figures in one operation.

Range : *Needle-point Outside the Figure :*

Circle 20" diam., Square, 14" $\times$ 14" sides, Rectangle 14" $\times$ 30" sides.

Needle-point Inside the Figure :

Circle 40" diam., Square, 30" $\times$ 30" sides, Rectangle 26" $\times$ 33" sides.

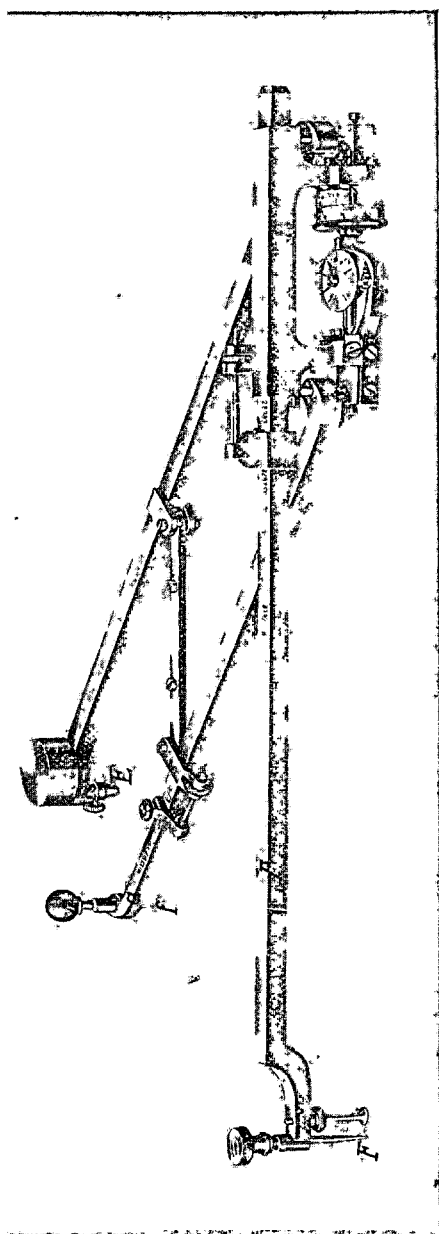


Fig. 535.—No. 5 Pantograph Planimeter.

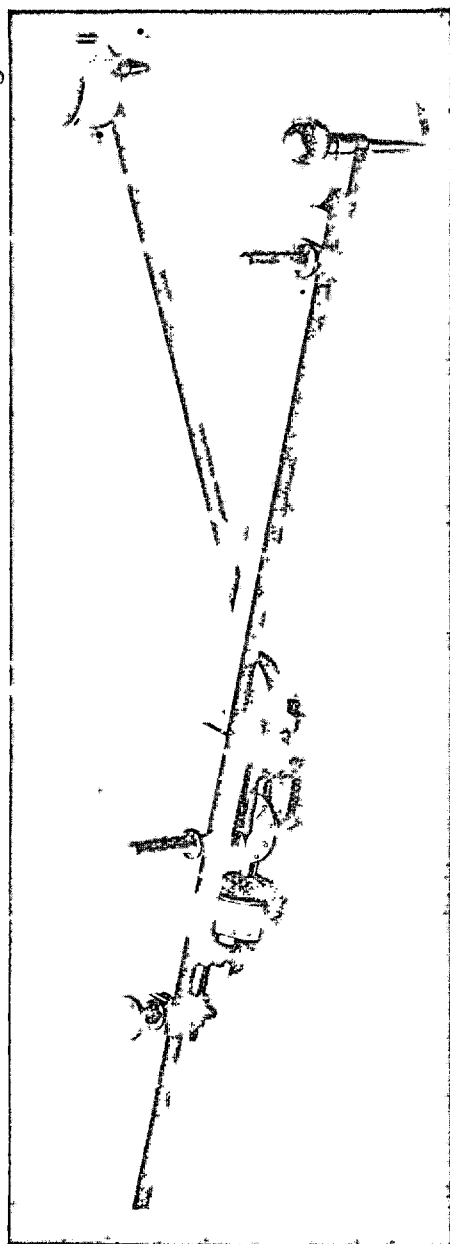


Fig. 536.—No. 6 Planimeter (Amsler's).

No. 8 Planimeter (brass and electrum).—The measuring roller of this planimeter works on a revolving disc and not directly on the surface of the figure to be measured.

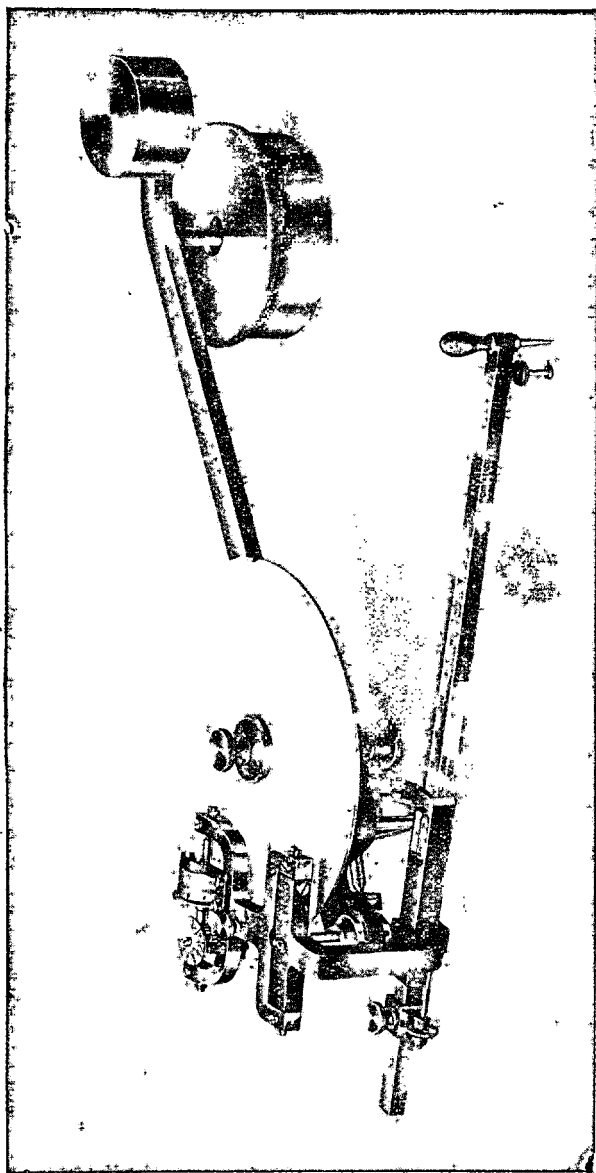


Fig. 537.—No. 8 Amsler's Disc Planimeter.

Range : *Pole-plate Outside the Figure to be measured :*
Circle of 10" diam. ; Square 9" $\times$ 9" sides ; Rectangle 6" $\times$ 18" sides.

Pole-plate Inside the Figure :

Circle 30" diameter ; Square 22" $\times$ 22" sides ; Rectangle 16" $\times$ 26" sides.

The instrument swings on a vertical pivot (pole).

Place the Planimeter on the drawing as shown in Fig. 536. Adjust the bar so that the index-mark on the slide coincides with one of the divisions on the bar. The unit of area is engraved to the right of the corresponding division. Shift the centre plate into such a position that the tracing point can go round the whole outline of the area. Now proceed as with the No. 3 planimeter. For putting a fresh paper on the disc, take off the screw-nut on the top of the disc.

No. 9 Planimeter (brass or electrum)—It is equally provided with a revolving disc. The instrument (Fig. 537) runs on a rail and its working is in no way affected by the condition of the drawing surface.

Place the rail on the drawing board as shown in Fig. 538, put the carriage upon the rail in such a way that the two wheels run in the groove on the

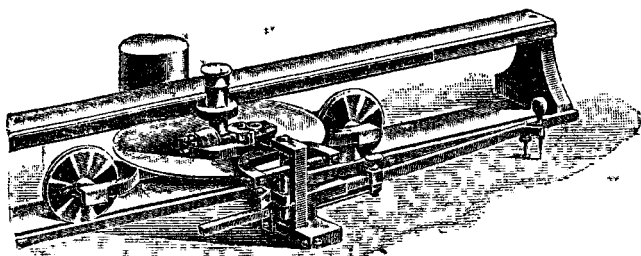


Fig. 538.—Amsler's No. 9 Planimeter.

under straight-edge, and that the pinion on the top of the disc gears into the rack of the upper straight-edge. The disc and part of the carriage will then lie between the two straight-edges. Put the counterweight into the rear end of the carriage standing out behind the rail. The adjustment of the divided bar and the measurement is done in the same way as with No. 3 planimeter. The counter of No. 9 Planimeter reads up to 100 turns of the roller. The reading is therefore a figure of 5 digits. It is the most accurate planimeter of all. For putting a fresh paper on the disc, swing back the little frame carrying the roller, take off the screw-nut at bottom of the disc, and unscrew the disc. Range : Rectangle 9" $\times$ 20" sides.

In consequence of the roller moving on a revolving disc the accuracy of the results obtained by means of No. 8 and No. 9 planimeters is greater than that obtainable by means of the other planimeters. On the other hand, the Nos. 1—7 planimeters are handier, less delicate, and cheaper than No. 8 and No. 9.

Amsler's Radial Planimeter (Fig. 539) is adapted for averaging circular diagrams drawn by dial instruments, and for computing any diagram plotted in polar co-ordinates.

Checking Circle (Electrum).—Any planimeter may be provided with a rule having at one end a pivot point, at the other end a fine mark upon which the tracing point of the planimeter is to be set for checking. The rule represents the radius of a standard circle the area of which is engraved on the rule. Move the rule once round about the centre and compare the reading of the planimeter with the area engraved on the rule.

General Remarks on Mechanism.—Avoid touching the rim of the measuring roller. It is made of steel and is liable to be spoiled by rust. Therefore do not attempt to set the roller to zero as this would involve more time and trouble than taking the reading as it stands. The roller must rotate very easily and must have a little shake between the pivot centers. The edge

of the roller should not touch the vernier. The vertical axle of the pole-arm should be without back-lash. Occasionally oil the pivot centers with fine oil. Any alterations of the tracing-arm or of the tracing-point disturb the adjustment of the planimeter and should therefore be avoided. There is no harm in taking out the measuring-roller, provided that it be carefully handled. The

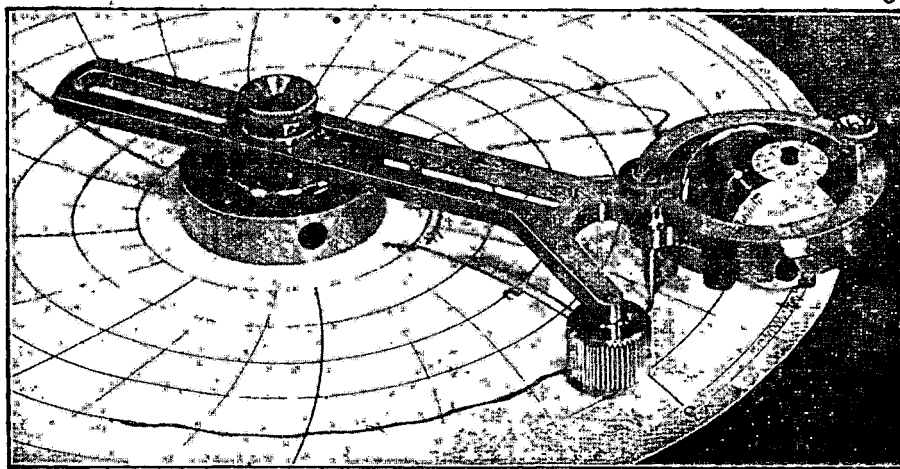


Fig. 539.—Amsler's Radial Planimeter of Electrum.

rim of the roller and the pivot points of the axle are particularly subject to injury. If the needle-point, E, must be replaced, slacken the little clamp-screw at the holder, press backward the broken needle, put another at its place, clamp it and break off the portion standing out on the top of the bar. Suitable needles are R. Hemming & Son's blunts 7.

728w. Principle of the Planimeter*—There are many proofs of the principle of this instrument, but all that are accessible seem a little beyond the grasp of many students who use the instrument. Let A be a pivot, round which the jointed rods, AB, BC, (Fig. 540) turn, and let AB', B'C' be the positions of the rods after C has moved to C', CC' being a small arc of the curve, whose area is required; and first let us suppose that this curve does not include the point A. The element of area, ABCC'B'A, consists of three parts, namely: (1) The triangle, or sector ABB'; (2) The parallelogram BCC'B', where B'C' is drawn equal and parallel to B'C. (3) The triangle or sector CBC'. Now the sum of the elements of area ABCC'B', taken with positive or negative sign, according to the direction in which AB, BC are rotating, is the area of the closed curve traced out by C; and when the circuit has been completed, the instrument will have returned by a reciprocating motion, i.e., without a rotation to its original position. Hence of the three parts

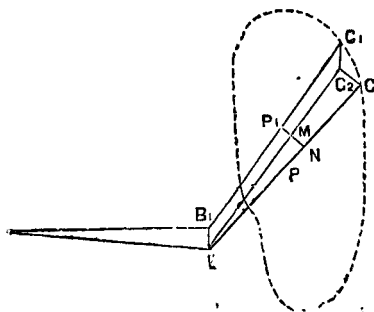


Fig. 540.—Principle of Amsler's Planimeter.

* By permission from the Engineering, London, of Dec. 17, 1897.

into which the element $ABCC'B$, has been resolved, the sum of the areas of (1) will be zero; that of (3) will also be zero; and the area of the closed curve reduces to the sum of such parallelogram as $BC^2C'B'$ and this sum must be their common base BC , multiplied by their aggregate heights.

Now let P be any fixed point on BC ; a rolling wheel at P , free to revolve on BC as axis, will register the total motion of P at right angles to BC . The element of motion of P in this direction is obtained by drawing $P'MN$ perpendicular to BC and (ultimately) to $B'C'$, P' being the position of P when C has moved to C' . The sum of these elements is registered by the wheel. But the sum of the elements MN vanishes just as in the case with the areas (1) and (3) from the reciprocating motion of BC ; and, therefore, the wheel registers simply the sum of the elements $P'M$, i.e., the aggregate heights of the parallelograms like $BC^2C'B'$. Hence the area of the closed curve is equal to the length of BC multiplied by "the registration of the wheel" the last phrase meaning the number of its revolution times its circumference, and hence returns after one rotation to its original position.

If the curve encloses the point A , and observing that the instrument does not from its construction permit BC to cover over AB , we must note that our curve now includes, besides the parallelogram areas equal to those of two circles where radii are AB , BC , while the registration gives us the heights of the parallelogram added to the circumference of the circle where radius is BC . Thus we must add to our result obtained from the registration the areas of the first two circles, and subtract the area of a parallelogram where base is BC , and height the circumference of the third, i.e., we must add $\pi AB^2 + \pi BC^2 - 2\pi BPC$. If a circle of known area be described about A , this constant is at once found for any particular instrument.

728x. Theory of Amsler's Polar Planimeter.—Zero circle.—If the tracer, F , be made to follow a circle with its centre at E , the $\triangle FEC$ will be

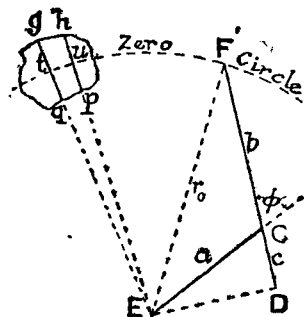


Fig. 541.—Principle of Polar Planimeter.

immutable, Fig. 541; and hence any point on EC or FD will describe a circle with its centre at E . When the tracer moves on the circle such that the plane of the wheel, D , passes through E , the wheel will describe a circle whose tangent will take the direction FD , so that the motion of the wheel will be in a direction $\perp$ to its plane and it will not rotate. This circle, described by the tracer without rotation of the wheel, is called the *zero circle*, and its radius, $r_0 = \sqrt{a^2 + b^2 + 2bc}$. Where $a = EC$, $b = FC$, and $c =$ projection of a on $b = CD =$ distance from C to the plane of the rim of the wheel. For $EF^2 = FD^2 + ED^2$; $CE^2 = CD^2 + ED^2$. $\therefore EF^2 = FD^2 + CE^2 - CD^2 = FC^2 + CD^2 + 2FC \cdot CD + CE^2 - CD^2 = r_0^2 = a^2 + b^2 + 2bc$ (1)

$\therefore$ the area of the zero circle $= \pi(a^2 + b^2 + 2bc)$. The area of the zero circle is computed by the maker for each instrument, and is engraved on the top of the bar over each index or setting, and nearest to the setting for which it is intended. The distance $FC = b$ may be changed but a and c are constants; hence the radius of the zero circle will vary with b .

728y. Direction of Rotation.—In Figs. 542-543, let the $\angle FEF' = d\theta$, FF' and IJ being circular arcs with their centre at F , and let $EL = r_0 =$ radius of the zero circle.

When FF' is outside the zero circle (Fig. 542) the tracer moving over the infinitesimal arc FF' from F to F' (clock-wise) will cause the wheel to move from D to D_1 . This motion may be resolved into DM_1 $\perp$ to the plane of the wheel, and not causing it to rotate and MD_1 , parallel to the

plane of the wheel and causing rotation. Moving the wheel over MD' will cause it to rotate clock-wise (looking in the direction DF), and will cause the reading of the wheel to increase. If the tracer were moved over $F'F$ from F' to F (counter-clock-wise) the wheel would rotate by the same amount, but in the opposite direction.

When FF' is within the zero circle (Fig 543), the motion over $F'F$ will cause the wheel to move from D to D' resolvable into DM and MD' . The motion of the wheel over MD' will cause it to rotate anti-clock-wise (looking in the direction DF). When the tracer is moved from F' to F the wheel will rotate clock-wise. Hence, when the tracer is moved clock-wise (*C. W.*) over a circular arc with its center at E ,—arc outside the zero circle, rotation is *C. W.*; arc within the zero circle, rotation is anti-clock-wise (*A. C. W.*).

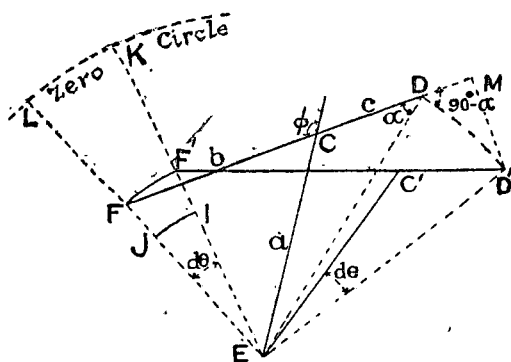


Fig. 543.—Principle of Polar Planimeter.

consider the effect upon the wheel of the tracer over the perimeter of $FF'IJ$, i.e., over the two radial lines $F'I$, and JF , and over the arcs FF' and IJ .

I. If the tracer be moved over $F'I$, the wheel will rotate by a certain amount, and if the tracer be now brought back over IF' to F' , it will rotate by an equal amount in the opposite direction, thus bringing the wheel to its original position. Also if the tracer be moved from F to I , and if the wheel be then lifted from the paper to prevent rotation while the tracer is moved to J , the angle between the arms a and b of the planimeter will be the same when the pointer is at J as it was when the pointer was at I . Hence moving the tracer over JF will have the same effect on the wheel as the motion over IF' . Therefore the rotation of the wheel produced by moving the tracer over $F'I$ will be neutralised by that produced when the tracer is moved over JF , so that in considering the rotation of the wheel caused by moving the tracer around the figure $F'F'IJ$ we may neglect the radial lines and consider only the circular arcs. Hence $F'J$ and $F'I$ are not necessarily infinitesimal so that we may treat any plane area as being made up of an infinite number of infinitesimal parts, each bounded by radial lines from E enclosing an angle $d\theta$ and by arcs with their centers at E , as $ghpq$ in Fig. 541.

II. The arc FF' may be outside of the zero circle or within it. Suppose the motion of the tracer over $F'F$ to be *C. W.*, and let the distance $EF = r$.

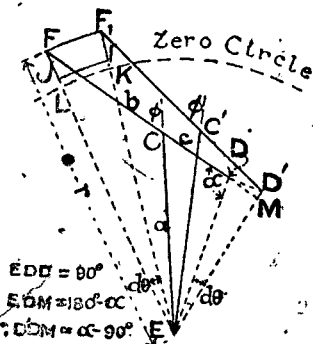


Fig. 542.—Principle of Polar Planimeter.

When the tracer is moved *A. C. W.*, the direction of the rotation of the wheel is reversed. Call *C. W.* rotation *positive*, since it increases the reading of the wheel, and hence *A. C. W.* rotation will be *negative*.

728z. Connection between Rotation and Area.—Since any area may be considered as being composed of an infinite number of elements like $FF'IJ$, polar element of area, in Figs. 542, 543, we shall con-

(a) The arc outside the zero-circle (Fig. 542). When the tracer is moved over the infinitesimal arc FF' , the wheel will describe a circular arc with its centre at E and with radius ED ;

$$\therefore \text{arc } DD' = ED d\theta.$$

The portion of this movement of the wheel that causes rotation is

$$MD' = DD' \sin. D'DM = DD' \sin. (\alpha - 90^\circ) =$$

$$= -DD' \cos. \alpha = -ED \cos. \alpha d\theta.$$

From the $\triangle s EFD$ and ECD we have

$$r^2 = (b+c)^2 + ED^2 - 2(b+c)ED \cos. \alpha$$

$$\text{and } a^2 = c^2 + ED^2 - 2c.ED \cos. \alpha.$$

$$\therefore ED \cos. \alpha = \frac{1}{2b} (a^2 + b^2 + 2bc - r^2) = \frac{1}{2b} (r_o^2 - r)$$

$$\therefore MD' = +\frac{1}{2b} (r^2 - r_o^2) d\theta, \quad \dots \dots \dots (2)$$

where $r > r_o$.

(b) The arc within the zero circle (Fig. 543). The $\angle D'DM$ is here equal to $90^\circ - \alpha$.

$$\therefore MD' = DD' (\sin. 90^\circ - \alpha) = DD' \cos. \alpha = ED \cos. \alpha d\theta;$$

and this by the method above gives

$$MD' = +\frac{1}{2b} (r_o^2 - r^2) d\theta, \quad \dots \dots \dots (3)$$

where $r < r_o$.

(c) These values of MD' give the amount of rotation. Considering the direction of rotation (Art. 728y.) we have for the rotation of the wheel when the arc is either outside of or within the zero circle :

$$\text{Tracer moved } O. W., \text{ rotation} = +\frac{1}{2b} (r^2 - r_o^2) d\theta; \quad \dots \dots (4)$$

$$\text{Tracer moved } A. C. W., \text{ rotation} = -\frac{1}{2b} (r^2 - r_o^2) d\theta; \quad \dots \dots (5)$$

where the relative values of r and r_o will determine the direction of rotation.

In Figs. 542, 543, area $FF'KL = \frac{1}{2} (r^2 - r_o^2) d\theta$;

hence the area between the arc FF' and the zero circle is equal to the product of b by the amount of rotation caused when the tracer is moved over the arc FF' . Eqns. 4 and 5 are the essential equations of the instrument.

III. The rotation produced by moving the tracer over the periphery of any figure will be the sum of the rotations produced by moving the tracer around the infinite number of its infinitesimal portions



(gh pq , Fig. 541). For in moving the tracer around the figure $abcd$ $hgfe$ (fig. 544), if we represent the rotation due to ab by $\overline{ab}$,

the total rotation will be $\overline{ab} + \overline{bc} + \overline{cd} + \overline{dh} + \overline{hg} + \overline{gf} + \overline{fe} + \overline{ea} = (\overline{ab} + \overline{bf} + \overline{fe} + \overline{ea}) + (\overline{bc} + \overline{cd} + \overline{dh} + \overline{gh} + \overline{gf} + \overline{fb})$,

Fig. 544.

Since $\overline{bf}$ will be neutralised by $\overline{fb}$.

$$\therefore \text{Total rotation} = \overline{abfe} + \overline{bcdhgf} + \overline{b}$$

$= \overline{ab} + \overline{cd} + \overline{hg} + \overline{fe}$, the same as if the tracer were moved only over the circular arcs.

728a'. Area of any Curve.—In using the planimeter the needle-point, E , may be placed either outside of the area to be measured, or within it, there are 3 cases in each position :—

(1) Periphery entirely outside of the zero circle.

(2) " " within " "

(3) " partly outside of and partly within the zero circle.

(A) Needle-point outside the Area.—Considering the tracer as starting from F and going around $FF'IJ$ from F to F_1 to I to J to F we have the following results :—

1. The periphery entirely outside of the zero circle (Fig. 542).

Rotation corresponding to $FF' = +(EF^2 - r_o^2) d\theta/2b$;

„ „ „ „ $IJ = -(EJ^2 - r_o^2) d\theta/2b$;

∴ Resultant Rotation $= (EF^2 - EJ^2) d\theta/2b$.

But area $FF'IJ = (EF^2 - EJ^2) d\theta/2$;

∴ Area $= b \times \text{rotation} \dots \dots \dots (6)$

2. The periphery entirely within the zero circle (Fig. 543).

Rotation corresponding to $FF' = +(EF^2 - r_o^2) d\theta/2b$;

„ „ „ „ $IJ = -(EJ^2 - r_o^2) d\theta/2b$;

Resultant Rotation $= (EF^2 - EJ^2) d\theta/2b$.

Area $= b \times \text{rotation} \dots \dots \dots (7)$

The periphery partly outside of and partly within the zero circle (Fig. 545.)

Rotation corresponding to $FF' = +(EF^2 - r_o^2) d\theta/2b$;

„ „ „ „ $IJ = -(EJ^2 - r_o^2) d\theta/2b$;

Resultant Rotation $= (EF^2 - EJ^2) d\theta/2b$.

Area $= b \times \text{rotation} \dots \dots \dots (8)$

4. Hence, in all cases when the needle-point is outside of the area, we have, $\text{area} = b \times \text{rotation}$. If the circumference of the wheel be denoted by s and the number of turns by n , we have $\text{rotation} = sn$, and $\text{area} = b \cdot sn$. (9) where n is always positive if the tracer is moved C. W. around the figure.

(B) Needle-point within the Area.

(1) The periphery entirely outside of the zero circle (Fig. 546).

Moving the tracer from F and around $FF'LK$ we have

Rotation corresponding to $FF' = +(EF^2 - r_o^2) d\theta/2b$

„ „ „ „ $KL = 0$ (Zero);

∴ Resultant Rotation $= (EF^2 - r_o^2) d\theta/2b$;

But area $FF'LK = (EF^2 - r_o^2) d\theta/2$,



Fig. 545.

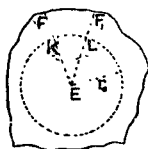


Fig. 546.

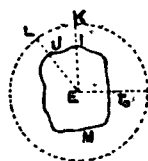


Fig. 547.

∴ Area $FF'LK = b \times \text{rotation}$.

Hence the area between the curve and the zero circle $= b \times \text{rotation}$, and to get the total area we must add πr_o^2 , the area of the zero circle.

∴ Area $= \pi r_o^2 + bsn \dots \dots \dots (10)$

When n will be positive for C. W. motion of the tracer.

(2) The periphery entirely within the zero circle (Fig. 547.) Moving the tracer clock-wise around JIM (or A.C.W. around J.I.LK) we have

Rotation corresponding to $JI = +(EJ^2 - r_o^2) d\theta/2b$

„ „ „ „ $KL = 0$ (Zero)

Resultant rotation $= (EJ^2 - r_o^2) d\theta/2b$, a negative quantity, for $EJ < r_o$.

But Area $JIKL = (r_o^2 - EJ^2) d\theta/2 = -b \times \text{rotation}$; But the rotation = sn where n is a negative quantity, the reading of the wheel having decreased.
 $\therefore$ Area $JILK = -b sn$

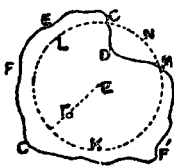
Hence the area between the curve and the zero circle = $-b \times \text{rotation} = -bsn$, and to get the area JIM we must subtract the area between the curve and the zero circle from the area of the zero circle.

$$\therefore \text{Area JIM} = \pi r_o^2 - (-bsn).$$

$$\therefore \text{Area} = \pi r_o^2 + bsn. \quad \dots \quad (11)$$

Where n will be negative for clock-wise motion of the tracer.

3. *The periphery partly outside of and partly within the zero circle* (Fig. 548) Moving the tracer from M to C along MGFF'C will give a positive rotation equal to area MGFF'C $LK \div b$. Moving the tracer from C to M along CDM will give a negative rotation equal to area CDMN $\div b$.



The resulting rotation will be positive or negative according as the area MGFF'CLK or the area CDMN is the greater; and the area of the curve will be for clock-wise motion of the tracer.

$$\text{Area} = \pi r_o^2 + bsn. \quad \dots \quad (12)$$

Fig. 548. (C) *Summary*: Hence we have for clock-wise motion of the tracer around the figure: (1) Needle-point outside of the area.

$$\text{Area} = bsn \quad \dots \quad (13)$$

here n is always positive

(2) Needle-point within the area

$$\text{Area} = \pi r_o^2 + bsn \quad \dots \quad (14)$$

Where n is positive or negative according as the reading of the wheel has increased or decreased.

728b'. Value of b .—The following dimensions of a planimeter were given by the makers:

$$S = 2.370''; a = 6.214''; c = 1.343''.$$

In this instrument the zero of the graduated tracer-arm, CF, is found at F and the index-mark on the frame that carries the wheel is opposite to C, so that the setting of the arm is the value of b .

(1) To find b so that the area of the diagram may be found to hundredths of a sq. in. Then 0.01 sq. in. will correspond to one unit on the wheel vernier ($n = 0.001$). Since $A = bsn$ (Eq. 13), we have $0.01 = b \times 2.370 \times 0.001$. $\therefore b = 4.219'$. Hence, if the bar CF is set at 4.219' and if the tracer is then made to pass over the periphery of an area, the rotation of the wheel will give the area in hundredths of a sq. in., each vernier unit representing 0.01 sq. in.

(2) Suppose that the diagram represents a figure on the scale of 1:96, and to find the area of the figure in sq. feet, set the arm at 4.219', as before; find the area of the diagram in sq. in. and then compute the area of the figure.

1 sq. ft. of the actual area corresponds to $1 \div 96^2$ sq. ft. of the diagram or to $144 \div 96^2$ sq. in. Hence, since we wish each vernier unit to correspond to 1 sq. ft. of the figure, $144 \div 96^2$ sq. in. of the diagram must correspond to one vernier-unit.

$$\therefore 144 \div 96^2 = b \times 2.370 \times 0.001. \quad \therefore b = 6.593.'$$

In using the planimeter a mark is made on the periphery, the tracer is placed there, and the reading of the wheel is found. The tracer is then moved around the periphery until it returns to the starting point, when a second reading of the wheel is made. The first reading was 2.403 turns and the second 5.677; the rotation would then be 3.274 turns, or 3274 vernier-units.

$$\therefore \text{Area} = 3274 \times 144 \text{ sq. in.} = 3274 \text{ sq. ft.}$$

(3) Let the Scale = d , and suppose that each vernier-unit is to correspond to y sq. in.² of the actual figure. Then y sq. in. of the figure will correspond to yd^2 sq. in. of the diagram. $\therefore yd^2 = bs \times 0.001$;

$$\therefore b = (1000/s) yd^2 \quad \dots \quad (15)$$

Having found the value of n with this setting, multiply the number of vernier-units by y to get the area of the figure, since 0.001 of a turn corresponds to y sq. in.

Making $y = 0.01$ and $d = 1 : 1$, we have $b = 4.219$ as before.

Making $y = 144$ and $d = 1 : 96$ we have $b = 6.593$ as before.

From Eq. 15, as $1000 \div s$ is a constant, b will be the same as long as yd^2 is the same. Hence the same setting may be used for different scales by assigning proper values to y . Thus, if $d = 1 : 96$ and $y = 144$, we have $b = 6.593$. Here $yd^2 = 144/96^2 = 1/64$ or $d = 1/8 \sqrt{y}$, and $b = 6.593$ would be the setting for scales found by assigning values to y .—

$$\begin{array}{l} y = 1 \text{ sq. in.}, d = 1 : 8 \\ y = 4 \text{ ,, ,,}, d = 1 : 16 \end{array}$$

$$\begin{array}{l} y = 9 \text{ sq. in.}, d = 1 : 24 \\ y = 144 \text{ ,, ,,}, d = 1 : 96 \end{array}$$

and so on.

4. Find the mean pressure of a Steam Engine when the length of the diagram is 4.17" and the Vertical Scale is 40 lbs. to 1".

$$\therefore \text{M. P.} = (40 \times \text{area}) / 4.17.$$

If we desire to find the M. P. to tenths of a pound, each vernier-unit will correspond to 0.1 lb. The area of the diagram corresponding to a mean pressure of 0.1 lbs will be

$$1/10 = (40 \times \text{area}) / 4.17, \text{ or area} = 0.010425 \text{ sq. in.}$$

and we wish each vernier unit to correspond to this area

$$\therefore 0.010425 = b \times 2.370 \times 0.001, \therefore b = 4.399.$$

5. If the values of s , a and c are not given, the following method may be used when a trial disc or a circle with a known area is provided: Let Δ = area of the circle, d = scale, y = area of the figure represented by one vernier-unit, the areas being in square inches. The actual area of the figure represented by the circle is $\Delta \div d^2$, hence the number of vernier-units that would correspond to it is $\Delta \div d^2 y$. Thus, if the scale is 1 : 300, and y is 10 sq. ft. = 1440 sq. ins., the number of vernier-units corresponding to the circle should be $\Delta \div \frac{1440}{90000} = \Delta \div 0.016$.

By trial we would then find the setting so that when the tracer is moved around the circle the wheel will rotate through this number of vernier-units.

728c. Descriptions of the Coradi Planimeters.*—(1) The Rolling Sphere Planimeter (Fig. 549 in about half size) rests on the plan by two cylindrical rollers, R' R'' , of equal size, and can be moved forward in a straight line to any desired distance, causing the rotation of the rollers which impart motion to the axle of the sphere and the spherical segment, K , thereon; the latter actuates the cylindrical measuring roller which remains parallel to the tracer-arm, and is provided with a graduated circle of celluloid; the measuring roller is pressed against the segment by means of a spring. The left end of the frame which carries the axle of the sphere can be raised by turning the eccentric screw marked with an arrow, so that the small wheel on the axle of the sphere becomes disengaged and during the trial tracing of the figures the travelling roller only turns. The frame of the measuring roller is further provided with a set screw, to separate the cylinder from the sphere. In the frame B is

* Taken by permission from "The Coradi Planimeters", Zurich.

a brake-screw which acts on the roller, R, and enables the instrument to be fixed on the plan. The tracer has a turning winged handle

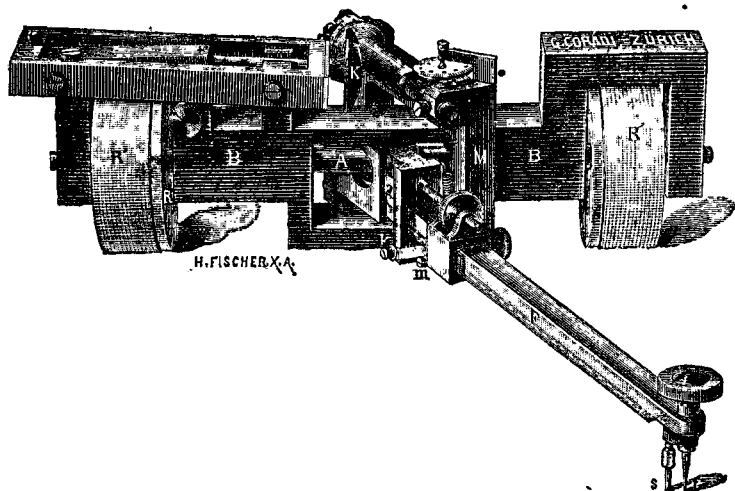


Fig. 549—Rolling Sphere Planimeter.

with an adjustable elastic support, as shewn in the compensation planimeters. The tracer-arm can describe an angle of 30° , left and right, so that areas of any desired length and a width not exceeding the length of the adjusted tracer arm can be measured in one operation.

The rolling planimeters are made in two sizes: The larger size has a roller 16 cm in length, the smaller one of $12\frac{1}{2}$ cm. The whole length of the tracer-arm is 30 cm. for the larger instrument and 24 cm for the smaller.

The rolling planimeter must not be shifted sideways on the paper, to avoid a gradual smoothening of the rough surface of the cylinder whereby the accuracy of the parallel motion would be impaired.

With the rolling sphere planimeter the measuring roller performs exclusively rolling movements on the surface of an accurately ground spherical segment; the result of the turning of the roller is therefore unaffected by the slipping or the condition of the paper whereon the figures to be measured are drawn.

(2) **The Precision Disc Polar Planimeter.**—Fig. 550 represents the instrument in about $\frac{1}{3}$ of its natural size. It consists of two separate parts, the pole disc P with a finely toothed rim and the planimeter proper, AFHSM. The latter is connected with the disc by simply placing the bearing, p, over the centre sphere of the pole disc and allowing the tracer, f, and running roller, L, to rest on the plan. By moving the tracer round the pole the small wheel, r, which always engages in the toothed circumference of the pole disc and the disc, S, on the axle of the small wheel is caused rotate. The disc, S, is made of aluminium, ribbed underneath and covered with

paper at the top. On the disc rests the measuring roller with a nickel steel disc. Graduation and counting wheel as in the rolling

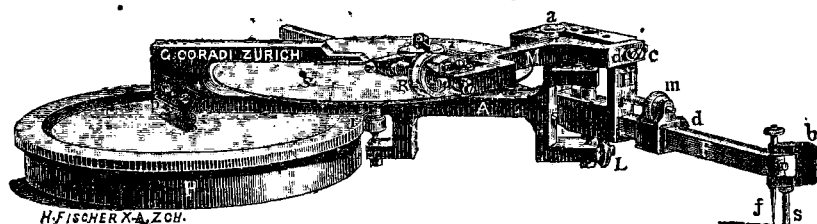


Fig. 550.—Precision Disc Polar Planimeter.

planimeter. In the frame, M, of the measuring roller is a screw by which the measuring roller can be raised from the disc.

The frame folds so that the disc can be easily cleaned. At the bottom of the frame M is a spring to prevent the whole weight of the frame resting on the axle of the measuring roller; the tracer has a turning winged handle with a support, s, which can be so adjusted that the point of the tracer moves just clear of the paper; by means of a spring the support keeps the tracer so suspended that its point can at any time be pressed into the paper, whereby the tracing and accurate adjustment of the initial point is considerably facilitated, without preventing the use of a rule for tracing.

(3) Rolling Disc Planimeter (Fig. 551) is a combination of

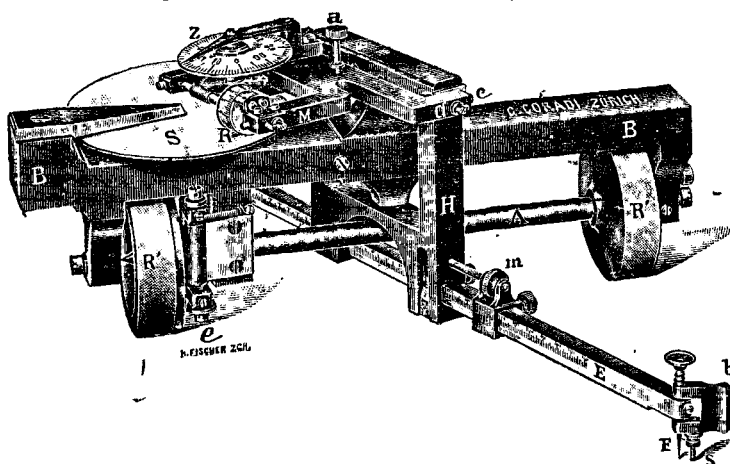


Fig. 551.—Rolling Disc Planimeter.

the rolling sphere planimeter and the disc polar planimeter, the integration parts (sphere and cylinder) are replaced by somewhat less intricate parts (disc, s, and roller, R). Range: Rectangle at any desired length and a width not exceeding the length of the adjusted tracer-arm.

(3)a. **Rolling Disc Planimeter with Measuring Roller-guide** (Fig. 552) is like the rolling sphere planimeter. The measuring

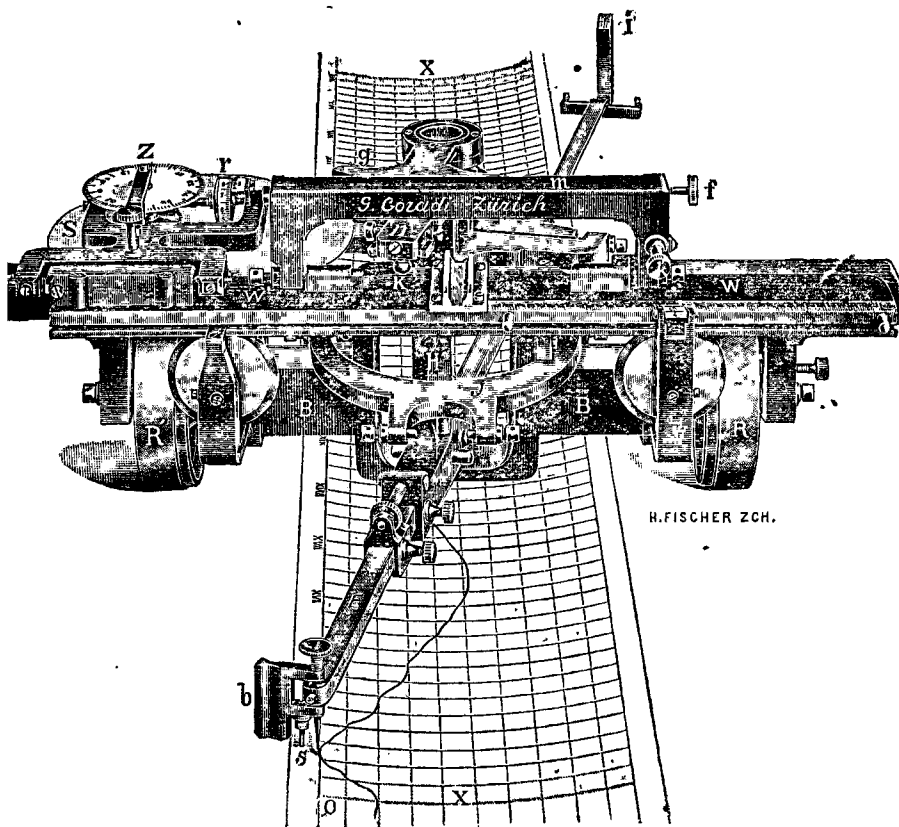


Fig. 562. Rolling Disc Planimeter with Measuring Roller-guide.

roller, r , with counting wheel, Z , which indicates 100 revolutions of the former, is fixed on the carriage W . It is provided with a device which makes it possible either to move the carriage W and the measuring roller proportionally to $\sin a$ (angle formed by the tracer-arm with the X -axis of the instrument) by means of the *sine* lever C , or to move the carriage proportionally to $\sin \text{arc } a$ by means of a toothed segment, centrically fixed on the rotating axle of the tracer-arm, which engages in a rack on the carriage W . When the segment is thrown into gear the lever C is lifted. This instrument is used for the ordinary determination of areas and for determining the contents of diagram cards with arc ordinates and arc-like abscissae distances, *i. e.*, for the two integrals $\int \sin a. dx$ and $\int \text{arc } a. dx$.

(3)b. **Theory of the Rolling Disc Planimeter.**—This will be developed by a system of rectangular co-ordinates, the path

of the fulcrum of the tracing-arm being taken as the axis of abscissæ. The path of the tracing-point it will be considered as made up of two motions, one parallel to the axis of abscissæ and the other at right angles to it. The elementary area considered will be that included between the axis of abscissæ and two ordinates drawn to the extremities of an elementary portion of the path. But since this element of the perimeter is supposed to be made up of two right lines, one perpendicular to the axis of abscissæ and the other parallel to it, our elementary area must also be divided in a similar way. It will at once be seen that one part of this area is Zero, since the two ordinates bounding it form one and the same line. This is the part generated by the motion at right angles to the axis of abscissæ. Now, the wheel-record for this part of the path is also Zero. (This is not strictly correct, although it leads to correct results. The components of motion are really one parallel to the axis of x , and one about the pivot, g , as a centre. This latter movement makes no record on the wheel, but it does generate an elementary area included between this element of the path, the ordinates to its extremities, and the axis of x . These areas are either added to or cut from the areas made by the onward element of motion in such a way as to exactly balance when the tracing-point starts from and returns to the axis of x , or when it closes on the initial point, wherever it may be). Hence all components of motion of the tracing-point at right angles to the axis of abscissæ have no influence upon the result. We will therefore only discuss a differential motion of the tracing-point in the direction of the axis of abscissæ.

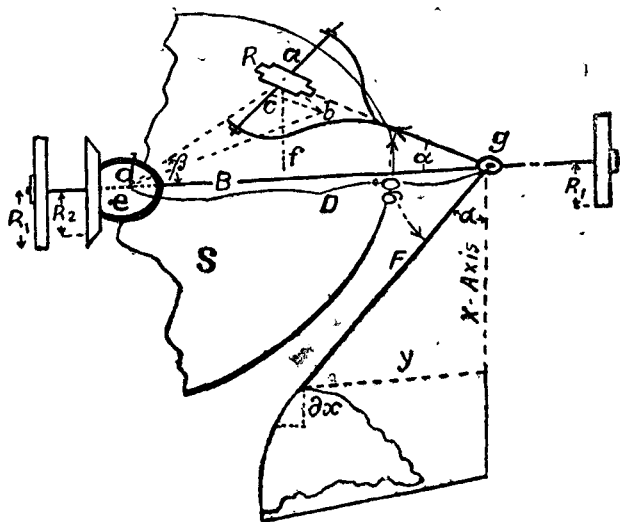


Fig. 553.—Theory of the Rolling Disc Planimeter

In Fig. 553, which is a line-sketch of the instrument shown in Fig. 551, with the corresponding parts similarly lettered, it is to be

shown that the motion of the wheel R caused by motioning the tracing-point over the path, dx , is equal to the corresponding area, ydx , multiplied by some constant which is a function of the dimensions of the instrument. A motion of the tracing-point, F, in the direction of the axis of abscissæ can only be obtained by moving the whole instrument on the rollers R_1 R_1 by the same amount, and hence when the point moves over the path dx the centres of the rollers R_1 have moved the same amount. This causes a movement of the pitch circle R_2 of $dx \times R_2/R_1$. This motion is conveyed to the disc through pinion e , so that any point on this disc, as a , distant ad from the axis, moves through a distance equal to $dx \times R_2/R_1 \times ad/e$. Let ab (Fig. 553) be this distance, then

$$ab = dx \times R_2/R_1 \times ad/e \quad \dots \dots \dots (1)$$

The motion of that portion of the disc on which the roller rests equal, to ab , causes the circumference of the wheel R to revolve by an amount equal to the component of the distance ab perpendicular to the axis of the wheel. This component part of the disc's motion is bc , and this is the measure of the wheel's motion. It remains therefore to show that $bc = ydx$ multiplied by an instrumental constant.

$$\text{Now, } bc = ab \sin. bac \quad \dots \dots \dots (2)$$

but $\angle bac = \alpha + \beta$, $\therefore gac$ and bad are both right angles. Also, $bac =$ supplement of $dag = \alpha + \beta$. Also, from $\triangle dag$,
 $\sin. dag : \sin. agd :: D : ad$, or $\sin. (\alpha + \beta) = D \sin. \alpha / ad \quad \dots (3)$

$\therefore Fga$ is also a right angle, and the angle formed by Fg and the x -axis is equal to α , whence $\sin. \alpha = y/F$.

$$\therefore bc = ab \sin. (\alpha + \beta) = ab \times D \sin. \alpha / ad = ab \times Dy/F \times ad \quad \dots (4)$$

Now, substituting the value of ab from (1) we have

$$bc = ydx \times D \times R_2/F \times R_1 \times e \quad \dots \dots \dots (5)$$

Since D , R_2 , F , R_1 , and e are all constants for any one instrument, we see that the wheel-record is a function of the area generated by the tracing-point, and the instrumental constants. It now follows that the summation of all these elementary areas included between the path of the tracing-point and the axis of abscissæ, is represented by the total *wheel movement*, or the difference between its initial and final readings, provided the tracing-point starts from and closes on the axis of x , or closes on the starting-point wherever that may be.

(3c. Comparison Between the Rolling Disc and the Polar Planimeters.—The axis of x corresponds to the Zero circle; the unrecorded motion about the pivot, g , (Fig. 553), corresponds to the balanced record of the motion about D (Fig. 542); and the significant forward motion of the former to the motion about E as a centre, in the latter. As in the case of the polar instrument, the proper length of arm F , to be used with, the rolling planimeter to give results in any desired unit, depends on the other instrumental constants. These being known the value of F may be computed in the same manner as with the polar planimeter.

(4) The **Compensation Polar Planimeters** (Fig. 554, 555, 556).—These planimeters replace the old constructions of the simple

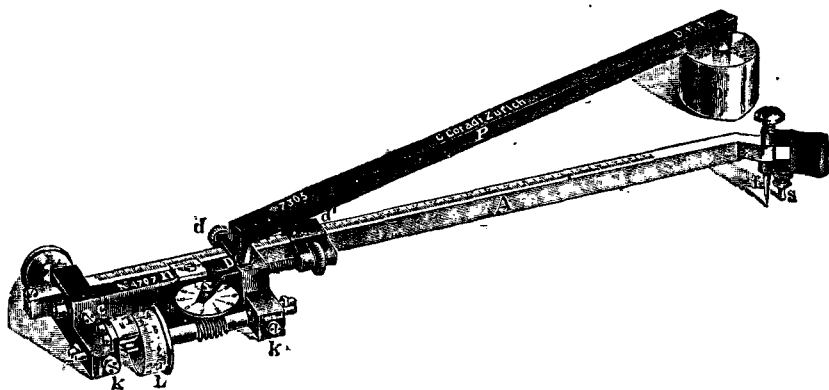


Fig. 554.—Compensation Polar Planimeter.

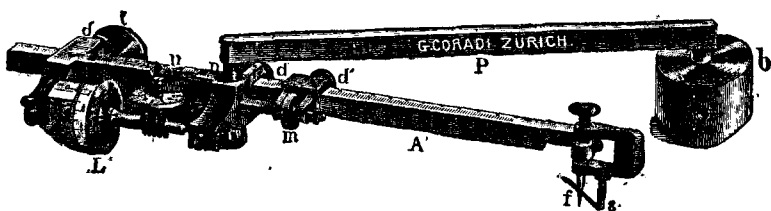


Fig. 555.—Compensation Polar Planimeter.

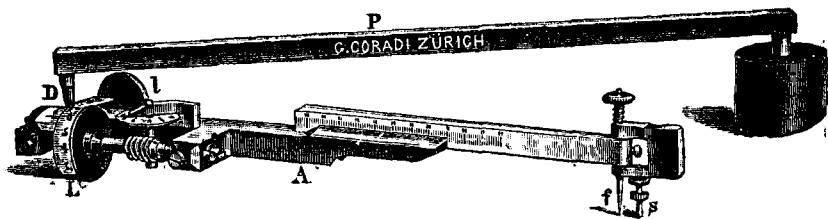


Fig. 556.—Compensation Polar Planimeter.

polar planimeter. They differ from the latter especially in that their pole-arm can be placed on *both sides of the tracer-arm*. It is thus possible to eliminate the error in the turning of the roller *u*, due to the nonparallelism of the axis of the roller and the tracer-arm, by a double tracing, one with the pole on the left and one with the pole on the right of the tracer-arm, as the error enters in the result once positively and once negatively.

If the axis of the measuring roller forms with the tracer-arm the angle δ , the turning of the roller is no longer $= x \sin a$, where x = distance traversed by the point of contact of the roller, and a = the angle formed by the axis of the roller with the direction

of motion of the point of contact. But $u = x \sin a \pm \delta$, therefore no longer proportional to the surface turned through by the tracer-arm; according as the sign of the angle is $+$ or $-$, the error in the turning of the roller will reach its positive or negative maximum and gradually decrease, as the pole-arm and tracer-arm approach the same straight line; now if the pole-arm can be still further turned so that its position changes over to the left side of the tracer-arm the sign of the angle δ is reversed, and the error still further decreased so that the turning u of the roller reaches a *minimum* in a position close to the pole, and on the left of the tracer-arm, if it formed a *maximum* in a position close to the pole and on the right of the tracer-arm—and *vice versa*.

The arithmetic mean of two tracings of a figure, each with a symmetrical position of the pole on the *left* and *right* of the tracer-arm, will therefore yield a result completely freed from the error due to the position of the axis of the roller. Two symmetrical positions of the tracer-arm with respect to the figure, with the pole on the right and left respectively of the tracer-arm, will at once be obtained if the pole is left in its place, and the tracer-arm, after the first-tracing, is brought into the same line as, and pushed underneath, the pole-arm to the opposite side, as the telescope of a theodolite.

Fig. 557 shows two symmetrical positions of the tracer-arm and the figure with the pole on the *left* and *right* of the tracer-arm

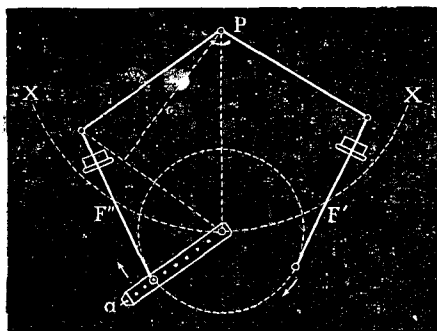


Fig. 557.—Two Symmetrical Positions of the Tracer Arm.

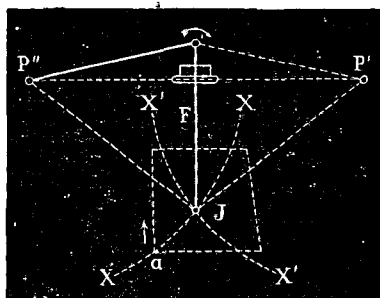


Fig. 558.—Two Symmetrical Positions of Area.

from the same pole position, P. Fig. 558 represents two symmetrical positions of the area, J, and the tracer-arm, F, from two pole positions, P' and P''. The error can be eliminated in either way.

In Fig. 554, about $\frac{2}{3}$ of its full size, the compensation planimeter consists of two parts, the tracing-frame, A L I, and the pole-arm P, which can be put in the case separately (Fig. 559). the tracing-frame rests on three points: the measuring roller, L, The tracer, J, or its support, s, and the roller I. The connection between the pole-arm and the tracer-arm is effected by a ball-and-socket

joint, placed in the tracer-arm as close as possible to the plane of the plan. The steel ball in the pole-arm is simply placed in the

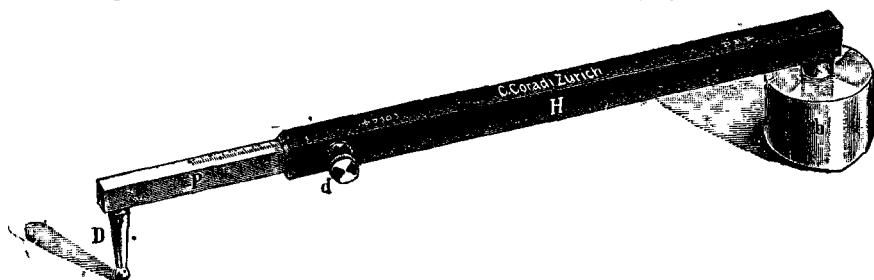


Fig. 559.—Adjustable Pole-arm.

opening, D, in the tracer-arm. This ball-and-socket joint forms the axis of rotation of the tracer arm which by means of the pole-arm moves on a circle as guide-line; at the same time it enables the tracing-frame always to rest with its three points on the plan, even if this is not flat. The pole consists of a brass cylinder, *b*, screwed in at the right end of the pole-arm (Fig. 559). Its lower surface is shaped like a gable forming an edge at right angles to the pole-arm which by turning about this edge can be lowered until its left end which carries the ball is firmly secured in the socket at D. In the centre of the cylinder, *b*, a small steel pin is inserted, kept in place by a set-screw at the side. This pin terminates on both sides in a fine hardened point, which slightly projects under the lower edge of the cylinder, *b*. *This point must not be pressed into the paper*, but only be placed loosely on it; the instrument will thus be better secured and the plan not be spoilt by pin-pricks. When the point has become blunt the pin can be reversed (the upper end be turned downward) and finally both points can be reground or renewed. If the pole-arm be slightly inclined sideways, without lifting it out of the ball-bearing, D, the pole point does not touch the paper and the pole can be moved till the zero of the roller graduation coincides with the zero of the vernier, whilst the tracer-point is pressed into its starting point. The tracer-arm is provided with a vernier and micrometer screw. Its whole length is $8\frac{3}{4}$ " (22 cm), that of the pole-arm 8" (20 cm). The tracer-arm can be adjusted in its sleeve and thus the values of the vernier unit can be varied from $0.016 \square$ " to $0.005 \square$ " (10 sq. mm to 2 sq. mm); with $0.016 \square$ " (10 sq. mm) the distance between the point of the tracer and the ball-and-socket joint (actual length of the tracer-arm) is $6\frac{1}{2}$ " (166 mm); with $0.005 \square$ " (2 sq. mm) it is 2" (33 mm).

In order to correct the error δ , above referred to, the tracer-bar can be slightly moved sideways in its sleeve by an adjusting-screw and a pressure-spring. The adjusting-screw is close to the left of the vernier holder and has its thread in the sleeve of the tracer-arm. It can be fixed in the sleeve by a brass set screw and presses with its flat end against the side face of the tracer-arm, on the opposite side of which a spiral spring taking the place of the set-screw, *d*, acts;

thus by turning the adjusting screw the position of the tracer-arm in its sleeve and, consequently, with respect to the axis of the roller,

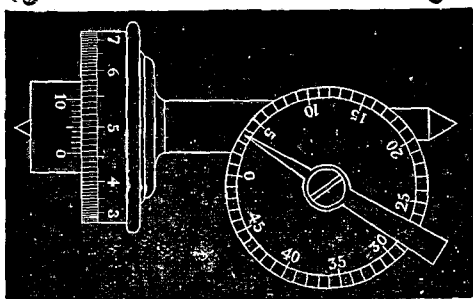


Fig 560.—Measuring Roller and the Counting Wheel

can be slightly varied without detriment to the position of the roller itself. The contact surface of the tracer-arm opposite to the right hand set-screw, *d*, is slightly bevelled, so that by tightening this screw the action of the spring is increased, and the tracer-arm perfectly secured in its sleeve in the required position. Before making and adjustment the set-screw must be loosened.

The roller and its graduations are shown by Fig. 560. The dial is made of celluloid and has 10 divisions. In order to measure a figure, the winged handle close to the tracer is grasped with the thumb and middle finger and, by pressing the index finger on the knob of the tracer, its point can at any time be pressed into the paper in order accurately to mark the initial position of the tracer or to fix the position of the tracer, whilst the hand, resting on the paper, is moved forward.

The Compensation Planimeter represented by Fig. 555 only differs from the preceding in the omission of the adjusting arrangement. The tracer-arm and the axis of the roller are permanently fixed parallel to each other. In this case the tracer-arm is secured in its sleeve by two set-screws.

The Planimeter, Fig. 556, is only constructed for a fixed vernier unit, which can be made to order anything between $0.016 \square''$ (0.40 sq. mm) and $0.0125 \square''$ (8 sq. mm). The part carrying the tracer is made of German silver, and provided with a short graduation on which the index-line on the support, *A*, indicates in $\frac{1}{2}$ mms. the distance of the tracer, *f*, from the ball-and-socket-joint, *D*.

In these polar planimeters their durability, easy handling and accuracy have been considerably increased. In addition to the advantages of former constructions of which none have been abandoned, following are the advantages:—(1) The ball-and-socket joint can by reason of its construction never become loose; it is only necessary to keep the recess, *D*, in the sleeve of the tracer-arm perfectly clean. (2) As the instrument is placed in the case in two separate parts, the joint of the pole-arm and tracer-arm cannot be loosened during the transport. (3) As the tracer-arm can perform an angular movement of nearly 180° left and right of the pole-arm, without deviating from its normal position with respect to the plane of the plan, larger areas can be traced than is possible with older makes, or with the ordinary Amsler planimeter in which the

total angular movement of the tracer-arm is no more than 100° to 120° . (4) In order to examine the parallel position of the axis of the roller the largest areas can be used, round which the tracer-arm can be moved as the same result must be obtained with the pole to the right or the left of the tracer-arm. (5) The reading on the roller is perfectly clear and visible from above, and as the pole-arm is not suspended from the tracer-arm the handling of the instrument is both easier and less liable to damage the instrument.

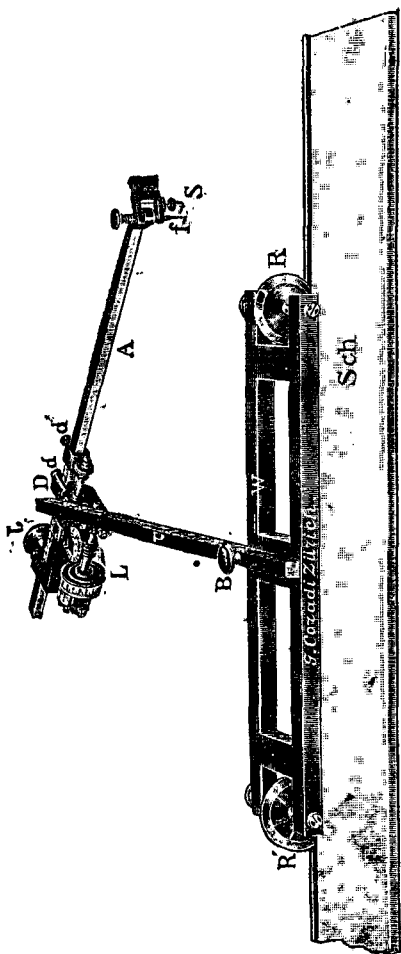


Fig. 561.—Carriage Planimeter.

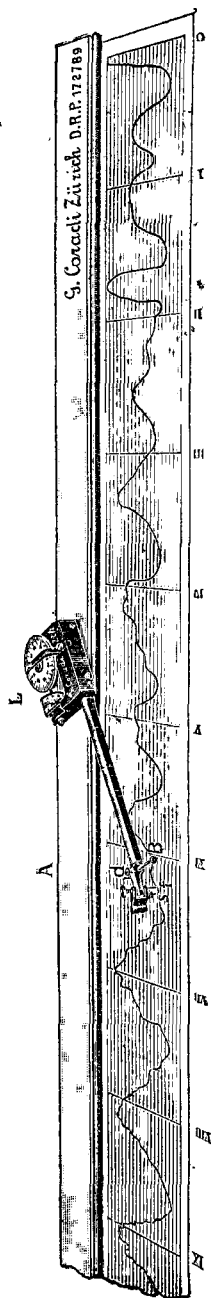


Fig. 562.—Rule Planimeter.

(4)a **The Carriage Planimeter** (Fig. 561) consists of a carriage connected as a pole-arm with a compensation planimeter (Figs. 554, 555, 556) and of a steel rule with a groove in which the carriage with the planimeter can be moved.

(5) **Rule Planimeter with compensation in one Tracing and with artificial surface of Rotation for the Measuring Roller** (Fig. 562).—These planimeters have no pole-arm. The tracer-arm carries at the front the tracer-point, and at the other end the spherical axis of rotation of chilled steel. This sphere can be moved in the groove of a steel rule, thus causing the measuring roller to rotate on the paper covered surface of the rule. In Fig. 562 the tracer-arm is 32 cm (13") long, fixed for a value of the vernier unit of 20 sq. mm (0.03 sq. in.). The counting wheel indicates up to 100 revolutions of the measuring roller. The movable point near the tracer-point can be so adjusted that the point of the tracer is at Zero, of the diagram (indicator curves) when the pin and the card touch the edge of the rule. This line can hence be traced without it being necessary to observe the point of the tracer. This line must be traced to the last ordinate which can be reached by the rule, the ordinate and the diagram curve and the initial ordinate then to be traced returning to the initial point. This instrument may be useful in areas of offset-pieces between the traverse lines and the boundary in a cadastral or other survey.

728d' The Hatchet Planimeter, invented by Capt. H. Prytz, a Dane, is the simplest of all area-measurers. It consists of a single rigid U-shaped rod (Fig. 563). The one end T is the tracer, the other Q has a sharp hatchet-like edge. If this is placed with QT on the paper and T is moved along any curve, Q will follow, describing a *curve of pursuit*. If the point of contact of the hatchet with the paper is marked at the beginning and at the end of the operation, then this distance between these two points multiplied by the length of the rod gives the area of the figure. The curve of pursuit may be made visible by putting a piece of "copying paper" under the hatchet. The greatest dimension of the area should not exceed $\frac{1}{2} QT$, otherwise the area must be divided into parts the area of which should be determined separately.

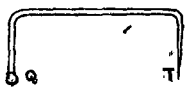


Fig. 563. — Hatchet Planimeter.

729. Amster's Integrator or Moment Planimeter.—This instrument besides computing areas of greater magnitude than the polar planimeter gives also *statical moments*, and the *moments of inertia* of a plain figure taken about any axis lying in the plane. This instrument (Fig. 564) has one tracer but three recording wheels. It is mounted on a carriage moving in a groove upon a straight rail. To the carriage are jointed the tracer-arm and three toothed wheels the centres of which are in a line perpendicular to the rail. If the radii of the outer toothed wheels be a , the radius of the middle toothed wheel for more than half the circumference will be $2a$, and for the other part $3a$. With the tracer-arm is connected the roller, A , similar

to the one of the planimeter. The smaller toothedwheels have also rollers, M and J. By means of the rollers the instrument is directed in the plane of the figure to be measured.

730. Integragraphs : Abdank-Abakanowicz System Coradi Construction.—*

The integraph (Fig. 565) is an integrator which not only indicates the final result of the integration, but also gives a graphic representation thereof; whilst the tracer travels round the outline of the figure, the integraph automatically draws a curve (integral curve) the ordinates of which are proportionate to the area of the figure travelled around ($y^1 - \int y. dx$).

If this first integral curve is traced by the tracer of the integraph as

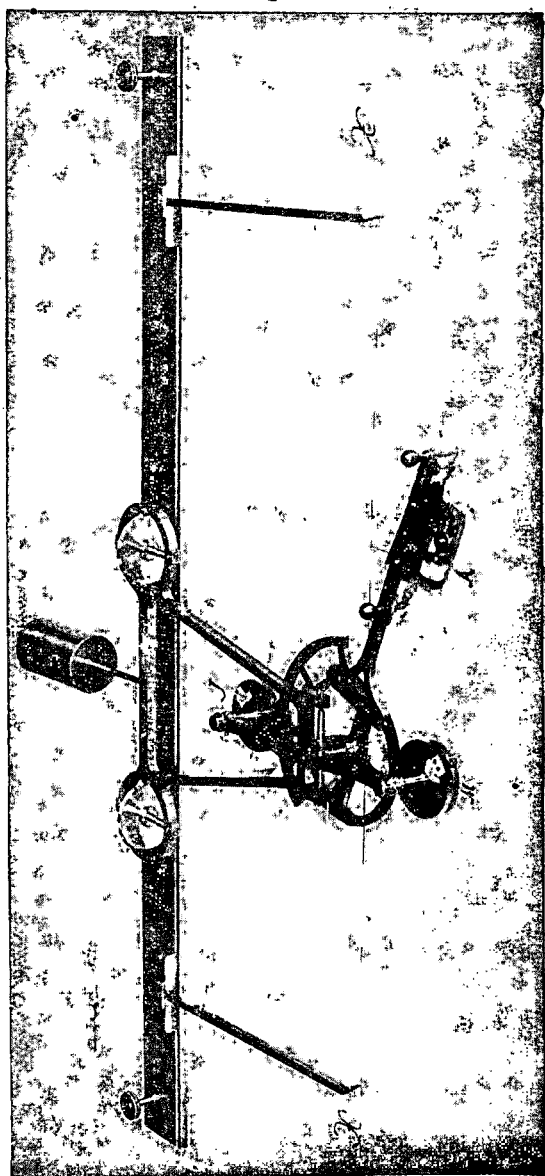


FIG. 564.—Amsler's Integrator.

* From Pamphlet by M. Coradi of Zurich.

a differential curve, the tracer draws a second integral curve, the ordinates of which are proportional to the moment of stability ($y^2 - \int y^2 dx$). By tracing this second integral curve we obtain a third integral curve the ordinates of which are proportional to the moment of inertia ($y^3 = \int y^2 dx$) etc. The y-axis for the moments can be chosen anywhere on the curve, *i.e.*, it can be shifted. By means of the integrator many of the most difficult calculations and problems occurring in the practice of an engineer can be solved in a simple and reliable manner and with much saving of time, and the operator need not possess any knowledge of higher mathematics.

The applications of the integrator are very numerous. With it we calculate areas, divide areas, determine centres of gravity; calculate moments of stability, load and resistance, solve algebraical equations, draw parabolas, etc.

The above illustration shows the latest construction of the integrator in about $\frac{1}{3}$ of the full size of the large pattern and about $\frac{1}{3}$ of the small pattern. The integrator rests on the plan, in the same way as Coradi's rolling planimeter, with three points, the two rollers, *rr*, on a common axle and the tracer. The rollers impart motion to the instrument so that it can be moved in a straight line on a horizontal table over distances of any length. The frame in which the running roller, *rr*, is pivoted between needle points consists of two rules, *L* and *L*₁, exactly rectangular to the direction of motion of the whole apparatus. In the grooves of the front rule rolls the carriage, *W*, carrying the base rule, *B*, with the tracer and the front vertical axle of rotation of the direction rule, *D*. In the grooves of the back rule rolls the integrating carriage, to which the integrating roller and drawing pen are fitted. In the centre of the large frame is the second axis of rotation of the direction rule; this carries a sleeve in which the direction rule slides: on the latter rolls a carriage connected by means of a parallelogram with the frame of the integrating roller, to keep the plane of the integra-

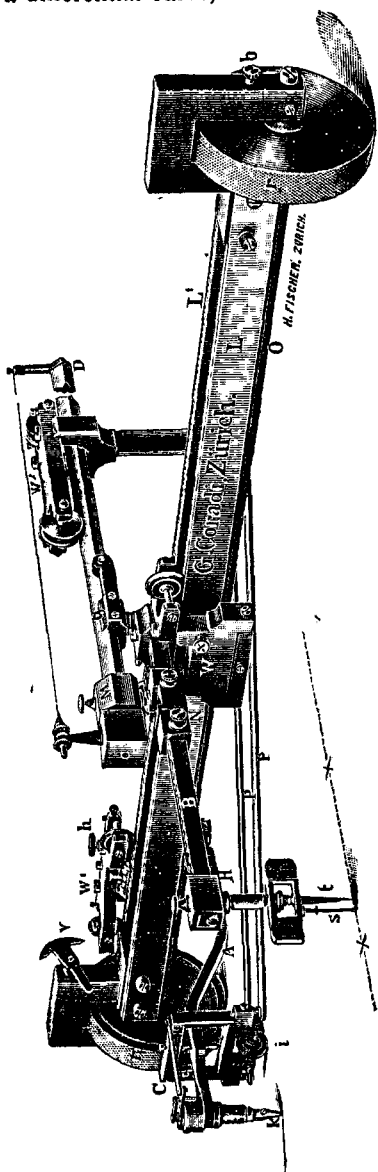


FIG. 565.—Coradi's Integrator.

of the direction rule; this carries a sleeve in which the direction rule slides: on the latter rolls a carriage connected by means of a parallelogram with the frame of the integrating roller, to keep the plane of the integra-

ting roller always parallel to the direction rule. The vertical distance between the two axes of rotation of the direction rule forms the base of the apparatus (corresponding to the tracer-arm in a planimeter). The front axis of rotation of the direction rule is in a sleeve adjustable on the base rule. The base rule has a graduation in $\frac{1}{2}$ mm (or $\frac{1}{10}$ "): the sleeve has a vernier for $\frac{1}{20}$ mm (or for $\frac{1}{100}$ "), and is provided with micrometer adjustment. The base can therefore be accurately set to $\frac{1}{20}$ mm ($\frac{1}{100}$ "), and in the large pattern be varied between 20 cm and 10 cm (8" and 4"); with the small pattern the base can be varied between 12 $\frac{1}{2}$ cm and 5 cm (5" and 2 $\frac{1}{2}$ "). The position of the vernier on the base rule for the most suitable values of the curve ordinate is given in a table in the case. If the direction rule is perpendicular to the large frame, i.e., in the X-direction, the plane of the integrating roller is also in the direction parallel to X. If the whole apparatus is moved forward with this position of the direction rule no lateral motion of the integrating-carriage will ensue. This position of the guide-carriage (zero or normal position) can be fixed by screwing the steel screw in the guide carriage and shewn by X into the hole marked with X, in an adjustable steel piece on the front rule. In this fixed zero position the tracer is placed on the line taken as the X-axis of the figure to be integrated, and the instrument is turned round the tracer (by grasping it on the left and right under the large frame and slightly raising it, so that the over-balance of the whole apparatus rests on the point of the tracer), until the rules L of the frame are parallel to a Y—ordinate which is easily ascertained by sighting over the front vertical surface of the rule or over the direction rule in the direction of an abscissa (X—direction). The set-screw X is now unscrewed and the tracing of the figure on the X—axis, from left to right commenced. Before commencing the tracing the integrating carriage is placed as far as possible to the left. The greater the distance to the left or right between the guide carriage and the normal position, the greater will be the angle, which the direction rule and the plane of the integrating roller always parallel thereto forms with the X—axis, and the greater will be the lateral motion of the integrating carriage in the Y—direction, on the whole apparatus moving a certain distance in the X—direction. The arm of the integrating carriage carrying the integrating roller and drawing pen can be raised by means of a screw on the integrating carriage to prevent the integrating roller resting on the drawing; in this case the integrating carriage can be moved by hand left and right, or by slightly lifting it by hand at the sleeve of the drawing pen.

The extreme positions of the two carriages, i. e., the one on the extreme left and the other on the extreme right, is especially impossible with a short base and should be avoided. With a proper use of the apparatus this position will not occur.

Fitted to the turning handle of the tracer is an elastic support which keeps the point of the tracer just clear of the paper; by pressing with the finger on the tracer the point can be pressed into the paper for the purpose of marking the initial point of the tracing.

The tracer is fixed in a sleeve of the base rule in which it can be adjusted so that its distance from the drawing pen (in the X-direction) is a round measure (5 cm, 4 cm, or 3 cm, or 2 $\frac{1}{11}$ " to 2") according to the base set, so that the corresponding Y-ordinates of the differential curve and the integral curve are displaced with respect to each other by that amount. With a short base (10 cm or 4" downwards) the tracer can also be placed on the same Y—ordinate as the drawing pen, but in this case the path of the integrating and guide carriage is obstructed. The maximum lateral motion of both carriage is 52 cm (20") with the large pattern, and 27 cm (10") with the small pattern.

The maximum area which can be traced is a rectangle of the above width and any desired length. If the movement of the integrating carriage is not sufficient to sum up the whole differential curve and the integrating carriage has reached the end of its movement to the right the whole instrument is

arrested by means of the brake-screw on the right in the large frame which acts on the large roller; we then mark the point of the differential curve where the tracer stops, as also the point of the integral curve where the drawing pen has arrived, raise the integrating roller from the paper by means of the raising screw and push the integrating carriage as far as possible to the left and the tracing can be continued. Before drawing an integral curve, it is necessary to roughly trace the differential curve with the tracer, allowing the integrating roller to rest on the paper, and without inserting the drawing pen; after convincing ourselves that we have chosen the suitable position for the two carriages and that the figure can be traced without difficulty the drawing pen is inserted. Instead of using the drawing pen we can insert a pencil holder for Fabé leads.

The direction rule can be shifted in the sleeve of the front axle of rotation and be fixed in any desired position by tightening the set-screw. With a short base the overbalance of the instrument can thereby always be made to rest on the side of the tracer. The shifting and fixing of the guide rule can, if required by the position of the carriages, be effected during the tracing without affecting the result, if sufficient care is taken.

On the back rule of the large frame is a graduation in mm (or $\frac{1}{10}$ "); the integrating carriage carries a vernier for $\frac{1}{100}$ mm (or $\frac{1}{1000}$ ") on which the final result can be read in the same way as with a planimeter.

The value of a centimeter or an inch of y-ordinate with 10 cm of 4" base is:—For the first integral curve 10 cm² or 4 square inches. For the second integral curve 100 cm² or 25 square inches $\times$ inch. For the third integral curve 1000 cm² $\times$ cm² or 64 sq. ins. $\times$ sq. ins. For a base of half the above size the values are $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ respectively of the above; if the base is twice as large as the above the values are 2, 4 and 8 times respectively as large as those given for the base of 4" or 10 cm. If the drawing is on a reduced scale, i.e., not in full size but in the proportion 1 : n the above values must be multiplied by n² for the area, n³ for the moment of stability and n⁴ for the moment of inertia.

The direction rule and base rule must be protected from distortion. The instrument must not be touched at the direction rule or base rule when raised or moved, but only at the rules of the large frame.

The table, on which the integrator is used, must be quite smooth and placed approximately horizontally as otherwise the whole apparatus would easily roll away. In order to prevent the latter a brake-screw is arranged in the frame on the right: by slightly tightening this screw the apparatus can be fixed in a perfectly immovable position. This brake-screw must of course not touch the roller when the instrument is used. Before returning the instrument to its case the two carriages are pushed as far as possible to the left and kept in this position by a swivel bolt.

Care should be taken that all the movements of the instrument are easy; the guide roller of the carriages must turn freely but not have any appreciable play: this can be remedied by the centre screws after raising the corresponding brass screws which after adjustment must again be tightened: on the wearing surfaces of the axes of the roller a little very fine oil may be occasionally put. The direction rule must always be kept clean and not be oiled. In the carriage on the guide rule the bearings of the axes of the guide rollers must not be altered as the result would be affected thereby.

731. Coradi's Ourvimeters are used (Fig. 566) for measuring horizontal lengths on maps and plans. The axes of the two measuring rollers and the guide point, *c*, lie in one line and the edges of the two rollers, with which the instrument rests on the plan, are exactly equidistant from the point *c*. The circumference of each roller is 40 mm, and is divided into 20 parts and numbered twice from 0 to 9 so that sum of the readings of both rollers give whole millimeters.

Both the rollers are numbered in the same direction so that on turning the instrument round the point *c*, without moving it forward, the sum of both turnings = zero; if the instrument is moved forward in a straight line each of the rollers shews half of the distance traversed by *c*. If we trace any given curve by placing the axis of the rollers perpendicularly to the elements of the curve (a deviation of 8° from the perpendicular produces a difference of $\frac{1}{100}$ only) the sum of both readings will shew the distance traversed by the point *c*. As regards accuracy this curvimeter exceeds all others. In experiments an accuracy of about $\frac{1}{2000}$ is obtained for straight lines.

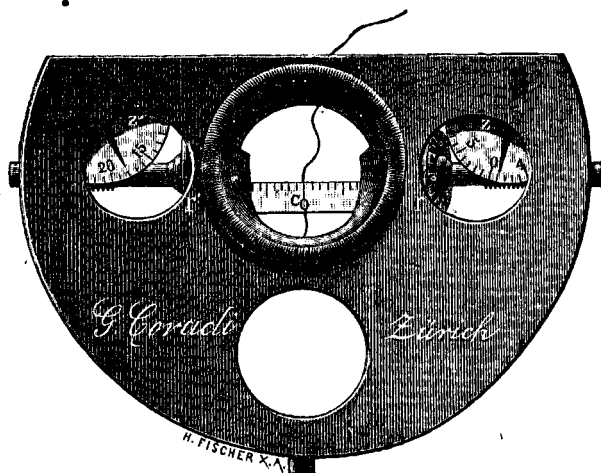


FIG. 566.—Coradi's Curvimeter.

732. Opisometer.—The length of any water-course, road, rail-

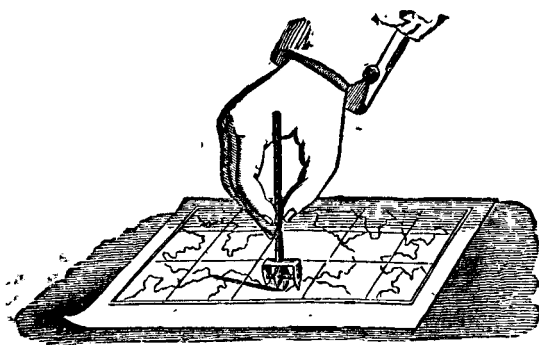


FIG. 567.—Opisometer.

way line, boundaries, etc., may be measured with this instrument (Fig. 567) on any map provided with a graduated scale. It consists of a milled-wheel mounted in a forked handle on a steel screw with a very fine thread. The wheel, being turned up to one end of the screw, is put down on the map with the handle held vertically over the point at which the measurement is to commence, and is run over the road, or line until the point is reached at which the measurement is to stop; it is then lifted off the paper, placed on the scale of the map, and run on a scale divided into feet, inches and tenths, so that the measurements may be taken with direct.

way line, boundaries, etc., may be measured with this instrument (Fig. 567) on any map provided with a graduated scale. It consists of a milled-wheel mounted in a forked handle on a steel screw with a very fine thread. The wheel, being turned up to one end of the screw, is put

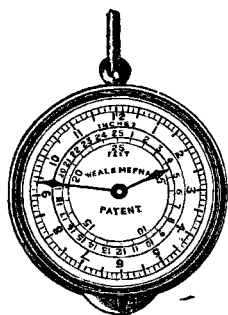


FIG. 568.—Wealemeffna.

733. Wealemeffna: Rotameter or Universal Map Measurer.—It is simply a clock-work measurer to feet and inches (Fig. 568). To measure a line, the instrument is set to zero, and the line is followed in one direction, by the small projecting tracer wheel, holding the instrument vertical, the large hand will then register the inches and fraction of an inch up to 12, and the small hand the feet up to 25. This instrument is made also in gold and silver, to be used as a pendant to a watch chain.

734. Other Useful Formulas of Mensuration.—

REQUIRED.	GIVEN	FORMULAS.
Area of		
Trapezoids ...	Parallel sides = a and b Perp. dist. bet. them = h	$\frac{h}{2} (a+b)$
Regular Polygons ...	Length of side = l Number of sides = n	$\frac{nl^2}{4} \cot. \frac{180^\circ}{n}$
Circles ...	Radius = r	$\pi r^2 (\pi = 3.1416)$
Ellipses ...	Semi-axes = a and b	πab (approx.)
Parabolas ...	Base = b , height = h	$\frac{2}{3} bh$
Surface of		
Cones ...	Radius of base = r Slant height = hs	$\pi rs + \pi r^2$
Cylinders ...	Radius = r , height = h	$2 \pi rh + 2 \pi r^2$
Spheres ...	Radius = r	$4 \pi r^2$
Zones ...	Height = h , Radius of its sphere = r	$2 \pi rh$
Volume of		
Prisms or Cylinders ...	Area of base = b , Height = h	bh
Pyramids or Cones ...	Area of base = b , Height = h	$bh/3$
Frustum of Pyramids or Cones ...	Area of bases = b and b' Height = h	$\frac{h}{3} (b+b' + \sqrt{bb'})$
Spheres ...	Radius = r	$\frac{4}{3} \pi r^3$
Prismoids ...	Areas of two ends = e and e' " middle section = m , height = h	$\frac{h}{6} (e+e'+4m)$

CHAPTER XI.

ON COPYING, REDUCING, AND ENLARGING PLANS AND MAPS.

735. Tracing Frame.—It consists of an open square wooden frame (Fig. 569) into which a sheet of plate-glass is fitted to the level of the surface forming a drawing board for copying drawings.

Another similar frame, but without glass, is hinged to the first and forms a bed to rest on any flat surface. The tracing frame is held at any angle (25°) convenient to receive the necessary light before a window or a lamp by means of a pair of struts abutting on the rack notches on the lower frame.

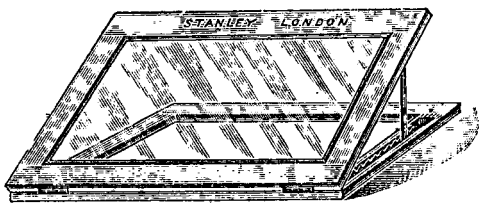


FIG. 569.—Tracing Frame.

736. To use the Tracing Frame.—The original drawing is first put over the glass with another sheet of paper fixed over it to receive the copy. The frame is then held inclined to the light so as to make the whole transparent. A mapping or drawing pen may now be used to trace the lines of the original drawing for the copy. The inclined position of the drawing renders the frame somewhat awkward to work upon.

737. Copying Table or Glass.—It is a drawing table with surface level (Fig. 570) and made of a strong sheet of plate-glass with a mirror below placed at an angle of 45° , if necessary, to reflect light through it,

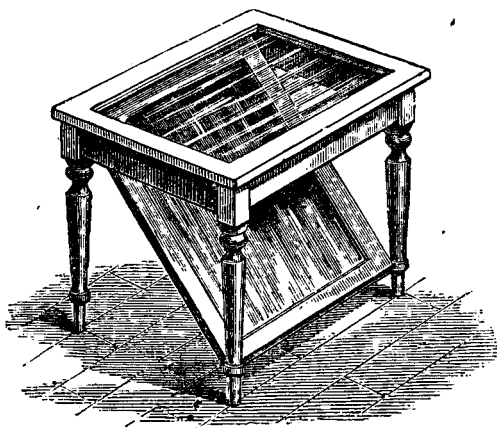


FIG. 570.—Copying Table or Glass.

738. Sun Printing Frame.—The apparatus (Fig. 571) used for all sun printing consists of a printing frame of a wood perfectly dovetailed at the corners. The front should be of plate-glass of about $\frac{3}{8}$ in. in thickness. The back should

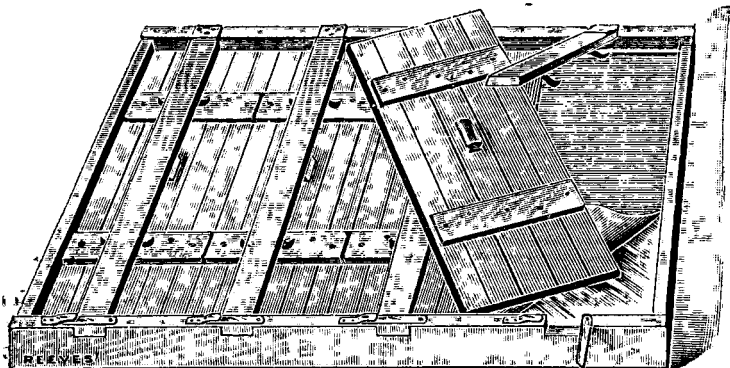


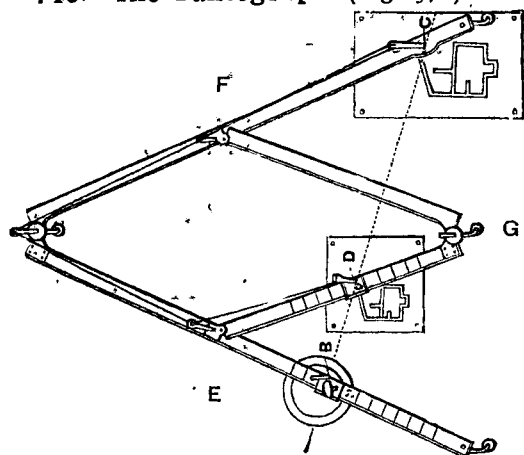
FIG. 571.—Sun Printing Frame.

be made to hold the negative and sensitive paper close up to the glass. The best form is to have narrow boards firmly sprung by steel springs and again over-sprung by ash battens which are held down by strong hinges and clips. Sometimes the tracing frame is supported on a portable frame on casters.

A zinc bath with plug to empty for washing the print, felt sheets to suit the sizes of the frames for equalising the pressure exerted by the springs, and a tin canister to keep sensitive papers in, ready for use, are also required, as also nonactinic arrangements (yellow blinds by day and a ruby lantern by night).

739. Reduction, Enlargement, or Copying Drawing by Mechanism.—This is effected by means of Instruments called the Pantograph and the Eidograph.

740. The Pantograph (Fig. 572) consists of four flat rulers



of stout brass, AC , FG , GE and AB , joined to each other at A , F , G , and E , so that $FA = FG = GE = EA$, and the figure $AFGE$ is always an exact rhombus, its opposite sides and all of them being equal. Several casters support the instrument, parallel to the paper, and allow it to move lightly over it in all directions. The long rule, AC , has a socket fixed near

FIG. 572.—Pantograph (Casella's).

the end, which carries a tracer when the instrument is used for reduction. The other long rule, AB , and one of the shorter rules, GE , have each a sliding index which can be clamped in any part of the rule by a clamping-screw. Each of the sliding indices carries a vertical socket which is placed over the edge of the rule in a line with the joints. The sockets are constructed to hold either a pencil holder, tracing-point, or fulcrum pin, as may be required. The rules, AB and EG , are graduated with a scale of proportions: $1:2, 1:3, \dots 11:12$, which indicate as one is to two, as one is to three...as eleven is to twelve. A brass weight which firmly supports a pin that fits into either of the sockets, D and B , forms the fulcrum (in figure B) upon which the whole instrument turns when in use. The pencil-holder is loaded at the top with shot or brass rings to cause the pencil to mark with the required distinctness. There is an arrangement to lift the pencil off the paper when required by pulling a string. This is effected by fixing the string at

the pencil-holder at D , turning it round the angles, E , A , over small pulleys and pulling it near the tracer by the hand to raise the pencil point. One thing is essential in every construction of the instrument, *i. e.*, that the *fulcrum*, *tracer*, and *pencil*, from the property of similar Δ s, should always be in a true line, as shown by the dotted line in the figure, when the instrument is set for use. In this position of the instrument the small ΔBDE will always be similar to the large ΔBCA and then, if the fulcrum be placed at B ; the pencil at D , and the tracer at C , when the instrument is moved round the fulcrum as a pivot, the pencil and tracer will move parallel to each other through distances which will be respectively in the proportion of BD to BC ; thus the pencil at D draws a reduced copy of the map under the tracer at C . If the pencil and tracer were interchanged an enlarged copy would be drawn; if the fulcrum and pencil were interchanged, and the slider set for D to bisect BC , the map would be copied exactly. The rhombus may be in any position on the instrument to the fancy of the maker.

***741. To Use the Pantograph.**—There are two ways of using the Pantograph, the Erect and the Reverse Manner.

For Erect copies the fulcrum is placed on the long arm B and the pencil on the short arm D , while for Reverse copies these positions are simply reversed. The tracer is always placed on the arm C .

Table of Reductions in the
Erect Manner.

Readings upon the Scales.	Reduce in the proportion of	Readings upon the Scales.	Reduce in the proportion of
1—2	1 to 2	3—7	3 to 10
1—3	1 " 3	3—5	3 " 8
1—4	1 " 4	2—3	2 " 5
1—5	1 " 5	3—4	3 " 7
1—6	1 " 6	4—5	4 " 9
1—7	1 " 7	5—6	5 " 11
1—8	1 " 8	6—7	6 " 13
1—9	1 " 9	7—8	7 " 15
1—10	1 " 10	8—9	8 " 17
1—11	1 " 11	9—10	9 " 19
...	...	10—11	10 " 21
...	...	11—12	11 " 23

Table of Reductions in the
Reverse Manner.

Readings upon the Scales.	Reduce in the proportion of	Readings upon the Scales.	Reduce in the proportion of
1—2	{ 1 to 1 } (full size)	3—7	3 to 7
1—3	1 to 2	3—5	3 " 5
1—4	1 " 3	2—3	2 " 3
1—5	1 " 4	3—4	3 " 4
1—6	1 " 5	4—5	4 " 5
1—7	1 " 6	5—6	5 " 6
1—8	1 " 7	6—7	6 " 7
1—9	1 " 8	7—8	7 " 8
1—10	1 " 9	8—9	8 " 9
1—11	1 " 10	9—10	9 " 10
...	...	10—11	10 " 11
...	...	11—12	11 " 12

***742. By the Erect Manner** of setting the pantograph, the reduced copy will appear erect, *i. e.*, the same way as in the original. The general disposition of the various parts of the instrument set in this manner and of the copies is shown in the figure. The instrument, thus set, will reduce in any of the given proportions not exceeding half size, technically from 1-2

***743. By the Reverse manner** of setting the instrument, the reduced copy appears reversed, or up-side down to the original, but for large drawings this method is most convenient, as the original, and copy come edge to edge and need not overlap each other, which is often compulsory in the erect manner; the range of scale is also much greater, as the proportions include the unit proportions of the erect scale.

The defects of these pantographs are excessive friction, vibration and contracted action in reductions below one-fifth due to the interference of the bars and the weight. These defects are considerably reduced by making the rules tubular. The weight, thus reduced, decreases the friction on the surface of the drawing and simultaneously prevents the vibrations caused by the thinness of the rules. To remedy the other defect, contracted action, the fulcrum and tracer are placed on the same side of the rules. Another improvement is made by making the weight triangular, instead of circular, which occupies less useable space. An improved pantograph, with square tubular bars is illustrated by Fig. 573 as made by Messrs Cooke & Son of York.

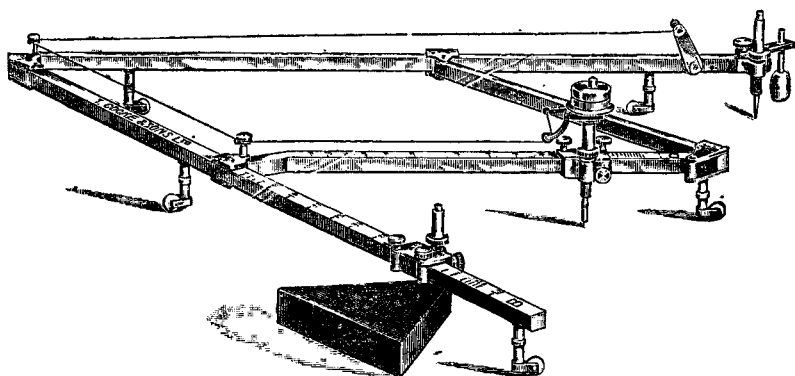


Fig. 573.—Improved Pantograph (Cooke's).

744. Fig. 572 shows the instrument in the act of reducing a plan. The method of procedure in reducing a plan is as under:—If it is required, for example, to reduce a plan to a scale of half the original. To effect this, the sliding indices are first clamped at the divisions marked $\frac{1}{2}$; the tracer is then fixed in the socket C over the original drawing. The pencil is next fitted to the tube at D over the paper to receive the copy, and the fulcrum is fixed to that at B. The instrument is now ready for use, and if the tracer C is now passed delicately and steadily along the boundary and other lines of the original drawing, a true copy but of one-half the scale of the original will be produced by the pencil for the copy.

If the drawing is to be enlarged, which the pantograph will do but very imperfectly, to double the size, then the tracer must be placed at D and the pencil at C, and if the copy is required on the same scale as the original, then the sliding indices still remaining at the same divisions upon EG and AB, the fulcrum must take the middle station and the pencil and tracer on the arms AB and AC of the instrument.

746. Coradi's Freely Suspended Precision Pantographs.—Fig. 574 shows the instrument. It consists of four square brass tubes connected by fine pivot joints and forming a parallelogram. They are suspended by metal rods h'' k'' and h''' k''' from the solid standard, K, to avoid the friction caused in the ordinary instruments by the rolling of the casters on the drawing. The standards are provided with spirit-levels and levelling-screws, and the length of the supporting rods can be regulated by adjusting screws to bring the bars into a horizontal plane. The instruments are set to the ordinary proportions by adjusting the sliding sockets to the respective index-marks engraved on 3 of the bars marked 1, 2

and 3 in the diagram. These bars are also fully graduated in millimeters with verniers for 0.1 mm on the sleeves with micrometer adjustment like the beam and arms of the Eidographs. The sliding stockets are provided with verniers and micrometer adjustments. The instrument (Fig. 574) is shown with the pole within the parallelogram, supported by an extra tube crossing the parallelogram diagonally, and suspended from the standard at its free end h'' , by rod $h'' k''$, while the near end revolves around the fulcrum. The ratio from 2 : 3 to i : 20 are set with the pole at the end, the ratios from 2 : 3 to 1 : 1 to 3 : 2 are set with the pole at the center. For this purpose the pole and pencil point are interchangeable in their sleeve. The instrument is set up in the same position for enlarging as for reducing, except that the tracing and pencil points are interchanged. The pencil point, p , can be raised or lowered by a lever at the tracing point, F .

By interchanging the fulcrum with the pencil point, the instrument can also be set up as in Fig. 575, this position dispenses with the extra supporting bar but exclusively used for reducing and enlarging in all ratios from 1 : 20 to 4 : 5 (with pole at the end only). In all other respects as in Fig. 574. Fig. 575 has not the contrivance for erecting the instrument with the pole within the parallelogram, which limit its range of action,

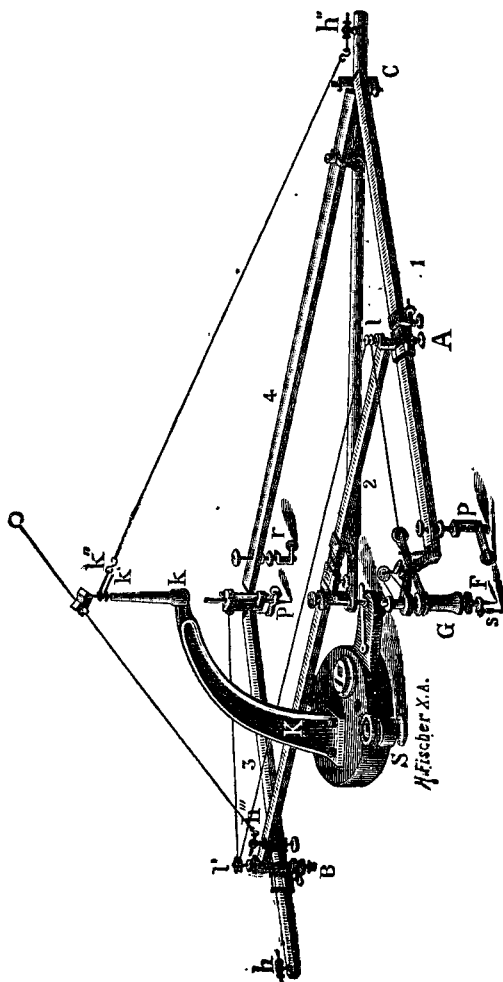


Fig. 574 —Coradi's freely Suspended Precision Pantograph with Diagonal Supporting Tube.

747. Raising and Lowering the Pencil Point.—In Coradi

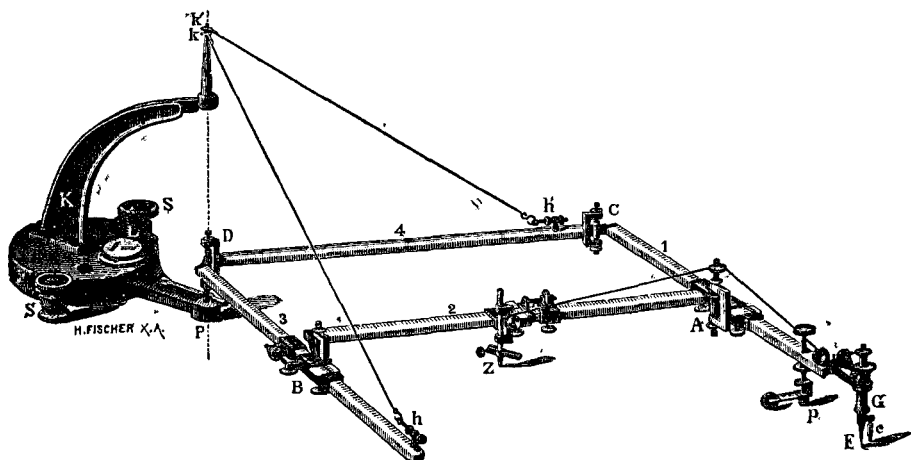


Fig. 575.—Coradi's Freely Suspended Pantograph without Diagonal Tube, pantographs, the guiding of the instrument and handling of the releasing

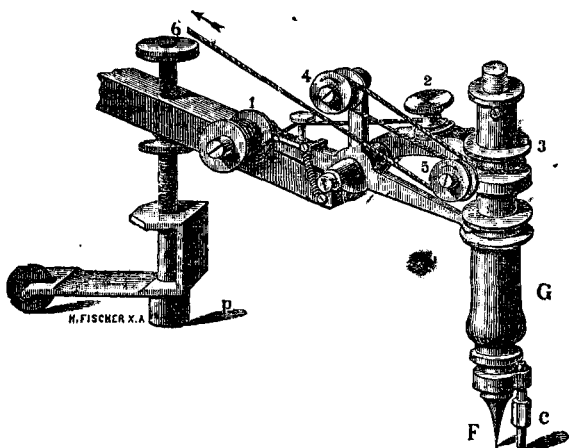


Fig. 576.—Raising and Lowering the Pencil Point.

by turning the handle, G. If the handle G is raised, the pencil point is also raised and kept in this position by a spiral spring on the releasing lever.

748. Other Advantages of these Pantographs are :—(a) These instruments are set up in the same way for enlarging as for reducing, and are equally accurate in both operations. In enlarging the error only increases proportionately to the unavoidable error of the setting and tracing. (b) On the pressing point a cap is screwed which prevents the point penetrating too far into the paper, enables the size of the prickings to be regulated and protects the point not in use. (c) The tracing point is furnished above its socket with a nut and spring sleeve, by means of which it can be adjusted to any height and used directly for pricking when enlarging. (d) The bars, the tracing, pencil and pricking points are nickelled to prevent rust.

Fig. 749. The Eidograph, shown by Fig. 577, is a far superior instrument to the ordinary pantograph, being more exact in its work, its motion being delicate and regular, and it covers a larger surface than a pantograph of similar size.

The details of construction are as under:—

The point of support is a heavy brass-covered leaden-weight, *H*, from the under side of which 3 or 4 projecting needle points keep it firm on the drawing. A pin, termed the *fulcrum*, is erected on the top of this weight, and upon this the whole instrument revolves horizontally. The fulcrum fits a socket attached to a sliding-box, *K*, in which slides the main beam, *C*. The sliding-box may be clamped to any part of the main beam, by a clamping screw shown on the engraving. Under each end of this beam is a pulley wheel, *J*, *J*, and the two are connected by an encircling steel-band, *I*, *I*, so that either pulley transmits simultaneous motion to the other. The wheels are of exactly equal diameters, having centre pins which revolve in sockets at the ends of the beam. The steel-band is adjustable, by means of a screw adjustment, *L*, to secure equal motion to both wheels. To the under surface of each pulley a sleeve is attached, through which slides an adjustable arm, *A* or *B*, to be clamped where required. The arm, *B*, carries a tracing point, *G*, and the arm, *A*, carries a pencil point, *D*. The pencil socket

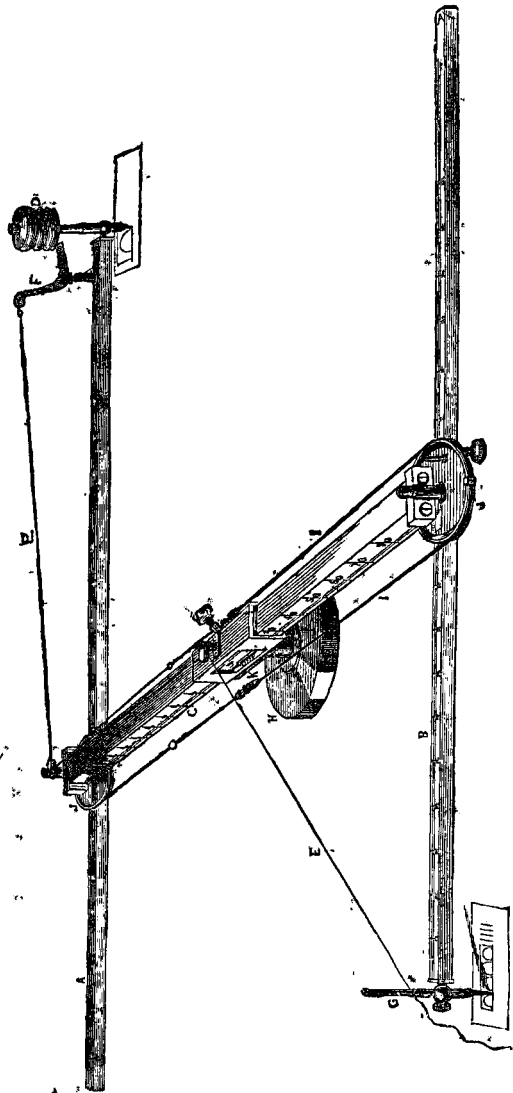


Fig. 577.—The Eidograph (Davis's).

may be raised by a lever, *F*, attached to a cord *E*, which passes over the centres of the instrument to the tracer, *G*. The two arms and the beam are made of square brass tubes, and are divided into 200 equal parts, which are figured so as to read 100 each way from the centre, and may be read to 1000th by means of the verniers on the sliding-boxes. There is a loose weight which fits upon any part of the main beam, packed in the box with the instrument. The object of putting on the weight is to steady the instrument when it is set to proportions which would otherwise tend to overbalance the fulcrum weight.

750. To adjust the Eidograph, place all the verniers at 0, which will bring them to the exact centres of the arms and the beam; place the arms at about right-angles with the beam, then make a mark simultaneously with the tracer and pencil point turn the instrument half round upon its fulcrum, so that the pencil point is brought into the mark made by the tracer; then, if the tracer fall into the mark made by the pencil, the instrument is in adjustment. If it should not fall into the same mark, the difference should be bisected, and the adjusting screws on the bands should be moved until the tracer falls exactly into the bisection, o make it in perfect adjustment.

When the eidograph is in adjustment, if the three verniers be set to the same reading on any part of their scales, the pencil point, fulcrum, and tracer will be in a true line. If it should not be so, it would show the dividing and centring of the instrument to be inaccurate. The divisions upon the eidograph do not positively indicate the reductions required to be performed by the instrument, but merely give a scale, which, with the assistance of the vernier, divides the beam and arms into 1000 parts. The setting is under :—Let a/A be the scales of the original and reduced plan, and x the reading of the graduations, then for the similar Δs we have :

$$\frac{100-x}{100+x} = \frac{A}{a}, \text{ or } x = \left(\frac{A-a}{A+a} \right) 100.$$

751. To obtain the quantity to which the reading is to be set for certain proportions, a printed table is pasted inside the lid of the instrument box which contains part of the following proportions :—

TABLE FOR REDUCING OR ENLARGING PROPORTIONS.

Proportions.	Divisions on Bars.	Proportions.	Divisions on Bars.
As 1 is to 2	33·333	As 2 is to 3	20
„ 1 „ 3	50	„ 2 „ 5	42·857
„ 1 „ 4	60	„ 3 „ 4	14·285
„ 1 „ 5	66·666	„ 3 „ 5	25
„ 1 „ 6	71·428	„ 4 „ 5	11·111
„ 1 „ 7	75	„ 5 „ 6	9·09
„ 1 „ 8	77·777	„ 6 „ 7	7·692
„ 1 „ 9	80	„ 7 „ 8	6·667
„ 1 „ 10	81·818	„ 8 „ 9	5·882

The following Rule* is sometimes pasted inside the case :—

To find the quantity equal to any given proportion for the setting of the eidograph :—Subtract one sum of the proportion from the other, and multiply this difference by 100 for a dividend. Add the two sums of the proportion together for a divisor. The quotient from the working of this will give the number to which the arms and beam are to be set.

* Taken by permission from Mr. Stanley's "Mathematical Drawing Instruments,"

Ex. 1. Let it be required to reduce a drawing in the proportion of 3 to 5, putting the problem arithmetically :—

$$\begin{array}{r} 5-3=2 \\ \times 100 \\ 5+3=8 \overline{) 200} \quad (25. \end{array}$$

The centre beam, C, is to be set to 25 on the side nearest the pencil point, D, the pencil arm, A, is also set to the 25 nearest the pencil point, D, and the tracer-arm B is set to the 25 farthest from the tracer, G. If it were required to enlarge in the same proportion, each side would have to be set at the opposite 25.

Ex. 2. Let it be required to reduce a plan of 5 feet to the mile to a Scale of 3 chains to the inch.

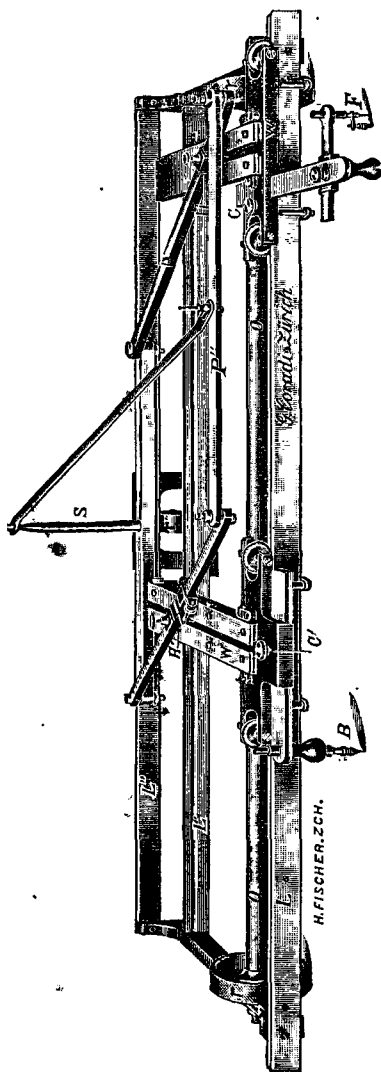
First, we must have like terms, therefore to reduce both proportions to feet to the inch will, in this instance, be the most simple way ; thus—
5 feet to the mile = 88 feet to the inch.
3 chains to the inch = 198 " " " "

$$\begin{array}{r} 198-88=110 \\ \times 100 \end{array}$$

$$198+88=286 \overline{) 11,000} \quad (38.461$$

If the slides of the instrument be set to 38.46, it will be practically near.

752. Improved Eidographs have been made with a movable caster supporting the main beam instead of balancing it by weight when using the instrument for reductions below one-third. The roller of the caster is 2" in diameter and fixed on the beam with its axis in a true line with the fulcrum. This causes very little friction. Another improvement is made by making the pencil and tracing points interchangeable. By this arrangement one arm of the eidograph may be reduced to about half its length. The fulcrum weight is made triangular. There are also other minor improvements.



753. The Affinograph (Fig. 578.—Affinograph-Ellipsograph. 578) is intended for reducing and enlarging the ordinates of curves in given ratios. The instrument is also an ellipsograph ; if the one pin describes a circle then the other pin draws an ellipse in proportion to the reduced or enlarged ordinates. In the X-direction the instru-

ment can travel to any desired length; in the V -direction the guide pin travel to a maximum length of 33 cm. (13").

754. Copying Plans.—Plans may be copied employing geometrical methods to fix the points of the plan by intersections or by co-ordinates. The process is tedious and therefore the following mechanical methods should be resorted to when the copy is to be of the same size as the original.

(1) **Copying by Tracing.**—The *tracing frame* (Fig. 569), or the *Copying glass* (Fig. 570) may be employed in the operation. The plan to be copied is covered with drawing paper, and placed on the pane of glass. The lines of the original are seen distinctly through the fair sheet, and can thus be traced, at once in Indian ink, light underneath enabling the tracing to be taken more easily, especially if the original be crumpled or dirty.

(2) **Copying on Tracing-Paper or Tracing-Cloth.**—In practice, original finished drawings are seldom worked from, as they would speedily become dirty and obliterated, and dimensions would be taken from them with difficulty. To obviate this, copies of plans are necessary. Tracing upon a sheet of thin semi-transparent paper or tracing-cloth laid on the original drawing is the most accurate method of taking a copy. A sheet of tracing-paper or tracing-cloth is attached with drawing-pins over the plan to be copied. The lines are then copied in Indian ink, a set-square being used for ruling the straight lines. The tracing-paper is then mounted on white paper and colour applied.

If tracing-cloth is used, colour should be applied to the back. In copying on tracing-cloth, difficulty is experienced, owing to the greasy nature of the surface, in getting the ink to run freely. This can be obviated by sprinkling chalk-powder on the surface of the cloth or a little ox-gall in the ink or colour will remedy it (*q.v.*, p. 456).

(3) **Copying by Transfer.**—The ordinary method is to fix a sheet of black-lead or carbonic paper, with the blackened side downwards, on the intended copy. Over the carbonic paper is placed the original to be copied and all the lines are gone over with a tracer. In this way, a copy of the original is obtained in black lines, which may afterwards be inked in.

Black lead paper is prepared by rubbing thin paper with a soft block of Cumberland lead or with black-lead powder, smoothly spread with a piece of cloth.

Carbon Paper is a blue paper which has one side coated with lamp-black ground to perfect fineness in slowly drying oil, and left a considerable time to season.

(4) **Duplicating Drawings by Pricking-Through.**—A plan may be copied by placing a clean sheet underneath it on the drawing board and by pricking-through the angular points of the original plan with a pricker or a fine needle. The points thus obtained on the clean sheet are then joined up and the plan inked in. This method is generally applied to plans of which the outlines are mainly straight lines and the damage to the original copy is unimportant.

All these systems of copying hurt, in a more or less degree, the original drawing.

(5) **Engraving.**—When a plan is to be engraved on copper or wood a copy of it is laid on the copper plate or wooden block with its face downwards, and the outlines scratched on the copper or wood with a point which cuts through the copy. When great accuracy is required in engraved plans, the principal stations are plotted on the copper, and the details laid down on it by tracing. Impressions from copper usually shrink, as they are taken off the copper when damp, all measurements should therefore be taken with the scale drawn on the sheet.

(6) **Reducing plans by Photography.**—Small scale maps are reduced from large scale ones by photography. The necessary details are

traced from a silver print of the photograph, and the tracing is placed on a copper plate. The lines are then passed over with a pointed instrument which cuts through the copy and scratches the copper underneath it. A process for photographing the details directly on to copper or zinc has also been introduced.

(7) **The Ferri-Cyanide or Ferro-Prussiate Process of Copying Plans.**—In this process, *the lines appear white on a dark-blue ground*. The chemicals used in the preparation of sensitising solutions are:—

Sensitising Solutions.					No. 1.	No. 2.
A	{	Ammonia-citrate of iron (Ferric salt)	...	...	100 grains	1½ ozs.
	{	Water	...	...	1 oz.	6 "
B	{	Potassium Ferri-cyanide (Red Prussiate of Potash).	...	...	70 grains...	1 "
	{	Water	...	...	1 oz.	6 "

Method of Sensitising the Paper.—Mix equal quantities of A and B—i. e. No. 1 solutions immediately before use, and apply to one side of the paper with a sponge, or each of the No. 2 solutions should be laid on separately with separate broad brushes or sponges having even surfaces. The sponge should be as full as it will hold of the solution, which should be liberally applied to the paper for about 2 minutes (3 minutes in the hot dry season and 1 minute during the rains). Then squeeze out the sponge and wipe off all the solution from the surface of the paper, care being taken to use the sponge lightly without abraising the surface. The paper, which is now of a bright yellow color on the prepared side should be hung up to dry in the dark, and in the latter case the paper afterwards suspended in the dark to dry between each coating. It is better to use fresh solutions, but they will keep good a long time in glass-stoppered bottles in a dark cool place. Ready sensitised paper is now an article of commerce, and is obtainable from the photographic dealers.

Printing.—The tracing is placed against the glass of the printing frame, face next the glass, and at the back of the tracing the prepared sheet is placed with prepared surface next the tracing. The frame is then exposed to light until the prepared sheet attains a deep bluish tone.

Fixing.—The sheet is then removed from the frame, and thoroughly washed in clear water in a zinc tray till the lines appear white.

If the paper is old, light deficient, chemically weak and the print becomes too light, it may be intensified slightly by immersions, in a weak solution of hydrochloric acid, 10 drops of HCL to 1 ounce of water, or 100 grains of citric acid to a pint of water.

Additions and Erasures.—A white line may be taken out by going over it with a quill pen or brush dipped in the sensitising solution, exposing to the sun and washing as before. Additions or corrections in white may be made with a quill pen dipped in a solution of 40 grains of carbonate of potash to 1 ounce of water. After using this solution the potash must be dried with blotting paper and washed or the lines will spread and become blurred.

The process depends on the actinic action of light reducing the ferric salts to the ferrous state under certain conditions, one of which is the presence of organic matter, such as the size contained in the paper. The ferrous salt then combines with the potassic salt to form an insoluble Prussian blue. For this and other processes the tracing has to be taken of the drawing in dense black lines, for the best result, formed of a good fluid Indian ink.

The tracing paper should be clear, transparent, and of a bluish tint. The tracing is termed a *negative*.

(8) **Blue Print Paper.** The following process, credited to Capt. Abney, yields a photographic paper giving blue lines on a white ground:

Common salt	... 3 ozs.	Dissolve the acacia in half the water and
Ferric Chloride	... 8 "	dissolve the other ingredients in the other half ;
Tartaric acid	... 3½ "	then mix. The liquid is applied with a brush
Acacia	... 25 "	to a strongly sized and well-rolled paper in a
Water	... 100 "	subdued light. The coating should be as even

as possible. The paper should be dried rapidly to prevent the solution sinking into its pores. When dry, the paper is ready for exposure. In sunlight, one or two minutes is generally sufficient to give an image, while in a dull light as much as an hour is necessary.

To develop the print, it is floated immediately after leaving the printing frame upon a saturated solution of *potassium ferro-cyanide*. None of the developing solution should be allowed to reach the back. The development is usually complete in less than a minute. The paper may be lifted off the solution when the face is wetted, the development proceeding with that which adheres to the print. A blue coloration of the back ground shows insufficient exposure and pale-blue over exposure.

When the development is complete, the print is floated on clean water, and after two or three minutes, is placed in a bath, made as follows:

Sulphuric acid	... 3 ozs.	In about ten minutes the acid will have
Hydrochloric acid	... 8 "	removed all iron salts not turned into the
Water	... 100 "	blue compound. It is next thoroughly

washed and dried. Blue spots may be removed by a four per cent solution of caustic potash.

The back of the tracing must be placed in contact with the sensitive surface. (Scientific American supplement, June 21, 1904).

(9) **Positive Cyanotype Process [Blue lines on a white ground].—**

The solutions required for sensitising the paper are as in the margin.

A	{	Gum arabic		1 oz.	Solution A can only be kept for a few days, solutions B and C will keep for several weeks in well-stoppered bottles. When required for use, they are mixed in the proportions of 20 parts of A, 8 parts of B, 5 parts of C, in the order given, or the
		Water		5 "	
B	{	Ammonia-citrate of Iron		1 "	
		Water		2 "	
C	{	Perchloride of iron		1 "	
		Water		2 "	

gum will be co-agulated. If this occurs, the mixture may again be rendered liquid by adding a few drops of glacial acetic acid.

Sensitising the Paper.—This process onwards, except exposure to sunlight, must be performed in a room illuminated by yellow light passing through yellow paper or calico fixed over the windows, or by candle or weak lamp light. The paper is pinned down to the drawing board and the solution applied with a sponge as evenly as possible to avoid streaks. The paper should dry quickly, so that the solution may not sink into its substance, and as soon as it is dry it should be stored away in an air-tight tin case.

The Exposure will vary with the intensity of light. In direct sunlight from 15 to 40 seconds will be enough; in the shade or dull light from 1 to 5 minutes, in rainy season 5 to 15 minutes; in dull foggy weather 15 to 30 minutes or longer may be required.

The Development of the print must be performed in yellow light in a saturated solution of ferro-cyanide of potassium; which should be poured into a lead tray in wood frame so as to fill it to a depth of about an inch. The print, with its edges turned up about an inch in depth so as to form a sort of tray, is now floated face downwards on the solution, the turned-up-edges

preventing any of the solution from getting on the back of the copy, which would cause blue stains to appear. After floating the copy on the solution for about half a minute. remove it carefully, hold it up, and allow the action of the developer to continue but only so long as the yellow ground remains free of blue spots. The longer the film of prussiate can be kept on the print, the stronger and darker will the lines come out. As soon as any sign of a blue spot begins to show, immerse the print at once face downward, in a tray of clean water and wash it in several changes. After washings immerse it in a bath of hydrochloric acid, 1oz. to 10ozs. of water; keep it in this bath from 5 to 10 minutes after which remove and place it face upwards in a leaden tray in wood frame of clean water and well wash and rub the surface with a sponge to remove the blue mucilage; then flush with clean water. The lines of the print will now be found to stand out blue on a clear white ground. The inside of the trays to be coated with Bate's black photographic or shellac varnish.

(10) Aniline Printing Processes (Dark lines on a clear ground from a Tracing).—Sensitising.—Well-sized paper is sensitised by economically floating or brushing over it with a large flat brush, either 1 or 2 solution of the bichromates of potassium or ammonium. The solution

1.	Potassium bichromate*	...	160 grains
	Phosphoric acid solution	...	2 ozs.
	Distilled water	...	5 "
2.	Ammonium bichromate*	...	160 grains.
	" chloride	...	160 "
	Sulphate of copper	...	30
	Sulphuric acid	...	$\frac{1}{2}$ oz.
	Water	...	10 ozs.
*Bichromates of potassium and ammonium are poisonous.			

should be applied as quickly and evenly as possible so that it may not sink into the body of the paper; surface impregnation only is required, otherwise the ground would be more or less discolored, and the lines and figures being imbedded would not be sharp. The operation may be carried out by lamp-light, or subdued day-light,

or yellow light, and the paper is dried quickly in the dark, or in a photographic dark room, and should be used on the day it is prepared, or at most the next day, as it does not keep.

Printing.—The paper being more sensitive than ferro-prussiate paper, is then exposed to light under a tracing, till the lines and figures of the drawing appear of a faint yellow color on a greenish-white ground. In bright sunlight the exposure is generally from 1 to 2 minutes. It is then exposed to the action of aniline vapour which deeply colors the parts protected from light by the lines and figures of the tracing, the vapour reacting on the chromic compound not reduced by light, and causes no action with the reduced chromium salts, as these acquire a neutral reaction and will not assimilate bases.

Development:—The print is inserted at the bottom of a shallow box with a close fitting lid. On the lid is fastened a piece of damp flannel, and on this piece is attached another bit of flannel on which a little of the marginal developing solution is sprinkled: development is completed in from 5 to 10 minutes dependent on the dura-

Aniline solution	...	1 oz.	tion of the exposure to light. Long
Benzole	...	12 ozs.	exposure and long development give the best results. When a good dark

tone is produced, the print is washed in clear water and then immersed for a short time in a bath of sulphuric acid $\frac{1}{2}$ oz., water 50 ozs. It is again washed in water and immersed for a few seconds in a bath of liquor ammonia $\frac{1}{2}$ oz., water 40 ozs., and then removed, washed and hung up to dry. The acid and ammonia baths are not essential but they give better results.

Development may also be made by pinning the exposed print to the lid of the box, and having the damp flannel, with the pice on which the aniline solution is sprinkled, at the bottom. Also, instead of sprinkling the solution in the flannel, the solution may be placed in a shallow dish at the bottom of the box, and when the development is complete the solution may be put in a well-stoppered bottle for future use.

(11) **Ferric Chloride & Gelatine Process (Dark lines on a clear ground from a Tracing).**—This process is dependent on the property of ferric chloride rendering gelatine insoluble. 300 grains of gelatine is first soaked in 5 ozs. of water until it is fairly soft, then 5 ozs. more of water is added, and the mixture stirred and dissolved by means of a gentle fire. Indian ink or any suitable pigment is then added.

The paper is coated with the hot pigmented gelatine solution by brushing, care being taken to make the coating thin and uniform. The paper when dry should present a uniformly colored surface, and is then sensitised by immersion in a solution as in margin.

Ferric chloride	... 2 ozs.	After sensitising, the paper is dried
Tartaric acid	... 300 grains.	in the dark or in a photographic
Water	... 20 ozs.	dark room. Printing is carried
		out as in the ferro-gallic process ;

the exposure to sunlight being about 5 to 15 minutes according to the intensity of light.

Development.—The part exposed to light, *i. e.*, the surface of the drawing, becomes soluble in hot water, while the parts covered by the lines and figures of the tracing remains insoluble ; hence development is effected by immersion in hot water ; a print is obtained according to the pigment used, showing dark or colored lines on a clear ground.

(12) **Acidified Ink Process (Black lines on a clear ground).**—The paper is uniformly and thinly coated by brushing with gelatine 300 grains, soaked in 10 ozs. of water. When dry it is sensitised by brushing on the back a solution of—Bichromate of potassium, 150 grains, ammonia $\frac{1}{2}$ dram, water 10 ozs. The solution should be allowed to thoroughly imbue the paper, to effect which a second brushing should be resorted to, after which the paper is dried in a dark room. When dry it is exposed to light under a tracing in a pressure frame until the lines and figures of the drawing are well-defined. The paper is then immersed in hot water and the bichromatised gelatine not rendered insoluble by the action of light, *i. e.*, those parts covered by the lines and figures of the tracing, dissolved out, leaving bare paper at these parts. The paper is then blotted and brushed over with the marginal solution : The ink solution fixes itself on the bare

Liquid Indian ink	... 1 oz.	parts which show the lines
Sulphuric acid	... 30 minims	and figures of the drawing, and
Caustic potash	... 12 „	a copy showing black lines on a
		clear ground is the result.

(13) **Modified Carbon Process.—Black lines on a clear ground.**—Paper is coated with gelatine and sensitised with bichromate of potassium as in the Acidified Ink Process. It is dried in a dark room, and then exposed, in a pressure frame, to light under a tracing, until the lines and figures become visible. The paper is then immersed in cold water and sponged, the water being changed several times to remove the chromium salt not reduced by light. The paper is now blotted fairly dry and its surface brushed over with—Liquid Indian Ink 1 oz., Bichromate of potash 10 grains. It is then dried in the dark or in a photographic dark room, and then placed in a pressure frame, with the back of the paper next to the glass and exposed to sun-light for about 2 minutes. The paper is then immersed in clear water and the surface brushed or sponged until the black lines are visible on a clear ground.

Theory.—In the first exposure to light under a tracing the surface of the drawing was, by the action of light, rendered insoluble and practically non-

absorbent; while the parts covered from light (lines and figures of the drawing) remain unchanged and absorbent, hence the lines and figures only take up the bichromatised Indian Ink and thereby become re-sensitised. When the back of the copy is exposed in the pressure frame the light acts through and on the lines and figures, rendering these insoluble, but practically causing no action on the ground of the print, which is easily brushed to show black lines on a clear ground.

(14) **Platinotype Process.**—It gives *white lines on a black ground*, and is based on the action of a ferrous salt, when exposed to actinic light, on platinum chloride. The sensitising solution is composed of 60 grains of Potassic-platinous chloride, 60 grains of ferric oxalate, and 1 oz. of water. The exposure lasts until the paper becomes of a dull orange tint. It is then developed in non-actinic light by floating it for 4 seconds in a solution of 130 grains of Potassic oxalate and 1 oz. of water at a temperature of 150° to 200° F. When properly developed, the print is washed for 10 minutes in 1 part of hydrochloric acid with 60 parts of water, and finally washed in relays of fresh water, for 15 minutes.

(15) **Ferro-gallic Process.**—In this process *the lines appear black on a white ground.* (a) The sensitising solution is made up of either No. 1 or No. 2 solution.

	No. 1.	No. 2.
Water ...	15 ozs.	40 ozs.
Gelatine ...	$\frac{1}{2}$ "	1 "
Perchloride of Iron	1 "	2 " (solid)
Tartaric Acid	$\frac{1}{2}$ "	10 drams
Persulphate of Zinc	$\frac{1}{2}$ "	...
Ferric Sulphate	...	10 "

The exposure to sun light is about the same as for ferro-prussiate prints, *i. e.*, from 5 to 10 minutes according to the intensity of light.

The operation of developing must be carried out in yellow or red light. The copy should be floated face downwards on the following solution:—Gallic acid 1 oz., Oxalic acid 5 grains, Methylated alcohol 10 ozs., water 50 ozs. Floating should

be done carefully by holding two opposite corners of the papers and bringing the hands closer together. The middle convex shape thus given to the sheet, should first touch the solution, and the corners held by the hand are then gradually lowered till the sheet floats on the solution. The formation of air bubbles is thus prevented; should, however any be formed they can be removed by means of a glass rod. The copy should be allowed to remain on the solution till the lines appear uniformly black. It is then removed and washed with clear water and hung up to dry. A great merit of this process is that the print may be tinted in colours as required.

(b) Other Satisfactory Sensitising Solutions are:—

A {	Gum Arabic ...	1 oz.
	Water ...	10 ozs.
B {	Tartaric Acid ...	1 oz.
	Water ...	4 ozs.
C {	Solution of Ferric Chloride	
	45° Beaumé ...	2 "
D {	Ferrous Sulphate ...	5 drams
	Water ...	4 ozs.

Solution D is added to B, and the mixture added slowly with constant stirring to A; finally solution C is gradually added stirring well the whole time during admixture.

Exposure is complete when the ground of the print is nearly white, the parts protected from the light being of a light yellow colour.

Development is effected by floating the print for about one minute face downwards in the following solution :—

Gallie Acid	...	...	1 dram	The print is then removed from the developing bath and washed within clean water.
Oxalic	...	...	3 grains	
Water	...	...	50 ozs.	

Ready-sensitised Ferro-Gallic paper is an article of commerce. The following is a good developing solution for ready sensitised paper :—

The acid and alum should first be dissolved in 2 pints of hot water after which the remaining water should be added. To develop the prints the copy is plunged in the developing solution till the drawing shows up clear and the lines black, after which it is washed and hung up to dry.			
Gallie Acid	...	1 oz.	
Alum	...	1 "	
Water	...	8 pints	

List of Apparatus and Chemicals for Ferro-prussiate and Ferro-gallate Printing Processes with Approximate Cost :—

1 Printing frame of Mahogany. Imperial size	...	...	Rs. 16	0	0
1 Plate glass $\frac{1}{4}$ " thick for do	...	...	12	0	0
1 Pad for printing frame, 3 thicknesses of flannel with $\frac{1}{2}$ a quire of paper	...	...	2	12	0
3 Tin trays in wood frame	...	...	9	0	0
12 American clips for drying paper	...	...	0	8	0
12 Drawing pins	...	...	0	8	0
1 Sponge of best quality	...	...	1	4	0
1 four-ozs. Measuring glass	...	...	0	12	0
4 yds. of yellow cloth for windows in sensitising room	...	...	0	9	0
1 lb Ammonia citrate of iron	...	...	2	8	0
1 lb Potassium ferricyanide	...	...	2	8	0
1 four-ozs. bottle citric acid	...	...	0	12	0
1 one oz. " Carbonate of soda, in powder	...	...	0	4	0
1 four—ozs. " Oxalate of potash	...	...	0	8	0
1 lb Gallie acid	...	...	4	8	0
2 lb Alum	...	...	0	8	0
1 Roll Schleicher and Schull's paper, 55 yds. long by 58 $\frac{1}{2}$ " wide, No. 51	...	...	25	0	0
Highly transparent tracing parchment, does not change color, in rolls 33 yds. long by 39 inches wide per roll	...	...	24	0	0

(16) Collo-chromate Printing Processes.—These processes are used for map printing and depend on the reaction of the chromium salts (alkaline bichromates) on gelatine, gum, albumen, or other colloid substances, which, in proportion to the amount of the action of light upon them, become more or less insoluble in and unabsorbent of water, and acquire the property of taking up greasy ink and not attracting plumbago or other fine pigment powder. When the map is in line the print is taken on paper coated with a preparation of gelatine and bichromate of potash, coloured with Indian ink or any other pigment; after a few minutes exposure in the copying frame the paper is plunged into tepid water, which dissolves the unaltered gelatine in the blank parts of the print—they have been protected from the light under the dark parts of the negative—leaving a clear image in pigment on a white ground.

(17) Apparatus for Copying Tracings by Artificial Light (Fig. 579), designed to meet the growing demand for rapid and inexpensive photo-copies of drawings or photographic films and tracings at any hour, in any weather, is now extensively used.

The Machine consists of a cylindrical glass frame (A) mounted on stand (B). The tracings and photo-paper are wrapped around

the outside of the cylinder and, excellent contact is obtained by means of suitable covers and provided with eyelets to slip on to pins

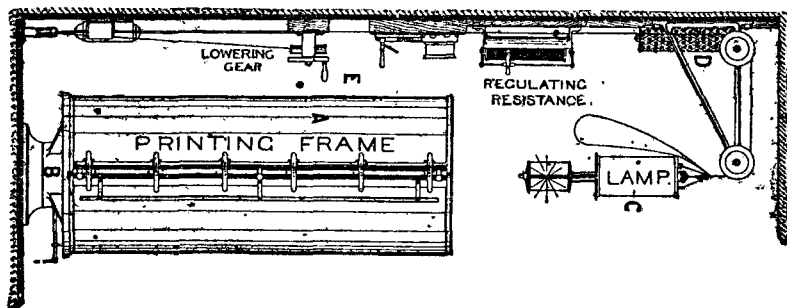


FIG. 579.—Electric Copying Frame.

projecting from a vertical steel rod, by revolving which the covers are drawn tightly around the glass. A Special Photo-Printing Arc-Lamp (C) is suspended by wire-ropes passing over the pulleys (D), and is gradually lowered throughout the cylinder, either by stages of about 8 at a time, with an interval of rest between each, or by means of the Patent Automatic Gear (E). Small prints can be taken on large frames without waste of light. On an Antiquarian Machine 2 Imperial Blue Prints can be produced in 2 minutes, 2 Double Elephant in 3, 2 Antiquarian in $3\frac{1}{2}$ minutes, and other prints in the same ratio, according to the sensitiveness of the paper used. An Antiquarian frame, arranged to admit 2 tracings, can be fitted up and worked in a corner 2' square.

(18) **Preparation of Drawings for Photo-Mechanical Reproduction.**—Maps, plans, engravings, pen and ink sketches, wash drawings, photographs, etc., are reproduced by the various photo-mechanical processes, which are economical and rapid in proportion to the complexity of the drawing, or the amount of detail in it, or when the drawing has to be enlarged or reduced to scale, or to a particular size. The reproduction is also produced with great accuracy and all personal error is eliminated. To obtain the best results by these processes, certain essential conditions, as shown below, should be observed in the preparation of the original drawings.

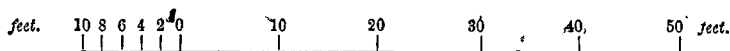
Reduction of Drawings.—Drawings intended for reproduction to a reduced scale should be drawn in an open and bold style, with lines, printing and figures proportionately thick.

Paper and Inking.—Good white or bluish paper should be used, and good black, stick Indian ink, freshly made, so that lines, printing and figures may be reproduced to the proper strength.

Washes.—Washes or shading are inadmissible for drawings in line; cross hatching should be used to distinguish different materials or parts of structures.

Tracings.—Tracing paper or cloth should be white or of a bluish tint. Discoloured or yellowish-tracing papers are quite unsuitable.

Scales.—Scales on drawings intended for enlargement or reduction should generally be shown in terms of a single unit of measurement, and not as relative to any second unit, as—"Scale, 20 feet to 1 inch," so that the scale can be reduced or enlarged with the drawing.



Pen and Ink Sketches.—Effects of light and shade should be produced by making the lines thick, or thin, and increasing their distance apart, and not by the use of pale ink.

Photographs.—Photographs should be clear and bright with full gradations of tone and detail in the shadows.

Wash Drawings, Print, etc.—Wash drawings or prints of any kind should be quite free from markings or stains. Notes of reference to be made on the back and not on the face of drawing or print, unless absolutely necessary, and then pencil only should be used.

Direct Zinc Printing Process.—A new process, by means of which drawings or tracings in black ink can be transferred to zinc without first making a negative, has been recently worked out in the Survey of India Office, which, for certain classes of work, has many advantages over the ordinary process of photo-zincography. By this process work is executed at less than one-quarter of the rate charged for photo-zincographic work, and with better results, provided that the original drawings sent in conform to the following conditions:—(1) the process is only adapted to the reproduction of drawings to the same scale and drawn on a single sheet. Two or more sections cannot be joined together as in ordinary photo-zincography. (2) Tracing paper or cloth is the most suitable form of original. Failing this, any smooth surfaced paper may be used. Rough surfaced drawing papers are unsuitable and should not be used. (3) It is necessary that the work should be drawn in good, black, freshly prepared Indian ink. With this proviso, any drawing will reproduce satisfactorily no matter how fine the lines may be. Color should not be used. (4) Nothing whatever should be pasted on either the front or back of the original. Slips of type or other matter pasted on the front will not reproduce, and consequently have to be drawn on the zinc afterwards. Nothing should be written on the back of original. (5) Headings, footnotes and other type matter should be neatly printed on the original in their proper places, as the rates charged do not admit of such matter being put on zinc by means of type transfers. (6) Originals should not be folded or creased if it can be avoided, but either left flat or rolled up when despatched.

General.—Printed matter for reproduction should, when practicable, be kept flat, or rolled, but not folded. Original drawings should be well finished with good lines, printing and figures, so that after being copied the results may be fit for immediate issue, and not require alteration and working up, which add to the cost, cause delay, and deteriorate the work generally.

13. Lithography.—The art of lithography is based on the circumstances that greasy substances do not mix with water and are repelled by gummy substances. A greasy ink is made to stick to a kind of limestone; the parts thus covered receive the printing ink; the other parts are prevented from doing so by the interposition of a film of water; then impressions are obtained by pressing the paper upon the stone. Compact homogeneous limestones of an even grey or drab colour should be used for this purpose. Lithographic ink is a mixture of tallow 4 oz., wax 5 oz., soap 4 oz., sellac 3 oz., and sufficient quantity of Paris black which being greasy, is readily absorbed by the stone or zinc. The inks of Lemer cier and Vanhymbeeck are the best. Lithographic chalk is made in the same way as the ink but burnt hard for drawing. Lithographic crayons of several degrees of hardness are also available. Transfer paper for writing and drawing is prepared by coating the surface of the paper with a composition of size made from parchment cuttings and flake white. The coating is also made of starch and glue. The following is an excellent recipe for making transparent paper for litho map work; it assists the tracing to be done considerably. Boil $\frac{1}{2}$ oz. of gelatine or isinglass in about a pint of water and coat twice ordinary tracing paper. Boil $\frac{1}{2}$ lb gamboge and $\frac{1}{2}$ lb arrowroot separately in about $\frac{1}{2}$ a pint of water each.

Mix together and strain through a thin cloth. If found too thick, add a little water and give one or two coats to the paper already coated with gelatine. The paper must be allowed to dry between the coatings. After the paper has been passed through a litho-press on a clean stone face downwards or otherwise rolled, it is ready for use. Lithographic pens, brushes, scrapers, crayon holders, hand-board, ruling* and circle pens, parallel rules, tracing paper, tracer and red tracing paper for transferring tracings to paper, or stone are also other requisites.

Drawing on Stone: Chalk Method.—For artistic work this method is employed. In preparing the stone for chalk drawing the surface should be "grained" by sprinkling a small quantity of finest sand, and a few drops of water over the surface, and by moving a small stone of the same quality over it with a circular motion. If the surface becomes too coarse or too fine or if scratches are found, the graining should be repeated. The drawing is then traced upon the stone with the tracing placed face downwards; red tracing paper is interposed and the outline carefully gone over with a tracer or hard pencil. The tracing papers are then removed and the surface covered with plain paper. Great care should be taken that no saliva drops on the stone nor the greasy hand is placed on it. In the former case white spot will appear, and in the latter the form of the hand and fingers will appear black on the print. The drawing is then outlined with chalk. The "tinting" or shading follows; lights may be picked out with the scraper, and ink used when sharp, dark touches are desirable. When completed the drawing is "etched" by applying acid in two ways—one by flooding the stone with diluted nitric acid, the other with acid diluted with gum-water, applied with a flat, soft brush, about 4 or 5 inches in width. After etching the water is drained off and the stone gummed and allowed to dry. It may then go to the printer for proving.

Pen and Brush Method.—The surface of the stone is ground and afterwards polished with Water-of-Ayr stone or snake stone. The drawing or writing is traced in the manner already described. The great disadvantage in this method is the necessity of reversing writing and lettering. The stone is then etched as described above before going into the printer's hands.

Lithography on Paper.—By this method the work is done on paper and afterwards transferred to the stone. The ink is prepared by rubbing a small quantity in a China saucer, the saucer being heated to make the ink adhere; water is then added, and the ink rubbed with a cork till it dissolves. The ink should be made of proper consistency. If thin it will not transfer properly; if too thick, it will not flow fully from the pen or brush, and will spread in transferring. The paper should only be held at the edges, and while working a clean piece of paper is placed under the hand. The stone should be polished free from scratches, and is generally warmed to make it more susceptible of receiving the ink. The transfer is placed face downwards off the stone, pulled repeatedly through the press, and afterwards removed to the trough, where hot water is poured over it. It is then removed leaving the ink and composition on the stone; the latter is washed off, and the stone gummed and allowed to dry. The work is then *proved* by rolling up, cleaning, etching, and taking the first impressions.

(20). **Zincography** nearly resembles lithography in principles. Zinc-plates are less costly and more portable than lithographic stones and are easily cut to convenient sizes. They are similarly "*grained*," a muller of zinc or pumice stone and steatite being used. Drawing on zinc is done exactly in the same way as on stone; the etching solution is applied with a camel-hair brush. It consists of a decoction of nut-galls; a solution of gum and phosphoric acid is sometimes added. During printing the plate is mounted on a block of hard wood.

Printing.—After the stone is etched it is put into the press and fixed. The gum is then washed away with a soft sponge and water; the drawing or writing is obliterated with a clean sponge and oil of turpentine and a few drops of sweet oil. Then the stone must be cleaned with water. The stone is then slightly wetted with an entirely clean sponge and the printing roller

charged with proper ink is passed repeatedly over the whole stone until the writing or drawing will appear in full strength. A sheet of damped paper is put on the stone, and the impression is taken from the stone by working the press. The stone, is again wetted, inked, the paper put on the stone and further impressions are taken. The printing ink is a mixture of lamp black—with a little Prussian blue added—and linseed oil varnish.

The transfer paper being damp during the process expands to a certain extent, so that the drawing on the stone is somewhat larger than the original; and this expansion is to a certain extent counteracted by the shrinking of the paper on which the impressions are taken. As with engraved plans, each sheet should have its own scale by means of which all measurements upon it should be made.

(21). **Photo-lithography and Photo-zincography** are the processes much used for map printing. The prints for transfer from the negatives are taken on paper coated with a mixture of gelatin and potassium bichromate, as in the pigment printing process, except that the greasy ink or colouring matter is not mixed with the gelatin, but applied evenly over the surface of the prints after exposure to the light. The inked print is immersed for a few minutes in tepid water to soften the gelatin still remaining soluble in the parts not acted upon by light, and is then laid on a sloping plate and washed with a soft sponge until all the unaltered soluble gelatin and the ink overlaying it are removed. The lines on which the light has acted remain insoluble and retain the ink forming a clear image of the subject in a greasy ink. By this process photographs of several sections of a map may be easily combined without showing lines of junction.

(22). **Photo-typography.**—By this process zinc blocks may be produced for use with type and printing in the same way as a wood-cut. The picture may be transferred on a zinc-plate in the same way as for photo-zincography or it may be printed directly from a reversed negative. In the latter case the zinc plate is coated with a thin layer of bitumen which becomes insoluble under exposure to the light, so that when after exposure the plate is washed with turpentine or benzole the picture remains on the zinc, while the ground is washed away. In both these cases the picture is strengthened by careful inking, applying powdered resin, which the plate is heated sufficiently to melt. The image is then etched with nitric acid which should bite the plate sufficiently deep to give good impressions.

The three processes above described are only suitable for maps in line.

(23). **Reducing and Enlarging Drawings by Hand.**—This is done in the case of plans by drawing Δ s to connect stations and other principal points on the original, measuring their sides and plotting them on a smaller or larger scale on the reduced or enlarged plan. The details may be reduced or enlarged by squares as described below. The only really accurate method of reducing and enlarging plans is to replot them from notes in the field-books to the required scale.

(24). **Reducing by the Lens.**—A suitable lens fitted to a camera can be used to reduce plans by replacing the ground glass with a sheet of clear glass covered by tracing paper. The drawing to be reduced is pinned on a vertical stand in strong sun-light and when focussed to the intended reduction the lines thrown on the tracing paper can be traced with a fine pencil.

(25). **To Make a Proportional Copy of any Irregular Figure such as a Plan by Squares**—The best method is by dividing the larger map into squares by faint lines, with a very soft lead pencil, and then drawing the reduced map upon a sheet of smaller squares. A pair of proportional dividers will assist much in fixing points intermediate to the sides of the squares. If the larger map would be injured by the drawing and rubbing out of the squares, threads may be stretched across it to form squares or to avoid defacing valuable plans the following process may be adopted. Rule a sheet of glass or a tracing paper with an inch or $\frac{1}{2}$ -inch squares in ink and lay this on the original, the copy being prepared in pencil

with squares similar, or in proportion to the ment. Two pencil dots on the original, corresponding to the corners of the glass, enable one to replace it correctly if required.

In Fig. 580, let the plan within the square, CB, be that of an estate which it is required to copy on a reduced scale of one half. The copy will, therefore, be of half the length and breadth, and contain $\frac{1}{4}$ of the original area. Draw lines, AB and AC, at right angles to each other from A towards B and C, set off a equal spaces as A—1, 1-2, 2-3, 3-4, 4—5, etc., on the line AB, and A—1, 1-2, 2-3, 3-4, 4-5 etc., on the line AC; and rule parallels forming a net work of small and equal squares as shown. Next draw upon the paper for the copy, a similar set of squares but having

proposed reduction or enlarge-

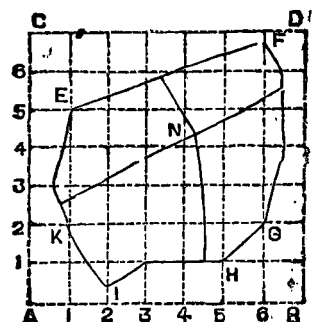


FIG. 580 —Reducing or Enlarging by Squares.

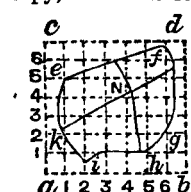


FIG. 581.—Reducing or Enlarging by Squares.

each side only one-half the length of the former (Fig. 581). It will now be clear that if the lines EF, FG, GH, etc. (Fig. 580) be drawn in the corresponding squares in Fig. 581 as ef, fg, gh, etc., a true copy on half the scale will be obtained. With the pencil draw the outline of the estate carefully as in Fig. 581, taking care that each portion of the figure occupies a corresponding square on the original. For example, the point G being at the junction of vertical 6 and horizontal 2 on the original drawing, it must occupy a similar position on the copy, and so on for other points. The process of enlarging plans, by means of squares, is similar to the above excepting that the points on the smaller squares are to be determined first and transferred to the larger squares for the copy. The process of enlarging, under any circumstances, does not, however admit of the same accuracy as reducing. The student should remember that when a plan is reduced to $\frac{1}{2}$ the scale the area is reduced to $\frac{1}{4}$ th, or if the scale is $\frac{1}{3}$ of the original, the area will be $\frac{1}{9}$, and *vice versa*, if the drawing is enlarged.

Ex. 1. A plan on a scale $\frac{1}{4800}$ is to be copied to $\frac{1}{1800}$.

Draw squares on the original with sides of, say, half an inch. For the copy draw the same number of squares: suppose, there are twelve such on the original copy occupying 6", therefore these will occupy $(4800 \times 6") \div 1800 = 16"$. Divide this into twelve for the size of the squares.

Ex. 2. A French plan, the scale of which is 6' long and consists of 600 metres, is to be copied to the scale of 8' to a mile. The metre is 39'37043".

$$\text{R. F. of plan} = \frac{6''}{600 \text{ metres}} = \frac{6}{600 \times 39'37043} = \frac{1}{3937'043}.$$

$$\text{R. F. of copy} = \frac{8}{63360} = \frac{1}{7920}.$$

If we draw $\frac{1}{2}$ squares on the original; 12 squares on the copy will occupy $\frac{3937'043 \times 9}{7920} = 4'473''$. Divide this into twelve for the size of the squares on the copy.

CHAPTER XII.—THE SLIDE RULE.

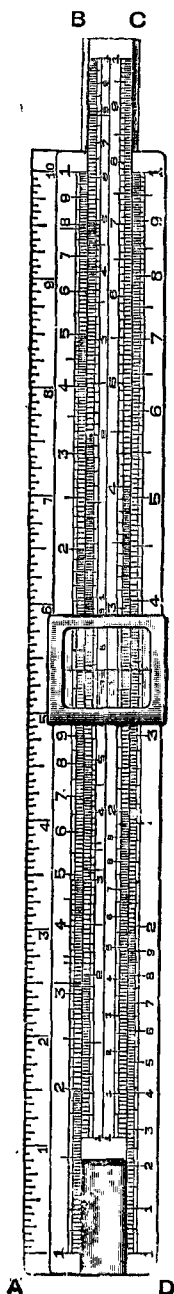


Fig. 582.—Slide Rule (Halden's) Showing Minuteness in Graduation.

755. (1) Slide Rules are instruments for performing logarithmic calculations mechanically, and are extensively used in Europe and the United States, specially where only rough approximations are required. It consists of three parts. The **stock or body** : AD (Fig. 582). 2. The **slide**, BC, which slides in the groove of the body. 3. The **cursor or runner**, E, arranged to slide freely to and fro, and which is used to locate any reading on the body or slide. This is effected by means of the black line marked on the glass. There are scales on both sides of the slide. The body scale, A, and its adjacent slide scale, B, are similar, as also are the slide scale, C, and its adjacent body scale D. They are known as A, B, C and D scales. The slide rules are made of mahogany or box-wood, about 10" long $\times$ $1\frac{1}{4}$ " wide $\times$ $\frac{3}{8}$ " thick, faced with celluloid strips on which the scales are engraved. These celluloid scales present the advantages of remaining unchanged, or at all events without the slightest relative change in varying temperature, and as the graduations are in back lines on a white ground, reading is not tiring to the eyes. Pocket rules 5" long, and office rules 20" long are also made.

Students must have slide rules by them as they read through the following explanations. The most elaborate diagram is not to be preferred to the rule itself.

(2) General Theory.—The slide-rule rests upon two simple principles : first, that magnitudes may be represented by the length of lines ; second, that these lines, when measured off upon one another may represent by the length of a resulting line, either a summation or a difference of the magnitude which the lines represent. The first principle is made use of in the logarithmic graduation of the scales ; the second principle finds application in the sliding motion which we impart to the scales. Slide rules have been constructed of many kinds and for many purposes, but they will all be found to reduce to these two elementary principles. The method in which the logarithmic graduation is carried out is explained as under :—

The numbers and their logarithms between 1 and 10, etc., are :

Numbers	...	1	2	3	4	5	6	7	8	9	10	11	12	etc.
Logarithms	...	0	.301	.477	.602	.699	.778	.845	.903	.954	1.000	1.041	1.079	

The upper part of the rule, *i.e.*, the part above the slide consists of two scales perfectly alike. That part of the scale between 1 on the left-hand and the middle 1 constitutes the left-hand A scale, and that between middle 1 and right-hand 1 the right-hand A scale (Fig. 582). Similarly the corresponding scales on the slide are the left-hand B scale and the right-hand B scale. The left-hand A scale is assumed as a *unit of length*, and the distances 1—2, 1—3, 1—4, etc., are laid off to scale to equal the mantissæ of the logarithms of 2, 3, 4, etc., to 10, or in other words, if the whole length between left-hand 1 and middle 1 be imagined divided into 1000 equal parts, the graduation marked 2 would be placed at the 301st division, since the logarithm of 2 is 0.301. Similarly division line 3 would be placed at the 477th, and line 4 at the 602nd division, and so on. For the sake of convenience in calculation, the number itself is stamped on the rule instead of its logarithm which the mark really represents. The same principle determines the positions of the other divisions of this, and also of the other scales, while it applies equally to the several subdivisions of all.

With such a contrivance as this, suppose it is required to find the result of multiplying 2 by 4 mechanically. The logarithm of 2 is .301. Adding this to .602 the logarithm of 4, we get .903, the logarithm of 8. With the slide rule this addition may be mechanically effected by drawing the slide B to the right until the left-hand index (1) of scale B coincides with 2 on scale A. Then over 4 on

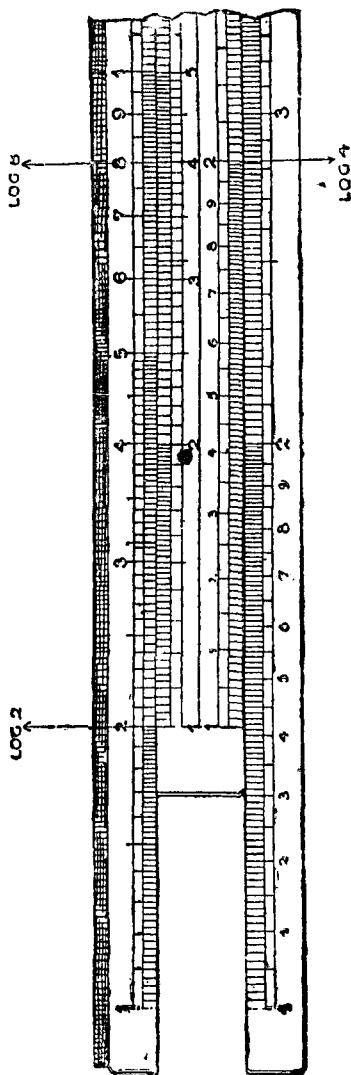


FIG. 583.—Portion of Thornton's Rule showing Theory of Multiplication and Division.

B is found 8 on A. With the slide so set, it is clear that to a length 1—2 on A, there has been added a length 1—4 on B, the result of which addition is at once read off on A over 4 on B. The same setting of the rule shows that in dividing 8 by 4, we take from a length 1—8 (representing 8) on A, a length 1—4 (representing 4) on B, the result, 2, being read off on A over the left-hand index 1 on B (Fig. 583). Instead of the anti-logarithms, 1, 2, 3, etc., there might have been written at the divisions the logarithms of these numbers, in which case a table of logarithms would have had to be employed. The use of the device here described not only enables logarithmic computations to be performed, but also dispenses with the use of a table of logarithms, as it is apparent that our scale represents graphically, a table of logarithms for all whole numbers between 1 and 10, with suitable sub-divisions.

(3) **Construction and Notation of the Slide Rule.**—The slide rule is shown in Fig. 582. The working parts are as follows: A thin slab of box-wood, called the *slide*, the upper surface of which is level with that of the rule, and upon the edges of which two scales, *B* and *C*, are engraved. The slide being fitted with tongued-and-grooved edges at its sides, is free to move between two other box-wood surfaces also bearing scales *A* and *D*. The latter are parts

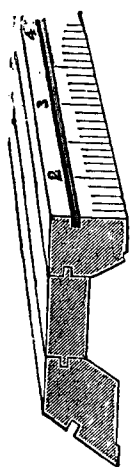


FIG. 584.

of the same piece of wood, being connected with each other under-neath the slide, (Fig. 584), and both of these together with the connecting boxwood member, form the "rule" into which the slide is recessed laterally while left perfect freedom of motion lengthwise, in both directions. The scales *C* and *D*, as has been said, are exact duplicates of one another, as are also scales, *A* and *B*, thus forming two pair of scales. The latter pair, in principle of graduation correspond to the former pair entirely, but are graduated to one-half the scale (or length of unit) of *C* and *D*. This reduction* in scale would make each of the upper scales one-half the length of the lower pair, were it not that we utilize the remaining half by engraving thereon another duplicate set of smaller scales, placed alongside of the former; thus making the total length of the upper *double pair* exactly that of the *single* lower pair. Each half of either "double scale," *A* or *B*, is not to be regarded as separate from its neighbor, but as joined to it, so as to form one continuous scale; the idea being to allow the double scales to represent all numbers for an interval of two whole powers of 10, while the lower scales represent all numbers for half that interval; if the lower scale embraces, for instance, the period from 1 to 10, then the upper similarly represents the period from 1 to 10 on the first (or left-hand) halves, while the second (right-hand) halves simultaneously represent the numbers from 10 to 100. The only other lines on the face of the rule are on

the left-hand *A* and *B* scales, indicating the ratio of the circumference of a circle to its diameter :— $\pi \doteq 3.1416$; and another special division is on the right-hand *A* and *B* scales, marking the place of $\pi \div 4 = 0.7854$, and is used in finding areas of circles.

Reverting again to the several graduations of *A* and *B* scales, it will be seen, that all the major spaces 1-2, 2-3, 3-4 to 90-100 are each divided into 10 minor spaces each reading .1 (Fig. 582), but since the lengths of the successive major divisions rapidly diminish it becomes impracticable to sub-divide each of these to the same extent as is possible with 1-2 or 10-20. The minor spaces within 1-2 are each divided into 5 parts, each must therefore read .02, and that between 2-3, 3-4, 4-5 into 2 parts, each representing .05. Now assigning to the left-hand (1) of *A* or *B* scale the value of unity or 1, the sub divisions represent 1.02, 1.04, 1.06, 1.08, 1.10, 1.12, 1.14, 1.16, 1.18, etc. to 2.00, 2.05, 2.10, 2.15, 2.20, 2.25, 2.30, etc., to 4.95 and 5.00. From 5 the readings are :—5.1 5.2, 5.3, etc., to 9.9 and 10.0. Precisely the same plan of sub-division is adopted in the right-hand half of the *A* and *B* scales, but in this case every line of graduation is of tenfold value as compared with those just mentioned ; so after the centre 1, which now reads 10, the suc-

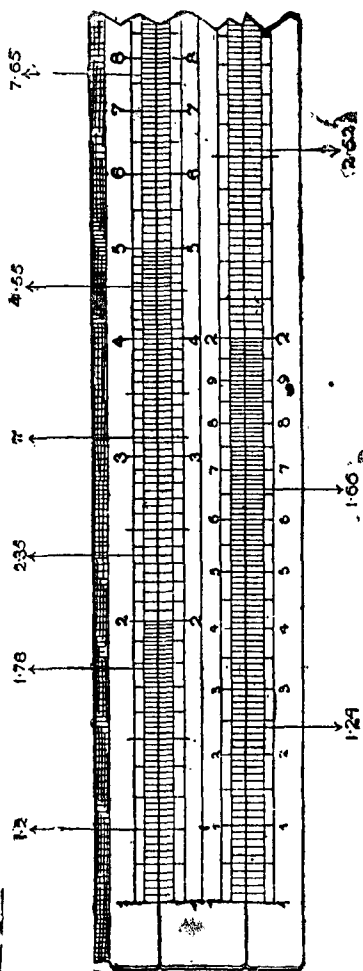


FIG. 585.—Thornton's Rule showing some Marked Graduations.

cessive sub-divisions are 10.2, 10.4, 10.6, 10.8, 11, 11.2, 11.4, 11.6, 11.8, etc., to 20.0, 20.5, 21.0, 21.5, 22.0, 22.5, 23.0, etc., 50. The remaining sub-divisions are read 51, 52, 53, 54, etc., 100 (the right-hand 1). Although further sub-division of any of the spaces is not practicable with this length of the rule further sub-division of the several spaces can be easily effected by the eye. With a little

practice the student will soon acquire expertness in correctly reading off any intermediate value by estimation. In this connection he will find Figs. 585 and 586 of material value. In these positions of 1.2, 1.78, 2.35, π , 4.55, 7.65, 14, 19.4, 28.5, 49 and 76 are clearly indicated.

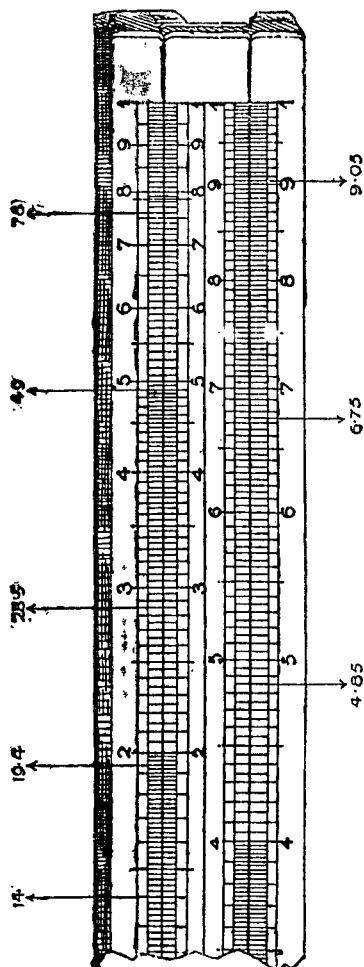


FIG. 4

Fig. 586.—Thornton's Rule showing some Marked Graduations.

Turning attention now to the *C* or *D* scale, it will be seen that it resembles the left-hand half of the *A* scale, but as it is of twice the length of that scale, the length of the main space, 1-2, is here sufficient to divide each of the 10 sub-divisions to be again divided into 10 parts, each showing .01. In the main spaces, 2-3 and 3-4, the sub-divisions are divided into 5 parts, each showing .02. The remainder of the major divisions are divided into 20 parts, each showing .05. Giving the left-hand index (1) of *D* scale, a value of 1 the successive sub-divisions would be read 1.01, 1.02, 1.03, 1.04, 1.05, 1.06, 1.07, 1.08, 1.09, 1.10, 1.11, 1.12, 1.13, 1.14, 1.15, 1.16, 1.17, 1.18, 1.19, etc., to 2.00, 2.02, 2.04, 2.06, 2.08, etc., to 3.98 and 4.00. From 4 the readings are 4.05, 4.10, 4.15, etc., to 9.95 and 10. On Figs. 585 and 586, the values of 1.24, 1.66, 2.52, 4.85, 6.75, 9.05 are marked for illustration. Either the upper scales, *A* and *B*, or the lower pair, *C* and *D*, may be employed for multiplication, division, proportion, etc. The upper

scales are to be preferred as they provide a much larger range of values to be read off without a second movement of the slide being necessary, but they give approximate results, while the lower ones being of double the length of *A* and *B* give more accurate results. The value of a division depends on the value assigned to the first 1 on either pair of scales. If we had given it the value of 10 or 100 we would have altered all the values of the

numbers represented on the scales. We would, in fact, have multiplied them all by 10 or 100 respectively. If the first 1 is made equal to 10^{-1} or 10^{-2} (that is $\frac{1}{10}$ or $\frac{1}{100}$) we divide all the values by 10 or 100 respectively. But in calculations involving the combined use of the upper and lower scales the value assigned to first 1 of the *D* scale fixes the value of all the divisions on the rule. If we assign to the first 1 the value of 10, the indicated numbers, in Figs. 585 and 586, would be 12, 17.8, 23.5, 10π , 45.5, etc.; if 100 then 120, 178, 235; 100π , 455, etc.; if $\frac{1}{10}$ or $\frac{1}{100}$ then the numbers indicated would be .12, .178, .235, π etc., or .012, .0178, .0235, .0 π , etc.

(4) **The Mathematical Relation between the Upper and Lower Scales.** A and D, of the slide rule depends on the following properties of logarithms: If the logarithm of a number is multiplied by 2 the product is the logarithm of the *square* of the number, and that, *vice versa*, if the logarithm of a number be divided by 2 we get the logarithm of the *square root* of the number. It has been said that each of the *A* scales is only one-half the length of the *D* scale. Therefore starting at the first 1, the lengths representing the logarithms of the numbers on the *A* scale are equal to those on the *D* scale multiplied by 2, and therefore equal to the logarithms of the squares of the numbers on the latter scale. Therefore opposite any number on *D*, we find its square on *A*, and, conversely, that under any number on *A* is its *square-root* on *D*. The cursor or runner fitted to the rule will show that 2, 3, 4, 5, 6, 7, 8, 9 are exactly below 4, 9, 16, 25, 36, 49, 64, and 81. This holds good for all intermediate values on the rule, *e. g.*, the *square* of .37 is 1369, and the *square root* of 10.8 is 3.28. In $37^2 = 1369$, the last figure 9 is got not from the rule but by noticing that $7 \times 7 = 49$. The last digit is generally mentally fixed. See Fig. 587.

(5) **Divisions in Millimeters, etc.**—The scale (Figs. 582-587) on the feathered-edge, divided into millimetres, runs up to 260 millimetres, on the edge of the rule; while the straight-edge has a scale of inches divided into eighths or tenths. Either one or other of these scales is engraved at the bottom of the groove in such a manner that by withdrawing the slide to the right and using the scale at the bottom of the groove, in conjunction with the corresponding scale on the edge it is possible to measure 20" on the one hand, or nearly 510 millimetres on the other.

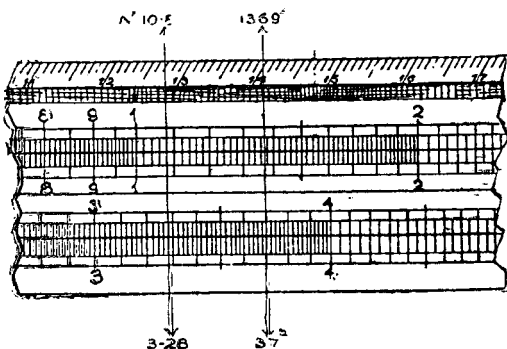


Fig. 587.—Portion of Thornton's Slide Rule showing Mathematical Relation between of the Upper and Lower Scales.

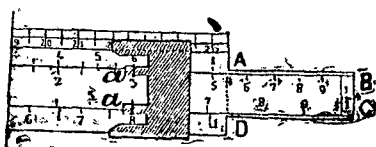
(6) **Construction of the Cursor or Runner.**—In some old

FIG. 588.—Chisel pointed Cursor.

line of the scales as required.

The modern form (Fig. 589) consists of a light square frame of metal, *E*, containing a piece of glass or mica, about 1 inch square, across the centre of which is a fine black line.

The cursor slides freely along the rule in grooves on its outer sides. The index line accurately transfers points from one scale to another. If it moves irregularly, the outer grooves should be examined and any obstruction removed. Occasionally a little tallow may be applied to the retaining edges of the cursor, and also to the grooves in which the slide moves.

Broken Hair-line Cursor.—With the full line cursor (Fig. 589) it has been found difficult to fix the decimal point. To remedy this defect the broken line has been designed; this enables the cursor be fixed with certainty on any reading.

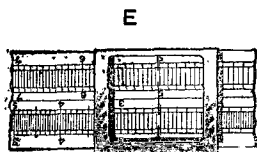


FIG. 589.—Thornton's Broken Hair-line Cursor.

Davis-Goulding's Improved Cursor.—This (Fig. 590) is a device which enables an operator to mechanically sub-divide a space within which the reading falls, and which would otherwise have to

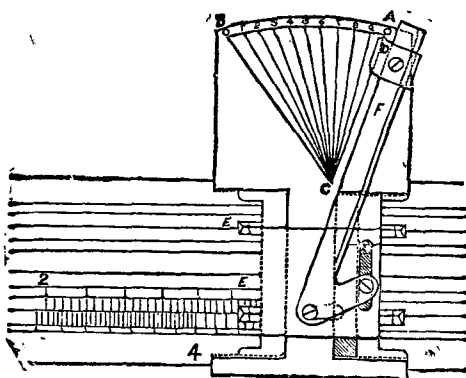


FIG. 590.—Goulding's Cursor (Davis').

be estimated by the eye when the third or fourth figure of a reading is required. It consists of a metal frame fitting in two of the grooves of the rule, and is kept square with, and in position on the rule by a small spring bearing on the edge above the upper scales of the rule; in this frame is another slide having four projections with chisel-edges and index-marks, *E*, *E*; this inner slide can be

moved forward by a lever *F*, carrying a pointer, *D*, underneath which is a triangular scale, *ABC*, divided into 10 equal parts at the base, *AB*. The line *BC* being parallel to the lever *F*, the pointer *D* will be on this line at any point from *B* to *C*; if at *B* and the lever, *F*, be

moved until the pointer, *D*, arrives at *A*, then the pointer, *D*, will have passed over 10 of the largest spaces, and the side will move through a distance equal to the largest space; we shall require to sub-divide, *viz* : 4 to 4'1 on the lower scale of the rule. Now, as the 10 divisions at *AB* vanish at *C* it is clear that the nearer *C* is approached with the pointer, *D*, the smaller will be the space passed over in 10 divisions, thus we may sub-divide and space on the rule. In using the cursor on spaces representing $\frac{3}{10}$ ths on the rule, we must call our divisions on the cursor $\frac{2}{10}$ ths each, as we are subdividing $\frac{3}{10}$ ths into 10 parts, and not $\frac{1}{10}$ as in the other case. Examples will make this clear :—

Ex. 1. The product of $167 \times 25 = 4175$ is required. To illustrate the method of use, set 1 of the slide to 167 on the lower scale and read the product under 25 on the slide. Now this is seen to be between 4150 and 4200, so set the pointer, *D*, on the cursor to the line *BC*, and bring the reading edge *E* to 42 on the rule; now move the lever *F* until the reading edge is at 41, and set the pointer, *D*, to the line *AC*, draw the lever back until the reading edge *E* is at 25 on the slide, then the pointer *D* will be at 75 on the cursor, which with 4100 on the rule, will give 4175, the product required.

Ex. 2. The product of $647 \times 18 = 11646$ is required. Proceed as before and read the product on the rule under 18 on the slide, set the cursor to 11700 on the rule, and move the lever, *F*, until the reading edge *E* is at 11600. Set the pointer *D* to line *AC* (in this case it will be much nearer to *O*), and draw lever *F* back until the reading edge *E* is at 18 on the slide, then read 46 on the cursor, which with 11600 on the rule will give 11646.

Care must, be taken to always start with the pointer over the line *BC*. With practice this may be carried to as many as 7 figures in the product.

Davis' Magnifying Cursor.—

In this the cursor line is set in a plano-

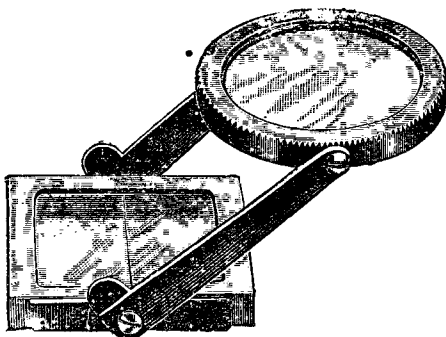


FIG. 592.—Thornton's Magnifier.

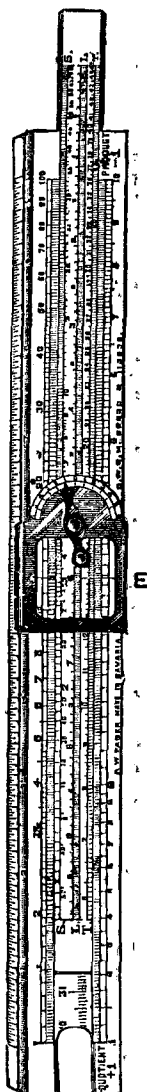


FIG. 591.—Slide Rule, Slide reversed showing Digit registering Cursor and Scales S. L. T.

convex glass, the magnifying effect of which is certainly helpful.

A. W. Faber's Digit-registering Cursor is supplied with a semi-circular scale, on the right side of cursor frame, (Fig. 591), running from 0 at the centre upward to -6, and downward to +6.

A small hand registers the number of digits to be added or deducted at the end of a lengthy computation.

(7) **Magnifier.**—Fig. 592 illustrates an adjustable magnifying glass with metal rim and arms which are riveted to the sides of the cursor for more accurate reading of the line on the cursor and exact reading of the decimal point on the rule.

(8) **Logarithmic Scales of Natural Sines and Tangents, etc., on the Back of the Slide.**—The slide can be completely run out of the rule and re-inserted, when so desired, reverse side up. The reverse side of the slide bears three scales (Fig. 591) one marked S, another marked T, and a third marked L:—

(9) **The Scale of Sines.**—The upper of these, marked S, is the logarithmic Scale of Natural Sines and is used to determine the natural sines of angles of from $35'$ to 90° . The graduations equal the mantissa of the log sin. of the several minutes and degrees. The first and 2nd numbers marked are $40'$ and $50'$. The main divisions 1, 2, 3, etc., represent the degrees of angles. The main spaces 1–2, 2–3, 3–4, 4–5...to 9–10, are divided into 12 parts, each of these will be read as $5'$. In the main spaces 10–15 and 15–20 the degrees are sub-divided into 6 parts, each giving $10'$. From 20 to 40, degrees are sub-divided into 2 parts, each giving $30'$. From 40, only degree spaces are marked. This *sine* slide is drawn to the same scale as A and B on the front of the rule. In order to determine the sine of an angle the S scale is placed adjacent to A scale with the right and left-hand 1s (indices) coinciding.

(10) **The Scale of Logarithms.**—The evenly-divided scale, marked L, equal in length to that of the scale D, in the centre of the reverse of the slide, bearing equal divisions (each division = $\frac{1}{100}$ or 02) or (1–500 of the scale length) answers the purpose of a table of ordinary 3–place printed logarithms, in which the numerical value of any logarithm may be directly read off the scale. The figures on the scale increase in an opposite direction to those of the other scales. The *mantissa* of the logarithm only can be read off the L scale, the characteristic being determined by the following rule:—*The Characteristic of a logarithm is equal to the number of digits in the number, minus 1. If the number is wholly decimal, the index is equal to the number of ciphers following the decimal point plus 1.* In the latter case the characteristic is *negative*, and is so distinguished by having the minus sign written over it. Figs. 593 and 594 shows logs of numbers on D scale.

(11) **The Scale of Tangents.**—The lowest scale on the back of the slide, marked T, is used for determining the natural tangents of angles. The scale commences at about $5^\circ 43'$, or at the angle whose tangent is 0.1. The other end of the scale runs up to 45° , or the angle whose tangent is 1. In some instruments, the graduations commence with $34'$, then the T scale is to be used with the right or left A scale; when they begin with $5^\circ 53'$ it is to be used with D scale. In the former case the slide is to be placed in the

rule with the $\top$ scale adjacent to A scales, and, with the right and left is coinciding, when opposite any angle on $\top$ will be found its tangent on A (Fig. 595). It should be noticed that the tangents read on the left-hand

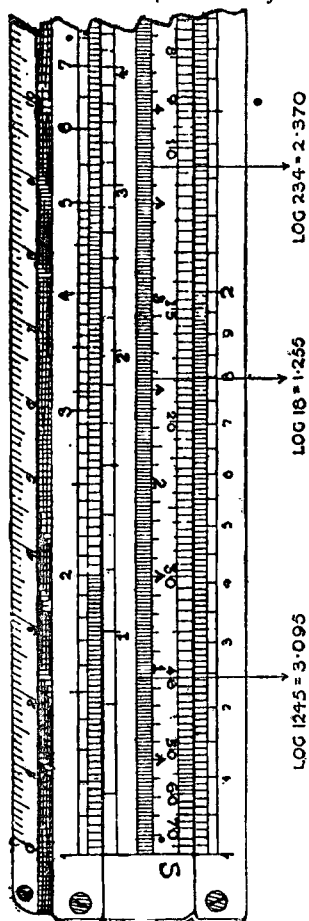


FIG. 594.—Slide Rule Showing Logs of Numbers.

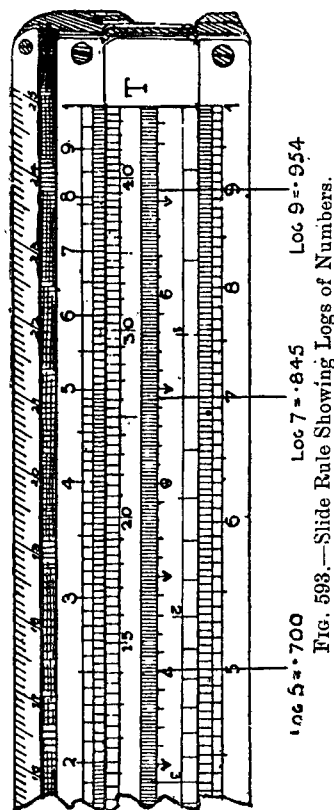


FIG. 598.—Slide Rule Showing Logs of Numbers.

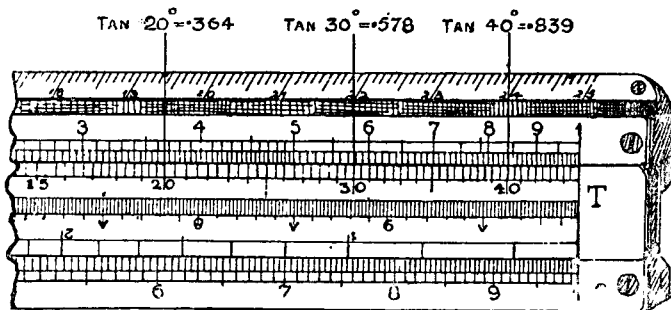


FIG. 595.—Slide Rule Showing Tangents of Angles.

A scale have values 0.01 to 0.1; while those on the right-hand *A* scale have values from 0.1 to 1.0. Otherwise expressed, to the values of any tangent on the left-hand *A* scale a cypher is to be prefixed; while on the right-hand *A* scale it is read directly as a decimal. The scale *T* is drawn to the same scale as that of *C* and *D*. From 5° 45' to 20° the degree divisions are sub-divided into 12 parts each showing 5', from 20° the degree spaces are sub-divided into 6 parts each giving 10'. All the numbers on this scale refer to degrees, the first one on the left being 6.

(12) Proportion.—The solution of problems involving proportion or ratio of quantities can be easily and expeditiously effected by the aid of the slide rule; either the upper or the lower pair of scales may be employed, depending upon the degree of exactitude demanded. Employing the *C* and *D* scales, it will be seen that with the left-hand indices of *C* and *D* coinciding, the ratio between all coinciding divisions of these scales is *unity* (1). If, however, the slide be moved to the right, so that the index of *C* coincides with 3 on *D*, it will be seen that the numbers on *D* and *C* will be in a ratio of 3 to 1. The same relationship will exist between other numbers on these two scales. The rule for proportion may be stated as follows:—

Rule for Proportion.—*Set the first term of a proportion on the C scale to the second term on the D scale, and opposite the third term on the C scale read the fourth term on the D scale.*

Ex. 3. What is the fourth term in the proportion

$$15 : 40 :: 16.5 : x ?$$

Draw the slide so that 15 on *C* coincides with 40 on *D*, and opposite 16.5 on *C* read 44 on *D*; or employing *A* and *B* scales, set the first term 15 on *A* scale immediately above the 2nd term 40 on scale *B* and the value of the fourth term $x = 44$ is on scale *B* underneath the 3rd term 16.5 on scale *A*.

This method may be modified so as to obtain from *any three* known terms of a proportion the remaining term of the expression. If the 2nd, 3rd, and 4th terms of the above example are given the procedure will be as under:—Set the 3rd term (16.5) on *C* to the 4th term (44) on *D*, and opposite the 2nd term (40) on *D* read the 1st term (15) on *C*. It should be borne in mind that the 1st and 3rd terms are always on one scale, and the 2nd and 4th on the other.

It should be noted that owing to the constant proportion existing between the numbers on the two scales, the fraction $\frac{\text{1st term}}{\text{2nd term}}$ may

be expressed as $\frac{1}{x}$, or as its reciprocal $\frac{x}{1}$, the value of x in the first

case being read on *D* under the index of the slide, and in the 2nd case on *C* over the index of *D*. This fact may be turned into practice in reducing vulgar fractions to decimals. Thus the decimal equivalent of $\frac{2}{16}$ is determined by placing 2 on *C* to 16 on *D*, when

over the index or 1 of D we read '125 on C . In this case the terms are $2 : 16 :: x : 1$. For the *inverse* process—to find a vulgar fraction equivalent to a given decimal—the given decimal fraction on C is set to the index of D , then opposite any numerator on C is found the corresponding denominator of the fraction on D .

If the index of C be set to 3.1416 on D , this ratio exists throughout between the numbers of the two scales. Hence against any diameter of a $\odot$ on C will be found the corresponding circumference on D . As exact setting can not be made owing to the π not being marked on the D scale, the slide may be set by some more exact convenient ratio. For example, instead of π , the ratio $\frac{22}{7}$ or a more correct value $\frac{22}{7} \frac{1}{10}$ is often employed by bringing 226 on C to 710 on D . The exact position of π is marked on the A and B scales. With the index of B scale placed under π on A , the diameters of $\odot$ s taken on B will have immediately above them their circumference on A .

Ex. 4. What is the circumference of a $\odot$ 2.55 feet in diameter? Draw the slide to the right until 1 on B coincides with π on A , and above 2.55 on B read 8 the required $\odot$ ce. The slide rule may be turned to practical advantage by determining in equivalent values of other denominations of various weights and measures by the aid of a table of conversion factors (see Table X in Part III of this work.)

Ex. 5. Convert 3.15 inches into Centimetres.

As 1 inch equals 2.54 centimetres, place 1 on B to 2.54 on A , when above 3.15 inches on B will be found 8 centimetres on A .

In the above example, the left-hand index of both A and B scales have a value of 1. Frequently, however, the two scales used in conversions of this kind differ considerably in value. Thus, in converting miles to metres, or *vice versa*, the left-hand index of B showing 1 mile would be set to 1610 on A , representing the equivalent number of metres.

Ex. 6. Convert 5.9 miles into metres.

Set 1 on B to 1.61 on A , then over 5.9 on B will be found 9499 metres on A . It should be noted that while the B scale is read as engraved, all the divisions of the A scale will have a thousandfold value.

(13) Proportion with the Slide inverted : Inverse Proportion.—If *more* requires *less* or *less* requires *more*, the case is one of inverse proportion. This form of proportion is easily dealt with by the preceding method, but the working is somewhat simplified by inverting the slide so that C scale is adjacent to A scale. With the runner the values on the inverted C (written C') scale and on the D scale can be read off. These will form a series of inverse ratios. Thus, in the proportion $3 : 4 :: 6 : 2$, the 3 on C' is placed opposite 4 on D , when under 6 on C' is seen 2 on D .

(14) Multiplication which is only a form of proportion in which one of the terms is 1, has now to be taken up. The rule is as under:—

Rules for Multiplication.—Set the index of the B scale to one of the factors on the A scale, and over the other factor on B read the product on A ; or, set the index of C to one factor on D , and under the other factor on C read the product on D .

Ex. 7. Multiply 4.5 by 1.4 . Using the *A* and *B* scales, 1 on *B* is set to 4.5 on *A*, and over 1.4 on *B* is read 6.3 on *A*.

Ex. 8. Multiply 125 by 28 . Using the *C* and *D* scales, and placing 1 on *C* to 1.25 (read as 125) on *D*, we read under 2.8 (read as 28) on *C* 3.5 (read as 3500) the required product on *D*.

If by using the left-hand 1 the factor on the slide is beyond the end of the rule, the right-hand 1 must be used.

Ex. 9. Multiply 72×47 . Set right-hand 1 on slide to 72 on rule, and read 3384 on rule under 47 on slide: the last figure is not marked on the rule, but we know it to be 4, because we can mentally multiply the last two factors $7 \times 2 = 14$, thus we get 4 for the last figure of the product.

These examples demand the necessity for taking care to assign the correct value to the result. The rule for the position of the decimal point is:—

Rule for the Number of Digits in a Product.—*The number of digits in the product is equal to the sum of the digits in the two factors if the answer is obtained with the slide projecting to the left; if, however, the slide projects to the right the number of digits in the product is equal to the sum of the digits in the two factors less 1, i. e., if the left-hand 1 of the slide be used, the number will be one less than the number of figures in the two factors; if the right-hand 1 be used, the number will equal the number of figures in the two factors.*

The application of these rules can best be indicated by some examples:—

Ex. 10. $25 \times 32 = 800$.

In this case the product is obtained by projecting the slide to the right; the number of digits in the product therefore equals $2 + 2 - 1 = 3$.

Ex. 11. $25 \times 75 = 1875$. Here the product is obtained by projecting the slide to the left, so that the number of digits in the product $= 2 + 2 = 4$.

Ex. 12. $3.2 \times 25 = 80$.

Here the slide projecting to the right the number of digits is $1 + 2 - 1 = 2$.

Ex. 13. $0.025 \times 0.75 = .01875$. Here the slide projecting to the left, the number of digits is therefore $-1 + 0 = -1$.

Ex. 14. $0.000125 \times 0.005 = 0.000000625$.

Here the slide projects to the right, to give the product, the number of digits is therefore $(-3) + (-2) - 1 = -6$.

The multiplication sums are better performed by means of scales *C* and *D*, because there, the divisions are much wider, but the graduations of the *C* and *D* scales do not admit of dealing with more than three figures in either factor; and for the same reason it is impossible to directly locate more than the 1st three figures of any product, although it is often possible, by mentally dividing the smallest space involved in the reading, to correctly determine the 4th figure of a product. This method can only be depended on in the earlier parts of the *C* and *D* scales. However, the last figure of a three-figure, and in some cases the last of a four-figure product, can be easily determined by an inspection of the factors.

Ex. 15. The product of $26 \times 29 = 754$ is required. Placing left 1 on *C* to 26 on *D*, we find opposite 29 on *C*, the product which lies between 750 and 755. By an inspection of the factors it is clear that the final figure is 4; since the product of $9 \times 6 = 54$, and the correct product is therefore 754.

Ex. 16. In such a case as $42.3 \times 3.7 = 156.5$.

We place left 1 on *B* to 3.7 on *A*, and find the result on *A* over 42.3 on *B*. This is seen to be more than 1560 and less than 1570, but it is clear that the final figure is 5, and the correct product is 156.5 or 156.51 the real value.

In cases where more than three figures form either or both of the factors,

the fourth and following figures must be neglected. If the first neglected figure is 5 or greater than 5, the third figure of the factor should be increased by 1. Generally this increase in one of the factors will be sufficient, but in some cases greater accuracy will be the result if both factors are increased in this way.

(15) **Continued Multiplication—Use of the Runner.**—With the runner the operation is continuous, so that not only is time saved but much greater accuracy secured, since the errors due to the several separate settings are altogether avoided.

Ex. 17. Find the product of $6 \times 2.5 \times 5 = 75$. Using the upper scales, set 1 on *B* opposite 6 on *A*. Slide the runner down until its line covers 2.5 on *B*. Slide *B* to the right until its division one comes under the runner. Slide the runner to cover 5 on *B* and over 5 will be found 75 on *A*.

Ex. 18. Find the product of $3.5 \times 85 \times 4.4 \times 2.5 = 3272$. Using the lower scales, set the right-hand index of *C* to 3.5 on *D*, and slide the runner down until its line covers 85 on *C*. Bringing the right-hand index of *C* to the runner line, the latter to 4.4 on *C*, and the left-hand index of *C* to the runner, we read under 2.5 on *C* the required product 327 on *D*, the last figure being estimated by the eye. There are $1+2+1+1=5$ digits in the factors and as the slide was moved to the right once there will be $5-1=4$ digits in the product, which will therefore read 3270. The true product is 3272.5 showing close agreement between the actual product and the result obtained by the rule.

(16) **Division.**—As division is merely the inverse of multiplication the following is the rule :—

Rule for Division.—Set the divisor on *B* under the dividend on *A*, and read the quotient on *A* over the index of *B*; or set the divisor on *C* over the dividend on *D*, and read the quotient on *D* under the index of *C*.

Ex. 19. Divide 4.65 by $3.5 = 1.33$.

Using the upper scales, place 3.5 on *B* under 4.65 on *A*, then over 1 on *B* is read 1.33 on *A*.

Ex. 20. Divide 5350 by 25.

Place 25 on *C*, over 5350 on *D*, and read 214 on *D* at left 1 of *C*; and as we are using the left 1 of the slide, we shall have one to add, so we get $4-2+1=3$ figures in the quotient, as the process is the inverse of that for multiplication.

(17) **Combined Multiplication and Division.**—The combination of the rules for multiplication and division as described before is very useful in solving problems of the following type :— $(a \times b) / c = x$. In such a case, x is divided by c by setting c on *C* to a on *D*. Then the multiplication of the quotient by b is effected by reading under b on *C* the required value of x on *D*.

Ex. 21. $(40 \times 16.5) / 33 = 20$.

Setting 33 on *C* to 40 on *D*, 20 is read off on *D*, under 16.5 on *C*.

More complicated cases are represented by the equation—

$$(a \times b \times c \times d) / (e \times f \times g \times h) = x$$

(18) **Advantage of the Cursor.**—It is in examples of this kind, the use of the runner saves much time, as by its aid, the result of such a lengthy calculation may be easily effected without looking up any of the intermediate values obtained from the successive steps in the operation.

Ex. 22. $(4 \times 3.6 \times 84) / (12 \times 2.7) = 37.3$.

Using the upper pair of scales, set 1 on *B* opposite 4 on *A*. Slide cursor to cover 3.6 on *B*. Move slide until 1 comes under the cursor. 84 on *B* is now found to have moved past the end of *A*. Therefore move *B* until right-hand 1 comes under the cursor. Slide cursor to cover 84 on *B*. Move *B* until 12 comes under the cursor. Slide cursor to cover 1 of *B*. Move *B* until

27 comes under the cursor. Slide cursor to cover 1 of *B*. Move *B* until 27 comes under the cursor. Opposite 1 on *B*, read on *A* the answer.

Ex. 28. $\frac{36 \times 68 \times 15 \times 44 \cdot 6}{44 \times 20 \times 4 \cdot 8 \times 9 \cdot 5} = 40 \cdot 8$. Using the lower pair of scales, set

44 on *C* to 36 on *D*, and slide the cursor to 68 on *C*. Bring 20 on *C* to the cursor, the cursor to 15 on *C*, 4·8 on *C* to cursor, the cursor to 44·6 on *C*, and 9·5 on *C* to cursor, then under the right-hand index of *C* read 40·8 on *D*.

Note.—The decimal point in the answer in such examples as these is best located after rough cancellation. Sometimes it will happen that when the slide has been finally set the result cannot be read off, for the reason that it falls beyond the end of the rule, either to the right or left. In such a case, the cursor is slid to whichever index of *C* is available and the slide moved until the other index coincides with the cursor, when the result can be read off. By changing the position of the factors, it will often be possible to avoid changing the position of the slide as would otherwise be necessary, owing to the reading falling outside the scale.

(19) **Multiplication and Division with the Slide inverted.**—By inverting the slide, so that *C* is placed below *A*, and setting the slide with the right and left-hand indices coinciding, it will be seen that the product of any number on *D* × the coincident number on the inverted *C*, (*C*¹) and which is readily shown by the cursor, is constantly 10. For this reason the numbers on *C*¹ are the *reciprocals* of those coinciding with them on *D*, and *vice versa*. Thus 2 on *D* is opposite 0·5 on *C*¹; 3 on *D* opposite to ·333; while opposite 8 on *C*¹ is 0·125 on *D*, etc. It will now be clear that if we apply the ordinary rule for multiplication, with the slide inverted, we shall be multiplying one factor on *D* by the *reciprocal* of the other on *C*¹. But multiplying by the *reciprocal* of a number is equivalent to *dividing* by that number. Again *dividing* a factor by the *reciprocal* of a number is equivalent to *multiplying* by that number. Hence it follows, that with the slide inverted the operations of multiplication and division are reversed.

In multiplying, therefore, with the slide inverted, place with the cursor one factor on *C*¹, opposite the factor on *D*, and read the answer on *D* under either index of *C*¹. It results that with the slide thus set, any pair of coinciding factors on *C*¹ and *D* will give the same constant product found on *D* under the index of *C*¹.

One useful application of this fact is found in selecting dimensions of given rectangular areas:—For by placing the index of *C*¹ to 14·4 on *D*, it is at once seen that an area of 1 bigha or 14400 sq. ft. may have sides 90 × 160 ft., 80 × 180 ft., 50 × 288 ft., 30 × 480 ft., with other innumerable values.

(20) **Squares and Square Roots.**—The relationship which exists between the upper and lower scales is such that just over any number on *D* or *C* will be seen its *square* on *A* or *B*, and conversely it follows that under any number on *A* or *B* is its *square root* on *D* or *C* (see p. 525).

Rule for Squaring a Number.—Set the cursor to the given number on *D* and read off the required square on *A* under the cursor.

Ex. 24. Find the square of 29.

Setting the cursor to 29 on *D*, the square is read on *A* as 841.

Ex. 25. Find the square of 76.

Setting the cursor to 76 on *D*, the square is read on *A* as 5776. In example

24 the result is read off the left-hand scale of *A*, while in example 25 it is read on the right-hand scale of *A*. In the 2nd case, the result is found to have *twice* the number of figures that the original number contained, while in the 1st case the number in the result is *one less* than *twice* that number.

Rule for Number of Figures in the Square of a Number is as under:—*If the result is read on the right-hand A scale, the number of digits is equal to twice the number of digits in the original numbers; but if the result is read on the left-hand A scale, the number of digits is one less than twice the number in the original numbers.*

The student should be careful in estimating the number of digits in the number and in the result when squaring decimals.

Ex. 26. Find the square of 0.0064.

Setting the cursor to 64 on *D*, the reading on *A* is seen to be 409 (4096); but to estimate the result correctly it should be noted that the number of digits in the original number is -2 , since 2 cyphers follow the decimal point. Therefore by the above rule the number of digits in the answer is $-2 \times 2 = -4$ (since the reading is taken on the right-hand *A* scale, so that the result is written .0000409 (.00004096).

Ex. 27. Find the square of .0125. Here the result read, 1562, is seen on the left-hand *A* scale, and the number of digits will be $(-1 \times 2, -1 = -3)$, the result is therefore read as .0001562 (.00015625.)

(21) Square Roots.—The operation is the inverse of that of squaring a number, and therefore it is only required to set the cursor to the given number on *A*, and to read off the required square root on *D* under the cursor.

It should be borne in mind that if the number consists of an *odd* number of digits, it is to be taken on the *left-hand A* scale, and the number of digits in the root is $(n+1)/2$, n being the number of digits in the original number. When, however, there is an *even* number of digits in the given number, it is to be taken on the *right-hand A* scale, and the root contains *one-half* the number of digits in the original number.

Ex. 28. Find the Square Root of 82369.

Set the cursor to 823 on *A* and read the result as 287 on *D*. The number of digits in the required root is $(n+1)/2 = (5+1)/2 = 3$ as shown above.

Ex. 29. Find $\sqrt{0.0035} = 0.591$.

Setting the cursor to 35 on the right-hand *A* scale (since -2 is an *even* number of digits) find 591 on *D*. The number of digits in the root will be $-\frac{2}{2} = -1$. The root is therefore read as .591.

Ex. 30. Find $\sqrt{0.035} = 0.187$. The number of digits is -1 , so on the left *A* scale, opposite 35, is found 187 on *D*. The number of digits in the root is $(-1+1)/2 = 0$. The root is therefore 0.187.

Ex. 31. Find $\sqrt{0.35} = 0.591$. The number of digits (0) is *even* and the cursor is set to 35 on the right *A* scale, showing 0.591 on *D*. By the rule, the number of digits in the root is $\frac{0}{2} = 0$, and the root will be 0.591.

The 2nd Method of Extracting Square Root is by using the *C* and *D* scales with the slide inverted. If the number has an *odd* number of digits, the *right* index, or if it has an *even* number of digits, the *left* index of the inverted *C* is placed in coincidence with the number on *D* of which the root is required. Then by sliding the cursor the number (the root sought) is read on *D* which is in line with the same number on *C*.

Ex. 32. Find the Square Root of 841 = 29. Setting the right index of *C* to 841 on *D*, 29 is the coinciding number on *D* and *C*.

(22) To Find the Cube of a Number.—In raising a number to its third

power, a combination of the preceding method of squaring and ordinary multiplication is used.

Rule for Cubing a Number:—Set the right or left index of *C* to the given number on *D*, and over the same number on the left *B* scale read the required cube on the left or right *A* scale.

Ex. 33. Find the Cube of $0.95 = 0.856$. Set the right-index of *C* to 95 on *D*, and over 95 on left *B* read 0.856 on *A* [0.856375..]

Ex. 34. Find the Cube of $1.35 = 2.460$. Set the left-index of *C* to 135 on *D* and over 1.35 on *B* read 2.46 on *A* [2.460375].

Ex. 35. Find the Cube of 3.5 . Set the left-index of *C* to 35 on *D*, and over 3.5 on *B* read 42.8 [42.875] on right *A*.

This method of cubing a number generally gives satisfactory results, although the operation may be carried out by continuous multiplication with the runner in the way already explained.

In many cases the number of digits in the product is self-evident, but the following rules will determine the number of digits in any cube, n being the number of digits in the number, and N the required number in the cube:—

$N = 3n - 2$, when the product is read on the left *A* scale, slide projecting to the right. $N = 3n - 1$, when the product is read on the right *A* scale, slide projecting to the right. $N = 3n$, when the product is read on the left *A* scale, slide projecting to the left. With the decimals the same rules apply, but, as before, number of digits must be read as -1 , -2 , etc., when 1, 2, etc., cyphers follow after the decimal point. Thus in Ex. 33, the result is read on the left *A* scale, with the slide projecting to the left. Here the number of digits will be 0 in the result which is therefore read as .856. In Ex. 34, the result is read on the left *A* scale, with the slide projecting to the right. Here the number of digits is 1 in the answer. In Ex. 35, the result is read on right *A* scale with the slide projecting to the right, the number of digits in the answer is therefore $3 + 1 - 1 = 2$.

● **(23) Cube Roots.**—The reverse process of cubing a number to obtain its cube root will now be clear using the same scales as before.

Rule for Extracting the Cube Root.—Move the slide, either to the left or right, until under the given number on *A* is found a number on the left *B* scale identical with the number simultaneously found on *D*, under the right or left index of *C*. This number is the required cube root.

By using the 4 scales mentioned before and in the way described it is possible to find 3 values for the root. To obtain the true value, it is necessary, as in arithmetic, to point off the given number into divisions of 3 digits, starting at the decimal point and proceeding to the left for numbers greater than unity, and to the right for numbers less than unity. Then, (1) if the 1st division of digits reading from the left consists of 1 digit, the number is to be taken on left *A* scale and the slide projecting to the right. (2) If of 2 digits the number is to be taken on right *A* scale with the slide to the right. (3) If of 3 digits, the number is to be taken on left *A* with the slide to the left.

Ex. 36. Find the Cube root of $18.6 = 2.65$.

According to the 2nd of the above conditions, slide the cursor to 18.6 on right *A*, move the slide to the right until it is seen that when the left-index of *C* is over 2.65 on *D*, 2.65 on *B* is also covered by the cursor.

(24) 2nd Method of Finding the Cube Root with the Inverted Slide.—The inverted slide is so set that what is now the right-hand index of slide coincides with the number of which the cube root is required; in this case 18.6. It is now necessary to find what number on *D* coincides with the same number on its adjacent inverted *B*, and this in the present case is found to be 2.65, which is therefore the root sought.

Ex. 37. Find by this Method $\sqrt[3]{140} = 5.195$.

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As the number contains 3 digits it is taken on the left *A* scale and to which the left-index of the inverted slide is placed, and 5.195 is the number (the required root) seen to coincide on the *D* and the inverted *B* scales.

(25) **Fourth Powers and Roots.**—To raise a number to its 4th power is to square the square of a number.

Rule for raising a Number to the Fourth Power.—Set the right or left index of *C* to the given number on *D*, and over the number on *C* read the fourth power on *A*.

Ex. 88. Find the value of 11^4 .

Setting the left-index of *C* to 11 on *D*, the result which is read on *A* over 11 on *C* is seen to be 14,650 [14,641]. The number of digits in the product may be ascertained by the rules for squaring a number.

The fourth root of a number may be extracted by reversing the process but it is preferable to extract the square root of the square root.

(26) **The Trigonometrical Applications of the Slide Rule.**—**The Use of the Scale of Sines.**—**First Method.**—When the *S* scale is placed adjacent to *A* scale, with the right and left indices coinciding, the sine of any angle may be read off on *A* immediately above the angle on *S*. It should be noted that the values of the sines of all angles read on the left half of *A*, i.e., between 1 and 10, are such that the decimal point is always followed by 0; while those read on the right half of *A*, between 10 and 100, are prefixed by the decimal point only. Thus:— $\sin.0^\circ40' = 0.0116$, $\sin.5^\circ40' = 0.0987$, $\sin.6^\circ = 0.1045$, $\sin.15^\circ = 0.259$, $\sin.21^\circ30' = 0.367$, $\sin.74^\circ = 0.961$.

(27) **Rule for the number of Integers in the Product, P, or Quotient, Q, is as follows, taking N = number of integers in the multiplier, M, or in the dividend, D, :—**

(a) When the result is obtained to the right of <i>M</i> or <i>D</i> , and in the same scale ...	$P = N - 2$	$Q = N$.
(b) When the result is obtained to the right of <i>M</i> or <i>D</i> , and in the other scale ...	$P = N - 1$	$Q = N + 1$.
(c) When the result is obtained to the left of <i>M</i> or <i>D</i> , and in the same scale ...	$P = N$	$Q = N + 2$.
(d) When the result is obtained to the left of <i>M</i> or <i>D</i> , and in the other scale ...	$P = N - 1$	$Q = N + 1$.

The method of calculating with the sine scale can best be illustrated by some examples :—

Ex. 39. Find the product of $25 \times \sin. 4^\circ20' = 1.89$.

Setting left 1 of *S* under 25 on *A*, the product 1.89 is read off on *A* above the line showing the angle $4^\circ20'$ on *S*.

Ex. 40. Find the product of $8.6 \times \sin. 21^\circ30' = 3.15$.

In this case, right-index of *S* is placed opposite 8.6 on left *A* scale when over $21^\circ30'$ on *S* is read 3.15 on *A*.

Ex. 41. Find the value of $175 \sin. 52^\circ = 137.9$.

Set the left end of *S* under 175. Next, on moving the cursor to cover 52° on *S*, that point is found to be beyond the right end of *A*. Therefore set the right end of *S* to 175 and opposite 52° of *S* read on *A* 138 [137.902.]

Ex. 42. Find the value of $25 \div \sin. 4^{\circ}20' = 330$.

$4^{\circ}20'$ on *S* is set to 25 on *A*, and over the left end of *S* is read 330.

Ex. 43. Find the value of $275 \div \cos. 75^{\circ} = 1061$.

Since $\cos. x = \sin. (90 - x)$, $\cos. 75^{\circ} = \sin. 15^{\circ}$; and the problem becomes $275 \div \sin. 15^{\circ}$. Bring the cursor over 275 on right *A*, and move under it 15° of *S*. Opposite the left-index of *S* read on *A* 1061.

Ex. 44. Given, two angles of a Δ , $A = 21^{\circ}30'$, $B = 42^{\circ}$, also the length of the side opposite to the angle B , $b = 700$: find the length of the sides a and c opposite to the angles A and C , $C = 116^{\circ}30'$.

The formulas are: $a = \frac{700 \times \sin. 21^{\circ}30'}{\sin. 42^{\circ}}$, and $c = \frac{700 \times \sin. 116^{\circ}30'}{\sin. 42^{\circ}}$.

To solve this problem bring 42° on *S* to 70 on *A*, and read off, on *A* over $21^{\circ}30'$ and $116^{\circ}30'$ the values of $a (= 383)$ and $c (= 936)$ respectively.

(28) 2nd Method of Finding the Sine of an Angle.—In this method, the slide need not be taken out of the rule and re-inserted, reverse side up. The back of the rule has a recess at the right end and upon the edges there are two index lines. With the upper one of these the sines of angles may be obtained thus:

Run out the slide to the right until the angle on the *S* scale coincides with the upper index line in the recess. Turn face of rule upwards, find the required sine on *B* under 100 of *A*. For example, $\sin. 1^{\circ} = 0.01745$, $\sin. 2^{\circ} = 0.0349$, $\sin. 2^{\circ}30' = 0.0437$, $\sin. 3^{\circ} = 0.0523$, $\sin. 4^{\circ} = 0.069$, $\sin. 5^{\circ} = 0.0872$, $\sin. 7^{\circ} = 0.122$, $\sin. 9^{\circ} = 0.1564$, $\sin. 13^{\circ} = 0.2249$, $\sin. 25^{\circ} = 0.423$, $\sin. 29^{\circ} = 0.485$, $\sin. 55^{\circ} = 0.819$, $\sin. 79^{\circ} = 0.982$.

Ex. 45. Find $\sin. 24^{\circ} = 0.406$.

The $\sin. 24^{\circ}$ is found on *A* over 24° on *S* of the reversed slide, the result being 0.406. The cursor is then slid to 0.406 on *D*, and the value of $\sin. 24^{\circ}$ is read off on *A* at the cursor line. This is seen to be 0.165.

(29) The Use of the Scale of Logarithms.—The logarithm of a number is found as follows:—Set the left-index of *C* to the given number on *D*. Turn the rule over, and read the mantissa of the logarithm on *L* at the lower index line in the recess at the right end of the rule

Thus $-\log 963 = 2.984$, $\log 96.3 = 1.984$, $\log 9.63 = 0.984$, $\log 0.963 = 1.984$, $\log 0.0963 = 2.984$.

Evidently, if the slide be taken out, and reversed, the logarithms can be readily read off. Thus in the above example, let the slide be taken out and replaced with its back to the front, and the figures on the *L* scale increasing in the same direction as those on *D*, the ends of the scales coinciding. Bring the cursor to cover 963 on *D*, and on the *L* scale the mantissa .984 is read off.

(30) The Opposite Process of Delogarithizing, i.e., to find a number having given its logarithm, is quite as simple. The rule is kept as before, and the process is reversed.

Ex. 46. Delogarithize the logarithm 2.468.

Set the logarithm 468 on the *L* scale to the index-line in the recess at the right end of the rule, and read 293.8 on *D* below left-index of *C*.

(31) Higher Powers and Roots.—This method of logarithizing numbers finds its practical application in determining powers and roots other than squares and cubes. The method to be adopted is shown by the following examples:—

Ex. 47. Find $\sqrt[3]{1893} = 2.94$.

Setting left-index of *C* to 1893, the index-line in the right recess at back of rule gives $\log 1893 = 3.277$. We have $\log 1893^{\frac{1}{3}} = \frac{1}{3} \log 1893 = 3.277/3 = 0.468$,

which, by Ex. 46 above, is found to be the log of 2.94. Therefore $\sqrt[3]{1893} = 2.94$.

Ex. 48. Find $\sqrt[3]{24,300,000} = 30$.

We find $\log 24,300,000 = 7.386$ by the above method, and $\frac{7.386}{3} = 2.477$, proceeding as above we get under left-index of C , 3. Therefore $\sqrt[3]{243,000,000} = 30$.

Ex. 49. Find $0.330^5 = .003914$.

Proceeding as above we got $\log 0.330 = 1.519$, and by the rule for multiplication $5 \times 1.519 = 7.595$. Setting 595 on L to the lower index-line on the right recess, the number corresponding to $\log 7.595$ is found to be .003914.

(32) **The Use of the Scale of Tangents.**—*1st method.*—The procedure adopted is similar to that used for finding the sines with the inverted slide. The tangents of the angles marked on T may then be read off on D . With the 10-inch rule only tangents between 0.1 and 1 corresponding to angles between $5^\circ 43'$ and 45° can be found. Thus— $\tan. 6^\circ 20' = 0.1109$, $\tan. 7^\circ = 0.1228$, $\tan. 9^\circ 50' = 0.1733$, $\tan. 20^\circ = 0.364$, $\tan. 30^\circ = 0.577$, $\tan. 40^\circ = 0.839$. For the tangents of angles greater than 45° the method of operation is shown below.

Ex. 50. Find $\tan. 75^\circ = .00373$.

$$\text{But } \tan 75^\circ = \frac{1}{\tan. (90^\circ - 75^\circ)} = \frac{1}{\tan. 15^\circ} = \frac{1}{.268} = .00373.$$

Ex. 51. Find $28 \tan. 60^\circ = 48.496$.

$\tan. 60 = 1 \div \tan. 30^\circ$. Set 30° of T opposite 28 of D . Then below the right-index of T read 48.5 (48.496).

The rules for the number of integers are the same as for the sines. Below $5^\circ 45'$ the tangents and sines are nearly equal, and as the discrepancy only affects the 4th decimal place, the S scale will serve for both.

Ex. 52. Find $\text{nat. tan. } 3^\circ 20' = 0.0582$.

Set the S scale adjacent to A scale, the left indices coinciding. Slide the cursor to cover $3^\circ 20'$ on S and read 582, on A . The correct answer is 0.0582.

When large numbers have to be multiplied, the discrepancy becomes serious but in the case of angles between $3^\circ 30'$ and $5^\circ 43'$, it may be corrected by adding to the sine of the angle, 3 per cent. of the value of the sine for the sufficiently correct value of the tangent.

Ex. 53. Log $\tan. 3^\circ 20' = 8.76492$.

Having found the nat. tan. as above, take the slide out and re-insert it to its normal position. Now set the left end of C to 582 on D . Look at the lower index line in the recess on the reverse of the rule and read .765, the mantissa required. The correct logarithmic value is 8.76492.

Ex. 54. Given the sides, $a = 35'$, $b = 21'$, the included angle $C = 50^\circ$, Find A and B and c .

The formula is $\tan. \frac{1}{2} (A - B) = \frac{(a - b) \tan. \frac{1}{2} (A + B)}{(a + b)}$. $a - b = 35 - 21 = 14$,
 $a + b = 35 + 21 = 56$; $\frac{1}{2} (A + B) = \frac{1}{2} (180^\circ - 50^\circ) = 65^\circ$. $\tan. \frac{1}{2} (A + B) = \tan. 65^\circ =$
 $\frac{1}{\tan. 25^\circ} = \frac{1}{0.4663} = 2.144$. We get 9.4660 on D by setting the cursor to 25° on

T . Then setting 0.4663 on C to 1 on D we get under right 1 of C 2.144 on D . Then set 1 on B opposite 14 on A . Slide cursor down until its line

covers 2'144 on *B*. Move *B* to left until 56 comes under the cursor. Read opposite 1 on *B* 536 on *A*, the value of the tangent which corresponds to an angle of 28° 12'. $A + B = 130^\circ$ and $A - B = 53^\circ 24'$, or $A = 93^\circ 12'$ (93° 11' 49") and $B = 36^\circ 48'$ [36° 48' 11"]. Finally, determine *c* as in example 44.

2nd. Method of finding Tangents of Angles may be used in which the reversal of the slide is avoided. There is a second aperture at the left end of the rule with a lower index mark.

Ex. 55. Find $\tan. 18^\circ 30' = 0.3846$.

Turning to the back of the rule, the slide is moved to the left until the angle 18° 30' on *T* agrees with the lower index in aperture. Reversing the rule, the tangent is obtained, over the left 1 of *D* to be .03346.

(33) Practical Illustrations of the Use of the Slide Rule in solving Problems of Mensuration, etc., are now given :

Ex. 56. In a Traverse Survey the Error at Closing is seen to be 14.36 feet to the north and 15.88 feet to the east of the starting point *A*. To find the closing error $L = \sqrt{(14.36^2 + 15.88^2)} = 21.4$ feet.

Slide the cursor to cover 14.36 on *D* and find its square 206 on *A*. similarly bring the cursor to cover 15.88 on *D* and find its square 252 on *D*. Adding these together we get 458; bringing the cursor to 458 on *A*, the square root is read off from *D* = 21.4.

Ex. 57. How many feet per mile are there in a grade of 1 in 20 ?

Set left 1 of slide opposite 20 on *A*, and opposite 5280 on the rule will be found 264 on the slide.

Ex. 58. Find the Area of a Circle 4.5 inches in Diameter.

$\text{Area} = \frac{\pi d^2}{4} = .7854d^2$. Set 4 on *B* to π on *A*, and over 4.5 on *C* read by the aid of the cursor, 15.9 [15.904] sq. in. on *A*, which is the area required; or set right 1 of *B* to .7854 line on *A* and over 4.5 on *C* read the area 15.9 on *A*.

Ex. 59. Find the Area in sq. ft. of a Circle 24 inches in Diameter.

Here the formula becomes : $\text{Area} = \frac{\pi d^2}{4 \times 144} = \frac{\pi d^2}{576} = \frac{.7854d^2}{144}$.

Therefore set 576 on the left *B* scale to π on *A* and read over 24 on *C* the area = 3.1416 sq. ft. on *A*; or set 144 on right *B* to .7854 line on *A*, and over 24 on *C* read area 3.1416 sq. ft. on *A*.

On the *C* scale of Faber's calculating rule a special point *C* is marked showing $\sqrt{4/\pi} = 1.128$ corresponding to 1.273 on *B*. This 1.273 = $\frac{4}{\pi}$ being reciprocal of $\frac{\pi}{4}$. In this case set this point *C* to the diameter on *D*, and over index of *B* read area on *A*. There is also another graduation *C'* on *C* scale, the distance between the marks *C* and *C'* being equal to that from 1 to 10 on *A*.

Ex. 60. Given the Chord *c* of a Circular Arc to be 8 ft., the vertical height, *h* = 3, to find, *d*, the diameter of the circle.

Here the formula is $x = c^2 / 4h$, $h + x = d$. Set *h* on *B* to *h* to half the chord on *D* ($c^2/4$ on *A*), and over 1 on *B* read *x* on *A* = 5.33.

Ex. 61. Given the Radius of a Circle, $r = 5''$, the angle subtended by the arc at the centre, $a^\circ = 90^\circ$, to find the length, *l*, of the arc.

The formula is $l = \frac{a^\circ}{360^\circ} \times 2\pi r = \frac{a^\circ r}{57.3}$.

Setting 57.3 on *C* to 9 on *D*, 7.85 is read off on *D* under 5 on *C*.

Ex. 62. The Area of a $\odot$ r Field is 3850 sq. chains, find its $\odot$ ce.

The formulas are: $\pi r^2 = A$; or $r = \sqrt{A/\pi} = \sqrt{3850/\pi}$; and $\pi d = c$; where A = area, d = diameter, r = radius, c = $\odot$ ce.

Setting π on B to 3850 on right A find $35 = r$ on D under 1 on B (1225 on A); then $2 \times 35 \times \pi = c$. Set 1 on B to 2 on A . Slide the cursor down until its line covers 35 on B . Slide B to the left until its division 1 comes under the cursor. Slide the cursor to cover π on B , and over π will be found 220 chains, the required circumference.

Ex. 63. Find the $\odot$ ce in feet of $\odot$ r plot of land $65'$ in diameter.

Here formula is: $-\pi d/12 = \pi 65/12$.

Set 12 on B to π on A , and over 65 on B read 17 (17.016) feet on A .

Ex. 64. Find the area of an elliptical field of which major and minor diameters, a and b , are 40 and 32' respectively.

Area = $.7854 ab$. Set 1 on B to 40 on A , slide cursor until its line covers 32 on B . Slide B to the left until its right 1 comes under the cursor. Slide cursor to cover .7854 on B , and over .7854 read 1005 on A .

Ex. 65. The Diameter of a Sphere is 9', Find the area of its surface.

Here Area = $d^2 \pi$. Set π on B to right 1 of A and over 9 on D read by means of the cursor the surface of sphere 254 (254.47 sq. feet).

Ex 66. The Chord of an Arc is 100 ft. and the angle subtended by it on the circumference is 150° , find the radius of the circle.

Here half the angle at the centre is 30° , half the chord = 50'. Therefore radius = $50 \div \sin. 30^\circ$. Set 30° on S to 50 on A and over left end of S read 100. Here the number of integer in the quotient is $N+1=2+1=3$, the result being found to the left of the dividend and in the other scale.

Ex. 67. Find the area of a $\triangle ABC = 43.3$ sq. ft., when $b=5$, $c=20'$, $A=60^\circ$. The formula is Area = $\frac{1}{2}bc \sin. A$. Set 60° on S to index on back of rule (see p. 538), slide cursor to 2 on B , 5 on B to cursor, cursor to 1 on B , 20 on B to cursor, and under 1 on A read area = 43.3 sq. ft. on B .

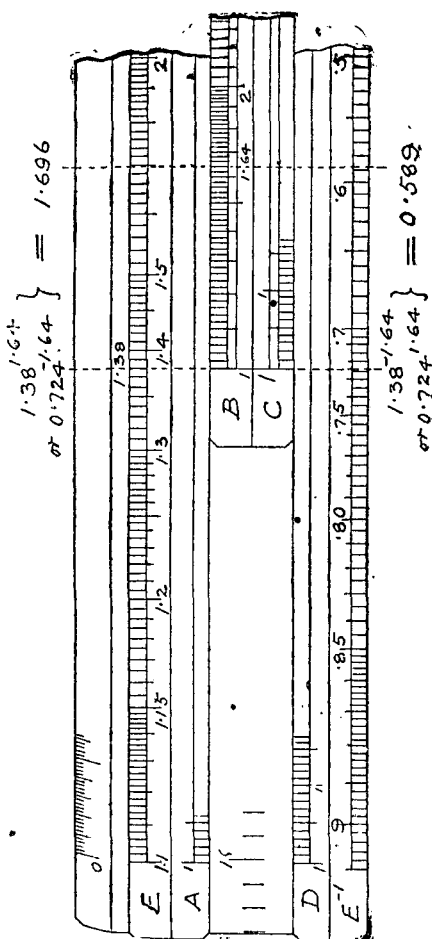


FIG. 596.—Part Thornton's Perry's Rule showing some calculations.

(34) Sums involving Screw cutting, Gearing, Delivery from Pumps, power of steam engines, etc., can be worked out with the aid of the slide rule.

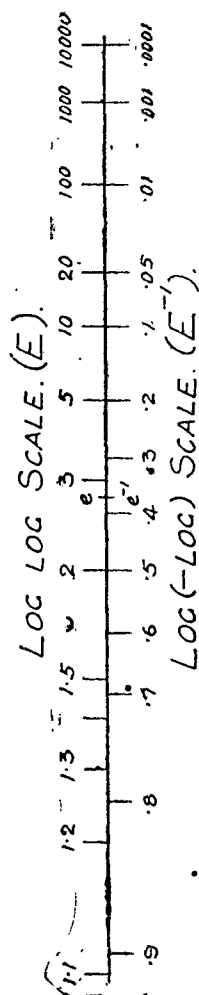


FIG 597.—Main Graduations of Perry's Rule.

756. (1) Improved Perry Slide Rule with Log.-Log. Scales.—Prof. Jno. Perry, F.R.S., of the Royal College of Science, London, designed an addition to the ordinary slide rule, being a ready method of evaluating expressions involving fractional indices, suitable, principally, in steam and gas engine, in hydraulic and electrical calculations, etc., and this result was accomplished by adding to the slide rule a scale graduated in divisions proportioned to the log log of the numbers from 2 to 1000, and so to read off the logs of numbers to any base, and this by simply replacing the scale of the rule. In practice this has answered well. The only objections being by some who have been accustomed to calculate with the C and D scales instead of the A and B scales, that the want of the D scale was a drawback, and to meet this difficulty, in this new rule not only has the D scale been retained, but the log-log scale has also been very much extended, and this improvement has been effected by making the rule a little wider and placing the log-log scale along the top and bottom edges, (Fig. 596).

In addition to the D scale, the log-log or E scale is accompanied by a log-(-log) scale, or scale of reciprocals, named E^{-1} , which addition enables numbers less than unity to be given, thereby practically completing the range of the log-log scale. At the same time negative indices can be directly dealt with. The scales are arranged to be used in conjunction with the B scale on the slide, and the range of numbers is from $\frac{1}{10000}$ to 0.91, and from 1.1 to 10,000, (Fig. 597, half size,) the main divisions only being there indicated. Note that the two scales, E and E^{-1} are mutually reciprocal. Thus, opposite 2 on the E scale is $\frac{1}{2}$ or 0.5 on the E^{-1} scale; or opposite 0.1 on the E^{-1} scale is $\frac{1}{0.1}$ or 10 on the E scale.

(2) Uses of the Scales, E and E^{-1} .—

Theory.—Let it be required to raise a number a to any power n . Put x for the answer. That is, $x = a^n$. Take logs, $\log x = n \log a$. Take logs again, $\log \log x = \log \log a + \log n$.

* Take from Instructions by Mr. A. G. Thornton, Manchester.

To find a^n , *i. e.*, to raise a number a to the power n , the operation is the same as that of multiplying a on scale E, by n on scale B, and reading off the answer on scale E. The index, n , may be greater or less than unity. To extract a root we divide instead of multiply. For a negative index we read off the answer on the reciprocal scale,

because $a^{-n} = a^{\frac{1}{-n}}$, *i. e.*, a^{-n} is the reciprocal of a^n . Thus to find a^{-n} . If a is greater than 1, we multiply a on scale E by n on scale B, and read off the answer on scale E^{-1} . If a is less than unity, then multiply a on E^{-1} by n on B, and read off the answer on E. Thus by using the rule verify that $2^2=4$; $2^{-2}=0.25$; $0.5^2=0.25$; $0.5^{-2}=4$.

Ex. 68. Find $1.38^{1.64}$. See Fig. 596.

Ans. By the aid of the cursor set 1 on B to 1.38 on E. Opposite 1.64 on B read off, by aid of the cursor, the answer 1.696 on E.

Ex. 69. Find $1.38^{-1.64}$. Fig. 596. Set 1 on B to 1.38 on E. Opposite 1.64 on B read off the answer 0.589 on the reciprocal scale E^{-1} .

Ex. 70. Find $0.724^{1.64}$. Fig. 596. Set 1 on B to 0.724 on E^{-1} . Opposite 1.64 on B read off the answer 0.589 on E-1.

Ex. 71. Find $0.724^{-1.64}$. Fig. 596. Set 1 on B to 0.724 on E^{-1} . Opposite 1.64 on B read off the answer 1.696 on the reciprocal scale E.

Ex. 72. Find $1.696^{\frac{1}{1.64}}$. Fig. 596. Set 1.64 on B to 1.696 on E. Opposite 1 on B read off the answer 1.38 on E.

Ex. 73. Find $1.696^{\frac{-1}{1.64}}$. Fig. 596. Proceed as in example 72, but read off the answer 0.724 on the reciprocal scale E^{-1} .

Ex. 74. Find $0.589^{\frac{1}{1.64}}$. Fig. 596. Set 1.64 on B to 0.589 on E^{-1} . Opposite 1 on B read off the answer 0.724 on E^{-1} .

Ex. 75. Find $0.589^{\frac{-1}{1.64}}$. Fig. 596. Proceed as in example 74, but read off the answer 1.38 on the reciprocal scale E.

Ex. 76. Find $1.38^{\frac{1}{1.64}}$. Fig. 596. Set 1 on B to 1.38 on E. Opposite 1.64 on B read off the answer 0.589 on the reciprocal scale E^{-1} .

Ex. 77. To find the log of any number to any base. Say find log. $_{1.38} 1.696$. That is, find x from the equation $1.38^x = 1.696$. Fig. 596. Set 1 on B to the base 1.38 on E. Opposite 1.696 on E read off the answer 1.64 on B.

Ex. 78. Find anti-log of 1.64 to the base 1.38, *i. e.*, find the value of $1.38^{1.64}$.

The answer is 1.696. See Example 68.

Note.—Hyperbolic or Napierian Logarithms.—The base of these logs is 2.718; it is marked e on the E scale. Its reciprocal 0.4343 is marked e^{-1} on the E^{-1} scale. Following are general exercises:—

EXERCISES FOR PRACTICE.

- | | | | |
|---|---------------------------|------------------------------------|--------------------|
| 1. Find $1.622^{.55}$. | <i>Ans.</i> 3.42. | 10. Find $1454^{1.26}_{78}$. | <i>Ans.</i> 3.76. |
| 2. Find $1.62^{-2.55}$. | <i>Ans.</i> 0.292. | | |
| 3. Find $0.6741^{.84}$. | <i>Ans.</i> 0.484. | 11. Find $0.0137^{.2.82}_{5.85}$. | <i>Ans.</i> 5.48. |
| 4. Find $0.674^{-1.84}$. | <i>Ans.</i> 2.066. | | |
| 5. Find $5.320^{.465}$. | <i>Ans.</i> 2.175. | 12. Find $0.246^{1}_{5.43}$. | <i>Ans.</i> 1.828. |
| 6. Find $0.127^{.595}$. | <i>Ans.</i> 0.293. | 13. Find $\log_e 3.5$. | <i>Ans.</i> 1.253. |
| 7. Find $0.127^{-0.595}$. | <i>Ans.</i> 3.414. | 14. Find $\log_e 10$. | <i>Ans.</i> 2.303. |
| 8. Find $35.8^{2.35}_{-1}$. | <i>Ans.</i> 4.58. | 15. Find $\log_e 1.5$. | <i>Ans.</i> 0.4055 |
| 9. Find $35.8^{.35}_{-1}$. | <i>Ans.</i> 0.2182 | 16. Find $\log_e 0.165$. | <i>Ans.</i> 1.802. |
| | | 17. Find $\log_e 0.636$. | <i>Ans.</i> 0.4526 |
| 18. Solve the equation $3.24^x = 7.58$. | <i>Ans.</i> $x = 1.72$. | | |
| 19. Solve the equation $0.265^{-x} = 1.265$. | <i>Ans.</i> $x = 0.177$. | | |
| 20. Solve the equation $\log_x 6.63 = 2.73$. | <i>Ans.</i> $x = 2$. | | |

NOTE.—The range of the two log log scales in the Improved Perry Rule is so extensive that most numbers which occur in practice will be found on one or other of them. But exceptional cases sometimes arise and are readily dealt with, and in several obvious ways, *e.g.*, we require the seventh root of 1.638. This, being less than 1.1, will be found to come off the E scale. But reading on E the number 1.152 opposite 2 on B (instead of opposite 1) we have the square of the required seventh root. We then use the A and D scales in the ordinary way to extract the square root of 1.152 and obtain for the required answer 1.073. Or again, required $12^{5.3}$. The answer is out of range, being greater than 10,000. But set the cursor opposite 0.53 on B (instead of 5.3), then use B to multiply by $\frac{1}{2}$, or $\frac{1}{3}$, or $\frac{1}{4}$, etc., and the number on E opposite the cursor is the square root, or the cube root, or the fourth root, etc., of the required answer. The latter can now be obtained in the usual way by the ordinary slide rule, by squaring, cubing, or double squaring, etc., the number.

757. Pocket Circular Calculators.—Boucher's Pocket Calculator (Fig. 598) is a convenient instrument, being only of the size of an ordinary watch.

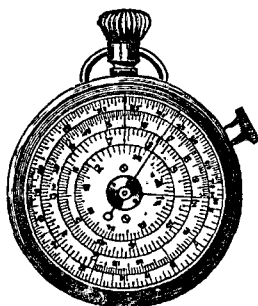


FIG. 598.—Stanley's Boucher Calculator

of a keyless watch. By means of the motions of the milled heads, any operations in multiplication, division and proportion, can be performed to four decimal places upon one face, and trigonometrical calculations upon the other.

The Scale on the front dial beginning from the outside):

1. Scale of Log. Sines
2. " Log. Numbers.
3. { " Log. (Numbers).²
4. {

On the back dial:—

1. Scale divided into equal parts.
2. {
3. { " of Log (Numbers)³
4. {

758. Stanley's Patent Calculator (Fig. 599) is an improvement upon Boucher's by the addition of extra gearing to a third index hand, which reads upon a central dial giving the number of revolutions of the Log. hand. By this arrangement continuous workings leave a final result + or -. The book of instructions gives Gauge points, from which calculations are made.

759. Thornton's Pocket Calculator (Fig. 600) consists of a revolving dial operated by the nut, A, and having on its face five graduated scales, marked, Nos. 1, 2, 3, 4, 5. No. 1 is a scale of logarithms. No. 2 is the calculating scale for multiplication and division. No. 3 and 4 is a scale of square roots.

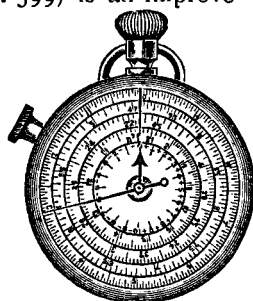


FIG. 599.—Stanley's Calculator.

No. 5 is a scale of sines of angles. The instrument also carries a movable finger, C, operated by the nut, B, and a fixed radial pointer D.

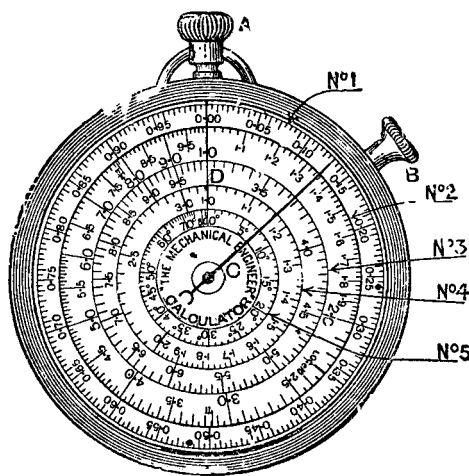


FIG. 600.—Thornton's Calculator.

Multiplication.—Use scale No. 2, and set one factor under the pointer, D, by turning nut, A. Set movable pointer, C, at 1 by turning nut, B. Turn nut, A, till second factor comes under pointer, C. Read off result under pointer, D.

Division.—Use scale No. 2, and set the dividend under the pointer, D. Set the pointer, C, to the divisor by turning nut, B. Turn dial, by nut, A, till I comes under pointer, C. Read off result under pointer, D.

Square Roots.—Use scales Nos. 3 and 4. Although there are two circles, one is really a continuation of the other. The numbers on scale No. 2 are the squares of those on scales Nos. 3 and 4. To find square root of a number set the number on scale No. 2, under one of the pointers (whichever is convenient). Ascertain whether the number of digits (neglecting decimal figures, if any) is even or odd. If number of digits is even, read off square root on scale No. 3. If number of digits is odd, read off square root on scale No. 4.

Squares.—Use scales, Nos. 3 and 4, and No. 2, and proceed as follows :—Set one of the pointers over the number on scale No. 3 or 4, whichever the

Logarithms.—Use scale No. 1. The readings on this scale are the common logarithms of the numbers radially opposite on scale No. 2, and can be read by the use of either the fixed or, movable finger, as happens to be most convenient. The characteristic of the logarithm is found in the usual way, and must be added by the operator just the same as when working from a book of tables.

number is on (scale No. 3 being simply a continuation of No. 4). Read off the number radially opposite, on scale 2. This is the square.

Sine of an Angle.—Use scale Nos. 5 and scale No. 1, and set one of the pointers over the angle on scale No. 5, read off value of sine radially opposite on scale No. 1.

Cosine of an Angle.—This can be easily deduced from scale No. 5, since the cosine of an angle is the sine of its complement. For example the cosine of 60° is the same as the sine of 30° .

Cubes and Cube Roots, and Miscellaneous Powers Generally.—These can be obtained by means of scales Nos. 1 and 2 just as readily as by the aid of a table of logarithms.

Hyperbolic Logarithms.—These may be found by multiplying the common logarithm on scale No. 1 by 2.30, which point is marked for convenience of reference on scale No. 2.

Areas of Circles.—Set pointer D to circle diameter on scale No. 3 or No. 4. Set movable pointer C at 1. Turn nut A till $\pi/4$ (i. e., .7854, and which, for convenience, is marked on scale No. 2) comes under movable pointer, C. Read off result on scale No. 2 behind fixed pointer, D.

Note.—In setting the pointers of the instrument to fractions of a division the judgment of the operator must be used, and the same remark applies to the reading of the results. The position of the decimal place can be determined by inspection without specific rules. With a very little practice the operator will find it possible to obtain results accurate to less than one per cent., which is closer than is required in most engineering calculations.

Fractions.—In working out long sets of fractions many operations and much tedious labour can be saved by the use of the instrument.

The general rule for dealing with fractional calculations is as follows: put the sum to be worked in the form $\frac{a \times b \times c \times \dots}{m \times n \times \dots}$, and disregard the decimal

point in the result until it is reached, then proceed as follows:—Set the dial by nut, A, till first multiplier (a) comes behind the fixed pointer, D. Turn the movable pointer, C, to the first divisor (m). Turn the dial by nut, A, till second multiplier (b) comes behind movable pointer C. Turn the movable pointer, C, to the second divisor (n), and so on, till the numbers are used up. Read off result behind fixed pointer, D, and estimate position of decimal place.

N. B.—If there are not enough divisors or multipliers in the fraction, use the number 1 instead, as often as may be required, that is until there is one more factor in the numerator than in the denominator.

Thus, $\frac{a \times b}{m \times n}$ would be worked as $\frac{a \times b \times 1}{m \times n}$; $\frac{a \times b}{m \times n \times p}$ as $\frac{a \times b \times 1 \times 1}{m \times n \times p}$; and $\frac{a \times b \times c \times d}{m \times n}$ as $\frac{a \times b \times c \times d}{m \times n \times 1}$.

The fixed pointer is only used to set instrument at first, and read results at finish. The movable pointer is shifted only for divisors. The dial is shifted only for multipliers.

760. The Halden Calculex (Fig. 601) consists of a disc within a ring, which together form a dial with logarithmic scales on both sides, surrounded by a metal ring and protected on each side by a glass disc with a cursor line marked radially thereon, which is capable of being revolved by the two thumbs when it is required to set the cursor line. The disc is also turned in setting the scales to the required position by holding the nuts on either side, between

the thumb and finger, the ring being fixed to the metal ring. It is made in nickel and has a face, front and back. By simply moving the head any calculation can be arrived at, full instructions, Pocket book, $2\frac{1}{4}'' \times 2\frac{1}{4}''$, being supplied with each sold.

761. Prof. Fuller's Spiral Slide Rule. (Fig. 602) consists of a cylinder, *d*, that can be moved up and down upon, and turned round, an

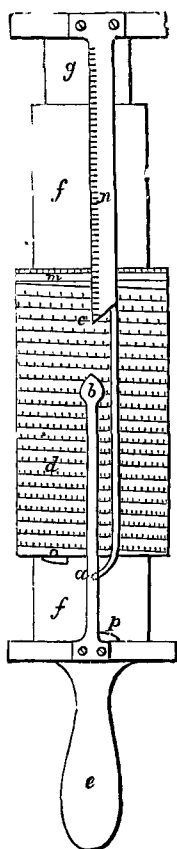


FIG. 602. — Fuller's Spiral Slide Rule.

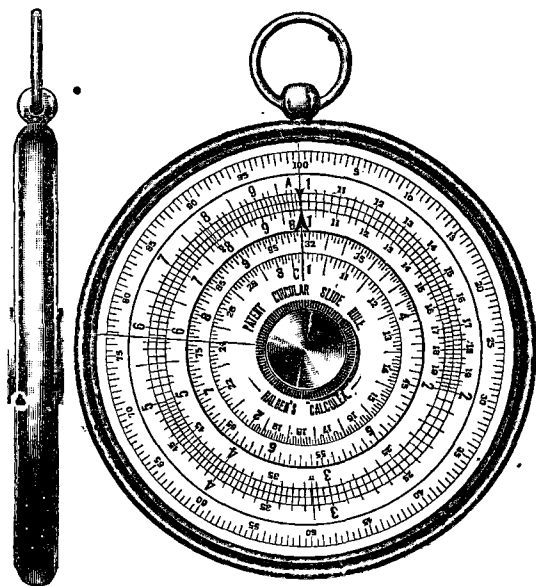


FIG. 601. — Halden Calculux.

axis, *f*, which is held by a handle, *e*. Upon this cylinder is wound in a spiral a single logarithmic scale. By this means a very long scale may be made use of. Fixed to the handle is an index, *b*. Two other indices *c*, and *a* whose distance apart is the axial length of the complete spiral, are fixed to the cylinder, *g*. This cylinder slides in *f* like a telescope tube, and thus enables the operator to place these indices in any required position relative to *d*. Two stops *o*, and *p*, are so fixed that when they are brought in contact, the index *b*, points to the commencement of the scale. *n*, and *m*, are two scales, the one the movable index, *n*, the other on the cylinder, *d*. The handle *e*, is now made to unscrew when not in use, thus making the case more compact. The use of slide rules has been confined to rough approximate calculations, as the length of scale hitherto made was sufficient only for about 160 divisions. In this rule the length of scale is 500 inches, and the number of divisions 7,250, consequently the approximation obtained by it is sufficient for most of the calculations. The same rule with extended trigonometrical scales, including Cos.^2 and $\text{Sin.} \times$

Cos., for giving horizontal distances from readings of the Tacheometer and Omnimeter, is designed by W. N. BAKEWELL M. I. C. E.

In using the rule a given number on d is set to the fixed index, b , and either a or c is brought to another number on the scale. This establishes a ratio; and if the cylinder is now moved so as to bring any number to b , the fourth term of the proportion will be found under a or c . In multiplication, one factor is brought to b , and a or c brought to 100. The other factor is then brought to a or c , and the result read off under b . By the aid of the scales, m and n , logarithms of numbers, powers and roots may be obtained.

762. The R. H. S. Calculator (Fig. 603) consists of a spiral logarithmic scale, 50 ins. long, wound upon a metal cylinder 8 ins.



FIG. 603.—Steward's R. H. S. Calculator.

long and $\frac{3}{4}$ in. in diameter. The spiral scale begins with 100 and ends with 1,000, and every number between 100 and 1,000 is represented by a line on the scale. A fourth figure in a number can be read by eye division of the space between the successive lines, the size of the divisions varying from 0.25 to 0.025 inch. A slotted tube which slides over the cylinder, and another small tube sliding over the former, are provided with the reading indicators. On one edge of the slotted tube are graduated and figured divisions, which are used in conjunction with the uniformly divided horizontal circular scale round the top, for finding the logarithm of any given number, and *vice versa*. The slotted tube has 4 indicators, these being formed at the ends of the two wide openings through which the scale is read. The small tube has two indicators. One of the indicators of the slotted tube being set to the first factor, and one of the indicators of the small tube with the 2nd factor, the slotted tube is moved until the 3rd factor comes under the same indicator of the small tube, when the 4th term will be read off under the same (right or left) indicator of the slotted tube at either end of the slot. In multiplying, 100 or the 1000 is taken for the 2nd factor in the above propor-

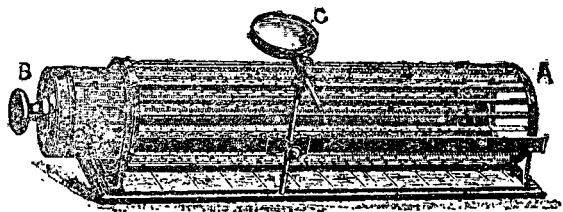


FIG. 604.—Thacher's Calculating Instrument.

tion, as already explained for Fuller's Rule; method of use is similar in both instruments.

763. Thacher's Calculating Instrument.—It consists of a cylinder

4" in diameter and 18" long, revolving in two vertical rings fixed on a wooden base (Fig. 604). Upon the cylinder logarithmic scales

longest ever made are arranged in 20 parallel lines. Joining the rings are 20 Δr graduated bars. Within the frame-work of the 20 Δr bars with duplicate logarithmic scale on their exposed sides, the cylinder, whose face they fit exactly, can be moved in the rings with either a longitudinal or rotary motion. The graduated Δr bars form the rule, and the cylinder the slide. Any graduation on the cylinder can be brought into coincidence with any graduation on any bar, and this rule is worked in the same manner as the ordinary straight slide rule, and gives results in 4 figures. The scale of logarithms corresponding with the scale on the cylinder is 30' long, and scale of square roots on the upper edge of the bars 60' long, rendering the instrument most accurate of all slide rules.

764. The Stadia Slide Rule is illustrated by Fig. 605 showing a scale very widely used in topographical work. With stadia measurements for direct readings of a vertical rod, we have the formulæ :

$$d = K a \cos^2 n, \text{ and } e = K a \frac{1}{2} \sin. 2n, \quad .$$

where n is the angle of elevation above the horizontal ; K , a constant dependent upon the construction of the telescope, and generally so adjusted as to be exactly 100 ; a , the reading on the vertical rod between stadia hairs ; d , the corrected distance of the observed point from the instrument ; and e , its elevation above the horizontal plane, through the horizontal axis of the telescope.

In this form of the slide scale, we have the slide bearing 2 scales ; the upper scale graduated to $\frac{1}{2} \sin. 2n$, the lower one for $\cos^2 n$. The rule carries two identically graduated scales of simple numbers representing the rod readings, a . Setting the index of the lower scale to coincidence with the rod reading, we read directly on the lower scale, opposite the observed angle, n , the corrected, *i. e.*, horizontal distance, d , also on the upper scale, the difference in elevation, e . This scale is serviceable, but too long to be convenient for anything but office use.

Some of the Other Special Rules are those of McPherson's calculating rule designed for use upon water supply, irrigation and drainage etc. ; Hudson's Horse-power computing scale ; Hudson's Shaft, Beam and Girder scales. Hudson's Pump scale and Ganga Ram's Slide Rules of which No. 2 gives thickness of retaining walls, and No. 3 gives strains on Girders of all kinds described in book of instructions.

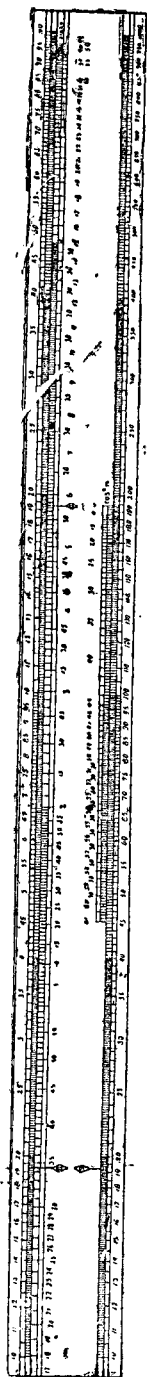


Fig. 605.—Stadia Slide Rule by Lietz (San Francisco).

CHAPTER XIII.—LEVELLING INSTRUMENTS FOR DETERMINING HORIZONTAL LINES, ETC.

765. Definitions—**Levelling** is the art of determining and representing the relative heights of points on the surface of the ground. Surveying may be taken as the horizontal and levelling as the vertical measurement of the ground.

A **Level surface** is a surface (not a plane) which is at every point perpendicular to a plumb-line at that point, *i.e.*, a vertical line pointing to the zenith, so that it might be the surface of a liquid at rest. Every such surface is approximately that of an oblate spheroid, as the sea-level, for example, is. A level is not a surface equidistant from the centre of the earth. A level is the surface of still water. A **level line** is a line lying in a level surface. A level surface is not a horizontal surface but a curved surface so that a level line is a curved line and would conform exactly to the earth's surface were that surface a perfectly regular one. It is like the surface of a still lake which is a level surface but not a horizontal one.

A **vertical line** is a straight line perpendicular to the surface which the earth would assume were it entirely liquid. At any point, a plumb-line coincides practically though not exactly, with a vertical line. A **horizontal line** is a line lying in a plane perpendicular to a vertical line.

A **plane at right angles to the plumb-line** is for that point a horizontal plane. As no two plumb-lines can be parallel, so no two planes, respectively horizontal at two different positions on the earth's surface, can be parallel.

Parallel horizontal planes can only be planes at different heights, all horizontal for a single position on the earth's surface.

The **Line of Sight** which passes through the observer's eye is a horizontal line and is parallel to a tangent to the level surface.

766. The method of finding how much one point is higher or lower than another on the surface of the earth owing to irregularity, may be (1) Geometrical, (2) Physical, (3) Trigonometrical.

Theory of Geometrical Levelling requires the establishment of a horizontal line or plane of observation tangential to the earth's



FIG. 606.—Difference of Level between
Two Points.

surface by the aid of a **spirit levelling instrument** as the earth's centre is an inaccessible point, from and to which measurements can not directly be made, and taking vertical measurements to points on the earth's surface from this horizon-

tal line. The distances of the points *A* and *B* (Fig. 606), for example, from this horizontal line, are measured directly by setting up vertical levelling staves. The difference of the two readings on the staves at *A* and *B* is the difference of level between the two points the horizontal line of sight, *ab*, being established by the levelling instrument at *C*. The height of the plane of sight which is equal for the two points *A* and *B*, and which is known as the **height of the instrument**, or *H. I.*, has nothing whatever to do with Levelling operations, except when it is used in the Level-book in which the height of instrument is used (q. v. Level-book No. 5, Chap. XIV).

Physical Levelling is based on the change of atmospheric pressure at different altitudes. The instruments used are the barometer and the thermometer.

Trigonometrical Levelling requires the measurement of distances and angles. By this method either the length of the line joining the two points of observation, or the horizontal distance between them is measured, and the angle between the two lines is taken with an angular instrument. The difference in height of the two objects can then be found by trigonometry, or by plotting the angle and measurement. For levelling by this method a clinometer, dial, theodolite, or in fact, any instrument having a vertical graduated arc may be used. To illustrate this Fig. 146 (p. 139) is selected. To find the difference of level between *a* and *c* or the height *bc*, the $\angle cab$ and one of the two lines *ac* and *ab* must be measured. The height is then found by trigonometrical calculations from the two magnitudes obtained. For instance, $cb = ac \sin. cab$, or $cb = ab \tan. cab$. The general principles of the Trigonometrical levelling have been described in Chapter III on Inaccessible Heights and Distances, q. v. Probs. XXVIII and XXXV (pp. 149-152), and Art. 485, p. 276.

767. The Mason's Level, Boning-Staves, and Spirit Level.—In Geometrical levelling operations, the plumb-line, mason's levels, boning-staves and most commonly the spirit-levels are used.

(a) **Mason's Levels.**—The **⊥-form of Brick layers' Level** (Fig. 607) is a thin horizontal rule with a vertical rule attached to it, in which a line and plummet are suspended; its use is to try the level of the wall at various stages of the building as it proceeds. If this the horizontal piece, being at right angles to the plumb-line, becomes horizontal, the plumb-line coincides with its mark.

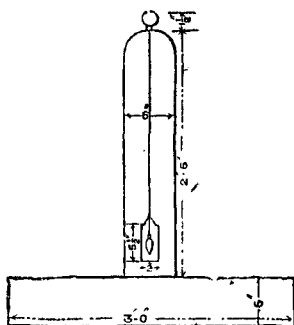


Fig. 607.—⊥ form Brick Layers' Level.

(b) The **A-form of Bricklayers' Level** (Fig. 608) may be used

either as a level or as a square to set out the sides of a building at right angles. It is so

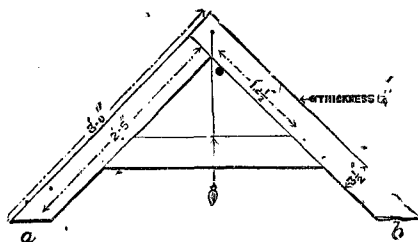


Fig. 608.—A-Form Brick Layers' Level.

a plummet swinging in the middle; its use is to guide the bricklayer in carrying up his wall perpendicularly by placing it vertically over a straight-edge placed on the wall.

(d) In forming the longitudinal section of a work, such as that of a road, where the slope will rarely exceed 1' in 30, a large mason's level shown in Fig. 610 may be used. The bar *ab* is placed strictly horizontal by raising or depressing one end till the plumb-bob suspended from *d* falls exactly on *c*. At the end of the bar *b* is fixed a gauge, *be*, at right angles to it, which by means of a screw can be raised or depressed. So if a slope of 1 in 60 is required and *ab* be 5' long, *be* will be

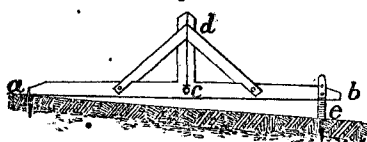


FIG. 610.—Large Mason's Level,

The level may then be carried on, and *a* placed over the peg which was driven at *e*, and so on for 20' or 30', and then a string pulled tightly over those pegs and produced, will give the slope for any length in advance.

(e) **Boning-Staves.**—Boning is a rough description of levelling, which is often performed by overseers in road and railway making. An experienced man, if given a couple of levels at the beginning and end of a given gradient, will bone the intermediate levels with great accuracy. Boning is done with boning rods about 5' high which resemble $\perp$ -squares, as follows:—Let *A* and *B*, Fig. 611, be two pegs driven to a certain depth, and according to a given inclination; if on both of these pegs boning rods, of equal heights, be held upright these will be parallel to the incline, and if a third rod be carried along the intended slope, the top of it, say at *O*, will

at right angles. It is so constructed that when the plumb-line coincides with the centre mark on the cross piece, the legs of the instrument *a* and *b* are at the same height, and a line joining them is horizontal.

(c) **The Plumbing Rule** (Fig. 609) is a thin rule 6" or 8" wide with a line and

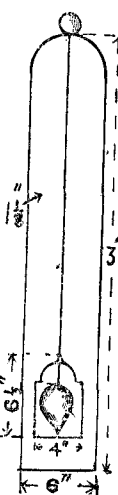


FIG. 609.—Plumbing Rule.

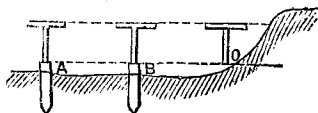


FIG. 611.—Boning-Staves.

be in a line with the top of the other two if the incline be correct ; if it is above, there will be more to cut away ; and if it is below, the excavation will have been made too deep : this method is certainly approximate, but quite sufficient to guide the excavators for a time.

(f) **The Spirit-Level or Level-bubble** (Fig. 612) consists of a round glass tube, with convexity upwards, nearly filled with pure alcohol for ordinary purposes ; for trade purposes with methylic alcohol which is cheaper. Very delicate tubes of astronomical instruments having air chambers are filled with sulphuric ether or chloroform and then sealed, a small quantity of ether or other vapor being left within, which rises as a bubble to the highest part of the tube. The sensitiveness of the bubble depends upon the mobility of the liquid with which it is filled, and to the amount of adhesion of the liquid to glass. The relative mobility of the above-named liquids has been found for small distances at a temperature of 60° F. and taking water as 100, methylic alcohol 22, alcohol 13, ether 5, chloroform 3. In field instruments ether is inadmissible owing to the high degree of expansion and contraction of that fluid with changes of temperature. By its movement the bubble will indicate any change in the position of the tube. In small and coarse levels the diameter of the tube is largest in the middle and decreases uniformly towards the ends, which are closed by the blow-pipe ; in long and delicate levels the tube is cylindrical, but with a longitudinal portion of the interior surface ground to the curvature of a circle of greater or less radius according as the level is designed to be more or less sensitive. It should be so mounted that the plane of the circle is nearly vertical. The grinding of the tube must be even so that the bubble will travel uniformly over the same distance. It is sometimes closed by circular glass stoppers cemented into the ends. When the tube is held horizontally, with the curved surface of the interior uppermost, the middle part is occupied by the vapour. Lines are etched on the outer surface at equal distances of $\frac{1}{10}$ " from the central point, to enable the tube to be set with the bubble exactly in the middle, and to enable any deviation from the centrality to be measured and the corresponding dislevelment to be calculated and allowed for in the reduction of observations afterwards. The glass tube is generally fixed in a metal tube with plaster-of-Paris for protection.



FIG. 612.—Bubble-Tube.

If the bubble (Fig. 612) by raising or lowering one end of the tube, is made to come inside the marks, *a* and *b*, which are equidistant from the highest part of the tube, or if the bubble expands from the influence of change of temperature and extends equally on each side of the marks, *a*, *b*, then if the glass tube be fitted into a metal tube and the bottom of which *AB* is parallel to a tangent to the centre of the tube, *AB* will be horizontal. In Fig. 612 the curvature of the tube is much exaggerated.

(g) **Principle of the Engineers' Spirit-Level.**—By the *tangent of the bubble-tube* is meant a straight line lying in the same plane with the tube's axis of figure, and tangent to it at the middle point. When the bubble is in the *centre of its run* this tangent line evidently becomes a horizontal line or the axis of the bubble-tube is horizontal. Now, if by any means we attach rigidly to the bubble tube a *line of sight*, and make that line of sight parallel with the tangent, then when the bubble is in the centre of its run, the line of sight will become a horizontal line. This is the fundamental theory of all spirit-levelling. The chief elements of a spirit-level are (1) a line of sight and (2) a bubble-tube. When the bubble tube is fixed to a telescope so as to be parallel to its collimation line it forms the **Engineer's Spirit-Level**. The line of sight in the telescope is capable of adjustment into parallelism with the tangent of the bubble-tube. The tangent permits of adjustment into perpendicularity to the vertical axis, around which it and the line of sight revolve, or in other words, if the bubble-tube be revolved about an axis in such a way that the bubble keeps a constant position in the tube, the axis of revolution is vertical.

(h) **Radius of Curvature of Level-tube.**—The radii of curvature of different level-tubes used for scientific purposes vary from about 30' to 1000' or more.

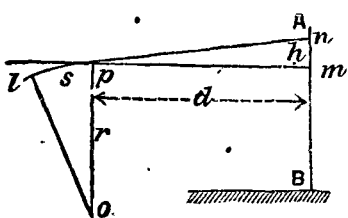


FIG. 613.

To find the radius of curvature of the tube, hold the staff AB (Fig. 613) at B , and adjust the bubble horizontally to a position pm and note the reading on the staff at m . Next move the bubble through the arc pl by raising or lowering one end, and note the distance $pl=s$ from the scale on the bubble-tube and the reading on the staff at n . The difference of the readings at n and m is thus found, and is called h say. The distance $pm=d$ is measured. Then the $\triangle smpn$ and opl are approximately similar, and $r/s=d/h$. $\therefore r=sd/h$; o being the centre of the circle of which the bubble is an arc. For example at 100' distance the difference of staff readings is 0.06ft. and the bubble was moved through 0.04ft. Then $r=(0.04 \times 100)/0.06=66.66'$.

(i) **The Sensitiveness of a Level-Tube.**—The interior of the glass level-tube should be ground to a circular curve, so as to secure both accuracy and sensitiveness in the bubble which should be cylindrical in bore. The sensitiveness of a bubble-tube is ascertained by the distance through which the bubble moves for a given change of inclination. The more sensitive the level the greater is the displacement of the bubble for any change of inclination, and the more accurate the work, but it should be remembered that the pitch of screws, quality of lenses or the optical power of the telescope, the graduations of the instruments all rule the sensitiveness of the bubble. The sensitiveness is in proportion to its length, a long bubble

settling much more quickly and accurately than a short one. Small tubes are sluggish and take time to work.

In Fig. 614, let s = length of arc through which the bubble moves for a change of inclination of $1''$, and r = radius of curvature of the level-tube.

$$\therefore \frac{s}{\text{Whole circumference}} = \frac{1''}{360^\circ} \text{ or } \frac{s}{2\pi r} = \frac{1''}{360^\circ} \therefore s = \frac{r}{206264.80625}.$$

The sensitiveness is therefore directly proportional to the radius. When the run is given, the radius of the curve of the tube may be found if desired. The distance of the run is required to be given in seconds of arc. *The run of a good sensitive tube is frequently made $\frac{1}{30}$ inch to the second.*

$$\therefore r = 206264.8 \times \frac{1}{30} = 572.9 \text{ feet nearly.}$$

For scientific purposes a millimetre run per second is commonly used, then $r = 206264.8 \times .001 \text{ metre} = 206.264 \text{ metres radius} = 680' \text{ nearly.}$ For an ordinary 12" dumpy the tube is divided into minutes at about $\frac{1}{30}''$ apart, radius 14 to 15 feet; for a sensitive 14" Y-level the same divisions may represent 5 seconds, radius of the bubble-tube about 170'.



FIG. 614.

Extra Bubble Tube.—When a level is to be used abroad one extra tube should be taken, as the level-tube is always exposed, and is more delicate than any other exposed part of the instrument, to replace an accidentally fractured tube. The tube may even burst by the heat of the sun's rays. It is convenient in replacing and mounting level-tubes to place white glazed paper under the bubble to reflect the light that passes through it to ensure better observation.



FIG. 615.—Chambered Bubble-Tube.

(j) The Divisions upon Ordinary Level-Tubes.

are made after the tube is finished, but with highly sensitive ones the divisions are made before the tube is finished. The run is taken from 10 to 30 divisions on each side of the centre of the tube, where it is lightly marked with glazier's diamond. These spaces are equally sub-divided and edged in with hydrofluoric acid, or marked with a hard edge dipped in turps.

(k) Chambered Bubble-Tube.—The size of the bubble varies with the temperature, the bubble contracting under heat, and in the midday it is sometimes half the length it was in the morning, hence, in high class levels the bubble-tube is chambered, *i.e.*, made so that the amount of liquid in the tube may be regulated according to temperature. The construction is shown in Fig. 615. A stopper is ground into one end of the tube, which in itself a small bottle, on the under side of which a hole is ground, so that by turning the tube over and raising it more or less, any amount of highly rarefied air it contains may be taken to form the bubble that may be desired.

The stopper is fixed with thin glue. The object is to keep the

bubble of almost equal dimensions for all measurements to be made with it.

768. The Accurate Measurement of Small Vertical Angles by means of bubble-tube readings is taken with greater accuracy than by any other device known. To effect this the bubble-tube should be ground so that its inside upper surface is circular on a longitudinal section with a long radius and uniformly graduated. Then a given movement of the bubble in the tube corresponds to a known angular change, when the angular value of one division of the graduated tube is known. These graduations are made on the outer surface of the tube. Bubble readings are taken from the centre to the ends. Then half the difference of the end readings is the distance of the bubble centre from the centre of the graduations. A small angle may be measured by noting the two end readings to divisions and tenths and taking half the difference. Move the bubble a given amount and take readings of both ends again, taking one half the difference. The difference of these two values in divisions of graduations of the bubble-tube multiplied by the angular value of one division of the bubble-tube is the vertical angle through which the tube was shifted.

The Angular Value of One Division of the Bubble may be found: (a) **By a Telescopic Line of Sight.**—Tape a line on level ground 200 to 500 ft. long. Set the bubble-tube mounted on a telescope at one end of the line and hold a staff vertically at the other. Bring the bubble near one end of its run by manipulating the instrument vertically and read both ends. Take also the staff reading. Bring the bubble near the other end of its run and read both the bubble ends and staff. Repeat this process many times, and take the half-difference of the end readings each time, thus finding the distance of the centre of bubble from the centre of tube for each position. Take the average of these values for each set of end readings separately. These mean values being for opposite ends of the tube, their sum gives the *average movement of bubble*. Similarly obtain the average of the upper and lower readings of the staff separately and their difference gives the *average movement of the line of collimation*. Now let B = bubble movement in divisions of graduations of the tube. S = movement on staff in feet, L = length of line in feet. Then, **Angular Value** of 1 division of

bubble in seconds of arc =
$$\frac{S}{(B \cdot L \cdot \sin. 1'')} \cdot \text{Log. sin. } 1'' = 4.6855749.$$

Ex. With a Dumpy-Level on a levelling staff 250' distant it was found that when the bubble moved over 20 divisions of the bubble-tube the staff reading varied from 5.65 to 5.30, or the subtended staff interval was 0.35'.

∴ One division of the bubble = $0.35 / (20 \times 250 \times \sin 1') \text{ or } \text{Log } 1.54407 - 1.30103 - 2.39794 - 4.168557 + 10 = 1.15953 = 14.44 \text{ seconds.}$

(b) **By a Level Trier or Tester.**—The bubble trier is a bar 12 to 20 inches long supported at one end on a knife edge and at the other by a micrometer screw. Attached to the bar are two

Y's in which the bubble-tube is placed and the bubble moved back and forth by means of the screw, each revolution of which gives a known angular movement to the bar. A pointer and scale is fixed on the bar by which the micrometer disc is read. The bar rests on a cast-iron or slate surface which can be made approximately level by means of 3 levelling screws without the aid of the micrometer screw. The Y's are adjustable for distance apart according to length of the bubble-tubes to be tried.

769. Circular Spirit-Level.—Mr. J. J. Hicks has patented a hermetically sealed circular level (Fig. 616) in which the upper surface of the level is spherical. It therefore indicates a level in every direction in place of only in one direction as in the case of ordinary spirit-level. These are not subject to evaporation and are very useful and portable for approximate levelling—as for plane tables, cameras, etc. The adjustment of this level is made in two directions at right angles to each other.

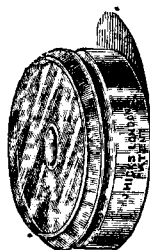


FIG. 616.—Circular Spirit Level.

770. Square Spirit-Level.—Fig. 617 illustrates a very useful level, $1\frac{1}{2}$ inches square only, for setting up plane table, etc., in which a pair of bubble-tubes are placed at right angles to each other.

771. Mechanics' or Masons' Spirit-Levels.—They are made from 6" to 18" long, generally of rose-wood, which is hard and stands well. The bubble-tube, (Fig. 618) which is of blown glass, is fixed in plaster-of-paris, and the upper plate of brass screwed down over it. Spirit-levels having top and bottom plates are also available. Fig. 619 shows a block level in gun-metal, with adjustable divided bubble fitted into a mahogany case. They are made from 9" to 18" long. These levels are commonly used upon a stout wooden straight edge of about 5' to 10' in. length. If the bubble retains the centre when placed upon the straight-edge then its ends are on the same level, if not, then the end which is lower is to be raised gradually until the bubble comes to the centre. The level is taken by stages upon the ground. This method of levelling is often used for putting on the same level different portions of a wall under construction or a small plot of



FIG. 617.—Square Spirit Level.

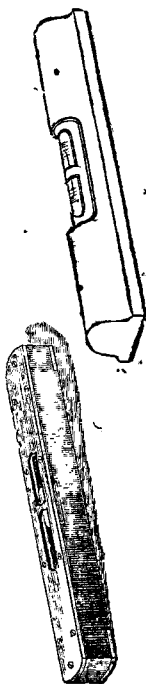


FIG. 618.—Masons' Spirit Levels.

ground. Corrections of error, both in level and straight-edge, may be made for any distance by reversing the forward and backward position of the level with its straight-edge alternately. Say the difference in level between two points A and F on a sloping ground (Fig. 176, p. 161) is required. The graduated straight-edge or staff is set on edge with one end on the ground, at A, the other end of it is raised until the spirit-level placed on its top, shows that the staff is horizontal; the length Aa and the vertical height aB are then taped and entered in the book. The point B is marked with a plumb-arrow (Fig. 620) where the inclination of the ground changes and the staff moved forward to the second position. The levelling thus proceeds, and from the datas obtained a section may be plotted, or the difference in level between any two points obtained from the figures.



FIG. 620.—
Drop-arrow.



FIG. 619.—Block Level.

772. Contractor's or Builder's Level.—Fig. 621 represents the cheapest form of level with a telescope and a tripod. The inner telescope tube is moved by a knob outside. The bubble-tube is solidly supported in collars. The

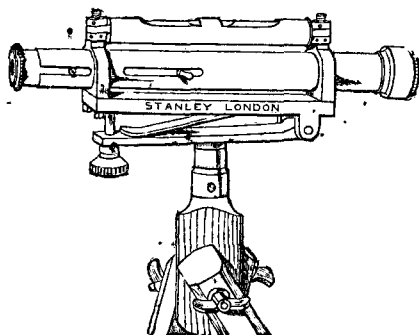


FIG. 621.—Builders' Level.

adjustment is in one direction only, so that the bubble must be adjusted and examined at the time of reading the staff. The instrument is supported on a *sprang* jointed at one end and held by a milled-headed screw at the other. The tripod is fairly firm in use, the legs being clamped by wing-nuts. The level is good enough for levelling operations connected with a building, bridge, lock, or drains, etc., within limited areas.

773. Thornton's Soffit and Suspension Level.—Fig. 622 shows the instrument. The bubble-tube is encased in a brass tube having openings at top or bottom for the bubble to be seen. The end pieces of the brass casing have hooks for suspending on line

and are formed with flat tops and bottoms all adjusted to bubble. It is portable, only measuring $6 \times 2\frac{1}{2}$ in. in case. It is suitable for

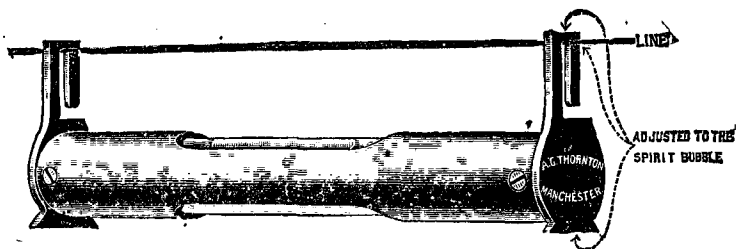


FIG. 622.—Soffit and Suspension Level.

levelling in all building works; for finding falls and levels of rivers; for testing levels of soffits read from under side (fig. 623) for road making and drainage works. Fig. 624 shows the method of obtaining difference in level between distant points.

774. The Cooke's Farm Irrigation Level (Fig. 625) is designed for the use of the farmers to carry out their irrigation schemes where it is hardly necessary to call in the aid of a highly paid surveyor. Its tripod legs are folded up over the metal top to protect the pin on which the level is attached to the tripod when not in use. First erect the stand with the projecting taper pin pointing upwards, whilst the legs should be well splayed out to make the tripod firm. Then take the level from the case and place it on the pivot of the tripod and tighten up screw, G. Move the tripod legs about

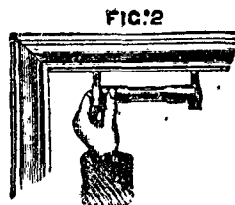


FIG. 623.—Testing Level of Soffits.

until the air space of the circular bubble, F, is about central. Next remove the protecting cap from the object glass (1" diameter) end of the telescope after directing the telescope in the required direction, move the eye-piece in or out until the object

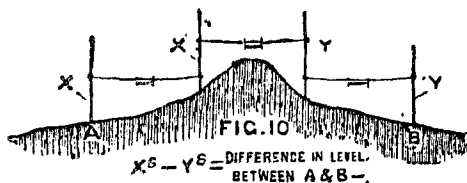


FIG. 624.—Finding Difference of Level.

looked at is clear and distinct. The screw, E, is used to bring the long bubble on top of the telescope to the center of its run. This bubble can be read from the eye-end by reflection from the hinged mirror, which when folded down also protects the bubble when the level is not in use. The divided head, D, used for accurately setting off a definite grade, rise or fall, fits merely

friction tight on screw E, and can be revolved upon it by a little pressure, taking care to hold the screw, E, with the other hand at the same time to prevent its turning and so disturbing the long bubble. The method of using the head, D, is as follows:—First set up the instrument and bring the long bubble to the centre of its run by means of screw, E. Next rotate the head, D, independent of screw, E, until the line engraved Level comes opposite to index, C, taking care not to disturb the long bubble in doing so ;

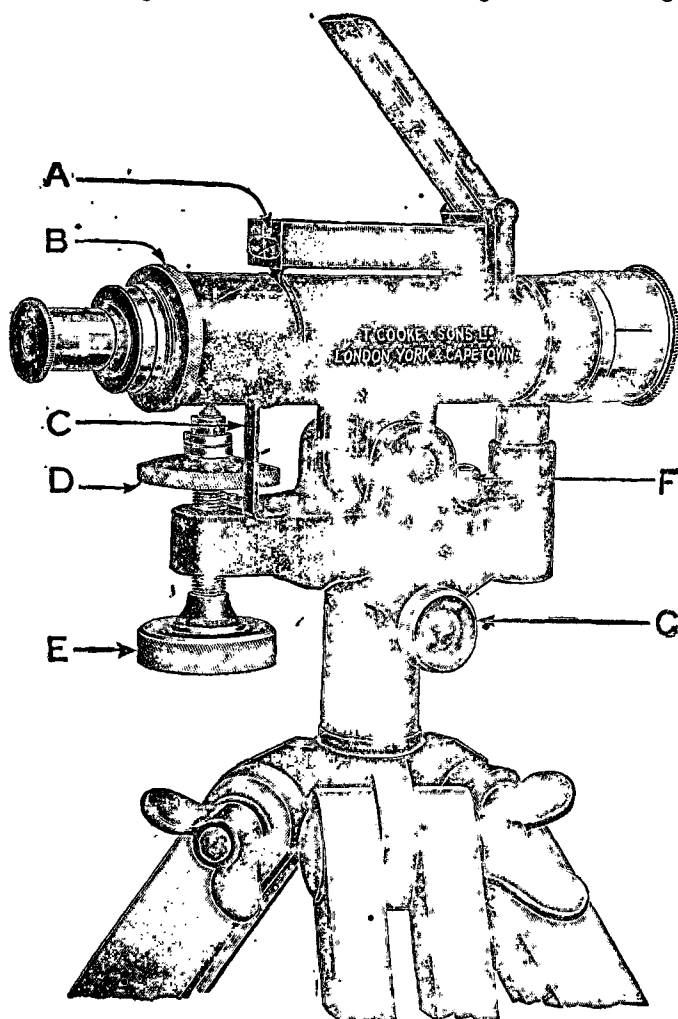


FIG. 625.—Irrigation Level.

if the bubble has moved, re-set it by screw, E, again, and bring Level

line on divided head-D correctly to index C. If the screw-head, E, be now revolved so as to bring one of the other engraved lines, say the one engraved, 300 opposite the index C, the telescope has been tilted up or down (rise or fall as the case may be, to a definite gradient of 1 in 300 or 1 ft. in 300 ft. Similarly for other gradients. The telescope gives about 6 magnifications:

775. The Kern's Pocket Level (Fig. 626)

is shown $\frac{1}{3}$ rd of the real size, objective $\frac{3}{4}$ inch, magnification 11 fold, with fixed stadia 1 : 100. Level is fixed above the telescope. Elevation screw with graduated drum and index serves for the vertical adjustment of the telescope and measure angles of $\pm 3^\circ$. The circumference of the drum is divided into 10 equal parts. Small horizontal circle division to 1° , and by means of an index to $\frac{1}{2}^\circ$. Two horizontal levelling screws and counter springs allow to rapidly fix the instrument. Weight of level is 2 lbs and of brass tripod 21" long, 3 lbs.

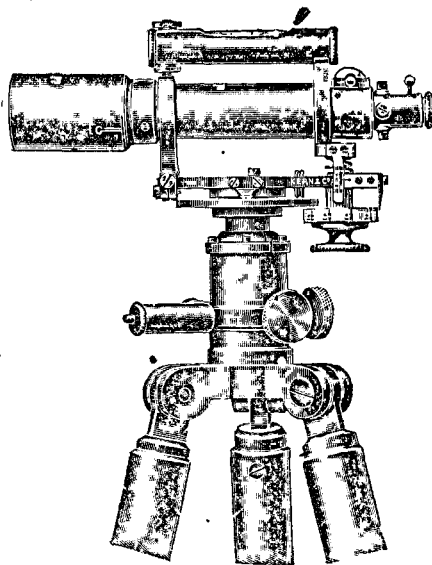


FIG. 626.—Kern's Pocket Level.

776. The Telescopes of Surveying Instruments whether of the Theodolites or of Levels consist of: (a) the object-glass, (b) the eye-piece, (c) the diaphragm, and (d) the Tube holding in place all the above: (a) **The object-glass** or *objective* or *image forming apparatus*, is an achromatic double lens which is of the double convex and plano-concave type (Fig. 627 is the longitudinal Section of an American Transit Telescope, and Fig. 628 of a Level Telescope). The outer one being double convex of crown-glass and the inner one being plano-convex of flint-glass. The refractive indices of both kinds of glass supplement each other, and the rays are brought to a focus and are rendered colorless. The objective is held in its cell by an internally fitting screwed ring with milled edge, so that the glasses may be taken out and separated to be cleaned and be easily replaced. Two notches or grooves are made in the edges of the glasses, each of which is deep enough to take a small brass pin which is soldered to the inside of the cell. The second notch indicates relative position so as to secure the glasses being replaced properly. The object-end of the telescope is enlarged so as to take the cell in which, the objective O, is placed, without cutting off any

part of the light or entailing the weight of larger tubes than is necessary to make use of the full field of the objective. The objective is placed at the end of a slide having two bearings, one at the end of the outer tube, the other in the ring, cc, which is

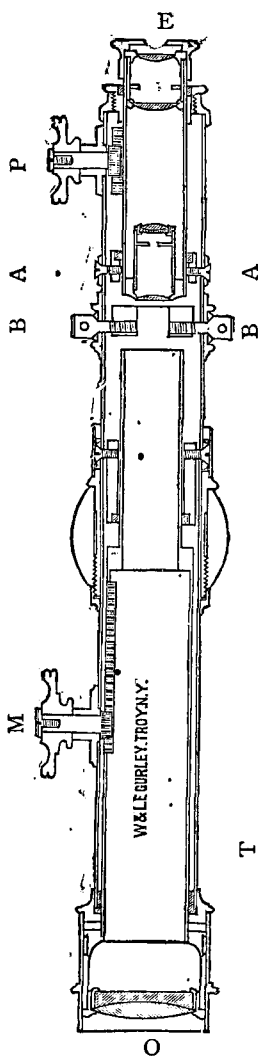


FIG. 627.—Longitudinal Section of the Telescope of an American Transit.

suspended within the tube by 4 screws only two of which are shown (Figs. 627-628). The nearest distance of the objective to the focal plane, BB, being the solar focus, or the distance at which a sharp image of the sun or star placed on the axial-line would be formed. The greatest distance of the objective from the focal plane in most instruments is such that a clear image will be given on this plane BB of an object placed about 20' from it. (b) *The Eye-piece or Image Magnifying and Viewing apparatus*, when a simple one, has two plano-convex lenses, the convex surfaces of which are turned towards each other and the plane surfaces are on the eye and object ends. The eye-end one is called the *eye-glass* and the further one *field-glass*. (c) BB is the focal plane of the objective where a metal disc is placed with an opening in its centre. This is termed the *diaphragm*. Both the objective and the eye-piece are fitted in sliding tubes in such a manner that they may be made to approach or recede from the focal plane BB. This eye-piece is sometimes called *inverting eye-piece*. It is not so; the objective inverts its image and the eye-piece picks up the image in its inverted position and magnifies it, but while this give more light it presents an inverted image of the object observed. This eye-piece is focussed by a spiral movement. *The erecting eye-piece* is composed of 4 lenses, which are called respectively, the *eye*, the *field*, the *amplifying* and the *object lenses*, the arrangement being such that the image is re-inverted and appear to the eye in its natural upright position, and the whole forming a *compound microscope*.

focussing on cross-wires attached to the ring BB (Figs. 627-628). Two stops are placed between the eye and field lenses, and the amplifying and object lenses to cut out extreme rays. The erecting eye-piece like the objective is moved out or in by pinions working in racks attached to their sides and are thus ad-

justed to proper focus. This eye-piece suffers loss of light at each of the 4 lenses, hence a telescope with it, for equally distinct vision to that obtained by using the 2-lens eye-piece, would require a larger objective. American Engineers prefer the erecting eye-piece.

(c) **The Reticule or Diaphragm** consists of cross-wires, as they are called, as well as of stadia-wires of silk, or cobweb, or fine platinum-wires stretched across a metal ring, or of lines ruled on glass. This ring can be moved vertically and horizontally by means of capstan-headed screws B, B. (Figs. 627—628). The wires or hairs are glued to the back of the ring.

(d) **Telescope Tube.**—Longitudinal sections of the complete Transit and Level telescopes are shown (Figs. 627; 628). The objective, receiving Rays of light from all points of a visible object (passing through the objective, O) converge them to a focus and form an inverted image of the object in the plane, B, B, of the cross-hairs, and this inverted image is magnified by the eye-piece, E. It is necessary that the image formed by the objective should be situated in the plane of the cross-hairs, and as the distance from the image

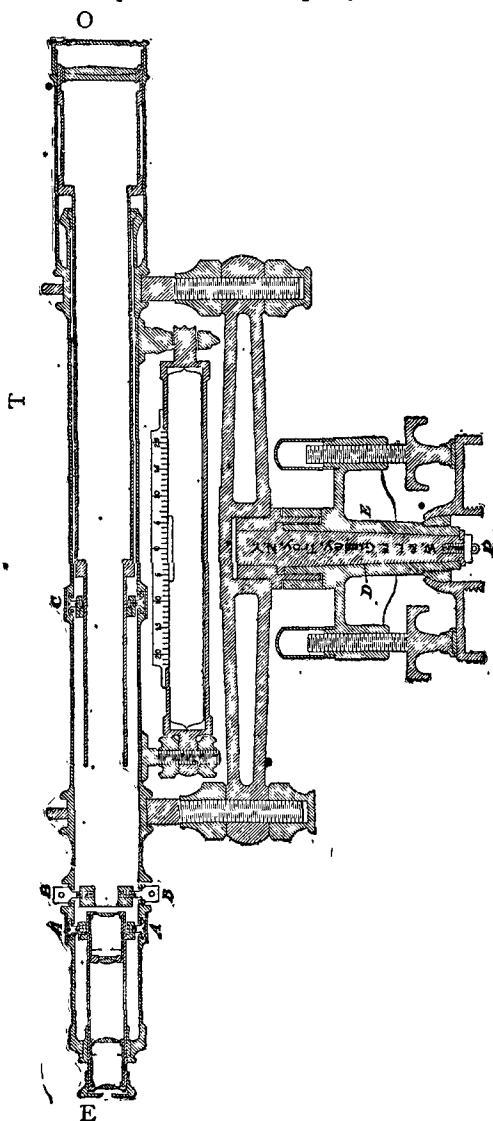


FIG. 628.—Longitudinal Section of an American Y-Level.

to the objective will change with the distance from the object to the objective, the plane of the cross-hairs being fixed, the objective held in the inside sliding tube must be movable. The movement of the sliding tube in and out of the main telescope tube, T, is effected by a rack and pinion motion worked by the milled-head, M, the slide being extended* for viewing objects near the observer and drawn in for viewing objects at a distance, so that the image will always lie in the plane, BB, of the cross-hairs. The operation of bringing the image by the movement of the object-glass is known as **focussing**.

In order that the image is to be magnified, it must lie between the eye-piece and its principal focus. In addition to the necessity for forming the image in the plane of the cross-hairs and between the eye-piece and its principal focus, the image and cross-hairs must be separated from the eye-piece by the distance of most distinct vision. This varies with different observers. Hence, the eye-piece must be movable and the two-lens eye-piece slide is moved in and out by hand simply, and the erecting eye-piece moved by the milled head, P, (Fig. 627).

The movable rings, one at A,A, and the other at C,C, are used in centering the eye-piece and in the adjustment of the objective slide. The lines passing through the optical centre are called the *lines of sight* as only those rays of light passing through the optical centre suffer no deviation. The use of the cross-hairs and the necessity for the formation of the image in the plane of the cross-hairs have now become apparent. If the image of a point is formed in the plane of the cross-hairs and the observer sees through the telescope the intersection of the cross-wires coinciding with this point, he knows that, except for the effect of refraction in the atmosphere, the intersection of the cross-hairs, the optical centre and the point are in the same straight line. If the image is not situated in the plane of the cross-hairs the intersection of the wires will appear to coincide with the image of the point only when the observer's eye is in line with these two, and *the eye moved to one side or the other can see between them*. Hence, if the image of the point is not in the plane of the cross-hairs, when the image of the point and the intersection of the hairs appear to coincide, the intersection of the hairs, the optical center of the objective, and the point may or may not be in the same straight line. *The condition*, resulting from the fact that the cross-hairs and image are not situated in the same plane, is known as *parallax* (c. f., Art. 168, p. 95).

776a. Defects of the Surveying Telescope in connection with the Object-glass are : 1. **Spherical Aberration.**—The rays of light which impinge on a lens near its edge undergo more refraction than those which pass through the lens near its center. The effect of this is that for every point of the object viewed there will be shown many points in the image formed, and the image hence will be hazy and blurred. The defect is remedied by making

the object-glass of two lenses (Figs. 627, 628) generally of crown and flint glass, having different dispersive and refractive powers, and the effect of this combination, when properly adjusted, is to remove almost entirely the blurring due to spherical aberration.

(2) **The Aberration of sphericity**, different from the first named, is due to rays of light proceeding from a flat object and passing through a simple convex lens and refracting in such a way that the image formed does not lie in a plane, but is concave towards the lens. In the case of the objective it is inappreciable, and will introduce no error when the ordinary surveying telescopes are used, as in these cases only one point is observed at a time. In the stadia measurements it is objectionable.

(3) **The Chromatic Aberration** is due to the fact that white light, in passing through a prism or simple convex lens, is separated into the colors of the spectrum. Different kinds of colored light are refracted differently. The resulting view is disfigured and indistinct. This error in the object-glass is corrected in the same way as (1). The object-glass (Figs. 627, 628) corrects both chromatic and spherical aberration, and is known as an *achromatic lens*. In the eye-piece, different combinations of lenses are used to correct the errors. (q. v., Fig. 628). In these the 2 lenses nearest the object-glass are inserted for the purpose of re-inverting the image, and showing it erect to the eye. Every extra lens put in a telescope causes a certain loss of light, and therefore most authorities prefer the telescope which shows the object inverted. The above defects in telescopes are inherent defects, and exist originally by no fault of the maker, though in a well-made telescope they are corrected.

776b. The Qualities of Telescopes, not inherent but dependent entirely on the way they are made, are 4 :—

1. **The definition of a telescope** is the clearness, distinctness, and sharpness with which it presents the image to the view. It depends on the accuracy with which the lenses have been *ground, polished and centered*. By the *centering of the lenses* is meant the placing in coincidence of the principal axes of those lenses which go to make up a compound lens, as the object-glass. The definition depends also on the spherical aberration, but lack of clearness due to this is an inherent defect.

2. **Illumination**.—By the illumination of a telescope is meant the relative amount of light conveyed by its lenses from a point to the eye, compared with the light sent from the point to the *naked eye*. An object seen through a telescope will appear brighter in proportion to the amount of light passing from it to the eye. The larger the objective, the more rays in the pencil of light which enters it from any point. Hence *the larger the object-glass, the higher the illuminating power of the telescope*. The *aperture* of the object-glass is the diameter of that part of it which transmits light to the eye. It is not easy to grind the edges of lenses

perfectly; so the practice is to place diaphragms in the tube to exclude rays passing through these edges. Generally, therefore the aperture of an object-glass will not be the same as its diameter. The lenses diffuse or reflect a certain amount of light. The best surveying telescopes transmit only 85% of the light impinging on the object-glass and the loss of light is 15%. In instruments of poor make the percentage falls to 70. Loss of light from this cause is an *inherent* defect, but is noted here owing to its affecting the illumination.

The eye-piece, as a magnifying glass, disperses the rays of light coming from any point of an object viewed and distributes them over a larger area, by which process the illumination is diminished. An object viewed through a telescope, except in the case of stars, will never appear as bright as when seen by the naked eye. At dusk, a staff, which can just be seen by the naked eye, cannot be seen through a transit telescope. An object seen through a telescope will appear at its brightest when the arrangement of the lenses is such that the pencil of rays issuing from the eye-piece is as large or larger than the pupil of the observer's eye. In this case the brightness of the object is the same, except for the losses due to loss of light in the lenses, as when seen by the naked eye. When the pencil is larger than the pupil of the eye the magnification will be proportionally small. The best arrangement of the lenses from the standpoint of illumination and magnification then, will be when the emergent pencil is of the same size as the pupil of the observer's eye. If this condition is fulfilled, and the lenses, as in the best instruments, transmit 85% of the light, the brightness of the object seen through the telescope will be 85% of its brightness as seen by the naked eye. If the emergent pencil is smaller than the pupil of the observer's eye, the brightness will be proportionally decreased. The illumination is, hence, seen to depend on (1) the aperture of the object-glass, (2) the light transmitting power of the lenses, and (3) magnification. The brightness of any object presented to the vision depends on the size of the pupil of the observer's eye, but this is not a part of the telescope.

3. The Magnification or magnifying power of a telescope is the ratio of the angular size of an object as seen through the telescope to its angular size as seen by the unaided eye, and hence depends upon the relation between the focal length of the object-glass and the focal length of the eye-piece considered as a single lens: thus, if F = focal length of the object-glass, and f that of the eye-piece, then F/f = magnifying power of the telescope = M_p .

The illumination is in inverse proportion to the magnification which, however, is an important factor also. It must be great enough to enable the observer to point his telescope accurately.

The magnifying power, M_p , may be increased or diminished by altering the focal length, f , of the eye-piece; but if the maker

increases the power too much, since only a fixed amount of light can enter the object-glass, this fixed amount of light is spread over too much surface in the field of view, and the object seen is therefore too faint. If the maker gets the magnifying power too small, then the surveyor has a difficulty in pointing his telescope accurately. The M_p of a telescope must be proportional to the aperture. If M_p is too high for the aperture, ordinary objects will appear too faint, and if the M_p is too low, the objects will appear so small that the surveyor cannot point upon them with sufficient accuracy. The M_p should be such that the least perceptible motion of the bubble of a level, or change in the reading of the verniers, should cause sufficient movement of the cross-wires over the object in the field of view to be readily noticeable. A higher power than this is worse than useless, since objects are less brilliant. A lower power would not develop the full capacities of the instrument. The M_p and the aperture of the telescopes should be adapted to the sensitiveness of the levels and the fineness of the graduations. A power of from 20 and 24 diameters in the telescopes of transits gives the best results, and that telescopes made for stadia work require between 20 and 35 diameters, and of 18" Y levels of 33, and of 15" dumpy 28 diameters (Lietz instruments).

(4) **Size of the Field of View.**—By the *field of view* of a telescope is meant the whole area seen by the observer when looking through the instrument. This area is a circle, and by the size of the field of view is meant the angle at the optical center of the object-glass subtended by a diameter of this circle. The size of the field of view (a) is independent of the size of the object-glass, (b) increases with the size of the eye-piece, (c) increases with the distance from the instrument, (d) increases as the objective is moved in towards the eye-piece, and (e) increases as the magnification decreases. The question of the proportion between the size of the field and the magnification comes in here as in the case of the proportion between the illumination and the magnification. Practice varies with different makers. A large size of the field is of course a convenience. Generally between the magnifications 20 and 30, the size of the field will vary from $1^\circ 30'$ to 1° and between 30 and 40 from 1° to $45'$ nearly.

776c. The Optical Tests of the Telescope.—A good surveying telescope should possess the following qualities: It should be free from: (1) spherical aberration, (2) the aberration of sphericity, (3) and chromatic aberration; (4) the definition should be good, (5) the illumination should be in suitable proportion to the other qualities of the telescope, and incidental to this (6) the aperture of the object-glass should be as large as practicable; (7) the magnification should be at its maximum compatible with a sufficiency of illumination, and (8) the field of view should embrace as large an area as possible. These qualities may be tested as follows:—

The Test of Telescopes in General.—If a telescope is to be tested for its qualities, make sure that all its lenses are perfectly clean,

1. To test for definition, use small, clear print, and view it from a distance of from thirty to fifty feet. If the print appears clear and well-defined, and fully as legible at this distance as if viewed with the naked eye at the distance of distinct vision, the surfaces of the object-glass are perfect and, well-finished and the definition of the telescope is good. If, on the contrary, the print appears dull and indistinct, and the finer details illegible, or even invisible, the lenses are faulty in centering, and their surfaces are imperfect and faulty in curvature, for the rays proceeding from the various points of the object are not refracted to their corresponding points in the image.

2. Indistinctness may be caused by spherical aberration, to test this, cover the object-glass with a ring of black paper, reducing the aperture to one-half; again focus small clear print to distinct vision; remove the ring of black paper and cover the center of the object-glass (previously left open), then mark how much the object-glass has to be moved in or out for distinct vision. If the spherical aberration has been reduced to a minimum, very little, if any, slide motion is necessary to obtain a distinct view under both tests. The amount of movement, however, constitutes a measure for the spherical aberration of the object-glass.

Another test, but not as good as the one just mentioned, is to focus an object to distinct vision; then slide the object-glass in or out, observing at the same time the quantity of motion necessary to render the object indistinct. If the spherical aberration is completely corrected, the object should, theoretically, be rendered indistinct by the slightest motion of the lens; but, practically, this is not the case, as the eye will accommodate itself in a measure to the difference of divergence of the rays, caused by the motion, in or out, of the object-glass, in the same manner as it will accommodate itself to near and distinct objects when viewing without the aid of lenses. So, if the image formed by a perfect object-glass is viewed by another perfect lens of long focal length, say six inches, the object-glass might be moved in or out one-fourth of an inch from the point of distinct vision, and the object will still appear comparatively clear, as the one-fourth-inch motion, with an eye-lens of such long focal length, cannot cause enough difference in the divergence of the rays to prevent the accommodation of most eyes to it. The shorter the focal length of the eye-lens, the more rapid will be the change of divergence or convergence of the rays with a certain amount of motion; therefore, the second test is only applicable with eye-pieces of very high power, which, at the slightest motion in or out, will cause a sufficient amount of divergence of the rays to prevent the accommodation of the eye to the change.

3. To test the chromatic aberration, either a celestial body or a white disc should be selected for an object.

Focus the object to distinct vision, thereupon move the object-glass slowly in and out alternately. If, in the first instance, a light yellow ring is seen at the edge of the object, and in the second one a ring of purple light, the object-glass may be considered perfect, as it proves that the most intense colors of the prismatic spectrum (orange and blue) are corrected.

4. To test for the Aberration of Sphericity or the flatness of field, take a square, flat object, the sides of which are about four inches long and perfectly straight—the best object is a heavily-lined square, drawn on white paper with India ink. Sight this object from such a distance that it will nearly fill the field of view of the telescope, and see if it still appears flat and its sides perfectly straight; if so, the telescope is a good one. If, on the contrary, the object appears distorted, i.e., if the sides, instead of being straight, form curves and the surfaces appear concave, instead of flat, the telescope is not good, for it shows that the proportions of foci, aperture and distances between the different lenses are not according to the laws of optics; owing, generally, to the attempt to force the magnifying power beyond its limits.

As all the refractions of light in the telescope are caused by flat and spherical surfaces, it is evident that the edge of a round flat object, when used for the above test, cannot be distorted, but that the surface only will appear concave to a keen observer's eye. A telescope which distorts the image to a perceptible degree will not, however, cause any errors in common use, if only one point in the lens is taken in all observations, but it is decidedly objectionable in stadia measurements, where two points in the field of view are used at the same time.

5. To Find the Magnifying Power of a Telescope.—A practical method for finding the magnifying power, available to anyone, which does not require any apparatus, taking up only a few moments' time, is the following: Set up the instrument, and about twenty or thirty feet therefrom hold up a graduated rod. Observe the rod with one eye by direct vision, and with the other through the telescope. Assume a certain space on the rod, say the height of a numeral, or two sharply-drawn lines, and count the number of divisions on the rod in that space; then observe the number of divisions that are seen by the naked eye in the same space enlarged. The ratio between the two is the power sought. It is the reading of a magnified space of known length on the graduated face of the rod. With a little practice both eyes will be able to distinguish the rod divisions at the same time. If what is known to be 0.1 of a foot, is enlarged by viewing it through a telescope so as to cover the space of 2.4 feet as seen by the unaided eye, the magnifying power is 24 for the distance in focus. The real power is somewhat less, for as the tube of the telescope is drawn out for near objects, the power necessarily increases. The magnifying power obtained by this method holds good for the distance that the rod can be read by the unaided eye, and it is always somewhat greater than the actual power.

It is well for the engineer to make a test of the power of a telescope himself, for it is to the interest of the maker to rate this higher than it really is. For a very accurate determination of the *magnifying power*, ascertain

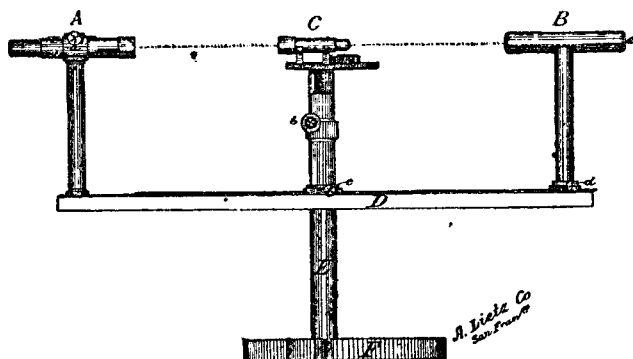


FIG. 629.—Apparatus for Measuring the Focal Length of Lenses and Lens-combinations.

the focal lengths of the objective and the eye-piece, in order, to compare them and to find their proportion. The former is obtained by a direct measurement from the objective lens to the cross-hairs, the latter, usually containing an entire system of lenses, presents many difficulties. For this purpose an apparatus which is so perfectly adjusted to do its work that it is of interest

to explain it as under:—Upon an iron base, F, (Fig. 629) stands the pillar, E, carrying cross-bar, D, upon which 3 standards are mounted. At one end of the bar the microscope, A, is held by a fixed stand. Its tube is adjustable to focus, by screw, *a*. At the other end collimator, B, slides upon the bar and is held by screw, *d*. Between the two is the holder for the eye-piece, C. The pillar slides upon the bar, and is clamped by screw, *c*. The screw, *b*, raises or lowers the small platform on which the eye-piece rests. To determine the focal length of any lens or combination, the eye-piece is placed on the platform, the collimator being turned with its small opening, *e*, towards the light (it may be used in the day time, or by lamp-light at night). The whole is properly adjusted by sliding the collimator and the platform on the bar. The eye-piece, which has been aligned between collimator and microscope, is then viewed by the latter, which is focussed by the screw, *a*, until the collimator opening, *c*, is distinctly visible as a round disc of light with a sharply-defined outline. In the field of the microscope a small graduated scale is seen, and by this scale the diameter of the light disc may be measured, by simply counting the number of lines that it covers. These give the focal length of the eye-piece directly and correctly in millimeters. Dividing the focal length of the objective, when the telescope is focussed to mean distance, in millimeters by the value just obtained, gives the magnifying power of the telescope. This apparatus measures focal lengths up to 8 centimeters or $3\frac{1}{2}$ inches. It is applicable to simple lenses as well as to any combinations of them. Even for concave lenses may be determined by it, but in that case the image lies behind the lens, and the device will measure until the lens touches the microscope and no further.

6. The Illumination.—There being no absolute unit brightness, the illumination is merely relative; and can be found only by a comparison between different instruments: the expression for the illumination is $I = LA^2/M^2$, where, *L*, is the proportion of light transmitted by the lenses, *A*=aperture of the object-glass and *M*=Linear Magnification. The brightness of an object viewed through a telescope being in inverse proportion to the area of the pupil of the observer's eye, if this is considered, and the diameter of the pupil=*e*, $B = LA^2/e^2M^2$. To test the illumination, at about dusk place two telescopes side by side. As darkness comes on, observe the same object through both telescopes. The telescope which makes the object visible later has the better illumination of the two.

7. The Aperture of the Object-Glass.—Focus the telescope for distant objects and point toward the sky. There will be formed by the eye-piece, and just behind it, a real image of the aperture of the object-glass. This image can be viewed by holding the eye some inches behind the telescope and looking through an ordinary magnifying glass. Hold the point of a pencil in contact with the outer face of the object-glass and move this in, if necessary, until it is just visible at the edge of the image formed by the eye-piece. To obtain the aperture of the object-glass from its diameter subtract twice the distance of the point from the edge of the lens.

8. The Size of the Field of View.—Focus the telescope on an object distant 300' from it and note two points, A and B, in opposite ends horizontally of the field of view. Measure the space, AB, between these points and the distance OG from the centre of the objective, O, to the point, G, half-way between A and B. OG will practically equal the distances OA and OB and measure OA or OB instead of OG. If a staff is held first on A and then on B, having focussed on A, B can be made the same distance from the telescope as A, by moving B to and from the telescope until it appears as distinct as A, the focus in the meantime remaining unchanged. For more accuracy, measure distances OA and OB making them equal. The size of the field of view equals the $\angle AOB$ and $\angle AOB = 2 \tan^{-1} (AG/GO)$. The size of the field will depend on the distance from the telescope of the objects observed. Focusing for 300' gives nearly an average size of the field.

776d. Totally Enclosed Telescope with Object-glass permanently fixed.—Some modern theodolites (of Messrs Carl Zeiss and of Watts) and levels (Casella's) are fitted with an extra internal lens between the object-glass and eye-piece which is used for focussing, when the object-glass is fixed. The advantages of this new form of telescope as made by Cassella (Fig. 630) are :

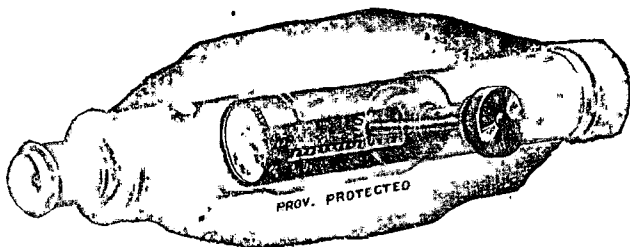


FIG. 630.—Casella's Totally Enclosed Telescope with
Object-glass fixed.

(1) the focussing screw is fixed to a pinion and the stem is carried through the hollow axis. Hence it is easily manipulated in every position of the telescope. (2) The telescope is hermetically sealed; hence it is dust-proof, and the lenses and diaphragm are free from the trouble of *dewing*. (3) It is of constant length and balanced at any focus. (4) The focussing adjustment is made by moving an internal lens on the Porro system, the object-glass being fixed permanently at the end of the tube.

This construction is free from the defects of the old system in which errors of collimation are possible owing to the dropping of the object-glass when focussed on near objects.

777. Three Principal Types of Engineer's Levels.—There are three principal types of levels in common use, the Wye or Y, the Dumpy level, and the Prism or Reflecting Level. The difference between the first two types is in the method of attachment of the telescope.

778. The Wye or Y-Level.—The oldest form of surveyor's level is the Y-level, so named from the telescope being supported in Y-shaped bearings. Fig. 631 shows the modern form of the instrument. The Ys are shown at Y, Y' edgewise. They are supported by the standards, S, R, upon the limb, L. Two collars are soldered round the telescope exactly fitting the Ys. The collars are perfectly cylindrical and parallel on the surface with the axis of the telescope, and ground in a gauge-plate to exact size so that the telescope may be turned end for end in the Ys without changing the direction of its axis in reversing it. There is a pair of flanges placed on the inside of the collars. They prevent the telescope from shifting longitudinally. The standard, R, is fixed firmly upon the limb, L, by a capstan-screw, R, while the stand S is fixed firmly by a

milled-head clamp, M. The telescope is secured in the Ys by strap-pieces, each of which has a joint at one end and a loose pin at the other, P, P. The pins are fastened by means of cords attached to parts of the instrument, and so prevent them from falling down on the ground. At the top of the innerside of the strap-piece under Y, Y" a piece of cork is inserted in a cave. The cork by its elasticity keeps an equal but light pressure upon the collar of the

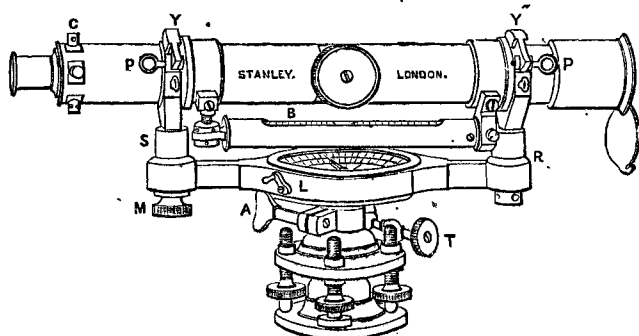


FIG. 631.—The Y-Level.

telescope. By the above plan of holding the telescope, it is free to revolve on its axis, by which perfect adjustment may be made in any direction. The Limb, L, carries a compass-box which is convenient for taking bearings and has a contrivance for throwing the needle off its centre when not in use. The spirit-bubble, B, is fixed under the telescope by a joint at one end, and a pair of capstan-screws at the other, to raise or depress it for adjustment.

Parallel Plates.—As these form part of many old instruments their construction is illustrated by Fig. 632, of which the left-hand

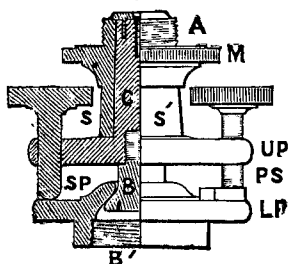


FIG. 632.—Half Sectional Elevation of Parallel Plates of Y-Level.

side is a half-section, and the right-hand side a half-elevation. The parallel plate foot is attached to the limb, L, of the instrument by the screw, A; M is a *milled-head*, by means of which the screw can be brought up firmly to its collar; SS', the socket which is ground to fit the cone C which forms part of the *upper parallel plate*, UP; B, a *ball-screw* which screws firmly into C. L.P., *lower parallel plate*, the upper part of which forms the ball socket, so that the whole instrument rotates about the ball, B, as a centre by the action of the parallel plate foot-screws, PS; B', the female screw for fixing this part, which is called altogether the *parallel plates*, to the *tripod-head* which is connected by brass joints with three mahogany or teak legs

so constructed as, when shut together to form one round staff secured for carriage by rings; a very convenient form for portability and when opened out to make a very firm stand, be the ground ever so uneven (Fig. 641). The parallel plate screws have female threads cut into the upper plate, UP , and their points press the lower parallel plate, LP , at certain points, there being a stop-piece placed round the point of one screw to prevent rotation. By turning these parallel plate screws the instrument may be made to stand on the parallel plates firmly and truly level for observation.

Clamping and Tangent Screws.—These are sometimes placed upon the axis for slow-motion. In some instruments immediately above the upper parallel plate is a collar, which can be made to embrace the conical axis tightly, by turning the clamping screw, A , (Fig. 631), a slow horizontal motion may then be given to the instrument by means of the tangent-screw, T , for the purpose of taking magnetic bearings, or to bisect any object.

The Diaphragm of the telescope of the Y-level is generally webbed with plain cross-webs—one horizontal and the other vertical stretched across a circular brass ring placed at the common focus of the object-glass and eye-piece. Four small capstan-screws, C , called the **collimating screws**, regulate the position of the diaphragm and wires in the telescope. The horizontal wire marks the intersection of the horizontal visual ray with the staff, the vertical wire enables the observer to see whether the staff is held vertically.

778 a. Watts & Sons' Y-Level (Fig. 633) has the Ys and the limb in one casting of gun-metal, the axis being of bell-metal. The

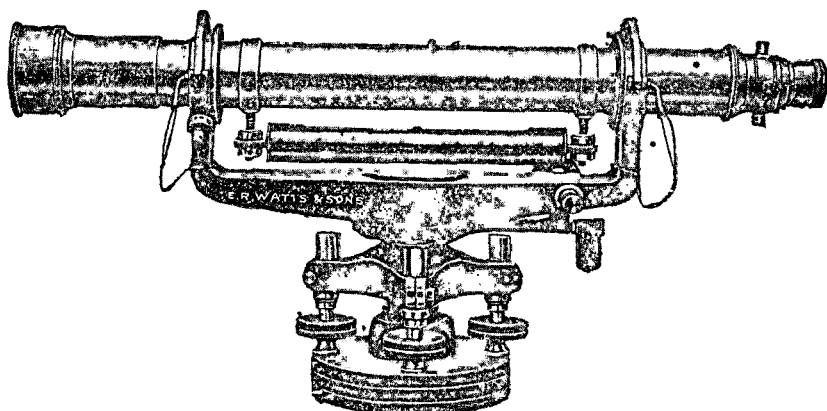


FIG. 633.—Watts & Sons' Y-Level.

Ys receive the telescope as in Fig. 631. The tribrach stage, or 4 parallel plate screws, forms the support of the instrument on the solid tripod stand head. Each of the levelling-screws is protected by a dust cap for its entire length, and has also an adjustment for

taking up wear. The telescope has a web stadia diaphragm, a clear aperture of $1\frac{5}{16}$ " and a focal length of 14". Two screw focussing inverting eye-pieces are supplied giving powers of about 20 and 40 diameters. The slow-motion screw clamp and a ray-shade are also provided.

778b. American Y-Levels.—The Y-levels are preferred by the American engineers. Fig. 634 shows the instrument as made by Messrs W. L. E. Gurley of Troy, N. Y. In Fig. 628 is given a longitudinal vertical section through the geometric axis of the telescope tube which rests in the Ys, in which it is fastened by the clips and their pins P, P, as in Fig. 631. The bar to which the Ys are attached is known as the level-bar. Connected with the level bar is the head of the levelling socket. The instrument is supported on a steel spindle P, whose axis of figure is the vertical axis of the

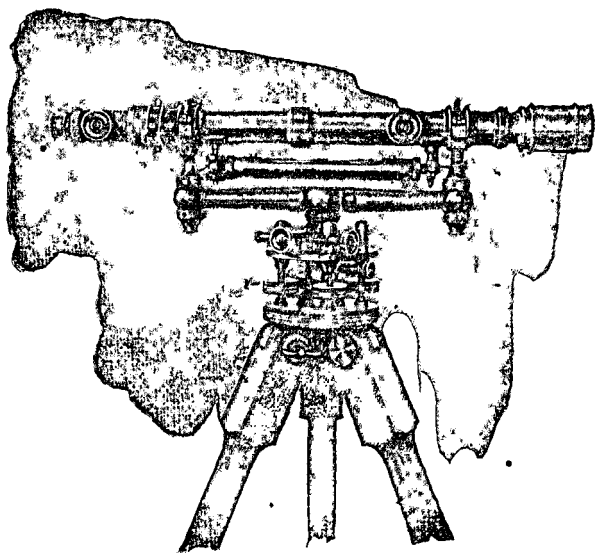


FIG. 634.—Gurley's Y-Level.

instrument and which is fitted in the socket extending entirely through the levelling head. Projecting from the socket are 4 arms, (only two E, D, are shown in the figure) heavily ribbed and tapped at the end which receive the levelling-screws, which are so threaded that they fit the long nuts accurately, securing the maximum wear and have caps protecting the threads from dust. Attached to the end of the socket by hemispherical nut is the bottom plate of the levelling head which is screwed on a tripod when the instrument is in use, and by the motion of the levelling screws the upper plate, level bar, bubble-tube, etc., are brought into any desired position,

The telescope has a rack and pinion movement to both objective and eye-piece, and the adjustment for centering the eye-piece, is shown at A, A. The arrangement for insuring the accurate projection of the objective slide is also shown at C. Both of these concealed from observation and disturbance by thin bands which screw over them.

778 c. The Reversion Levels, Fig. 635, have, by their easy adjustment and by their compensation of errors of the instruments, a great advantage to the ordinary Y-levels. These instruments can be adjusted from any place, without using a levelling rod. The

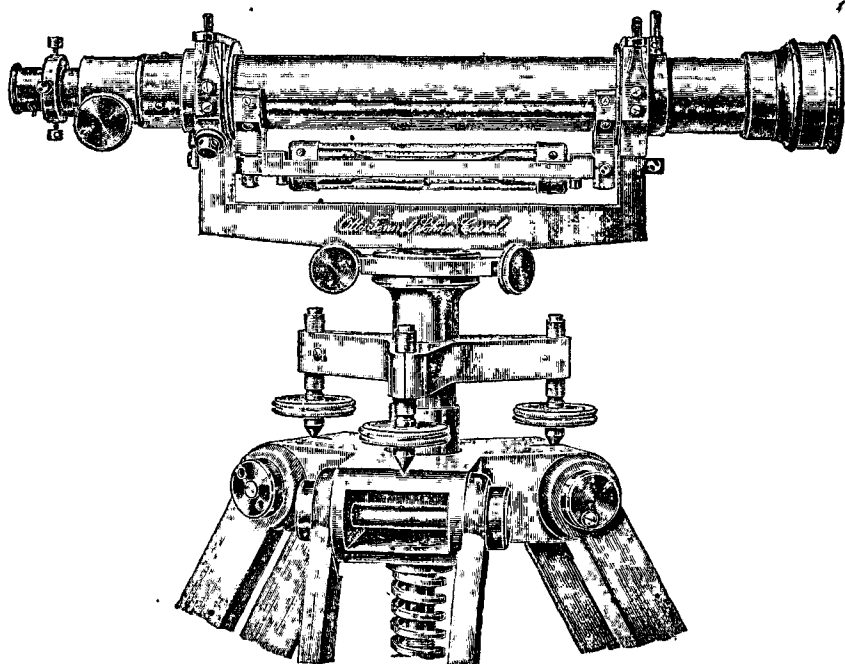


FIG. 635.—Reversion Level.

Spirit-Level used in this feature differs from the ordinary one in that it is ground to the true shape of a barrel so that the tangents to the level-bubble curves at the zero points of the scales, are parallel and diametrically opposite. By the use of the reversion-level attachable to any Y-level in place of a single reading level, it is possible to do good levelling, though the adjustment of the spirit-level and cross-wires are entirely deranged and the collars worn, by first making the level-bubble central and taking a reading, then by revolving the telescope 180° in its *P*'s, which point is indicated by an adjustable stop, making the bubble again central and taking another reading. The arithmetical mean is the

correct result. This device will, in an emergency, be appreciated when it is known that by the use of the above method, the work will average as good as that done with an ordinary good Y-level in adjustment. The adjustment of an instrument provided with a reversion-level is made in precisely the same manner as if the spirit-level was of the single reading kind, since the adjustment of the level when it is reversed will take care of itself.

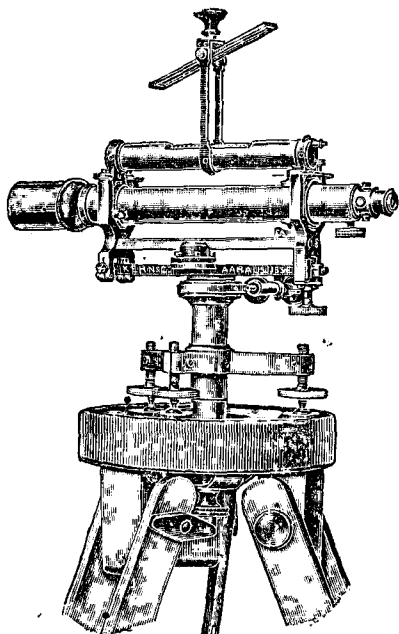
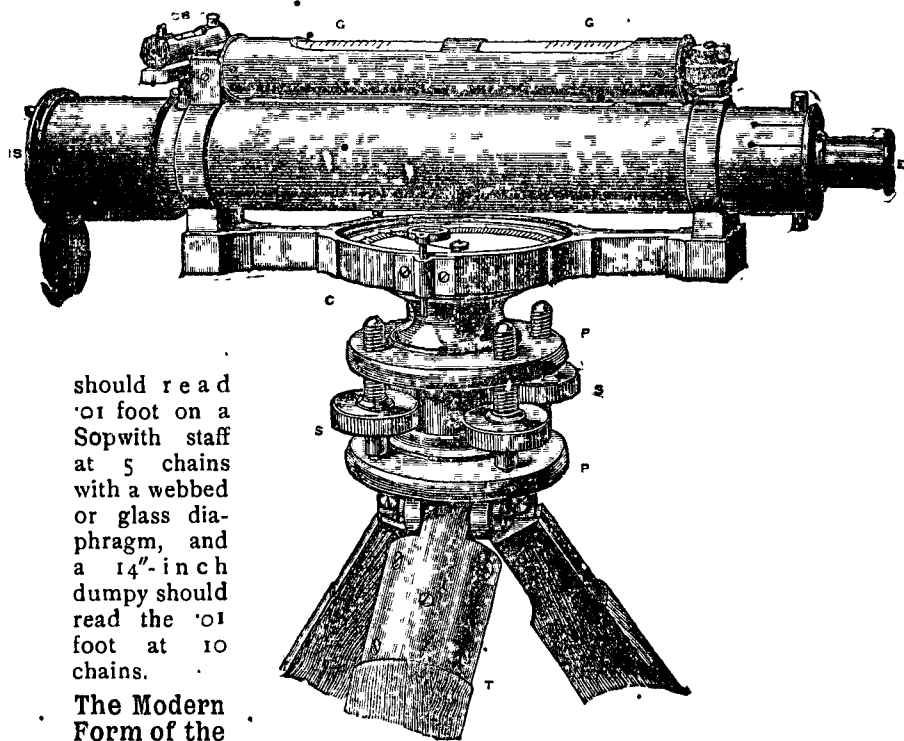


FIG. 636.—Kern's Level with
Elevation Screw.

The reversion-level is guarded by a revolvable outer tube (not shown in the cut) leaving a space of air, as a non-conductor of heat between it and the ordinary level mounting tube. This exterior tube serves both a protection against breakage, and sudden changes of temperature, and as its inner surface is painted white, it also acts as a reflector which facilitates the reading of the bubble.

778 d. The Kern's Level, (Fig. 636) has a telescope with orthoscopic eye-piece of magnification 20 diameters. The bubble-tube is supplied with a mirror. There is a micrometer elevation screw under one of the Ys; a circular bubble on the level-bar and a clamp and tangent-screw for horizontal motion.

779. The Dumpy Level.—One of the most important improvements made by the late W. Gravatt in his dumpy level was the addition of a transverse bubble, C B, (Fig. 637) by which the instrument can be set up approximately level without turning the telescope several times in two directions at right angles to each other, as was necessary in the setting-up of the *P*-level, thus effecting a great saving of time. The compact form, lightness and large field of view in the telescope of the dumpy made it a favourite among the civil engineers, and it has remained so up till now with a slight modification in its mechanical parts. It receives its name from its dumpy or compact form due to the large aperture and short focal length of the telescope which is generally inverting. The focal length is generally 9" to 14" in length. In Gravatt's time a 12" dumpy level was found to be quite equal in power and of less than half the weight and bulk to a 20" *P*-Level. A 12-inch dumpy



should read
'01 foot on a
Sopwith staff
at 5 chains
with a webbed
or glass dia-
phragm, and
a 14"-inch
dumpy should
read the '01
foot at 10
chains.

The Modern Form of the Dumpy (Fig.

637) consists of a telescope which carries a ray-shade, RS, at the object-glass end, to work in the field to eastward or westward facing a low sun. The eye-piece, EP, is adjustable to the webs in the diaphragm by a screwing motion in or out. The bubble-tube, G G, is supported upon the telescope by a pair of bands accurately fitted and soldered round it. One of the bands carries a hinge joint and the other a pair of capstan-screws for supporting the bubble-tube. The bottom part of each band-piece is a solid block of metal to support the telescope as it rests upon the limb, L, beneath. The limb may be either a casting with a socket screw only in its centre or a compass-box may be formed in the centre and a socket-screw be placed under this. The telescope support is attached to the limb (Fig. 638)

FIG. 637.—Barker & Son's Dumpy Level.

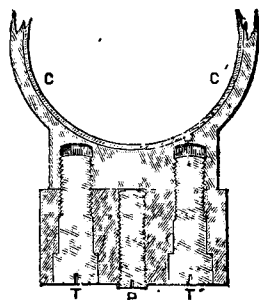


FIG. 638.—Attachment of
Telescope to Limb
of Dumpy.

by three screws two of which T, T, draw the telescope CC₁ down and one in the centre P, presses it upwards. This plan is moderately solid with the exception that a little torsion may be put on the telescope by unequal screwing otherwise the block is solidly fitted upon the limb. In cheap levels this solid ring-fitting which connects the limb firmly with the bubble-tube is replaced by blocks soldered on the telescope with soft solder: the method is very unsound from imperfect soldering as the blocks are liable to become loosened by a jar.

The Diaphragm (Fig. 639) of the dumpy level has generally two vertical and one horizontal webs. To show that the staff is held upright its image is brought between the vertical webs. The top of the portion of the horizontal web between the two vertical ones is the index of level to which all readings are taken for reading the levelling staff in the field. There are only two vertical collimating screws which adjust the diaphragm verti-

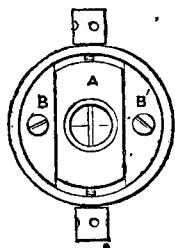


FIG. 639.—Diaphragm of Dumpy with Web.



FIG. 640.—Stadia web diaphragm.

cally in a dovetail slide. Fig. 639 shows the face of the diaphragm:—BB' slide-pieces, A—slide moved by the capstan-headed screws, Fig. 640 shows stadia-web diaphragm with two extra horizontal wires now generally used with the dumpy level.

Tripods or Stands.—The same form of tripod is used both for the *V*-level and the dumpy. In Fig. 641, the one marked, *a*, is a very strong framed mahogany stand for 8 to 12 inch theodolites; *b* is a rectangular sliding leg stand with *V* grooves for 4" to 6" theodolites and 12 to 18-inch levels. *c* is a round form sliding leg stand for the same purpose as for *b*. *d* shows the ordinary round solid form of stand for 10 to 18-inch levels and 4 to 6-inch theodolites. *e* shows the ordinary framed stand for 4 to 6-inch theodolites and 12 to 18-inch levels. The round solid form of stand is commonly used. When shut up it is a cylindrical pole which is bellied out at about one-third of its length from the top and diminished downwards and upwards from this point (fig. 641*d*). For a 14" level it is 3½" at its greatest diameter when shut up, tapering up to 2½" at both ends. For a 12" level the section is less. It is usually in the form of three legs, each of which being in the section of a V on two sides and in the form of an arc on the other. The limb is attached to the tripod either by means of parallel plates and four foot-screws or by means of the tribrach arrangement.

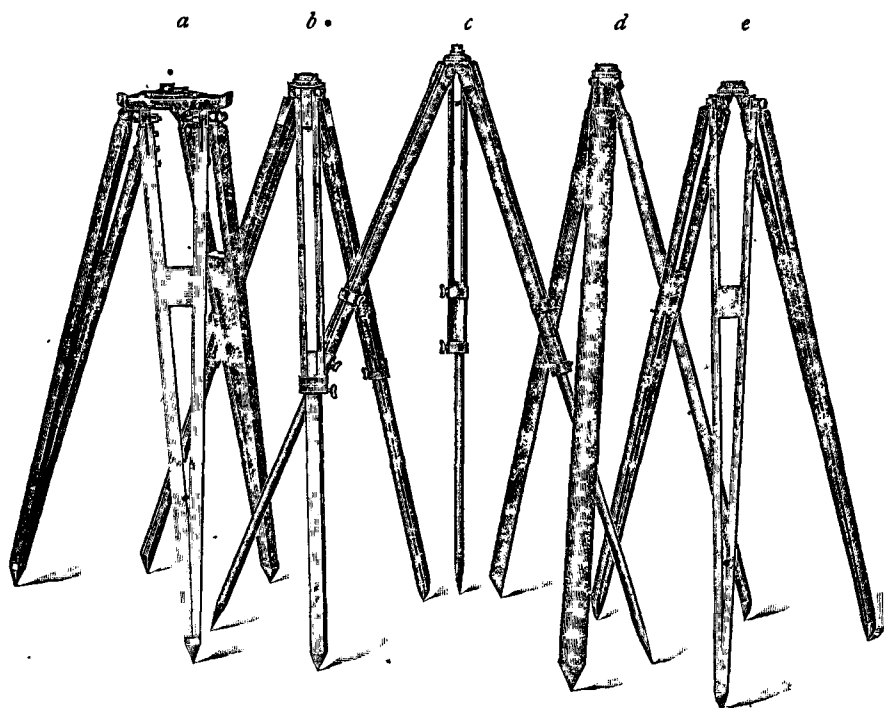


FIG. 641.—Stands of Theodolites and Levels.

779a. Improved Dumpy Level.—Fig. 642 shows a dumpy level with a much greater sensitive and longer bubble. The longer bubble-tube instead of being placed in a stiff joint at one end, or between rigid clamping nuts at both ends, has a barrel-fitting at one end which is ground into a parallel hole. This method enables circular self-ad-

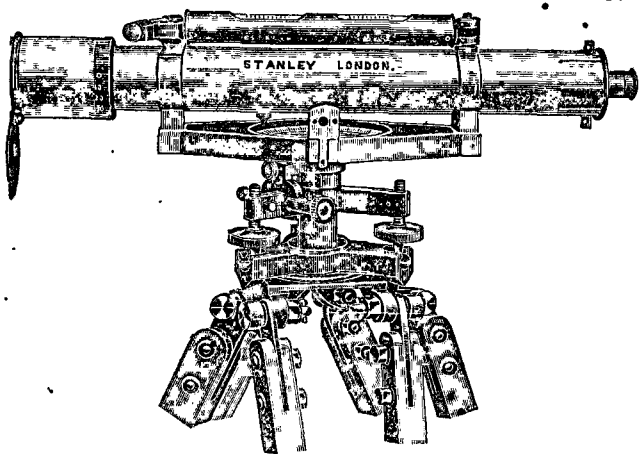


FIG., 642.—Improved Dumpy Level.

justment to be made to the bubble-tube, which the clamping of the nuts can never twist or strain during vertical displacement. The cross-bubble casing is bored entirely out of the solid. It is supported upon the side of one telescope-strap by three stout pins, the centre one fitting its hole, and the two outer ones are loosely held by cross-screws to permit a small amount of necessary adjustment. The level fixings are thus made in five pieces only including screws, instead of thirteen as usual. This arrangement makes the level solid and portable for hard wear. The limb is framed, having greater vertical sectional strength and resistance to torsion without increase of weight in the instrument. The compass is included in the frame of the limb, and is read with a prism, this being much more convenient and exact than looking down upon the divided $\odot$, the instrument being necessarily placed for use at nearly the height of the eye. The compass ring is made of aluminium. The stage and centre are in one casting of gun-metal, thus doing away with screwing together the most vital part of the instrument as in the ordinary dumpy. This adds greatly to the rigidity and general accuracy of the instrument with less weight.

Tribrach.—The above instrument has a tribrach stage instead of four parallel screws. These screws can never strain the vertical axis. In the old form of tribrach the knobs of the screws are held down by a spring plate placed above them. This plate, in carrying the load upon its stand, is very liable to strain sufficiently for the screws to get loose. In this instrument each screw has a ball at the lower end which is inserted in a tubular fitting formed in a solid tribrach, which is made of exact dimensions to take it. The tube is open on the upper side of the lower parallel plate which has a female central screw for fitting it on to the tripod. There are three pointed downward projections of the lower parallel plate which support the level upon a wall, or rock in cases where the tripod cannot be used—a most important advantage in town levelling. The tripod head is made more firmly than that of the ordinary construction thereby dispensing with shaky tripods. From long wear the levelling screws may become slightly loose

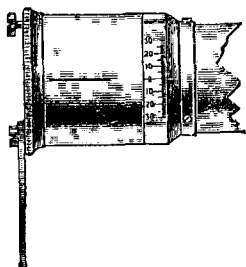


FIG. 643.—Ray-Shade.

in the tribrach arms. Cross-screws are provided for taking up this wear as it is essential that the levelling screws should be without shake in the screwed holes in the tribrach.

The Ray-Shade (Fig. 643). is pierced with two narrow sighting slits opposite each other at 180° . A zero line is carried from one slit to a line on the ray-shade-fitting when the slits are quite horizontal. Sights through the slits at zero enables an approximate cross-level to be taken. The edge of the tube of the ray-shade is divided for 30° on each side of the Zero line to 2° , for taking vertical angles of

cross-sights up to 30° up and down. This will be found useful when the vertical angle is only required fairly correct, for securing expedition in making the difference by hypo and base correction. The necessity for a ray-shade is utilized for other useful work such as for cross-section work in a hilly country.

Cooke-Deacon's Optical Square Cross-Sight Attachment, A, (Fig. 644) carrying a prism P, can be fitted to any level for ranging out lines at right angles to the main line, on flat or hilly ground. After closing the shutter, S, the line of sight is deflected through an angle of 90° , and by revolving the attachment

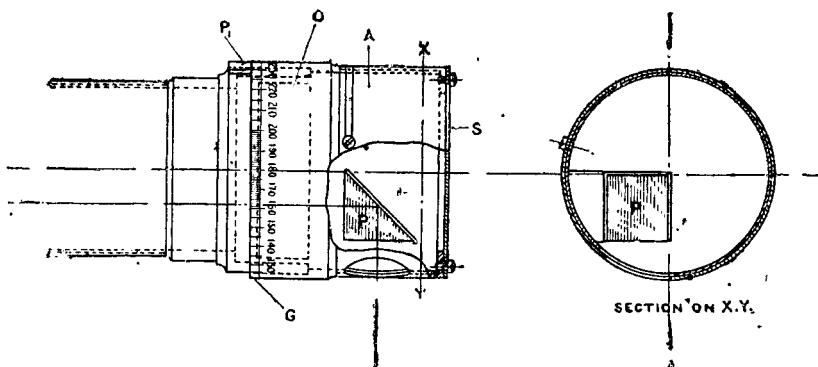


FIG. 644.—Cooke-Deacon's Cross-Sight Attachment.

about the object-glass O, any point in the vertical plane at right angles to the axis of the telescope, can be brought into focus without altering the position of the level. With the addition of stadia lines to the diaphragm tacheometrical readings may be taken in all directions at a single setting up of the level. The presence of the prism in no way interferes with ordinary staff readings.

The Index of Diaphragm.—Fig. 645 shows the index which replaces the web (Fig. 639), and is a finely-pointed needle formed of platino-iridium (platinum 75, iridium 25). This alloy has about the hardness of spring tempered steel and perfectly non-corrosive in air or moisture. The pair of vertical points indicate the position for holding the staff. The point reading is much more exact than with the web, as irradiation due to edge reading of the web is entirely avoided, and also the covering of the object as it would be intersected by the web due to the angle its thickness subtends upon the staff, which is very palpable at 10 chains distance. The iridium point is strong enough to be kept perfectly clean by touching it occasionally with the point of a camel-hair brush if it appear dusty. The point is practically more exact than the web. It is shown

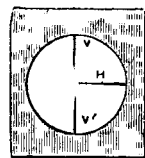
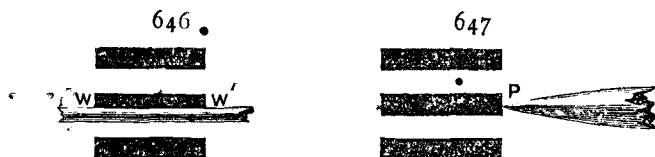


FIG. 645.—Points Diaphragm.

by Fig. 646 which illustrates the image of a division of the staff partially covered by a web, WW^1 ; and by Fig. 647 which



FIGS. 646, 647.—Diagrams showing Magnified Readings by a Web and a Point:

shows the magnified image of a point, P, brought towards a division for reading: It is clear from the illustrations that the fractional part of the $\frac{1}{100}$ foot block, which the point, P, cuts, is much more easily estimated than that in which the web, WW^1 , covers a part of a similar block. The diaphragms are interchangeable. In place of the web (Fig. 639), and ordinary platino-iridium stadia points (Fig. 645), stadia glass diaphragm (Fig. 648), or stadia points of platino-iridium for clamp and tangent levels (Fig. 649) can be fitted to all better class levels. The three latter are set for stadia reading, 1 : 100, so that the distance of the staff is read in the telescope without *Chain measurement*.



FIG. 648.—
Stadia Glass
Diaphragm.



FIG. 649.—
Stadia points for
Clamp & Tangent
Levels.

Tangent Screw Adjustment.—In Fig. 642 the tangent adjustment is shown. The old form of tangent adjustment by clamping on the axis, as this was found to disturb the centre, is avoided.

In the present illustration the clamp is left free by joining to the axis until it grips one of the arms of the tribrach upon a vertical surface, in this way it cannot disturb the axis. This tangent adjustment enables the index point to be brought exactly up to the edge of the line as it is shown in Fig. 647 at P.

Stadia Points.—The maker makes the points (Fig. 645) V, V', stadia points, by making the distance of the extreme ends of these or the distance between other two horizontal lines (Fig. 648) or points (Fig. 649) subtend an angle, equal to 10 ft. of the levelling staff at 10 chains distance plus a constant, (to be discussed in the Chapter on the Tacheometer Survey) by which the measurements of the distance of the staff can be taken or checked by observation through the telescope only. **Stadia measurements** are readily taken with either the Points or Glass diaphragm, and if care is used are more correct than chaining upon uneven ground. These are perfectly simple to use, being set at 1=100. All that is necessary is to read the staff with them. Say the upper line or point, coincides with 6.15 and the lower 8.90, then $8.90 - 6.15 = 2.75$, which is the space seen between the two points. Shift the decimal point two

places to the right and this gives 275, which is the distance in feet if a foot-staff is used. To this must always be added the constant of the instrument, which will be found marked in the lid of the case, say 1.75 feet, thus making 276.75, the correct measurement.

Stands.—The level (Fig. 642) is shown mounted on a framed stand. (Fig. 641) which is generally necessary for rigidity for large instruments of over 14".

779b. Watts' Dumpy Level.—Fig. 650 shows a dumpy, of good construction, with three foot-screws tribrach, cross-bubble,

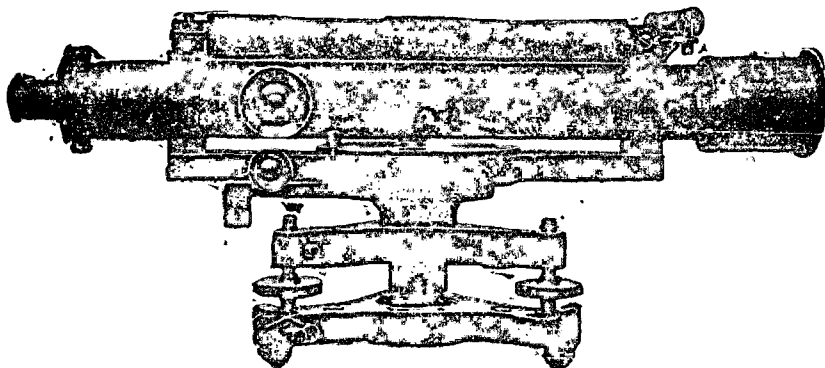


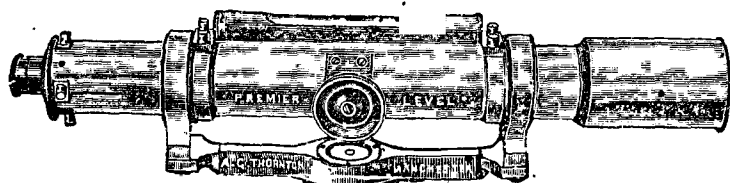
FIG. 650. — Watts' Dumpy Level.

clamp and tangent-screw, which is always in the same relative position to the observer, two inverting screw focussing eye-pieces, ray-shade graduations 20° on each side of the zero line. This level has an aluminium ring compass, reading to $\frac{1}{2}^\circ$ s, and is fitted with a prism-reader, and is made in sizes of 10", 12", 14", 16", and 18".

Size of Level.—The instruments are measured from the object-glass to the diaphragm only, instead of including the eye-piece to give them their size.

779c. In Thornton's Premier Level. (Fig. 651) the improvements made are: the focussing tube is entirely protected and covered by means of a cap fitted on end of telescope thus preventing the possibility of wet getting into the telescope, which frequently causes damage to tube and besides, dimness of glasses. (2) The hair-lines are drawn with a diamond point on a thin crystal plate. (3) The bubble-tube is enclosed in two brass tubes so as to protect the spirit-tube from injury. (4) A small level is placed below telescope for ready adjustment and to save the wear and tear of the parallel plate screws. (5) It is provided with a 15" telescope.

For a beginner a 14 in. dumpy without the compass would be most serviceable, especially if somewhat heavy tripod legs were used to promote steadiness in the field. A surveyor would get as good results from a 12 in. or 10 in. telescope with a lighter tripod.



779 d. Improved Engineer's Level.—

In this level (Fig. 652) the centre is cast in one piece of hard gun-metal with the telescope body and object-end and bubble fitting. The cast

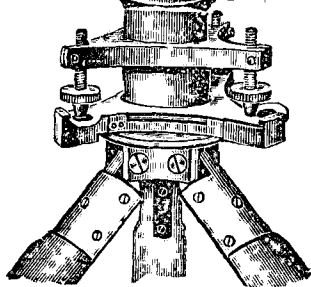


FIG. 651.—Premier Level.

telescope body is cored out to a suitable thickness, so that, although of vastly greater strength and rigidity, it does not weigh as much as the old form of tubular body, collars and stage, while it does

away with many separate pieces which are usually soldered and screwed together, and thus forms the strongest and most compact level yet made, which with ordinary care should last in perfect adjustment a lifetime. The pinion is fitted to the side of the outer cast body, which adds greatly to its strength. The level is fitted with a clamp and slow-motion screw to the horizontal movement, tribrach

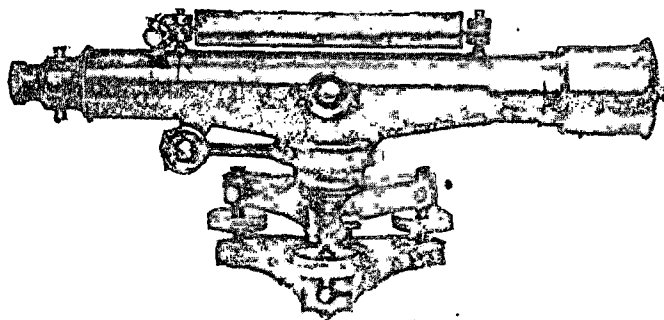


FIG. 652.—Watts' Improved Dumpy Level.

levelling, and locking plate, so that the level can be used on a wall as well as on the tripod, just as may be most convenient to the surveyor at the moment. The cap of the object-glass is provided with horizontal sight-slits and is graduated at one edge, so that side slopes can readily be read at the same time as the main level is accomplished. The instrument is fitted with a cross-level as well as a main level. The cross-level is very useful in setting up the instrument, the main spirit-level being so exceedingly delicate that it is inconvenient to use it till making the final adjustment. The diaphragm is fitted

with (a) webs (Fig. 640), or (b) Platinum-iridium points (Fig. 649), or (c) lines on glass (Fig. 648). These are set for stadia reading 1 : 100, so that the distance of the staff is read in the telescope *without chain measurement*. The points do not cover up the divisions of the staff, are permanent in all climates, may be dusted with a brush. A $2\frac{1}{2}$ " prismatic compass is packed separately in the case, and can be fitted to put on the instrument when required.

779 e. Improved Engineer's Level with Quick Setting Ball Joint.—This instrument (Fig. 653) is similar to the above;

but has the addition of the Stanley quick-setting ball-joint, by which the instrument can be levelled in half the usual time and at considerably less wear to the foot-screws. It allows of a much greater range of adjustment so that the instrument may be used on very uneven ground.

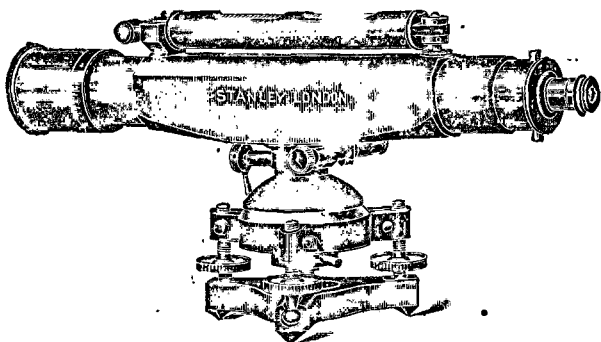


FIG. 653.—Improved Engineer's Level with Quick-Setting Ball-Joint.

779 f. Casella's Dumpy Level (Figs. 654-656).—The special features of this instrument are : (1) that focussing is obtained by an internal lens, moved by a rack and pinion (Fig. 656). This enables the optical parts to be enclosed completely and ensures that

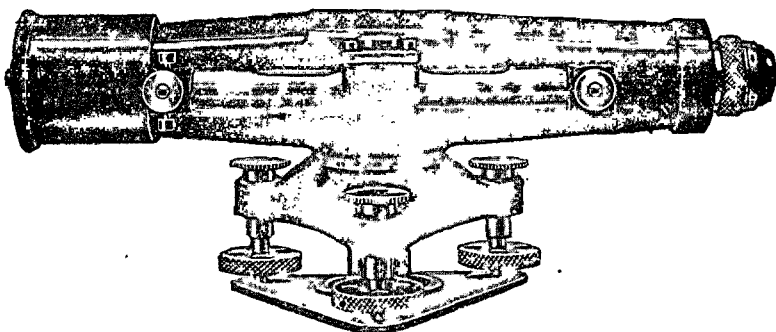


FIG. 654.—Casella's Dumpy Level showing Bubble-Tube on Side.

the line of collimation is not disturbed when the instrument is set to focus on any object from 6' to infinity. The fact that dirt or moisture cannot find its way into the instrument will be appreciated.

(2) The bubbles are fixed to the side of the body, the main bubble being fitted in a nickel-plated casing in order to obviate as much as possible changes due to temperature variation. The nickel-casing also carries a circular bubble which is used for approximate levelling

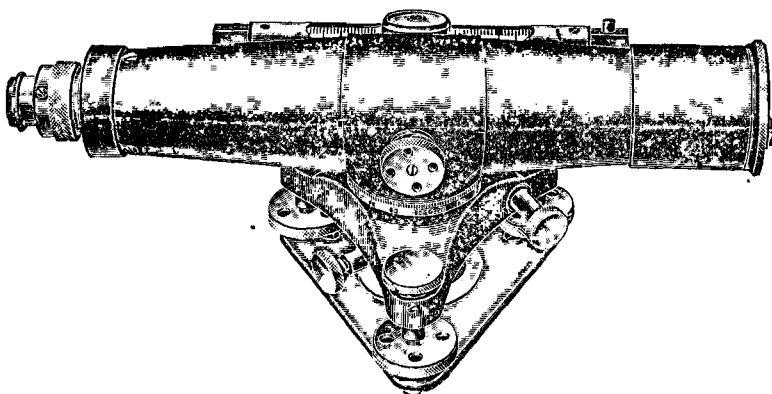


FIG. 655.—Casella's Dumpy Level showing another Side View.

of the instrument. This form of bubble (Fig. 655) will be found much more convenient in use than the usual type of cross-bubble. (3) The diaphragm is ruled on a disc of glass and can be adjusted when necessary by screws under the cover in front of the eye-piece, but no adjustment of the diaphragms is required unless repairs due to accidental damage have been rendered necessary. The eye-piece gives a magnification of 24 diameters. A graduated circle, (Fig. 655)

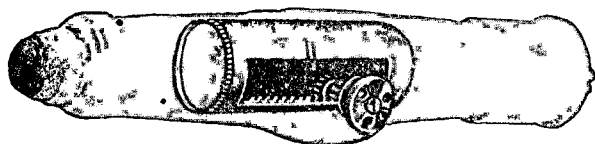


FIG. 656.—Telescope of Casella's Dumpy Level.

is of open frame pattern and shoes are fitted with steps for pressing the legs into the ground with the foot.

779 g. American Engineer's Dumpy Level.—In this instrument, Fig. 657, the telescope is permanently held by two vertical arms attached to the level bar, and cannot be taken therefrom. The level tube rests on these arms over the telescope, and is also fixed. The telescope tube is thereby brought as close as possible to the tripod head, which is a desirable characteristic. This dumpy has a bronze centre, a 15-inch telescope, a vial of such curvature, as to give for each inch of motion of the bubble an angle of three minutes. The bubble-tube is provided with a folding mirror, which protects the exposed spirit-level when shut down, or as an indicator

to the observer at the eye-piece, of the exact position of the bubble, when elevated. Stadia hairs are also fitted to the diaphragm.

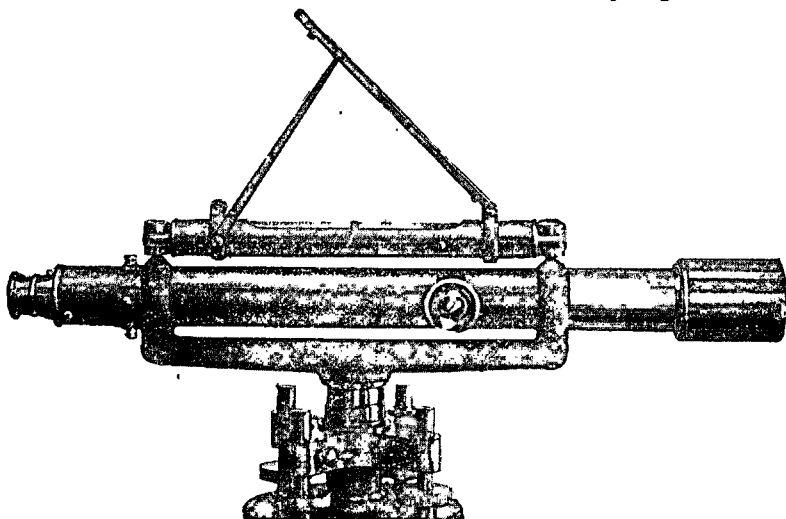


FIG. 657.—Litz's American Dumpy Level.

780. Comparison between the Y and the Dumpy Levels:—

From the opticians' point of view, Y-level is a very perfect instrument in its arrangement for adjustments, but it possesses a good *many loose parts* and the Ys and collars do not remain as perfect as when they leave the optician, for they are liable to wear by friction of constant movement in being carried about. The collars become corroded by exposure, and sand from flying dust fixes into the collars and Ys. The cross-bubble on the dumpy effects a great saving of time in setting up the instrument. The cross-wires of the diaphragm in the dumpy cannot be disturbed from their position by rotation of the telescope after the level is once set up and the vertical webs indicate at once if the staff is held vertically. The dumpy differs from the Y-level in being attached to the level-bar by immovable upright pieces, in having the level-tube firmly secured to the uprights of the level-bar, in being provided with an inverting eye-piece (unless ordered otherwise) and in the absence of tangent and slow-motion screws in most instruments (q. v., figs. 642, 650, 652, 653). Compactness is the object aimed at with the dumpy, hence superior to the Y-level in retaining its adjustments, and this must be secured at the sacrifice of the parts of the Y-level which may be considered more in the light of convenience than necessities. The methods of adjustment of the dumpy are not so convenient, resembling closely the adjustment of the transit telescope and its attached level. The peg-adjustment of the dumpy is a great bugbear to young engineers.

781. The Cooke's Engineer's Reversible Level (Fig. 658) is of the Dumpy Form.—This instrument has been specially designed with two objects in view; first to afford an efficient means of adjusting the line of collimation of the telescope to perfect perpendicularity to the vertical axis of the instrument; and secondly, to obtain a form of construction in which great compactness, strength and rigidity are secured to the instrument where most required, rendering it far less liable to be deranged by careless

handling, which is almost inevitable under some circumstances than the older well-known forms of reversible levels, notably the

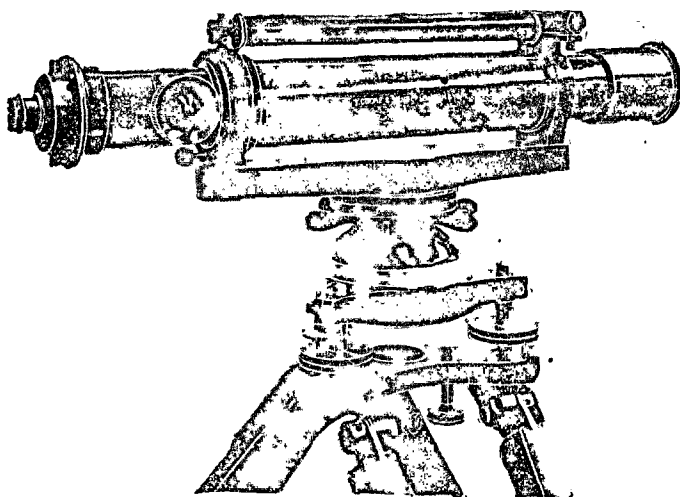


FIG. 658.—Cooke's Reversible Level.

Y-Level, all the advantages of which are retained in this instrument whilst its objectionable features are avoided.

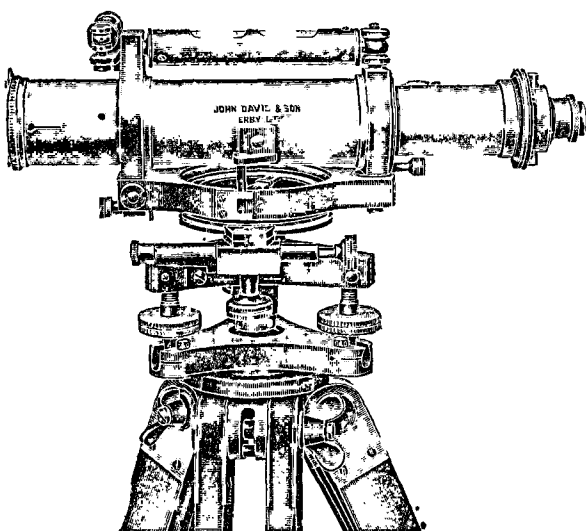


FIG. 659.—View showing Prismatic Reader for Compass, Clamp and Tangent to lock body and locking screw.

This instrument can be had in three patterns: (1) with 3 foot-screws and locking plate, main and cross-bubbles, high and low power and inverting eye-piece; ray-shade with cross-sight slits giving vertical angles up to 30° above or below the horizontal axis of the telescope, and compass and prismatic reader in the centre of the level bar; (2)

With 3 foot-screws on tribrach 'stage permanently attached to

foot-screws on split-leg tripod, slow-motion adjustment, and detachable compass screwed on at the end of the main bubble; other features are the same as in (1). (3) With 3 foot-screws and locking plate over framed tripod with rough levelling arrangement and detachable compass, other features are the same as in (1). A general view of this third pattern is given in Fig. 658.

FIG. 3.

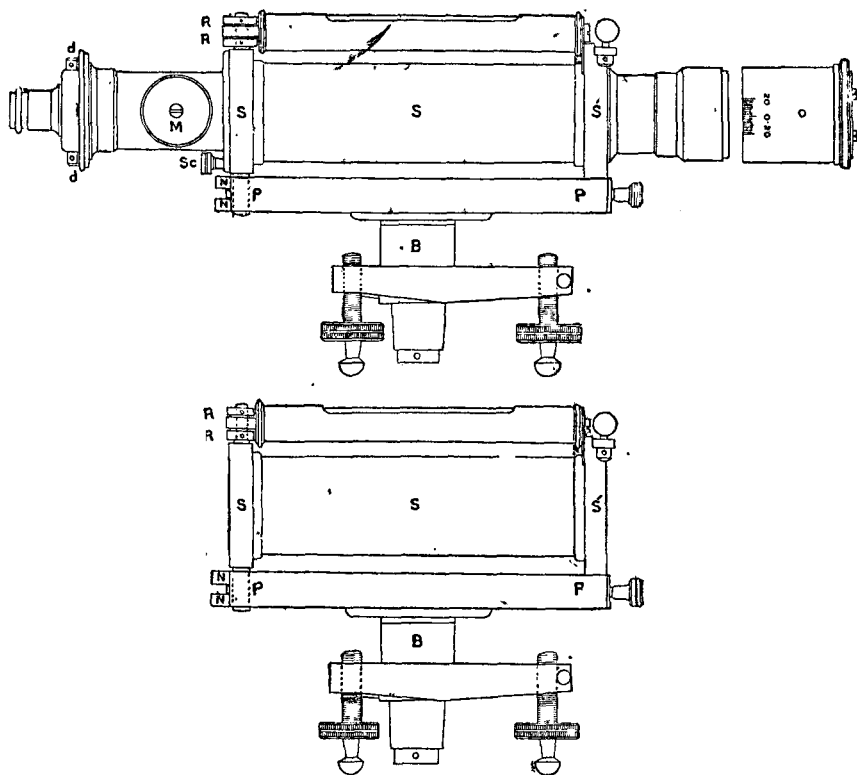


FIG. 2.

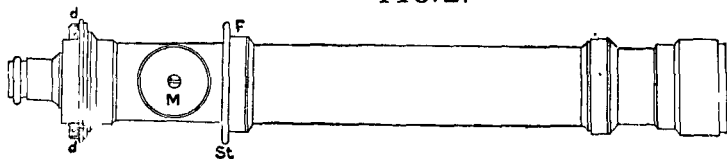


FIG. 1.

FIG. 660.—Construction of Cooke's Reversible Level.

781.a. John Davis & Son's Reversible Level (Fig. 659) is similar to Cooke's first pattern.

781b. The Construction of the Cooke's Instrument is shown in Fig. 660, where fig. 1 is the telescope (shown detached), which bears two perfectly equal cylindrical flanges, F and F' , turned on it concentrically with the tube, being essentially similar to the telescope of a P -Level. The cross-lines consisting of one horizontal line and two subsidiary vertical lines, are ruled upon parallel glass carried by a perforated block in the eye-end, which is adjustable vertically to correct collimation by the two antagonistic screws, d and d' . The object-glass is made to approach to, or recede from the cross-lines for focussing, by turning the milled-head, M . The two flanges, F and F' , instead of resting upon two P 's, fit perfectly within two corresponding collars, S and S' , which latter form the ends of the rigid socket, $S-S-S'$. The two flanges, F and F' and their corresponding collars S and S' being exactly equal, the telescope tube can be introduced indifferently from either end of the socket $S-S-S'$ (fig. 2), and pushed home until the stop-flange, St , comes in contact with the socket end. The telescope can be further secured in its socket by the screw Sc . One end, S , of the socket is furnished with a threaded bolt which passes through a hole in the base-plate, $P-P$, and is adjustable vertically, and locked by the two lock-nuts N and N' .

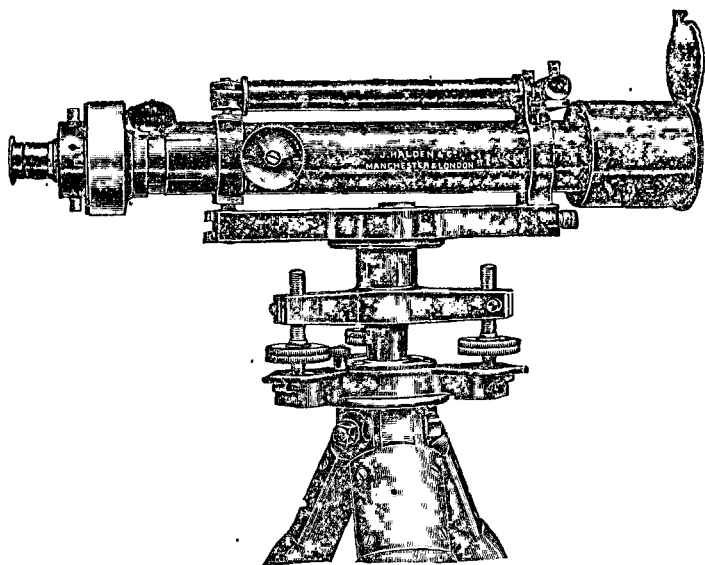


FIG. 661.—Cushing's Reversible Level.

782. Cushing's Reversible Level.—This instrument (Fig. 661) resembles the dumpy except that its telescope is of improved

form possessing the necessary adjustment of the *P*-level. The telescope is rigidly fixed in collars soldered to the tube, as in the dumpy. The ends of the tube form stout socket collars which are exactly alike, and are ground to fit either the objective or the eye-piece end of the telescope, so that these parts may be reversed, the one for the other. This reversing is almost equivalent to turning the telescope end for end in the *P*-level. The end also rotates in its fitting, which is nearly equivalent to rotating the telescope half round in the *P*-level. Each of the reversible ends of the telescope is secured in its ground fitting by studs and slides (*bayonet notches*). By this plan adjustments may be made of collimation and of fixing the line of collimation perpendicular to the vertical axis like the *P*-levels, if the object-glass is originally correctly centered. The diaphragm is fitted with fine lines on glass which resemble the webs in the eye-piece. As glasses become often bedewed in the telescope the eye-piece end may be removed from its ground fitting, and the glass cleaned and replaced without disturbing the adjustment in any degree. The supports of the telescope have a rocking axis at one end; and are adjusted by capstan-headed nuts at the other. The adjustable support for setting up the instrument is upon the tribrach system. The illustration given is that of a 14" level; with the 14", an open framed stand is also used in place of the solid tripod, as shown in Fig. 66t.

782 a. Orr's Reversible Level. is exactly similar in construction to the Cushing's Level and its permanent adjustments are also the same.

783. Notes on Diaphragms of Levels, Spider-webs.—These are clear and good, but very delicate, sagging with damp and breaking by tension, if long in the instrument to become very dry. Unless under special skill of the user, the re-webbing and adjustment must be done by an optician.

Lines on Glass.—These are accurate and permanent. The presence of the glass diaphragm obstructs a little light at all times, but the great defect of this diaphragm is that it becomes bedewed by changes of temperature and requires cleaning. This may be done effectively with soft silk or a cigarette paper, and the lines filled in by breathing upon the face and finely scraping some black lead from a soft pencil, and gently rubbing it into the lines with the paper. In Stanley's instruments the glass cell can be removed from its ground fitting (by simply withdrawing it with the fingers after unscrewing the tube holding the eye-piece) to be cleaned and can be replaced without interfering with the adjustment.

Points Diaphragm.—This gives an exact reading and is permanent, so that an instrument with points will remain in adjustment for many years. In case of dust on the points this can be removed gently with a camel-hair brush provided in the case. The only objection made to this diaphragm is that there is some difficulty in bringing the points up to the reading of a distant staff. This is entirely remedied by having tangent adjustment to the axis, which is always provided in a theodolite and is recommended for a point reading level. Points are the most satisfactory of all, but where they are objected to, a webbed diaphragm is recommended, and a spire glass diaphragm in case of accidents to the webs, and this glass diaphragm (stadia reading) in a gun-metal case is supplied at an extra cost of 12/6.

Object-Glass.—This should *never* be removed unless absolutely necessary for cleaning, and then the greatest care should be used. *Notice particularly*

the position of each lens and which face is forward and replace in the same way. An old silk handkerchief is the best for cleaning. After having the object-glass out for any purpose, the adjustment of the telescope will require checking and possibly correcting.

784. The Adjustments of the Level are divided into Temporary and Permanent Adjustments like those of the theodolite:

784. a. Temporary Adjustments of the Level.—These are adjustments which must be attended to each time the instrument is set up, and are as follows:—

1. **Setting-up of the Level.**—Open out the tripod legs so that they stand inclined, if on level ground, towards the centre of the instrument at an angle of about 70° to the horizon. The toes of the legs must then be set firmly in the ground, though, to insure this, they need not be pushed in with all the operator's strength. If on a floor, set the toes in cracks, on a hillside put two legs downhill and one up. Always make the tripod head as nearly level as may be. This will facilitate final levelling of the instrument. The instrument is then taken out of its box, (the foot-screws, S, S (Fig. 637) are then brought to the centre of their run,) and screwed tightly down upon the tripod head in case of levels with *parallel plates* (Fig. 632), or Stanley's improved tripod head (Fig. 642). In case of instruments with *Locking Plates* the conical feet of the levelling-screws fall into V-grooves in the head of the tripod. The locking plate which is a thin 3-armed metal plate is centred upon the hollow axis of the head so that it will move horizontally. It has a *hole and slot* at the end of each of its arms, the holes permit the feet of the levelling-screws to fall into the V-grooves in the tripod head. The locking plate when moved horizontally locks all the feet at once, and secures the instrument on the stand head from accidents. A milled-headed screw clamp fixes the locking plate in its locked position. The head of this screw is under the tripod head.

2. **Adjustment of the Eye-piece to the Wires and Adjustment to Focus of the Object-glass.**—**Elimination of Parallax.**—The foci of the object-glass and eye-piece must be made to coincide with the cross-hairs of the diaphragm. The eye-piece is adjusted by sliding it gently in and out in its tube with a slight screwing motion until the webs, lines on glass, or points are seen most clearly and distinctly on looking through the telescope. (The telescope of the 18" to 22" American Levels are furnished with rack and pinion movement to both objective and eye-piece). The eye-piece remaining in focus, the telescope is racked out or in by means of the milled-headed screw (focussing screw) at the side of the telescope (Fig. 660. M) until any well-defined object, such as a division on a staff, is distinctly visible. All errors of parallax must now be removed. If now the eye be moved up and down, and the object sighted appears fixed at the same position to the wires, the focus is perfect. If, in moving the eye, the object bisected appears to follow its motion about the intersection of the wires, the focus of the telescope lies beyond the wires; the object-glass must therefore be moved inwards by turning the focussing screw very gently in a left-handed direction. If, on the other hand, the object sighted moves in the reverse direction to the eye, the focus of the telescope is towards the eye-piece, and the object-glass requires to be moved outwards by the milled-head in a right-handed direction. After a few trials the object and the wires will appear to coincide however obliquely observed. The adjustment of the eye-piece need only to be made once, but the object-glass must be focussed for each new object sighted.

3. **To Level up the Instrument.**—(a) To set up a Four-Screw Level with Parallel Plates, having set up the tripod and screwed the level over it, turn the bubble-tube parallel to one pair of levelling screws and by turning them bring the bubble to the centre. Then turn it over the other pair and repeat the operation. If the level is in permanent

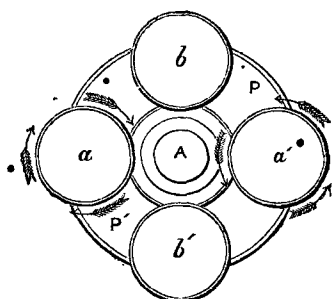


FIG. 662.—Plan of Parallel Plate Screws.

instrument would be raised. If turned in the opposite direction, the right-hand end would be raised. The opposite end always requires to be raised from that to which the bubble runs. When the bubble retains the centre over the screws, a, a' , it is then placed over b, b' , and adjusted in a similar manner, returning again to the position, a, a' , for final adjustment. If the level is in permanent adjustment the bubble will retain the centre in making a complete revolution of the instrument upon its vertical axis. Be careful not to work the screws too tight. If they become so, loosen all four and start again.

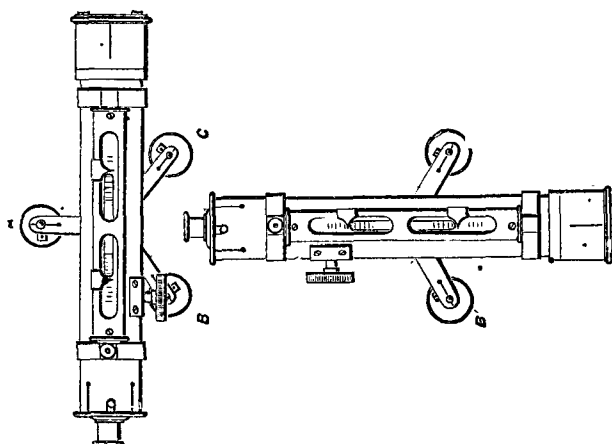


FIG. 663.—To set up a Tribrach Level.

(b) To set up a Tribrach Level.—Place the level lineally with either pair of screws, B, C, and bring the bubble to the centre of its run (Fig. 663). Then place the level at right angles to this, so that, the screw, A, comes under the telescope; bring the bubble to its centre; now return to first position to see that this is right. If not exact, repeat the operation. As there is no strain on the axis in a tribrach level, it is made much more sensitive than a parallel plate one.

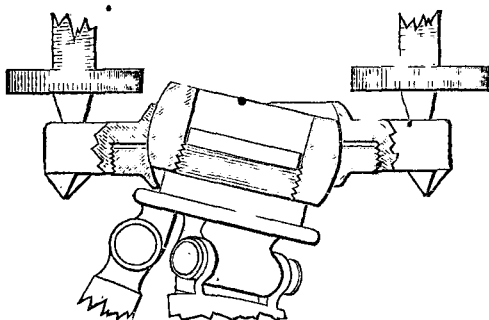


FIG. 664.—Quick Setting Locking Plate.

allows of sufficient rocking movement in any direction to compensate for the uneven setting up of the stand. It does not add to its height. By its use the instrument may be set nearly level, and in less than half a turn of the screws will bring it into true position. It will be a valuable addition to any levelling three-screw theodolite which has to be used on uneven ground.

Quick Setting Locking Plate.—For use on very uneven ground Stanley's Quick Setting Locking Plate saves much time and wear of the levelling screws. This improvement, Fig. 664, consists of a section of a ball (screwed to fit the stand top) fitted within the locking plate, and a simple means of firmly clamping it in any position, which when released

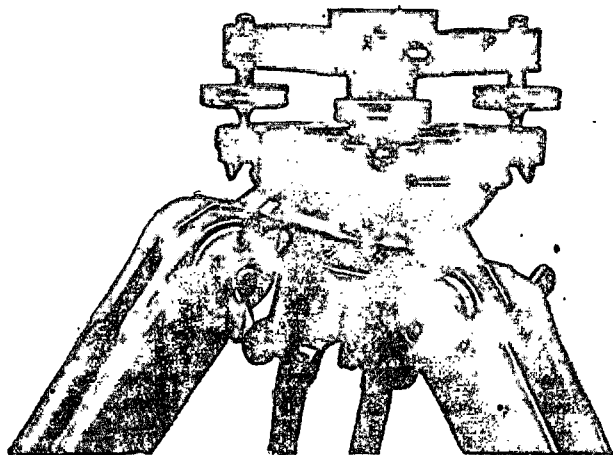


FIG. 665.—Quick Levelling Tripod.

Watts' Quick Levelling Tripod (Fig. 665) Stand will be found convenient for approximately levelling an instrument on rough ground or where the movement of the legs is restricted, through having to work in confined spaces, a great saving of time will be effected, and the levelling screws will be saved a large amount of unnecessary wear. The full range of motion allowed by this tripod, excluding the levelling screws, is 24°.

Quick Setting Ball Joint.—An improved form of ball-joint (Fig. 653) which gives all the advantages of the Hoffman-head combined with tribrach system. By its use the level can be set up in far less time than with the ordinary construction, and it also allows of a much greater range of adjustment so that the instrument may be used on very uneven ground. It can be instantly set up approximately and the levelling screws then used for a final adjustment which is effected with less than half a turn, thus saving wear on the screws (q. v., Fig. 658).

785. Adjustments.—*In a theoretically perfect Level the following points are established:—*(1) The object and eye-glasses are perpendicular to the optical axis at all distances apart.*

(2) The optical axis coincides with the axis of rotation of the Ys.[†]

(3) The axis of collimation coincides with the optical axis.

(4) The axis of collimation is parallel to the telescope level.

(5) The collars resting in the Ys are circles of the same diameter and concentric with the line of collimation of the telescope*.

(6) The Ys are exactly similar and similarly placed with reference to the line of collimation of the telescope.*

(7) The level-bubble moves over equal spaces for equal displacements of the telescope in altitude.*

(8) The level-bubble expands or contracts equally from the centre in both direction during changes of temperature.*

(9) The vertical axis of revolutions is perpendicular is to the line of collimation of the telescope.

* Of the above, the maker establishes points numbered 1, 2, 5, 6, 7 and 8. The remaining points 3, 4, 9 are established when the instrument leaves the shop, but being liable to derangement from rough usage they are made adjustable in the field.

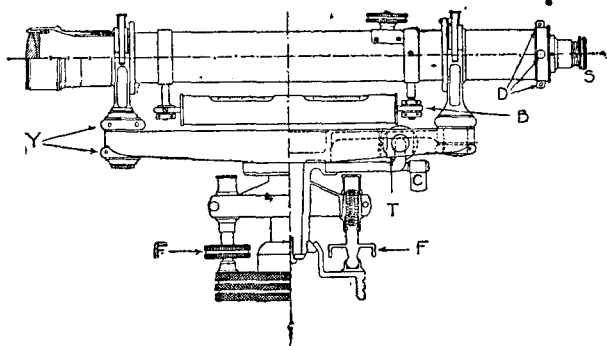


FIG. 666.—Watt's Y-Level Adjustment.

786. The Permanent Adjustments of the Y-Level, which must be made in the following order, are:—1. To Adjust the Line of Collimation of the Telescope, i. e., to bring the cross-wires into the longitudinal axis, so that their point of intersection will strike the same point of a distant object during an entire rotation of the telescope in its Ys.

—First some well-defined test-object, which need not necessarily be of exactly the same height or level of the instrument, is selected. The test-object should not be less than 20 feet away from the observer. See that the instrument is securely screwed home on its stand, FIG. 666, and that the tripod itself is quite firm and rigid; bring the foot-screws, F, F, to the centre of their run, and roughly level by the aid of the legs, and press the legs well into the ground. See that the telescope lays over two opposite foot-screws and clamp the instrument by the clamp, C. Bring the bubble to the centre of its run by means of the foot-screws, F, F. In this adjustment it is not necessary to level the instrument. Now the instrument is placed on its stand in such a

manner, that when the telescope is pointed towards the object, it lies directly over one of the foot-screws. The eye-piece, S, is carefully pushed in or out in its tube, until the cross-lines are seen as distinctly as possible, and the telescope is then focussed upon the test-mark. All error of parallax must now be corrected: If, when the eye is moved gently up and down or sideways behind the eye-piece of the telescope, the image of the test-object moves with respect to the horizontal cross-line, the object-glass must be brought nearer to, or taken farther away by the milled-head rack-screw, until no movement takes place. If the cross-lines are now undefined, and appear to flitter, the eye-piece must again be adjusted in its tube. If the object moves, the object-glass requires adjustment. If the cross-wires move, the eye-piece requires adjustment.

After all parallax has been corrected, the test-mark 100 to 500' distant, is carefully bisected by the tangent-screw, T, and, after the instrument is clamped with the horizontal wire on the selected point. Then gently, without jarring, the Ys are opened, and the telescope is carefully revolved or rotated half-way round in its Ys, bottom side up, till the cross-line is again horizontal. If the same spot is not now bisected, but falls above or below the cross-line, the error is corrected half by the foot-screw underneath the telescope, and half by the vertical antagonistic screws, D, at right angles to the cross-line at the eye end of the telescope, always remembering the inverting property of the eye-piece, i.e., the diaphragm ring must be moved in the direction opposite to what appears to be the right one; unless the telescope is an inverting one. Then turn the telescope back again to its former position, and again by the tangent-screw bring the cross-hair to coincide with the test-mark. Then again turn the telescope halfway round as before. The hair will now be found to be more nearly in its place, but, very likely, not precisely so. Hence the operation must be repeated until accuracy is obtained. Now revolve the telescope a $\frac{1}{2}$ turn and correct the vertical wire in the same way. Repeat this operation till the point of intersection of the wires remain stationary, i.e., strike the same spot when the telescope is revolved entirely round in its Ys.

II.—To Adjust the Axis of the Telescope, so that it may be exactly at right-angles to the vertical axis upon which the instrument turns. The telescope is focussed upon the test-mark, preferably upon a line of a levelling staff, placed over one pair of foot-screws, and the instrument is levelled, and the image carefully bisected by the horizontal cross-line, the telescope is then lifted out of its Y's and reversed end for end in them. If on re-directing the telescope towards the test-mark by turning it half a revolution on its vertical axis the image of the latter is no longer bisected by the cross-line, the error is corrected half by the foot-screw underneath the telescope, and half by the milled-head, M (Fig. 631), or by the circular lock-nuts Y (Fig. 666) which are placed on the adjustable Ys. The above operation must be repeated until the bisection of the image is as exact as possible, and remains quite undisturbed by the reversal of the telescope in its Ys. then turn the instrument 90°, so that the telescope lays over the other pair of foot-screws, F, F', and repeat this operation till bisection by the cross-line is correct, and the bubble remains correct in whatever position the instrument is turned on its axis.

III. The Adjustment of the Bubble-Tube.—The instrument is first set up as nearly level as possible by the foot-screws. The telescope is then turned in a direction parallel to one pair of foot-screws and by turning them the bubble is brought to the centre of its run. The telescope is then turned half-way round upon its vertical axis, and the bubble is brought to the centre, half by the foot-screws and half by the circular nuts, B, at the end of the bubble-tube; (Fig. 666). Next, turn the telescope only a quarter round, that it may lie over the other foot-screw, or other pair of foot-screws, then bring the bubble to the centre, by the aid of the foot-screw or screws. The telescope is again turned half-way round; and any error, which will be very small,

corrected half by the foot-screws and half by the circular nuts at the end of bubble-tube. The bubble should now remain in the centre of its run, while the telescope is turned all round upon its vertical axis, but if it does not, the same method must be continued, until the required exactitude is obtained. With a little practice and care the accuracy of the level's adjustments can any where be tested and effectually made in a few minutes.

Note.—It is sometimes convenient to perform the adjustments indoors, by day light a small cross may be marked with ink on a sheet of white writing-paper for the test-mark, which may be placed at as great a distance as convenient, say 20' or 30'. By night a pin-hole may be made through a piece of paper and a candle or a lamp be placed behind it. The bubble of a well-constructed level can be adjusted by night by placing a cross drawn on paper with a candle or gas-burner shining on it at 20', or so distance from the instrument.

786a. Some Levellers make the Permanent Adjustments of the Y-Level in the following order:—Before examining the adjustments, the sun-shade should be placed on the telescope, as it is only accurately in balance with this.

1st Adjustment.—To Set the Spirit-level parallel to the Line of Collimation, and, at the same time, place its axis in a plane with that of the Telescope. It is best to attend to the latter first.

Operation.—Turn the telescope so as to stand over two opposing foot-screws, clamp the instrument and bring the bubble to the centre of the tube; then rotate the telescope in its Ys by opening the clips, so as to put the level out of a vertical (say 15° or 20°). If the bubble changes its position, it shows that the axis is not in a plane with that of the telescope. Correct it by moving the two side screws of the level case, until one-half of the deviation has been taken up. A few repetitions will insure accuracy, and destroy the side motion of the level.

The Level must now be made parallel with the line of the bottom of the Collars.

Operation.—Bring the bubble to the centre of the tube; then reverse the telescope in the Ys end for end; do this carefully. The displacement of the bubble, if there be any, is the double error, which is corrected by taking up one-half of it by means of nuts, B; and the other half with the foot-screws, F, F. This operation is repeated until the bubble remains in the centre.

To accomplish a proper adjustment of the Level to the Line of Collimation, it becomes absolutely necessary that the Collars be of equal diameter, if not, a Y-level is not any better than a dumpy, and will have to be adjusted as such.

Providing the Ys are filed out to the same absolute angle, the instrument may still be adjustable in all its parts:—The spirit-level may be made parallel to the line of the bottom of the collars; the Ys may be so adjusted that the bubble will remain in the centre of the tube; the line of collimation may be brought to the centre of revolution of the telescope; and this reversed end for end in the Ys, leaving the bubble in the middle, even if there be some difference in the diameter of the collars. It is the general opinion that after level, Ys and cross-lines are adjusted, the instrument must be correct. This is by no means certain, as the least difference in the Size of the Collars will throw out the line of Collimation considerably. This difference is sometimes found in new instruments, and is also produced by unequal wear, denting, etc. It is, therefore, advisable that the equality of the collars should be tested from time to time, which is done by a method given further on.

2nd Adjustment.—To place the Level at right-angles to the Vertical axis of the Instrument.

Operation.—Turn the instrument so that the telescope shall stand over two opposing foot-screws, and bring the bubble to the centre of its tube; then turn the instrument 180° on its centre. If the bubble shows any displacement, correct one-half of it by means of the nuts, Y, under the bar at the Y-Supports and one-half by the foot-Screws. Repeat till perfect.

3rd. Adjustment.—To place the cross-wires in the optical axis of the telescope, so that the intersection of the cross-wires will remain on an object in revolving it.

Operation.—Set the intersection of the cross-wires on a point at about, 200' or 300' distant, then revolve the telescope in its Ys half-way, so as to have the level-case on top. If the wires have moved, from the point, bring them back one-half of the amount of the displacement. Try again, and repeat the operation if necessary.

The eye-piece may then be properly aligned and directed by the four diaphragm screws, so that the field of view shall appear evenly divided by the cross-hairs. In this, as well as in any other telescope, we assume that the tubes are straight, the object-glass well-centred, and the slide well-fitted.

The Collar Test.—After the instrument is properly adjusted, the equality of the collars may be ascertained in the following manner:—

Operation.—Make two bench-marks, place the instrument exactly midway between them, and find their true difference of level by reading level-staffs set upon them. Now place the instrument near one of the bench-marks and read the staffs again. If the difference of the reading is equal to the true difference of level, the collars are of equal diameter, and the line of collimation is at right-angles to the vertical axis of the instrument. This test, once made, holds good ever after, as it shows that the collars are true, and hence that a correct adjustment is assured of all its other parts. But denting, the settling of sand particles, and unequal wear will also affect the adjustments in the same way. If the test shows that the collimation line is *not* perpendicular to the vertical axis, then the collars are of unequal diameter, and the instrument is nothing more or less than a dumpy, as this defect deprives it of all the advantages for an easy adjustment which characterises the Y-level in comparison with the dumpy. This defect may, however, be temporarily adjusted in the same manner as the line of collimation in the dumpy level is adjusted, but it must ever thereafter remain permanently in its Ys, as it would, if reversed end for end, double the error which existed previous to this adjustment. The correction may also be made by displacing the horizontal cross-hair to the extent that the line of collimation shall be truly horizontal and, at the same time, parallel with the axis of the spirit-level; but in that event, there will be no longer any agreement with the optical axis, which again gives rise to a number of inaccuracies that can not be obviated.

A Y-level must have equal diameters of its collars, if not, it should be sent to the instrument-maker for remedy; for if one collar exceed the other in diameter the centred bubble, if reversed in the Ys, will indicate a displacement corresponding to 4 times the angle intercepted between the collar axis and that of the spirit-level.

787. The Permanent Adjustments of the Dumpy Level are: 1. To set the Bubble-tube at Right angles to the Vertical Axis of Motion or in other words to make the Instrument Traverse, so that the bubble remains at the centre of its run in every position of the telescope. Bring the bubble to the centre of its run approximately by the stand, place the telescope over two of the foot-screws, and by turning them inwards or outwards, as necessary, bring the bubble to the centre of its run. Now turn the telescope through 90°, so that it may be over the 3rd foot-screw, and by it bring the bubble to the centre of its run. Repeat the process till the bubble remains at the centre in two positions of the telescope at right angles to each other remembering that the telescope has not been turned end for end in the operations. This is a *temporary* adjustment which makes the bubble axis nearly horizontal and the vertical axis nearly vertical. Then turn it through a further 90° or 180° from its original position, and place it over the same two foot-screws again. If the bubble now retains the centre, the axis of the bubble is horizontal and the instrument is in adjustment; if not, correct half the deviation by the foot-screws and the other half by the opposing nuts at the one end of the bubble-tube by loosening one and tightening the other, by means of the tommy pin (provided in the case), or by the lower telescope

screws T, T', (Fig. 638) which carry the telescope on the lower bar. Repeat the whole process, if necessary, till the bubble retains the centre for a complete revolution of the telescope on its vertical axis of motion.

II. To Set the Diaphragm so that the Cross-hairs shall be absolutely Horizontal and Vertical.—Put on the sunshade, focus the eye-piece until the hairs are distinctly visible and the parallax destroyed; then proceed as under:—Loosen the capstan-head screws that secure the diaphragm at the eye-end of the telescope, and rotate the diaphragm slightly until a point or a small mark is bisected by the opposite ends of the horizontal web when the telescope is moved in azimuth (q. v., p. 78, Art. 135). The vertical wire is of no particular importance.

III. To Set the Line of Collimation parallel to the Axis of the adjusted Bubble-tube, i.e., to ensure that the Collimation line is horizontal when the Bubble is at the centre of its run or in other words, to bring the horizontal wire of the Diaphragm into Collimation, Known as the Peg Adjustment, or Running a Level Line in the Air.—

First Method.—(a) Select a piece of fairly level ground, and drive in 2 pegs 300' apart; set up the level exactly half-way between them so that the 2 pegs and the centre of the Dumpy are all in a straight line. Place a levelling rod upon one of the pegs, and take an exact reading with the level. Then place the rod upon the other peg, and drive it down further until the reading is identical with that obtained when the rod was held upon the first peg. We have now the two pegs exactly level. (b) Now set up the level at a point 15' to 20' beyond one of them but in a line with these two pegs. Place the rod upon the near peg, and take the reading, taking care that the bubble is in the centre of its run. Now place the rod upon the further peg, and if the reading is the same as before, the bubble being in the centre of its run, the collimation is correct. If the latter readings differ from the former, correct the greater part of the discrepancy by means of the top and bottom diaphragm capstan-head screws, and repeat the operation (b) until agreement is obtained. The level will now be in correct adjustment.

2nd Method.—There is no necessity to bring the two pegs to the same level, e.g., On a tolerably level piece of ground mark two points for setting

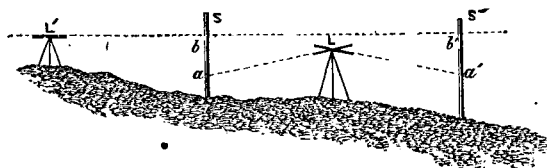


Fig. 667.—Adjustment of Dumpy Level.

up successively a levelling staff, S and S', say 660' apart (Fig. 667) either with Δ staff plates well-trodden down on the ground, or with pegs or stones driven firmly down in the earth to make a firm base for the staff, if the surface is not firm. Set the level up at L, exactly half-way between S and S', and in a line with them. Now go through the temporary adjustments and take the readings of staff in both positions, i.e., back-sight of S, and then the fore-sight of S'. Then, whether the level be in adjustment or not, the staff readings at S and S' will be similarly affected, and difference of these readings will be the correct difference of level of the points S and S' over which the staff is successively placed. If the reading a on staff at S be 3.25, and that a' on staff at S' be 5.75, the correct difference of level is $5.75 - 3.25 = 2.50$.

Next remove the level to L', on one side of both staves, say the left, 1 chain from S, and in such a position that both the staves at S and S' may be in the field of view. Now level up the instrument carefully and read the staff S. To this reading add or subtract, as the case may be, the true difference of level already obtained between a and a' , the result would be the reading that the leveller should read on staff at S', if the collimation line be in adjustment. Read the staff at S' now, and if the expected reading is not obtained, it shows that the collimation line is not horizontal or parallel to the bubble-

tube. If the reading be higher or lower than the expected reading it shows that the collimation line is elevated above or depressed below the horizontal position. The difference between the reading obtained and that expected on staff *S'* gives the amount of error due to the slanting position of the line of collimation. Take $\frac{1}{10}$ of this error and add or subtract it from the reading on staff at *S'* according as the collimation line is depressed or elevated, and by means of the collimating screws set the horizontal wire to the correct reading. This would result in a corresponding alteration in the reading on staff at *S*; and the difference between the new readings on staff at *S* and *S'* will be found to agree with the true difference of level previously obtained between the points.

Ex. 1. If the readings be 8.60 and 9.80 when the level was removed to *L'*, find to what reading must the collimation line be set on the distant staff by the collimating screws so that it may be made horizontal. The true difference of level between the points = $5.75 - 3.25 = 2.50$. Therefore for a reading of 8.60 on staff *S*, we should get a reading of $8.60 + 2.50 = 11.10$ on staff *S'*, and in lieu of that we get 9.80. Therefore the error due to the line of collimation being out equals $11.10 - 9.80 = 1.30$. Here the line of collimation is depressed. In 10 chains the error is 1.30, therefore, in 1 chain it will be 0.13 which is the correction for the nearer staff *S* to be added, or the new reading to which the collimation line must be set by the collimating screws on the staff *S* should be $8.60 + 0.13 = 8.73$. For 11 chains the correction will be $1.30 + 0.13 = 1.43$ on the distant staff *S'* or the reading to which the line of collimation must be set on the distant staff *S'* is $9.80 + 1.43 = 11.23$. The difference between the new readings on the staff at *S* and *S'* should now give the true difference of level = $11.23 - 8.73 = 2.50$ roman the same result as that when the level was at *L*.

Ex. 2. If the readings on the levelling staff at *S* and *S'*, 10 chains apart, be 7.50 and 4.50 when the level was at *L*, and 6.50 and 5.50 when the level was removed to *L'*, 1 chain outside the first reading. Find to what reading must the line of collimation be set on the distant staff by the collimating screws so that it may be horizontal.

Then the correct difference of level is $7.50 - 4.50 = 3'$, therefore for a reading of 6.50 on *S* we should get a reading of $6.50 - 3.00 = 3.50$ on staff *S'*, and instead of that we have 5.50. Therefore the error due to the collimation line being out equals $5.50 - 3.50 = 2$. Here the line of collimation is elevated. Hence, the correction for the staff *S* equals to $(1 \times 2)/10 = 0.20$, and for the staff *S'* equals to 2.20. Therefore the new readings to which the collimation line must be set by the collimating screws will be $6.50 - 0.20 = 6.30$ and $5.50 - 2.20 = 3.30$ and the correct difference of level will come to $6.30 - 3.30 = 3$ as before.

The 2nd method is geometrically correct and direct, requiring no approximate measurement from a peg to the centre of the lens, no trial setting of the telescope, no trials to drive a peg just enough and not too far,

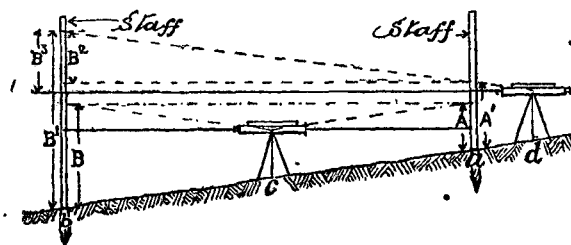


Fig. 668.—2nd Two-Peg Method of Adjustment of the Collimation Line in the Dumpy Level.

the instrument at *c*, take readings *A* and *B* on *a* and *b* respectively, carefully

and no auxiliaries except staff, and tape or chain. On slightly rising ground locate 4 points, *a, b, c, d*, on the same line making $bc = ca$, and ad any distance, preferably not much less than ca , and in some simple ratio with it, for ease of calculation afterwards. (Fig. 668). Set

levelling before each sight. Then, unless the instrument is otherwise much out of adjustment, $(B-A)$ is the *true* difference of level of a and b .

Next set up at d , level carefully, and take readings A' , and B' , on a and b respectively. [In strictness the centre of the instrument should not be set over d , but beyond, by an additional distance = principal focal length of the object-glass plus the distance from that lens to the centre of the tripod (q.v. Chapter on Tacheometry)]. Then $(B'-A')-(B-A)=B^2$ = *error of collimation* in the distance bx , i.e., the error due to the vertical angle between the line of sight and the axis of spirit-level. Now, by similar triangles we have $-B^2 : bx = B^3 : bd$, $\therefore B^3 = (B^2 \times bd)/bx$, which is the error in the distance bd , and is to be applied to the reading B' . Set the rod to read $(B'-B^3)$. Then for adjustment of a dumpy, having first adjusted the bubble so that it remains true in all positions about the vertical axis, point the telescope on the rod, held at b with target set to read $B'-B^3$. By means of the diaphragm screws raise or lower the horizontal line until it bisects the target. To test the adjustment, set the rod over a , with index reading $(B'-B^3)-(B-A)$, and see if the target is still bisected.

Adjustment of Attached Level of Transit Telescope (q.v., p. 74).—The staff being held plumb at b , with target set to read $(B'-B^3)$, move the telescope by *Vertical Tangent-Screw*, until the line of sight bisects the target; clamp securely. Then bring the bubble to the center of the tube by means of the *level-adjusting nuts* alone. Test as in the other case. *Remarks.*—The diagram shows a special case, viz., when $(B'-A') > (B-A)$, or the angle subtended by B^3 is one of elevation. If $(B'-A') = (B-A)$ the line of sight is already level and no adjustment is needed. If $(B'-A') < (B-A)$, B^3 subtends an angle of depression, and is to be added to B' . In the latter case, if the slope of the ground is slight, the difference $(B'-A')$ may be either zero or a very small quantity, positive or negative; but in all cases it is added algebraically to $(B-A)$ to obtain B^2 .

As in all other methods of adjustment, we assume that the maker has done his part so well that the line of collimation will not be disturbed in any movement of the objective for focussing. Let us suppose that the line of collimation is made truly horizontal, and that in its prolongation we have set the centres of two targets, one over a and one over b , the instrument being at d . If now we focus upon the farther target, the image will be bisected by the horizontal web. Then change the focus so as to view the nearer target. If the centre of the objective has not moved truly in the line of collimation, the new image will not be bisected at the focus, and the nearer target will appear to be out of level, when in fact it is not. Hence, since this adjustment requires change of focus, it cannot be made if the above defect, in the movement of the objective, exists. (q.v. p. 77, Art. 134). If, however, the distance ad be not too small and the defect alluded to be only slight, the error in changing focus for b and a may be hardly appreciable. The adjustment once made approximately, we need not remark that, in the field-work, any further error of objective is avoided when taking equidistant sights.

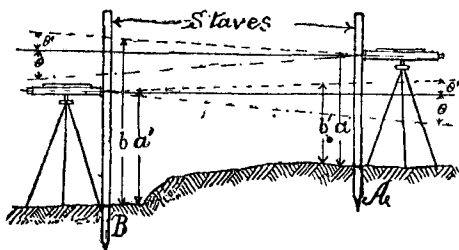


Fig. 669.—Third Method of Adjustment of the Collimation Line in the Dumpy Level.

3rd Method.—Drive two pegs, A and B , on a nearly level ground and spaced about 200' apart (Fig. 669). Set up the level so close to one of them A , that when the levelling staff is held upon it in a vertical position the eye-end of the telescope will be about half an inch from its face. Level the instrument, and look through the *object-glass* at the

staff, swaying the latter gently until it comes into the focus of the eye-piece about half an inch from the face of the staff, and set a pencil-point on the staff at the centre on the small field of view which should be from $\frac{1}{8}$ " to $\frac{1}{4}$ " in diameter. Read and book the height, which we call *instrument height*, a . Next, hold the same staff on distant peg B, add with the bubble in the centre, read the height there in the usual way, and call it *staff-reading* b . Now carry the level to the distant peg B, set it up, carefully levelled, close to the staff, taking a similar reading through the object-glass, which call *instrument height* a' . Remove the staff to the first peg and read it, calling it *staff-reading* b' . Now, if the line of sight had been parallel to the bubble-axis in each case, it would have been represented by two horizontal lines when the bubble was in the middle of its run, and hence the difference of the readings in each position would be the same, i.e., $b' = b - a'$. (This assumption neglects the earth's curvature. This is 8" 1 mile, and is proportional to the square of the distance. For 200' it will be about 0.001 of a foot, and twice this, or 0.002 of a foot is the error made in the above assumption). When, however, the line of sight is inclined to the bubble axis (Fig. 669) it makes a vertical angle of elevation or depression $= \theta'$ or θ . In the case of an angle of elevation, the first difference of readings is augmented and the second difference of readings diminished by that angle θ' .

Hence, when the difference of readings at a , i.e., $a - b'$ is less than the difference of readings at b , i.e., $b - a'$, the line of sight *throws upwards*, and when more it *throws downwards*. This is simply explained as below: "When the sum of the instrument heights, $a + a'$ is less or more than the sum of the staff readings $b + b'$, the line of sight throws upwards or downwards, and half the difference is to be deducted from or added to the second staff-reading."

The difference of the two differences $= (b - a') - (a - b')$ clearly measures double the angle θ' . This may be transformed into $(b + b') - (a' + a) = d$ say; and is twice the error. In Fig. 669 as the line of sight threw upwards, it, $\frac{1}{2}d$ has to be deducted. The test of this method is that wherever this level is placed when the adjustment is complete, whether touching either of the staves or midway between them, the difference of level is precisely the same. The angle θ' is caused by the line of sight not being parallel to the plane of rotation, and having determined its amount, *eliminate the whole of it* by the screw L (Fig. 637) or $\perp$, $\perp$ (Fig. 638) by screwing it up or down until the reading on the staff is corrected by the amount which measures the angle θ' on the staff. This will of course disturb the bubble. Bring the bubble back to the centre of its run entirely by capstan-screws at the end of the bubble-tube. We kept the plane of rotation horizontal all the time and we have now all 4 lines representing (1) the plane of rotation adjustable by the foot-screws, (2) Level bar adjustable by screw L (Fig. 637) or screws $\perp$, $\perp$ (Fig. 638); (3) Line of sight adjustable by diaphragm screws; (4) Bubble axis adjustable by the capstan-screws at the end of the bubble tube.

The *line of sight* could have been made *horizontal* by correcting the final error by the diaphragm screws, but the method as above given is the best.

When the screw L (Fig. 637) under the level bar is not provided as in the case with smaller dumpys, the parallelism of the Level bar and the horizontal plane of rotation can only be effected by the instrument-maker. (*q.v.*, p. 603).

4th Method.—When, however, the difference of level is known, the instrument need only be set up beside one of them, and thus a combination of the last two methods will shorten operations. Set up the level midway between two pegs and read the staff on peg, A, with a reading, say 5.42'; and remove the staff to peg, B, and read it, say the reading is 6.42', i.e., B is 1 ft. lower than A. The level is now carried near to A, and set up with its eye-piece $\frac{1}{2}$ away from the staff. Set up at A the telescope resting over one foot-screw. The instrument is levelled and bubble adjusted (Adjustment 1) and the staff at A read by looking through the object-glass with the value of the height of the line of sight of 4.62, and as B is 1' lower than A, the line of Sight to be horizontal must cut the staff held

on peg B at 5.62 ; if not, adjust to obtain this reading by diaphragm screws or screws $\perp \perp$ (Fig. 638). By this method the focus does not require altering and hence least liable to error.

When the adjustments are completed, particular care should be taken that all the screws are tight ; if not, the adjustment will not last long but if carefully made it will last a month's steady work. Every day before starting 5 to 10 minutes work will show that all the adjustments are in order by the above method. In levelling work either with the Y or with the dumpy, be careful to have the instrument set up just half-way between the last T.P. and the B.M. where the work is to be taken up and carried on. Then before starting again, set up near the B.M. and determine again the difference in elevation of the two points, this amounts to taking a test level and will serve as a check on the adjustment of the instrument

Examples of the 3rd Method.

READINGS AT STATION :		EXAMPLE 1.		EXAMPLE 2.	
		Line of Sight throwing upwards.		Line of Sight throwing Downwards.	
A	Ht. of Inst. at A = a	4.52		4 43	
A	Staff reading at B = b		5.16		5.25
B	Ht. of Inst. at B = a'	4.58		4.49	
B	Staff Reading at A = b		5.06		1 75
	Sum of Inst. Hts. = $a + a'$	9.10		8.92	
	Sum of Staff Readings = $b + b'$		10.22		7.00
	Difference	-1.12		+1.92	
	$\frac{1}{2}$ Difference	-0.56		+0.96	
	Reading of True Horl. Line of Sight " from B "	5.06 4.50		1.75 2.71	

To which readings, 4.50 and 2.71, the cross-hairs are to be brought by the screw L or $\perp \perp$.

788. Gravatt's method of adjustment of levels with 3 staves was commonly used before but is not now used for the method does not effect what it wants to do, i.e., to place the intersection of the cross-wires in the axis of the telescope which is not absolutely essential to accurate levelling.

What is wanted is only that the line of collimation should be horizontal and move in a horizontal plane, *i.e.*, it should be parallel to the bubble-tube adjusted

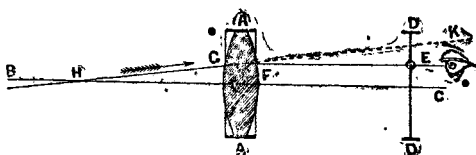


Fig. 670—To demonstrate that the Exact Coincidence of the horizontal cross-wire of the cross-wires with the Axis of the Telescope-*E*, instead of traversing the axis *BC* of the tubes, remains at a certain distance from that axis. Then, when the inner tube is drawn in and out, the cross-wire, *E*, will move along the straight line *EF* parallel to the axis *CB*. Let *H* be the *outer principal focus* of the object-glass situated on *CB*. Then it is known that, all rays of light whose paths within the telescope are parallel to *BC*, pass through the focus *H*, outside the telescope; so that, for example, a ray of light whose path within the telescope is *FE*, has for its path outside the telescope the straight line *HG*; and hence it follows that all possible positions of the cross-wire *E*, as the inner tube slides in and out, coincide with the images of points situated, in *one straight line GH*.

Consequently that line (or its prolongation within the telescope, *GK*) may be regarded as the *true line of Collimation*; and if the spirit-level is adjusted so as to be parallel to that line, correct results will be obtained in levelling, although the cross-wire may not traverse the axis of the telescope.

789. The Permanent Adjustments of the Cooke's Reversible Level are.—(1) The adjustment of the socket S.S.S., Fig. 660, so that its axis of figure may be exactly at right angles to the vertical axis *B*, is effected thus:—First, some minute and well-defined test-object, which need not necessarily be on the same level or height as the instrument, is selected. A small needle-hole in a piece of paper, fixed not less than 20 feet away, forms a convenient mark; but if the instrument requires adjustment in the open field, where perhaps the above-mentioned mark cannot be made use of, it will be most likely an easy matter to select some immovable and sufficiently well-defined test-mark from the features of the surrounding landscape. The instrument is mounted firmly upon its stand in such a manner that when the telescope is directed towards the test-mark, it may lie directly over one of the foot-screws. The screw-focussing eye-piece is gently rotated until the cross-lines are seen as distinctly as possible, and the telescope is then focussed upon the test-mark, and with the aid of the foot-screw the image of the latter caused to fall nicely upon that part of the horizontal line lying midway between the two vertical lines. All error of parallax must now be corrected. If, when the eye is moved gently up and down the eye-piece, the image of the test-mark does not move in the slightest degree with respect to the horizontal cross-line, then the telescope is correctly focussed; but if the image seems to follow the movements of the eye, that is to tend to move above the cross-line when the eye is raised, and to move below it when the eye is lowered, then the image is not focussed exactly upon the cross-line, but lies on the side of the cross-lines towards the object-glass. The object-glass must now be caused to approach the cross-lines by means of the milled head. If the image of the test-mark should apparently move in a direction contrary to that of the eye, that is to tend to move below the cross-line when the eye is raised, and *vice versa*, then the image lies nearer to the eye than the cross-lines, and the error is corrected by moving, with the aid of the milled head, the object-glass farther away from the cross-lines. After all

* Taken from Rankine's Civil Engineering.

parallax has been removed, and the test-mark carefully bisected by the cross-line, the screw, *Sc*, is taken out, and the telescope is withdrawn carefully from its socket, and when right out, the socket is turned round with the other hand, end for end, and then the telescope is carefully pushed home again. The telescope is now reversed in its socket, or more correctly the socket is reversed upon the telescope. Care should be taken that no dust is allowed to settle upon the flanges of the telescope during its insertion, and that the hole in the stop-flange, *S*, through which the screw, *Sc*, passes, is kept vertically below the centre of the tube when the telescope is pushed home, so that the single line in the eye-end may be as horizontal as possible. If on redirecting the telescope towards the test-object, the image of the latter is no longer bisected by the cross-line, but falls either above or below it, then the foot-screw below the telescope is turned until the image of the test-mark is brought as exactly as possible half-way back towards bisection by the cross-line. The lock-nuts, *N* and *N*, are then loosened sufficiently to allow them to be turned by a moderate pressure on the end of the lever used for their adjustment, and the end, *S*, of the socket is raised or lowered until the test-mark is again nicely bisected by the cross-lines, after which the operations of reversal of the telescope in its socket, and the correction of the error half by the foot-screw and half by the lock-nuts, *N* and *N*, are repeated until the bisection of the image is as nice and exact as possible, and remains quite undisturbed by the reversal of the telescope within its socket (Fig. 660, 2-3).

(2) If it be thought necessary to adjust or examine the collimation of the telescope, the test-mark is carefully bisected by the cross-line, and then the screw, *Sc*., is withdrawn, and the telescope is carefully twisted or rotated half-way round in its socket, so that the screw-hole through which *Sc* passes is brought to the top and the cross-lines is again horizontal. If the image is not now bisected, but falls above or below the cross-line, the error or deviation is half corrected by means of the foot-screw lying under the telescope, and then the image is caused to be bisected by the cross-line by adjusting the latter by means of the antagonistic screws, *d* and *d*. The telescope is twisted half-way round again and any remaining error corrected as before; if necessary, the operation is repeated until accuracy is obtained (Fig. 660, 1-2). This adjustment, if required, should precede the adjustment of the socket.

(3) In order to adjust the bubble-tube, the instrument is first set, as nearly level as the operator is able to judge, by means of the three foot-screws.

A
n |
|
 $\frac{d}{B} + \frac{e}{C}$
|
| o

Fig. 671.—
To Adjust
the Bubble-
Tube.

The telescope is then turned in a direction as nearly as possible parallel to the line joining two of the foot-screws. If *A*, *B*, and *C* (Fig. 671) represent the three foot-screws, then the telescope is directed along the line, *d—e*, which is parallel to the line joining *B* and *C*, and by means of the two foot-screws, *B* and *C*, the bubble is brought to the centre of its run, so that both ends of the bubble read the same. The telescope is then turned round, end for end, one-half revolution, and if the bubble now departs from the centre of its run, it is brought half-way back again by one of the foot-screws *B* or *C*, and the other half of the error is corrected by means of the lock-nuts, *R* and *R*. For example, if, on the telescope being reversed on its axis, the bubble comes to rest eight divisions away from its previous central position, then by one of the foot-screws, *B* or *C*, the bubble is caused to move back four divisions, and the remain

ing four divisions are corrected by means of the lock-nuts, *R* and *R*. Both ends of the bubble should now give the same reading. The telescope is now turned at right angles to its former position so that it lies directly over the foot-screw, *A*, along the line, *n—o*, and the foot-screw, *A*, is turned until the bubble gives similar readings at both ends. The telescope is then reversed, and any deviation (which will be very slight) of the bubble is half-corrected by the foot-screw, *A*, and half by the lock-nuts, *R* and *R*. The bubble-tube should now be approximately correct in its adjustment, and any further exactitude may be obtained by

a continuation of the above method, the telescope being directed along either of the lines $d-e$ or $n-o$, and care being taken to allow the bubble time to settle to its rest after each reversal of the instrument. After adjustment the telescope ought to lie in its socket with its stop-flange, St, against the end, S, of the socket, so that it may be secured by the screw, Sc.

790. **The Permanent Adjustments of the Cushing's Level are:** 1. **To Adjust the Line of Collimation.**—Set up the instrument with the telescope over one of the foot-screws, and having removed parallax, direct the telescope to some well-defined object at any convenient distance, and make the horizontal line bisect it: now carefully turn the eye-end in its socket from right to left, until the holes in the flange of the eye-end are opposite the heads of the screws, and remove it; replace it again, but in an inverted position, taking care to turn the eye-end from left to right until it comes to a stop, when the lines will be in their proper position; if the horizontal line still bisects the object, the line of collimation is in the axis of the telescope; but if it does not, correct half the apparent error by the collimating screws, and the other half by the foot-screw over which the telescope stands. Repeat the operation until the line of collimation continues to bisect the object when the eye-end is inverted.

2. **To Set the Line of Collimation at Right-angles to the Vertical Axis of the Instrument.**—Remove the dew cap, the object being bisected, and the telescope focussed, remove the eye-end and the object-glass cell from their respective sockets, and place them in the opposite ends of the telescope; then turn the instrument half-round on its vertical axis, and if the object is still bisected by the horizontal line, the line of collimation is at right angles to the vertical axis; if not, correct half the apparent error by means of the two large clamping nuts at one end of the horizontal bar, and the other half by the foot-screw under the telescope. Repeat this, until the eye and object ends can be reversed without any apparent change of position in the object. Care should be taken, in changing the object-glass from end to end, to keep the top uppermost.

3. **To Set the Spirit-level at Right-angles to the Vertical Axis of the Instrument.**—Level the instrument approximately by the legs, and turn the telescope until it is parallel to two of the foot-screws, and by their means bring the bubble to the middle of its run; now turn the instrument half-round on its vertical axis; if the bubble remains in the middle of its run, the adjustment is perfect; if not, correct half the apparent error by the foot-screws, and the other half by the two nuts at the eye-end of the spirit-level; turn the instrument a quarter round on its axis, and bring the bubble to the middle of its run by the third foot-screw; turn it through another quarter, and it will be over the same foot-screws as it was in the first instance, then bring the bubble to the middle of its run again. Repeat the foregoing operations until the bubble remains in the middle of its run while the instrument is turned on its vertical axis through a complete revolution. The cross-level at the object-end of the telescope should now be adjusted by the small screws at its ends provided for that purpose. The instrument will then be in perfect adjustment.

790 A. **Testing and Adjusting the Cooke's Farm Level** (Fig. 625):—*Farm Level* is thoroughly tested and adjusted before it leaves the factory, but before commencing any new work this can easily be done as follows:—First set up the Level on its tripod and bring the long bubble to the centre of its run, as already explained, on page 559. Next unscrew the ring (B).

At a convenient distance erect the staff, and clamp the sliding target at the height at which the pointer of the telescope coincides with the centre of the target. Note the reading on the staff. Then take firmly hold of the object-glass end (the large end) of the telescope, and without shifting the Level otherwise, pull the telescope tube out from the body casting. Turn the body of the Level half-way round upon the pivot of the tripod, so that the ends are reversed, and insert the telescope tube again into the body of the

instrument. The telescope will now have been reversed in its fitting; that is, the object-glass end instead of the eye-piece will now be placed over the large setting screw. Again bring the long bubble into the centre of its run by means of the large screw (E) and note whether the pointer of the telescope coincides with or overlaps the target again. If the telescope sights the target exactly as before then the instrument is in perfect adjustment.

If at the second reading it is found that the pointer of the telescope does not sight exactly on the centre of the target, then move the target on the staff until coincidence between the telescope pointer and the centre of the staff is effected. Note the difference of the readings and clamp the target half-way between the first and second positions. For example, say that the first reading was 4 ft. 6 ins. and the second reading 5 ft. 0 ins., then the difference would be 6 ins., and in this case the target would have to be clamped at 4 ft. 9 ins. Having done this, next sight the telescope on the target and then once more bring the bubble to the centre of its run, *but this time* by means of the small screw (A). The instrument will now be in correct adjustment, but a repetition of the readings should be made to correct the first observations. If found correct again, we know that the telescope and bubble are parallel to each other, and that both are truly horizontal when the long bubble is in the center of its run.

The adjustment can also be readily tested or corrected on a wall, a post or door, without the use of the staff, by fixing up a piece of stiff white paper and noting thereon by a pencil the two sights given by the telescope, before and after reversal. If the instrument is in correct adjustment the two levels will be identical, but if not, then, a mark made half-way between the two pencil marks on the paper will be the true level at which to set the instrument; this is to be done of course as before, first directing the pointer in the telescope to this third mid-way mark by means of the large screw (E), and then bringing the bubble to the centre of its run by the screw (A). *Fig. 672* clearly shows how the Farm Level should be adjusted.

791. Choice of an Ordinary Engineers' Level.—Levels of 14" or 16" focal-lengths will do for good work. The Y-level is universally preferred by the American and Continental Engineers on account of its accuracy and ease of adjustments. The dumpy is handier for small sizes. English Surveyors prefer it on the ground of its

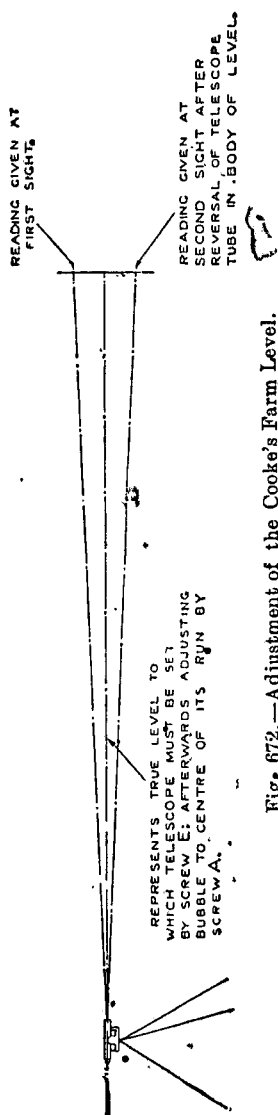


Fig. 672.—Adjustment of the Cooke's Farm Level.

supposed superiority to the Y-level in *retaining* longer its adjustments. The Y-level will retain its adjustment just as well, last longer without re-grinding the axis, and is adjusted in a few minutes. The Y-level has the advantage of requiring no peg adjustment for collimation, the peg-adjustment of the dumpy is a great bug-bear to young engineers. A dumpy possessing a good *powerful telescope, sensitive bubble, and stability*, will do just as good work as the more costly Y. There is a diversity of opinion amongst surveyors as to the respective advantages of the 3-screw or 4-screw arrangement. The chief advantage claimed for the 4-screw adjustment is the rapidity with which the instrument is levelled; it may be so to a beginner, but once a man is accustomed to the 3-screw adjustment, the instrument can be as quickly levelled with the latter as with the former. The great disadvantage of the 4-screw adjustment is the risk of straining the axis of the instrument if the screws are forced, but with the 3-screw, this is impossible as there is no counter resistance to the screws. In the author's opinion 3-screw type is the best, it at least releases one hand while turning the 3rd screw, but his advice is that the surveyor to have the pattern he gets accustomed to from the beginning, as if carefully used and not strained in any way, equally accurate results, can be obtained with either type. One objection has been made to the tribrach over the 4-screw type of adjustment that the latter admits of greater inclination to the tripod which is important in hilly countries. To remedy this a ball arrangement to axis which permits the level to be set 15° to the inclination of the tripod independently of the screw adjustment, so that the level, when the tripod is set at its best angle, may be brought in a minute to its final position (Figs. 658, 664, 665). The cross-bubble usually placed on the dumpy is not a necessity, but is useful in saving time in swinging the telescope backwards and forwards with every movement of the foot-screws.

The long bubble should be reversible, and placed over the telescope so as to bring the telescope tube as close as possible to the tripod head to make the instrument more steady. The bubble should be provided with a folding mirror, which will protect the bubble when shut down, and act as an indicator to the observer without changing his position (Figs. 636, 657) at the eye-piece, of the exact position of the bubble when elevated. Many Y and dumpys have the long bubble placed under the telescope as then it is less liable to be injured and more or less shaded from the direct rays of the Sun. The bubble should be sensitive so as to give for every 2 minutes of arc a one-inch motion of the bubble, *i. e.*, $15''$ seconds of arc for one division of $\frac{1}{8}''$. The instrument should be fitted with a clamp and tangent-screw to the horizontal movement. The object-glass focusing screw should be to the right of the observer and telescope and not on the top of it. The aperture of the objective should be $1\frac{3}{8}''$ and used to its full value, affords a high illumination with a magnifying power of 33, and as the diameter of the aperture of the objective divided by the power gives the diameter of the pencil of

light entering the eye, the diameter of the pencil should be $1\frac{3}{4} \div 33 = \frac{1}{24}$, which will give the power and brightness in accordance with the optical laws. The telescope should be supplied with a *slide protector* and with a *sunshade*. A cap should be provided for the objective and a shutter for the eye-lens. The foot-screws should be dust-capped. The telescope should be fitted with fixed stadia-wires, set to read 1 : 100, and preferably with an anallatic lens; and a prismatic compass. For all preliminary work these are necessary. The Longitudinal-section work may be checked with chain measurements, but all cross-section work, and other prominent side-shots may be taken with compass-bearings and stadia-readings on other staffs used for the purpose. The readings will be quickly taken, and the plot obtained from these will form a good guide showing the nature of the ground, for the final alignment to be made with a theodolite traverse. The instrument should have a tribrach stage to screw on to the tripod head, then the three points of the tribrach support the level upon a wall or rock when tripod cannot be used (Figs. 642, 650, 652, 653, 665). The feet of the tripod should be properly shod with small projecting pieces or stirrups to press home the legs with the foot as the pressure will then be in the direction of the grain of the wood. The box should be of seasoned mahogany with 3 hinges. It should be fitted with a spare bubble, a glass stadia diaphragm, a waterproof hood for the instrument in addition to the usual necessities.

792. The Kern's Precise Level

is simply a Y-level (Fig. 673). It is strongly constructed. It was used in the precise levelling work in Europe and by the Mississippi River Commission. The specially luminous telescope with opening

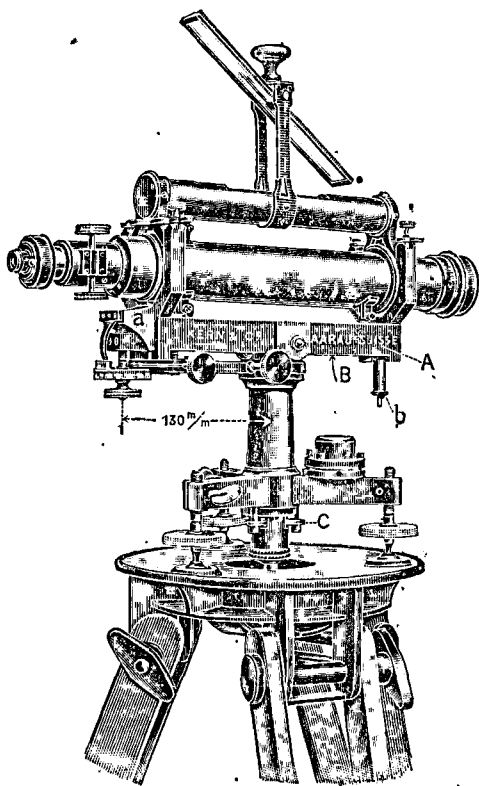


Fig. 673.—Kern's Precise Level.

of objective of $1\frac{5}{8}$ " (41 mm) and focus of $14\frac{1}{8}$ " (380 mm) with orthoscopic eye-piece having magnifications of 42 diameters can be turned and tilted in its bearings, and is exactly balanced on its centre. The arrangement of the cross-webs as desired; usually fixed stadia 1 : 100. There are three pins to bring the vertical diaphragm wire into the vertical position. The bubble has a sensitiveness of 8" per $\frac{1}{8}$ " (2 mm.) and is free from strain in its bearings. The casing is covered with an insulating substance for protection against rapid changes of temperature and carries a level mirror. The telescope support can be tilted on the middle of the axis, and can be brought by means of a fine micrometer elevation-screw lever, 130 mm, thread 0.3 mm, into normal position. The micrometer screw carries a drum with celluloid divisions of 100 parts, by means of which the level can be adjusted. A box-level is fixed on the triangular base for rapid bringing of the instrument into the horizontal position. The triangular base has accurately cut level-screws with 0.6 mm threads; and steel axis as hard as glass rests on a correctable steelplate, C. The instrument is also provided with central clamping and central tightener. The weight of the instrument is 13 lbs. Weight of case with all necessities is 18 lbs.

792A. Adjustments of Kern's Precise Level.—The stand should be placed in a horizontal position as exactly as possible, according to the eye, or by means of a spirit-level. Its position is made perfectly stable by pressing the feet well into the ground and by screwing the foot-screws firmly in.

The instrument is lifted out of the instrument-case without the telescope, placed on the stand, and fixed by means of its central screw without screwing the nut firmly on. The left hand, with fingers outspread, should now be placed on the tripod of the instrument and pressed against the stand, the right hand at the same time gripping the beam above the centre and lifting it upwards. It will be felt that the pivot is lifted slightly in its socket without it being possible to lift it out. This lifting of the pivot should be repeated a few times, and the beam at the same time turned to the left and right which will cause the oil to be distributed on the whole friction surface of the pivot. The telescope bearing should then be cleaned by means of a fine piece of linen or buckskin after which the telescope itself should be lifted out of the case, the level-rings well-cleaned and the telescope then carefully placed in its bearings. If the instrument is provided with a portable levelling level, the feet of the same should likewise be cleaned, after which the level should be placed on the telescope rings and one of the caps provisionally closed.

Before proceeding to adjust the horizontal position of the instrument it will be necessary to correct the levelling level. Let us presume that it has been placed on the telescope, one of the caps remaining open. We now turn the beam with the telescope so that the level in its first position is placed parallel to two of the levelling-

screws of the tripod; we then balance the bubble of the level by means of these two levelling screws, turn the beam with the telescope at a right angle, balance the level-bubble by means of the third levelling-screw, and immediately turn the level back to its first position, any slight deviation of the bubble being corrected by means of the two first levelling-screws. The level should now be carefully lifted off at the side of the open cap, and reversed and one half of a possible deviation of the bubble corrected by means of a corresponding movement of the glass scale and the other half by means of one of the two first mentioned levelling-screws. Two successive reversions of the level should suffice for correcting the bubble. Finally the beam with the telescope and level should be again turned at a right angle and the deviation of the bubble corrected simply by means of the corresponding third levelling-screw of the tripod.

We then proceed to the horizontal adjustment of the instrument which will depend upon the construction of the same :

Levelling Instrument with Elevation Screw.—Let us presume, that the instrument is placed on its stand, and that the levelling-level is exactly corrected. The telescope is placed exactly above one of the 3 levelling-screws of the tripod. At a distance of 30-50 metres we sight a levelling-staff or any other distinct point, remove the level, lift the telescope out of its bearings, at the same time maintaining its previous direction, and without any turning of its longitudinal axis; we then turn the beam (the bearing) 180° . We then replace the telescope and examine if the cross of the reticule still coincides vertically with the point first chosen. The position of the bubble of the level will likewise show if there is any deviation. If the level balances, a look through the telescope will convince us that the cross of the reticule likewise coincides exactly with the point first chosen. On the other hand, if the bubble shows a deviation, half of the deviation is corrected, by looking at the point through the telescope, by means of the elevation-screw on the beam, and the other half by the levelling-screw of the tripod, above which the beam or the telescope is placed. Needless to say in this case, there can be no question of a lateral correction. We repeat once more the turning of the beam of 180° after having lifted the telescope out of its bearings, and correct any small deviation of the bubble which may exist, one half by means of the elevation screw, and the other half by means of the levelling-screw situated under the beam. We then turn the beam at a right angle, i.e., parallel to the two other levelling-screws of the tripod and correct the deviation of the level bubble simply by means of these levelling-screws. After these manipulations the bubble will remain stationary no matter in which direction the instrument may be turned. Consequently the position of the instrument is exactly horizontal.

Finally we have to correct the central position of the reticule and adjust the vertical thread according to a plumb line. The correc-

tion of the central position of the reticule could of course be made before any of the other corrections are made, as it is not at all necessary for the instrument to be in an exactly horizontal position in order to carry out this operation. The adjustment of the vertical thread according to a plumb-line however can only be made when the instrument is entirely corrected and as both these operations, so to say, belong together it is better to wait with the central adjustment of the reticule to the last. As the telescope is movable in its bearings the correction of the reticule does not require much time.

By means of the telescope a distinct point about 30 metres distant is sighted, either on a wall or otherwise on the ground, the beam is then locked and the aim exactly rectified. The telescope is then turned on its mechanical axis 180° . If during this turning the cross of the reticule leaves the point aimed at, the deviation is corrected half vertically and half laterally by means of the corresponding reticule screws on the movable ocular tube, and with this new position of the reticule a new distinct point is aimed at. If after having again turned the telescope 180° on its mechanical axis the reticule again shows a deviation from this new-point (the first point no longer concerns) us, which in any event can only be slight; then this deviation is likewise corrected half vertically and half laterally until the reticule remains perfectly immovable and does not leave the point in question during the turning of the telescope. The instrument is then quickly placed perfectly horizontally; and the aim taken, at a plumb-line, after which first the front contact screw on the telescope is corrected by a corresponding turning towards the contact pin on the beam of the instrument until the vertical thread in the eye-piece covers the plumb-line entirely. If no plumb-line is at disposal any small distinct point can be aimed at, and the horizontal thread moved over this point from one end to the other by means of the horizontal micrometer screw, after which the deviation, if any, is corrected by means of the above mentioned correction screw until the point aimed at is exactly parallel with the thread. The telescope is then lifted out of its bearings and the beam turned 180° after which the front contact screw is corrected in the same manner as before until the vertical thread is placed in an exact position.

793. **The Invar Precise Binocular Level.**—Two views of the instrument are shown in Figs. 674, 675. It is designed by the U. S. A. Coast and Geodetic Survey (1901), and it is also used by the G. T. Survey of India in levelling operations where the greatest possible degree of accuracy obtainable is required. It is too delicate for the ordinary work of the Engineer, but is now used in precise levelling operations over long lines in geodetic levelling. It has the line of collimation independent of the vertical axis, has 3 levelling screws, and clamp and tangent adjustment, and a circular bubble for approximate levelling. "The aims in designing this level were to select a material with a view to the smallness of its expansion

coefficient, to protect the vital parts against sudden and unequal changes of temperature, to reduce to the smallest possible dimension the linear distance between the bubble-tube and the line of collimation, to insure stability by reducing the distance between the centre of gravity and the plane of support, and to enable the observer to obtain the staff-reading as nearly as possible, simultaneously with the setting of the level." The telescope and its

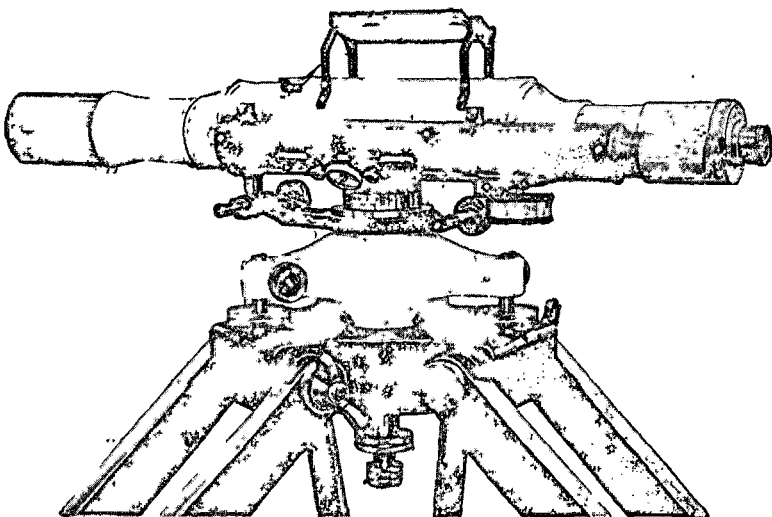


Fig. 674.—Watts and Son's Invar Precise Binocular Level.

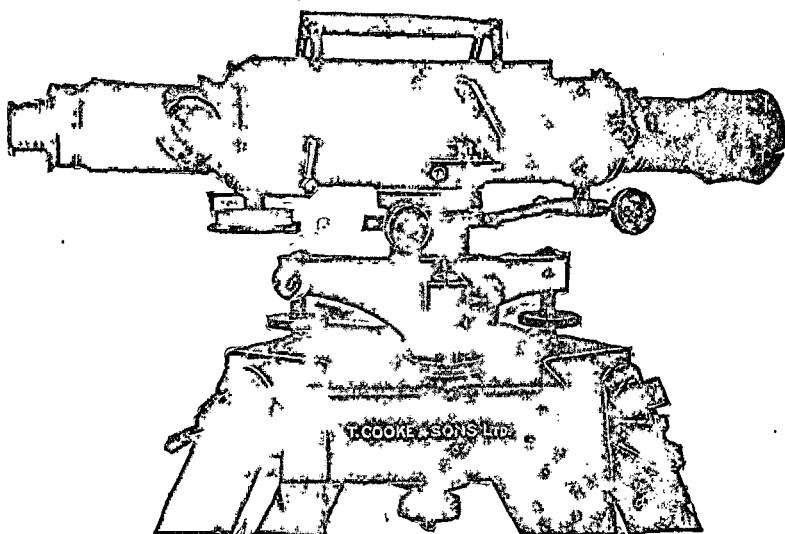


Fig. 675.—Cook's Invar Precise Binocular Level.

support, bubble mounting, and all main adjustments are of Invar (Nickel steel), having a small coefficient of expansion, the more important screws having a coefficient as low as 0.000001 for 1°C . Main vertical axis is of hard tool-steel, and the axis socket tribrach is of special close grained cast-iron. The object-glass has 1.62 ins. aperture and 18" focal length (Fig. 674), or 1.7 ins. aperture and 16" focal length (Fig. 675). The eye-pieces of the former 25 and 40 diameters and of the latter 32 and 44. The weight of the instrument is 14 lbs. The telescope is flexibly supported inside a fixed tube instead of in Ys. This tube is slotted at top, and a very sensitive spirit-level with chambered bubble is recessed into this fixed tube, so as to be in almost immediate contact with the telescope-tube, and as far as possible removed from external influence. It is divided in Fig. 675 into 2 mm. spaces the value of one division being 2 secs. of arc, and in Fig. 674 into 2 mm. width of division and a run of 1.8 secs. of arc.

There are two main distinguishing features: (1) The main chambered bubble readings are taken by an upper mirror throwing the images of the two bubble ends upon two prismatic reflectors set in an auxiliary telescope-tube or reader the eye-piece of which is mounted alongside, and on the left of the eye-piece of the main telescope-tube, and at binocular distance from it, and thence to the left eye of the observer. By standing erect the observer then sees both the staff and the bubble at the same time, and can hold the bubble to a central position while reading the staff. A simple lens at the eye-end of this tube, changed for different eyes, brings these images into the range of distinct vision. This contrivance enables the bubble to be read simultaneously with the staff without parallax, and with an erect position of the body. There is also an adjustment provided to vary the binocular distance within limits. This instrument is set in perfect adjustment far more rapidly than any other design of level, and its adjustments are more permanent and more easily corrected.

(2) Instead of the usual (in the case of a lengthy sensitive bubble) operation of accurately levelling up the instrument by the foot-screws and reversion of telescope, all that is needed in the *precise level* is to approximately level the instrument, so that the absolutely horizontal position of the telescope axis is within the range of the finely threaded micrometer screw, near the eye-end of the telescope, the latter being then used to complete the adjustment for any particular sight. This, of course, needs a stand slightly taller than is usual, but entirely dispenses with the stooping necessary with most forms of level. It also does away with the observer changing his position on the ground, a fruitful source of errors and readjustments, etc. The instrument is cloth covered to resist extremes of temperature and to reduce errors caused thereby. A small lever takes the weight of the telescope off the micrometer head when the level is not in use. The micrometer screw is used for setting the line of collimation. Its telescope sets very low on the tripod stand and has proved very stable in the wind.

794. Description of The Zeiss Level, Type No. 1, (Fig. 676).—It consists of two readily disjoining parts, viz.: the motion head and the telescope. When packed in the case the motion head lies parallel to the telescope, thereby reducing the case to a very convenient size.

The Motion Head comprises the stock-head with its fittings for mounting the instrument on the stand, the preliminary levelling device with circular bubble, slow horizontal motion, and fine elevation screw motion. The Level is mounted upon the stand by means of a cylindrical socket, T, the clamping-screw, V, serving to fix it upon the stockhead pin. The instrument is levelled preliminarily by bringing the bubble of the circular level, N, to the centre of its run by means of the screws, C, C, C. The vertical stem is made of steel and ground truly cylindrical, and

hence does not at any time require readjustment. It works exceedingly smoothly, yet shows no tendency to shake. The vertical spindle may be fixed in position by the clamp, M, whilst its slow-motion is effected by means of the micrometer screw, B, with the buffer spring. The slow-motion mechanism of the telescope about its horizontal axis consists of a forked lever, S, having attached to it a very accurately cut elevation and depression screw, A, by means of which, in conjunction with a buffer spring, the bubble can be rapidly brought to the centre of its run. A key is provided by means of which the working of the screws A, B and CC may be adjusted to a nicety. After the removal of the set-screw O, the entire revolving portion of

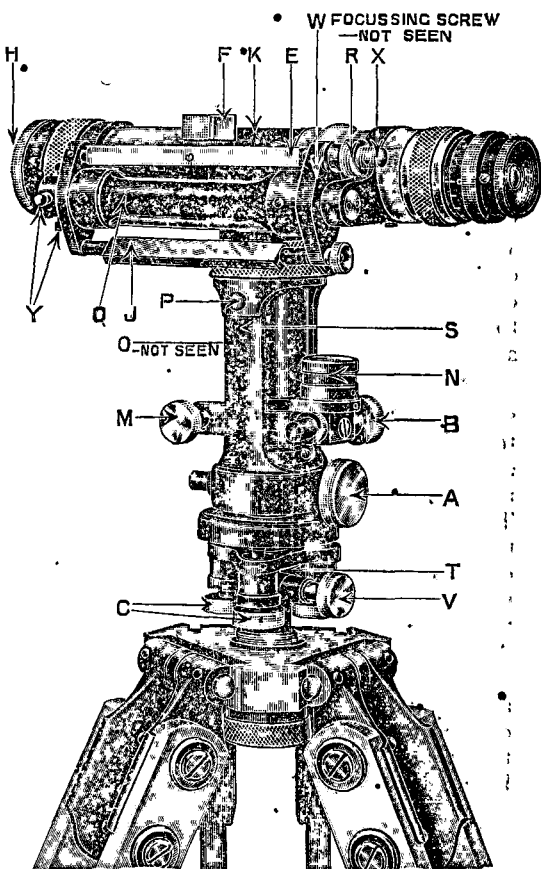


Fig. 676.—The Zeiss Level, Type No. 1, as made by Messrs. T. Cooke & Sons Ltd. of York

the motion head may be lifted off for the purpose of cleaning and oiling the vertical spindle. This should, however, be done on rare occasions only.

The telescope is mounted upon a cross-head bearing K, and rotates on its axis within the limits fixed by two stops. A reversible bubble-tube is attached to the telescope by means of adjustable screws. *Both the objective and the cross-webs are permanently fixed.* This ensures an invariable tube length, and affords the means of completely excluding the access of dust and water. The telescope is focussed for different distances by means of a sliding lens situated within the tube and actuated externally by means of the milled-head, W. By this method of focussing wear in the focusing tube causes only one-sixth of the error that would take place with the old style of sliding draw-tube. The final focussing of the eye-piece with respect to the cross-webs is effected by the aid of a screw collar graduated at its periphery in terms of diopters.

The reversible bubble is mounted in a manner which obviates straining, and is encased in a glass cylinder so as to protect it from the influence of changes of temperature. The bubble-tube *is in no way divided*. The movements of the bubble are viewed through a novel combination of prisms which is contained within the casing, E. This compound prism forms two tangent images of the two ends of the bubble, which may be viewed from the eye-piece end or from the objective end of the telescope. The bubble is brightly illuminated from below by light reflected at the mirror J. To bring the bubble to the centre of its run it is only necessary to turn the elevation screw, A, until the two images of the bubble-ends become coincident. This arrangement for viewing the bubble eliminates every trace of parallax, and renders it an easy task to bring the bubble accurately to the centre of its run. In the second position of the telescope (bubble tube on the right) the bubble is likewise viewed in the prism, F, from below and through the bubble-glass. The prism casing, E, may be displaced along the bubble-tube, which furnishes the means of adjusting the instrument.

794A. Directions for Using the Zeiss Level No. 1.—
1.—How to set up the Instrument.—Having released the milled-thumb-screws, and folded down the hinged fitting within the case, commence by withdrawing the motion head. Slip the socket surrounded by the tribrach screws upon the socket pin, and fix it thereon by tightening the screw-clamp, V. Next with both thumbs press back the two spring locks attached to the cradles in which the telescope is accommodated, which will cause the clips to open of their own accord. Turn the conical milled-head at the middle of the telescope in a clockwise direction up to the stop. To mount the telescope on the motion head, seize it with the left-hand by the objective, and with the right hand by the conical head and turn it so as to cause the bubble to be at the top. The eye-end should be on the same side as the granieter screw. In this position it will be found that the three gaps in the screw shanks of the motion head correspond to the interrupted screw threads in the telescope socket. To lock both parts firmly draw up the conical head. By means of the tribrach screws bring the bubble, N, to the centre of its run and set the mirror under the reversible level in such a position that the upper prism, F, as seen from the eye-end, may appear brightly illuminated. Set

the eye-piece to your eyesight by the scale of diopters. Remove the eye-piece cap and keep it in the case.

2. **How to adjust and use the Level.**—The telescope is available for readings in four positions, viz. :—

Position I: Eye-piece in working position, i.e., on the side of the scale of diopters, the bubble-tube being on the left of the telescope.

Position II: Eye-piece in working position, bubble-tube on the right of the telescope.

Position III: Eye-piece in objective cap for verifying the instrument; bubble on the right of the telescope.

Position IV: Eye-piece in objective cap for verifying the instrument, bubble on the left of the telescope.

When changing over into Position III the prism, F, should be turned round. When the eye-piece is in the objective cap (Positions III and IV) focus the cross-lines by sliding the objective cap in or out, and while so doing place the right-hand to the cap and hold the telescope with the left-hand.

Ordinary levelling operations of a practical nature are generally carried out in Position I only. In special cases requiring great accuracy it is advisable to take readings in both Positions I and II. Positions III and IV are exclusively intended for verifying the adjustment of the instrument. A complete adjustment can be obtained from a single station point.

The adjustment is made in the following manner: Using a properly divided staff set it up vertically at a distance of about 45 yards and take readings in Positions IV, III, II and I, the bubble being in each case brought to the centre of its run by viewing it in the prism, F, whilst operating the gradiometer screw A. The mean of the four readings so obtained gives a horizontal collimation which is entirely free from systematic errors. Now by means of the gradiometer screw, A, and with the telescope in Position I set the cross-webs to the computed mean of the staff readings. If it should be found that the bubble has travelled from the centre of its run, as seen in the upper prism, F, release the clamp, R, and displace the prism box, E, by means of the milled-head, X, until two halves as seen through the prism, F, appear to coincide.—N.B., the coarse adjusting screws, Y, should not be tampered with unless the required amount of adjustment cannot be obtained by moving the prism box as detailed above.

Where, in exceptional cases, it is proposed to employ readings in Position II as well, it is not sufficient to set the cross-webs in Position I to the mean reading on the staff, and it will be necessary to ascertain any difference which may exist between the readings obtained in Positions I and II and to ensure that the mean of readings taken in Positions I and II may agree with the readings obtained in all four positions. Differences between readings obtained in Positions I and II are not affected by the displacement of the prism box, E.

To eliminate errors caused by non-parallelism of the axis in the horizontal-plane it is generally sufficient to bring the bubble approximately to the centre of its run whilst the telescope is being turned through an angle of 90°, the bubble being at the top. Such errors may also be corrected by the following more accurate, but also more laborious method:

Place the tripod so that one leg points towards the staff, and one of the levelling screws lies vertically under the objective or eye-end of the telescope. Level the instrument by means of the circular bubble, bring the reversible bubble to the centre of its run, and take a reading. Now give a half or whole turn to one of the screws on either side of the instrument and restore the reading approximately by means of the other of these screws, completing its restoration by means of the gradiometer screw, A. Should this occasion a displacement of the reversible bubble from the centre of its run, as seen in the upper prism, F, this should be corrected by means of the horizontal adjusting screws.

The possessor of a newly acquired instrument is recommended to control its adjustment at first from day to day by taking the complete set of four readings. This will afford him an opportunity of realising how well the instrument retains its adjustment, and having thus familiarised himself with its qualities in this respect he will content himself to verify it on rare occasions only. If after continued use the vertical spindle should be found to work heavily, or if it be suspected that moisture has found its way in, release the screw on the vertical spindle, open the clamp, and remove the body of the instrument from the vertical spindle. Carefully clean the spindle and socket with a linen rag, and, having applied three or four drops of fresh oil, remount the motion head and replace the screw on the right. In the event of the tribrach screws moving too freely this may be remedied by slightly tightening the screw at the bottom of the socket, whereby all three screws are actuated in an equal degree. No other parts should be detached. This applies in particular to the telescope.

3. **How to pack up the Instrument.**—To pack the instrument in its case, release the conical milled-head under the telescope, holding the latter by the left hand and turning it until the bubble is at the top, and lift it off the motion head. Place the instrument in position in the following manner: Let the eye-piece be at the end where the handle is, place the reversible bubble on the hinge side of the case and the conical milled head at the bottom. At one end of the bubble see that the adjusting button may clear the spring lock of the cradle. Depress both clips until they snap into position. To put in the motion head, let the gradienter screw face the

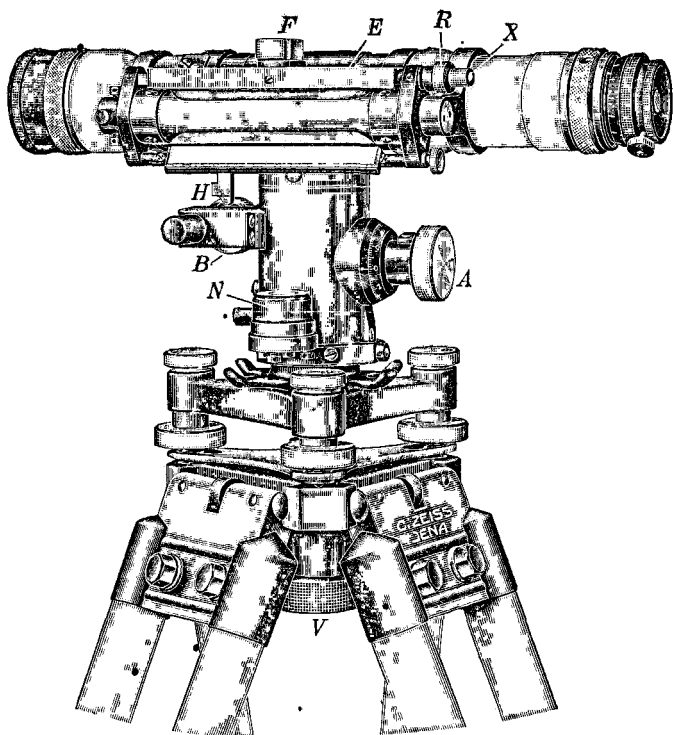


Fig. 677.—Zeiss Levels Nos. II and III,

looking side of the case, whilst the clamping-screw, A, on the socket should point obliquely downwards. Turn up the hinge and slip it over the upper end of the motion head, and finally secure the fitting in position by tightening up the milled screw.

4. **The Tripod.**—The tripod head is so arranged that all three legs can be drawn together simultaneously by a central screw. By means of this screw the freedom of their motion should be so regulated that when given a spread of 20 or 24 inches they will not close up of their own accord. This condition may be maintained for a considerable time.

795. **Directions for the use of the Zeiss Levels Nos. II and III (Fig. 677).**—1. **How to Set up the Instrument.**—First take out the levelling head from the case and place it on the tripod head with the points of the levelling screws resting in the grooves of the tripod head plate. By lightly drawing up the centre clamping screw V (which serves also for securing the levelling head within the case) attach the levelling head to the tripod. With the thumb press back the springs on the three bearing brackets, which will cause their covers to open of their own accord. Next place the body of the instrument (i. e., the motion head with the telescope) by fitting the cylindrical socket end of the vertical spindle upon the tripod stem and gently tighten the star shaped clamping wheel. Release the clamp H, which fixes the telescope during transport; the telescope can then be rotated about its longitudinal axis within the limits of two stops, and the instrument is now ready for use. Commenced by bringing the circular bubble to the centre of its run. Place the illuminating mirror under the reversible bubble in such a position that the upper prism F, as seen from the eye-piece, may appear brightly illuminated. Set the eye-piece, to suit your eyesight in diopters. The objective cap should meanwhile be kept in the case.

2. **How to Adjust the Instrument.**—Readings can be taken with the telescope in four positions, as follows :

Position I: Eye-piece in its working position (on the side bearing the scale of diopters), bubble on the left of the telescope ;

Position II: Eye-piece in its working position, bubble on the right of the telescope,

Position III: Eye-piece in its adjusting position (slipped into objective cap), bubble on the right of the telescope ;

Position IV: Eye-piece in its adjusting position, bubble on the left of the telescope.

When making the transition to Position III turn the prism, F, through 180°. When the eye-piece is in the objective cap (Positions III and IV) the cross-webs should be brought distinctly into view by sliding the objective cap, the right hand being applied to the cap, whilst the telescope is being held with the left hand.

In Ordinary Levelling Operations it is usual to employ Position I only, whilst for the purposes of more exacting geometrical surveys it is advisable to take combined readings in Positions I and II. Positions III and IV are solely employed for verifying and adjusting the instrument.

The level can be completely adjusted from a single station without reference to any previous adjustment.—The procedure is as follows : Setting up vertically a suitably and correctly divided staff at a distance of about 50 yards take readings in Positions IV, III, II and I with the bubble at the centre of its run throughout, the bubble being watched in the upper prism, F, and re-set by means of the grader screw, A. The mean of the four readings so obtained gives a horizontal collimation which is entirely free from systematic errors. Now, by means of the grader screw, A, with the telescope in Position I set the middle horizontal line of the cross-web to the mean of these readings on the staff. Should the bubble halves seen in the upper prism, F, appear displaced, release the clamp, R, and move the prism box, E, by means of the milled-head, X, until the two bubble halves appear to coincide.

When it is proposed to employ readings in Position II as well, it is not sufficient to set the cross-web in Position I to the mean reading on the staff, and it will be necessary to ascertain any difference which may exist between the readings in Position I and II, and to ensure that the mean of readings taken in Positions I and II may agree with the mean of readings obtained in all four positions. Differences, if any, between readings in Positions I and II are not affected by the displacement of the prism box, *E*.

To eliminate errors caused by non-parallelism of the axis it is generally sufficient to bring the bubble to the centre of its run with the telescope turned through an angle of 90° (the bubble being at the top). More accurately such errors may be corrected by the following, though rather more laborious, method:

Place the tripod so that one leg points towards the staff. Having levelled the instrument by means of the circular bubble bring the reversible bubble to the centre of its run and take a reading; now give a complete or half a turn to one of the levelling screws on either side of the instrument, and proceed to restore the reading approximately by means of the other of these two screws and complete the restoration by means of the gradienter screw, *A*. Should this operation cause a displacement of the reversible bubble, as seen in the upper prism, *F*, from the centre of its run the requisite correction should be applied by means of the horizontal adjusting screws.

The possessor of a newly acquired instrument is recommended to check its adjustment at first from day to day by taking the complete set of four readings. This should afford him an opportunity of realising how well the instrument retains its adjustment, and having thus familiarised himself with its qualities in this respect, he will be satisfied henceforth to verify the adjustment on rare occasions only. If after continued use the vertical spindle should be found to work heavily, or if it be suspected that moisture has found its way in, the screw on the right of the vertical spindle should be released, the clamp opened and the body of the instrument detached from the vertical spindle. Having carefully cleaned the spindle and the socket with a linen rag apply three or four drops of fresh oil, remount the motion head, and replace the screw on the right.

In the event of the tripod screws moving too freely this may be remedied by screwing up the milled heads above them.

3. **How to pack up the Instrument.**--Before returning the instrument to its case set the clamp, *H*, so that the lever prevents the telescope from turning. The level will then be above the telescope. Release the star-shaped clamping wheel at the lower end of the motion head, seize the telescope with the left hand at the objective end, with the right hand at the eyepiece end and, lifting it off the tripod, place it into the case. Press down the three holding clips until they snap into position. Release the centre clamping screw *V*, remove the levelling head, and slipping it upon the pin in the case between the eye end of the telescope and the motion head, fix it in position by means of the centre clamping screw.

4. **The Tripod.**--The tripod head is so arranged that all three legs can be drawn together simultaneously by a central screw. By means of this screw the freedom of their motion should be so regulated that when given a spread of 20 to 24 inches they will not close up of their own accord. This condition may be maintained for a considerable time. In the event of the wood contracting during dry weather it is easy to restore the original rigid connection between the wood and metal fittings by tightening the hexagonal screw heads.

795A. **Supplementary Directions for Using the No. III Zeiss Level with Micrometer Attachment (Fig. 678).**--To secure the highest attainable degree of accuracy in very exacting geodetic levelling operations No. III Zeiss Level is supplied with a device for optically displacing the line of sight parallel to itself (Wild's Method). In an instrument fitted with this device fractional divisions on the staff, instead of being estimated by the eye, are determined by displacing telescope parallel to itself until the centre of its field coincides with the nearest line

or the nearest middle of a division on the staff. This device called the Micrometer Attachment, consists of a glass plate having its sides

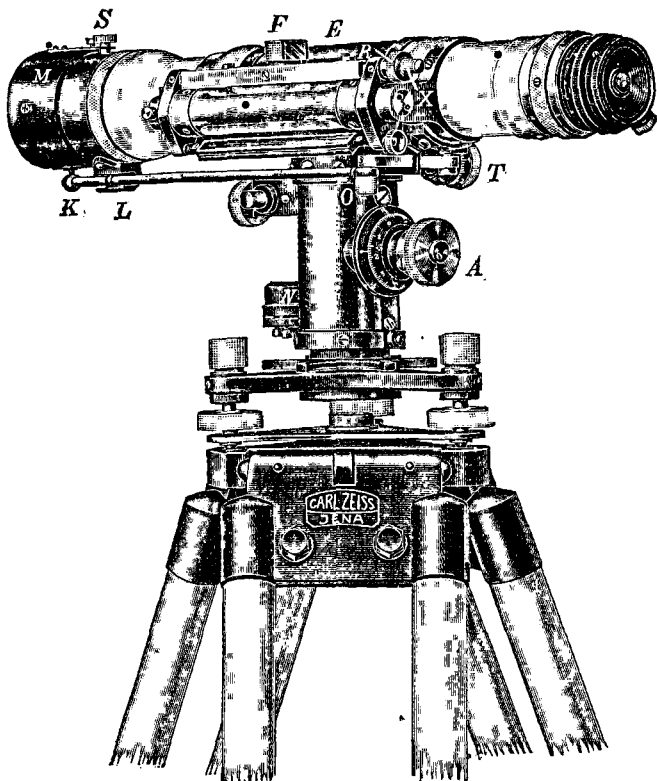


Fig. 678.—High Precision Zeiss Level No. III with Micrometer Attachment.

ground and polished accurately plane and parallel, and mounted within the casing, M, on a horizontal pivot together with a mechanism for rotating this plate and providing a reading of the altitudinal displacement of the sighting line.

The instrument is set up in the following manner: First, slip the sleeve fitting, M, upon the objective end of the telescope, attaching it in such a manner that the two marks situated at the top may accurately register. Clamp the sleeve with the looking screw, S, to prevent the sleeve fitting slipping off whilst the level is being carried from station to station. Next, attach the coupling rod, which is accommodated in the lid of the case, by its ball-end, K, to the projecting stud in the plate mount and with the thumb screw, O, secure it with its other end to the bell crank pivoted above the gradienter screw. The rotation of the milled-head, T, will then cause the plane and parallel plate to dip and thereby produce an altitudinal displacement of the sighting line. The altitudinal displacement can be read off the drum at T to $\frac{1}{100}$ mm, and $\frac{1}{100}$ mm may be estimated.

The drum is numbered at intervals corresponding to an altitudinal displacement of 0.5 mm. The zero point of the graduated drum is so arranged that all readings have the same sign. In consequence of this arrangement, when

the drum is in its zero position, the optically displaced sighting line rises 5.40 intervals of the drum scale or 2.70 mm above the true sighting line. The device will accordingly measure the small differences in altitude intervening between the optically raised zero horizon and the sighted division line or division centre on the staff. When the drum is set to 5.40 the plane and parallel plate will accordingly be normal to the sighting line, and in this position there is no optical displacement of the latter. When the staff has its zero point at the lower end it follows that the readings of the staff and those of the drum agree in sign, and in this case the required value is obtained by adding the staff and drum readings.

The Instrument should be adjusted in the following manner:—Having taken with the instrument four readings in accordance with the directions given respecting the Zeiss Levels Nos II and III, apply the Micrometer Attachment in accordance with the directions given above, and set the drum, T, to read 5.40. With the aid of the gradimeter screw, A, set the cross-webs to the mean of the four readings obtained in the four positions, and should the bubble fail to come to the centre of its run in this means position of the telescope release the lock-nut, R, and with the aid of the small screw, X, displace the prism casing, E, until the two ends of the bubble coincide as seen through the prism. When the cross-webs are set to the mean reading on the staff, whilst at the same time the bubble is at the centre of its run, the correction will include any error of deviation which may be due to a slight defect in the parallelism of the faces of the plate as well as any deviation due to the straining of the telescope occasioned by the weight of the sleeve attachment at the objective end; and, consequently the sight line and the bubble axis will be accurately parallel. Levels are taken with the instrument in Position I only.

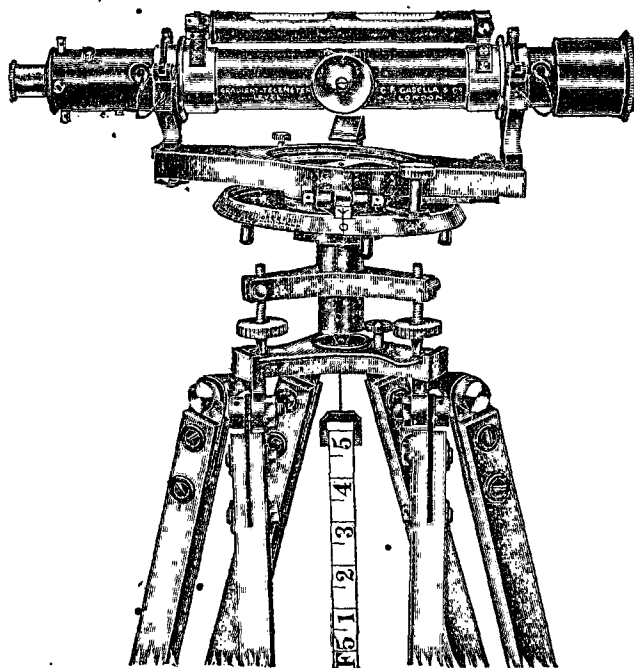


Fig. 679.—Casella's Gradient Telemeter Level, New Pattern.

796. The Gradient Telemeter Level is an ingenious instrument (Fig. 679) for taking gradients, measuring or setting out distances, and obtaining differences in level, all of which it does by one and the same observation. It performs those operations with singular accuracy and ease to the observer.

enabling a much greater quantity of work to be got through in the same time compared with the usual methods employed by engineers and surveyors. The linear distances can be obtained more accurately than with the land-chain and this regardless of broken ground, or the presence of streams, or other water, between the observer's station and the distant object.

A marked advantage with this instrument, and one that renders it particularly attractive when employed in the

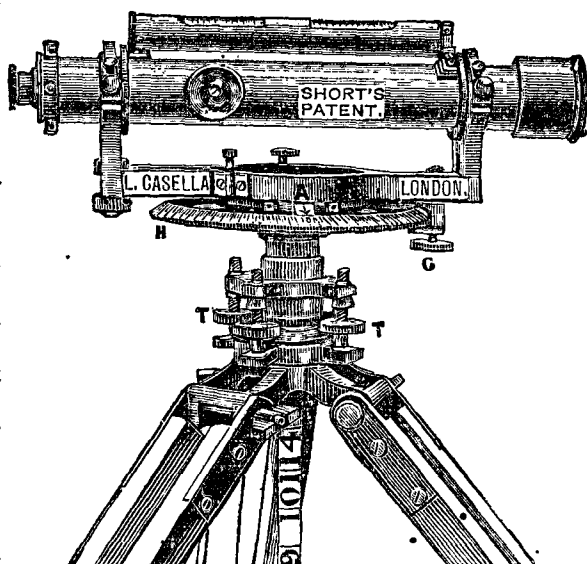


Fig. 680.—Old Pattern Gradient Telemeter Level.

field, is that there are *no calculations* to be made in connection with its use; there is *no fine micrometer-screw* to work; with its certain *errors* and tedious calculations; there are no moving etched or spiderlines which may get broken, thus rendering the instrument useless until it can be returned to the maker to be set right; but, by a single revolution or movement on the axis of the instrument, and an observation through the telescope, the gradient, distance or difference of level is at once obtained.

Description of the Level :—This is similar to the Y-Level, but with the addition a *horizontal limb or circle*, H, immediately below the stage on which are marked the gradients both of rise and fall, 1200 to 10 (in new pattern instrument 1 in 500 to 1 in 24). (Fig. 679.) The gradient marks are such a distance apart on the limb, that they can be read easily at the index, A, without a reading microscope, and being read by an index point (not a vernier) the gradient of the object is seen at a glance without the trouble of any calculations; this together with the fact that the same observation which gives the gradient also gives the distance of the observer's Station from the object enables accurate levelling being done in a short time. The index, A, is fixed to and moves with the telescope and by a special construction of the instrument the horizontal motion of the telescope is converted into vertical dip or elevation, so that the

gradient and distance of objects are obtained by the readings on the Horizontal limb, H.

Method of Taking Gradients:—Set up the level on its stand, clamp the index at zero of the graduated limb, and level it by means of the foot-screws, T, T, and level on telescope. While this leveling is being done, the axis or centre attached to the graduated horizontal limb, H, must alone be used, the instrument must not be rotated on the axis attached to the telescope. There are two axes to this level, an inner and an outer one as with a theodolite. The inner is fixed to the telescope and the movable index which reads the graduations, and the outer axis is fixed to the horizontal limb. There is a clamp, G, for fixing these two axes so that motion is got only by the axis of the horizontal limb, H. Supposing the gradient or slope is required between two stations (Fig. 681) O, where the instrument is set up and levelled, and B, where the staff, S, is held upright, measure the H. I. by the tape hanging below the instrument (suppose this height is 4' 9"), and direct the telescope and read the staff, S, at B. If the staff reading and the H. I. i.e., 4' 9" agree, then O and B are on the same level. If the two differ, loosen the index clamp-screw which fixes the two axes of the instrument and holding the telescope lightly with one hand, move the graduated limb with the other by means of ribs or radial bars under its surface until the cross-hairs exactly cut on the staff at S the same reading 4' 9" by the aid of a tangent-screw to clamp G. The index $\downarrow$ A, attached to the telescope will during this operation have moved on the graduated limb, and on the index being read, the gradient say, 1 in 150 between two stations, O and B, is given without any calculation whatever. In the example shown, the ground at B, falls from O, and hence a *falling gradient* is given. If the ground rise at B the telescope must be reversed in the Ys so that the object-end of the telescope is next the clamp-screw, G, when the result will be given that the gradient, a rising one of the station B will be read on the graduated limb, or a *rise or fall* is learned by noting the word RISE or FALL who is engraved at the ends of the stage on which Ys are fixed—the engraved word next the eye-end of the telescope always denoting whether the distant station has a *rise or fall*. The gradient of distant stations can be obtained at one setting up of the instrument and observation, even when the vertical interval, rise or fall, of the two Stations is as much as 90' or 100', or on the instrument being placed *between* 2 stations, the vertical intervals of nearly 200' can be obtained in a few minutes with an ordinary 14' or 16' staff divided to $\frac{1}{100}$ ths and $\frac{1}{1000}$ ths of a foot.

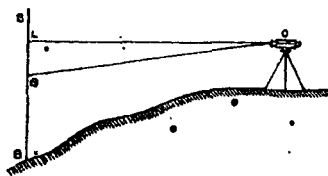


Fig. 681.—Use of the Gradient Telemeter Level.

Measuring Distances.—To find the distance between 2 stations, O and B, (Fig. 681), set up and level the instrument and clamp the index at zero.

From B take reading of staff at S, and suppose it to be 10'68"; then unloosen clamp G, move telescope until the index point reads exactly 100 on the limb, H, and fix it there by clamp, G, then move telescope round on the outer axis of the instrument until it again reads the staff, say 6'42". Then the difference of the two readings is 4'26", and the distance between O and B is 426'.

Another method is to fix the index at graduations 50, $33\frac{1}{3}$, 25, 20, $16\frac{2}{3}$, $12\frac{1}{2}$, 10 on the limb respectively, and divide the difference of the readings by 2, 3, 4, 5, 6, 8, 10 respectively. By the use of these gradients any error in distance resulting from incorrect reading of staff is reduced.

Method of Getting Horizontal Linear Distances, and also Levels up to 90 feet and 100 feet on Sloping Ground, when the Levelling staff is wholly below or above the horizontal line of sight, without moving the staff or the instrument.

—When the instrument is levelled, and the horizontal line of sight, AB, is above the top of the 14 feet staff, proceed as follows: Loosen clamp, move telescope until the index reaches gradient 100, and sight the staff; if the line of sight is still above staff, move the telescope and index farther until the upper part of the staff is visible through the telescope. Suppose the index then near gradient 28, move it again until it cuts exactly gradient 25, (Fig. 682) and take the staff reading at that gradient, suppose this to be 13.86 feet. Then move index on to G 20, and again read the staff; let this reading be 8.45 feet, and their difference is 13.8—8.45 = 5.41. Hence the horizontal linear distance, AB, as in above illustration, is 541 feet, without any calculation.

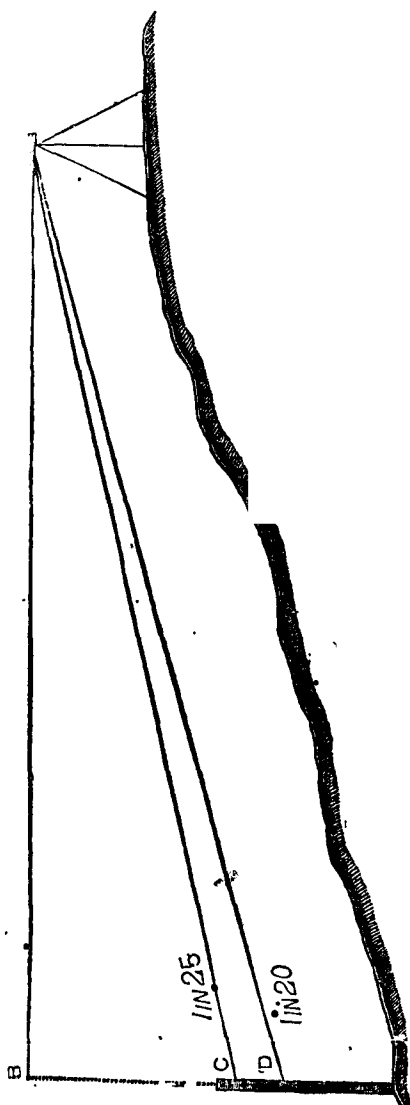


Fig. 682.—Use of the Gradient Telemeter Level.

Any pair of gradients in the list below will give by the difference of their readings multiplied by 100 the horizontal distance of the staff from the instrument, the choice of gradients being determined by the nature of the ground.—

The level of station B, is got by adding $\frac{1}{20}$ the linear distance (27.05 feet) to the staff reading at G 20 (8.45 feet). The level of the distant station is therefore 35.50 feet less the height of the instrument from the ground at station, A. The gradient of station, B, is got without calculation by reading the staff at that station, as explained before. If lower gradients than 20 and 25 are used, levels up to nearly 100 feet can be taken.

Gradient Pairs.

$$\begin{array}{l} \text{G100} \} \quad 66\frac{2}{3} \} \quad 60 \} \quad 50 \} \quad 33\frac{1}{2} \} \quad 25 \} \quad 20 \} \quad 12\frac{1}{2} \} \quad 11\frac{1}{9} \} \\ \text{G 50} \} \quad 40. \} \quad 37\frac{1}{3} \} \quad 33\frac{1}{3} \} \quad 25 \} \quad 20 \} \quad 16\frac{2}{3} \} \quad 11\frac{1}{11} \} \quad 10 \} \end{array}$$

Again any gradient and its half being used, gives the higher figure as multiplier for the difference of the two readings, thus giving in another manner the horizontal distance:—

$\left. \begin{array}{l} G\ 80 \\ G\ 40 \end{array} \right\}$	gives 80	" multiplier."	$\left. \begin{array}{l} G\ 70 \\ G\ 35 \end{array} \right\}$	gives 70	" multiplier"
$\left. \begin{array}{l} G\ 60 \\ G\ 30 \end{array} \right\}$	" 60	"	$\left. \begin{array}{l} G\ 50 \\ G\ 25 \end{array} \right\}$	" 50	"
$\left. \begin{array}{l} G\ 40 \\ G\ 20 \end{array} \right\}$	" 40	"	$\left. \begin{array}{l} G\ 30 \\ G\ 15 \end{array} \right\}$	" 30	"
$\left. \begin{array}{l} G\ 20 \\ G\ 10 \end{array} \right\}$	" 20	"			

Example: Let G 80 and its half G 40 be taken, assume the staff reading at G 80 to be 13.86' and the reading at G 40 to be 8.54', the difference of these readings would be 5.41', and this multiplied by the higher figure G 80 gives 432.80' thus : $5.41 \times 80 = 432.80'$ and the horizontal distance from the instrument to the staff is, in this case, 432.80'. It will be seen on inspection that the reading at G 80 gives a vertical depth from the horizontal line of $\frac{1}{80}$ part the horizontal distance, and in a like manner the reading at G 40 gives a vertical depth of $\frac{1}{40}$ ($\frac{2}{80}$) of the horizontal distance; their difference will necessarily be $\frac{1}{80}$ of the horizontal distance, and the higher figure 80 being used as a multiplier, will give the resulting distance as 432.80 feet.

The motive in this example is that $\frac{1}{40} = \frac{2}{80}$, and the difference $\frac{1}{80}$ requires 80 as a 'multiplier' to resume the full horizontal distance 432.80 feet.

Setting Out Distances by the Subtense Method: "Rule."—If any two integers whatever be taken, and used as divisors into the Distance required, the result will be a **gradient pair**, which, on being worked with, will give on the levelling staff a **subtense** in feet that is equal to the difference between the two integers selected.

“ Example—(selected integers 5 and 10) :—

Distance	$315 \div \text{integer } 5 = 63$	
	,, $315 \div$,, Difference=5, giving subtense 5 feet.	$10 = 31\frac{1}{2}$

Method of Work with gradients 63 and 31½.—Set up the instrument, send out the levelling staff in the direction of the required line, move the index to G 63, and read the staff; assume this reading to be 5·86', then move the index forward to G 31½, and again read the staff. If the distance, from instrument to staff is exactly 315 feet, the second staff reading will be 0·86 feet, i.e., give difference 5'00" between readings if the second reading more or less than 0·86' the staff must be signalled farther or nearer, until the difference of the two readings is exactly 5 feet.

The greater the subtense used, the more accurate will the 'setting-out' be, and in all cases the nearer to the foot of the staff the two readings are taken, the more correct will the line 'set-out' be. If the staff readings are carefully taken, distances can be 'set-out' by this 'subtense' method with at least as much accuracy as best chaining—of course much quicker, and across water or rough ground where the chain cannot be used at all. Distances thus 'set-out' are always horizontal distances, no matter what the rise or fall of ground may be. It is best to select such integers as divisors that give gradients below $\frac{1}{100}$. As a rule gradient between $\frac{1}{10}$ and $\frac{1}{60}$, give the best and quickest results.

Table with Examples of Subtenses from 4 feet to 10 feet, for Distance 315 feet.

For Subtense :	Integers as Divisors.	Examples Selected.	Resulting Gradient Pairs :	Giving Subtense on Staff.
4 ft.	Any two, 4th in succession	5 and 9	63 and 35	feet. 4'00
5 ft.	" 5th "	5 " 10	63 " 31½	5'00
6 ft.	" 6th "	9 " 15	35 " 21	6'00
7 ft.	" 7th "	5 " 12	63 " 26½	7'00
8 ft.	" 8th "	6 " 14	52½ " 22½	8'00
9 ft.	" 9th "	6 " 15	52½ " 21	9'00
10 ft.	" 10th "	10 " 20	31½ " 15½	10'00

Taking Levels and Gradients, and Measuring Distances at the same time:—The level having been set up on its stand and levelled by means of the four levelling screws, T T, and the height of Instrument at station O (Fig. 681) having been read at the suspended tape (take this height as 4 feet 9 inches) the usual method of observing station staves at any forward or back stations is then adopted, and their difference from 4 feet 9 inches (the height of the telescope at the observer's station), is consequently the difference of their levels from the station of the observer. The levels of any stations thus having been taken as usual with levelling instruments, the distance of any one or all of the stations is got by either of the two methods explained previously, and this can be done in a few minutes without the observer troubling to move from the instrument, and without the trouble of signalling to the staff-holder, except loosening the clamp G, and moving the telescope round so that the index reads the graduations on the limb, which give the distance in feet. The distance having thus been taken without any calculation whatever, the telescope can *instantly* be moved back to the zero point, and the instrument is once more level for all the usual purposes of levelling. The same observation which gives the level of a distant station, and one other observation which will occupy a minute or less while the clamp G is loose, will also give the gradient or inclination between the station of the observer and the distant station—as explained under the head 'for taking gradients.'

796 A. Adjustments of the Gradient Telemeter Level.—The adjustments common to all Y Levels for collimating the horizontal wire of the telescope by reversing the telescope in its Y's, apply equally to this instrument.

It is then necessary to set the Y's truly at right angles to the *outer* axis of rotation of the instrument in the following manner:—First clamp the index at zero, and having sighted a distant object, reverse the telescope end for end in its Y's, rotate the instrument on its *outer* axis until the same object is again visible, and cut by the *horizontal* wire. If the horizontal wire does not cut the distant object at *exactly* the same place, raise or lower that

Y, which has two counter-nuts for this purpose, until the same object is cut accurately alike on each reversal of the telescope (end for end) and this *most important* adjustment is correct. While this is being done the index must remain at zero.

The adjustment of the reading index to the gradient limb is effected thus: Let a line be *accurately* measured on level ground (say 200'), set up the instrument at one end of this line and a levelling-staff at the other end. Then take the staff readings as directed before using the zero and gradient 100. If the result differs from the length of the line (200'), move the index to the right or to the left by the two screws which bear on the side of the index until *exactly* 200' is given by the observation, when the index will be in perfect adjustment for gradient 100, and also for all other gradients marked on the limb.

It will be understood that the index being moved slightly to the right or left, increases or lessens the staff reading from which the linear distance is got.

797. Davis's Level Tacheometer (Fig. 683) has compound centres as in a theodolite with horizontal plates 6" diameter, divided into $\frac{1}{2}^{\circ}$ s, with a vernier reading to 15". The telescope has achromatic lenses $1\frac{1}{8}$ " diameter, eye-piece 24 diameters. Stadia lines on diaphragm

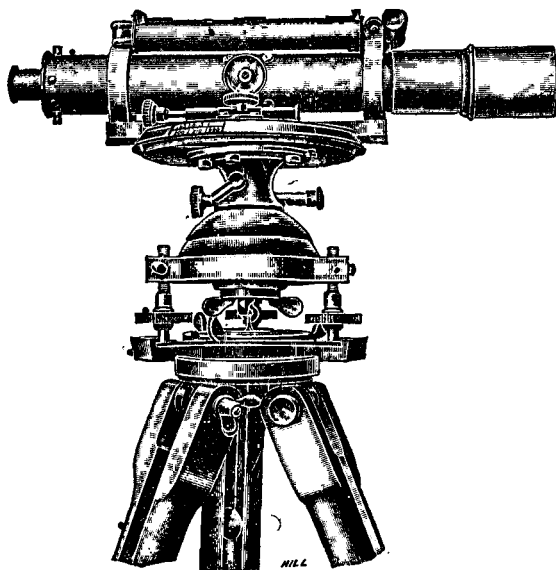


Fig. 683.—Davis's Level Tacheometer.

reading 1-50 and 1-100. The centres are fitted with clamp and slow-motion screws. The instrument is mounted on Hoffman joint with tribrach-stage and centering motion.

798. Stanley's Gradiometer provides means of extending and fixing a scale of gradients for a few degrees by means of an open reading upon a divided helix. The general construction of the instrument. Fig. 684 is similar to Stanley's level Fig. 642. The axis of

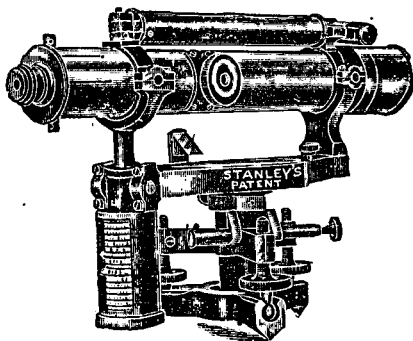


Fig. 684.—Stanley's Gradiometer.

the instrument is placed vertically. The screw gives an inclination, up or down, to $5^{\circ} 30'$. It carries a drum of about an inch in diameter at one point of support, made of aluminium. A helix is cut upon the periphery of the drum, which carries an index so that a single turn of the screw is indicated upon the divisions of the scale on the drum over a linear extent of 3 inches, the whole extent of the scale being about 63 inches for a movement of $5^{\circ} 30'$, which makes the scale very open.

Use of the Gradiometer.—To take a town survey, the altitudes of which were made with a Dumpy or Y level, or with this instrument working at its fixed level point of graduation, and we desire to work out the drainage inclination between two points. We will say the levels indicate a fall of 10 feet between these points. Then, if we measure the line, the drainage is intended to take, however angular or zigzag it may be, the length of this line will be that into which the 10 feet is to be divided. Say a line of streets measures 5000 feet. This gives 1 foot in 500. Therefore if we set the gradiometer to that proportion, all pipes may be laid directly by observation of the top edge of a boning rod of the length of the height of the axis of the telescope plus the intended depth of the drain. As the height of the gradiometer may vary in its setting up, a tape is provided with it to measure this height, and a special boning rod may have a sliding fitting to set off this height correctly upon it; so that there is no calculation but simple observation for any distance. If the depth of drain is higher or lower at the outfall, this difference must be considered in estimating the entire inclination, but the boning rod will still give the inclination above the fixed datum.

799. The Lister's Inclinator Theodolite was originally designed to set out upon the surface of land the widths of slopes or batters by pegs, as required in the execution of railway, canal, and other earthworks. In general construction it resembles a transit theodolite with the addition of an extra vertical axis to the telescope piercing the horizontal axis at right angles, (Fig. 685). In this construction the telescope upon the horizontal axis can be set by the vertical supplementary axis to any inclination, so that if the vertical axis is set to the slope of a railway cutting, any number of points or pegs may be set out continuously with the same setting by direct observation through the telescope across any irregularity or inclination of the land surface, resulting in saving of much labour over the ordinary system of pegging by calculation with the aid of a theodolite, where each peg requires a separate setting of the instrument. When the inclinometer theodolite

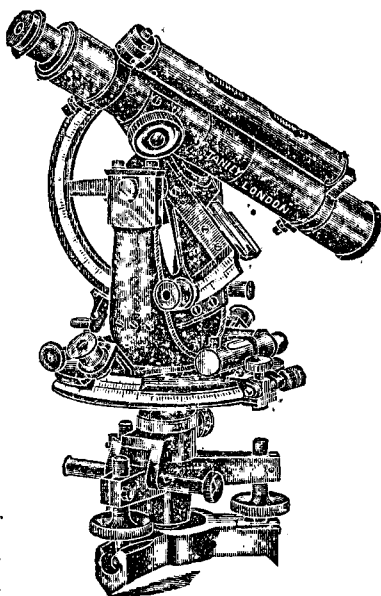


Fig. 685.—Lister's Inclinator Theodolite.

is used for surveying purposes, the telescope is fixed by a spring catch which places it firmly true to the reading of the ordinary vertical arc.

When the line of slope to be followed is a curve, each point or peg is determined in the ordinary manner by tangential angles. In this case the telescope must be depressed where the slope line assumes to chord at each point, that is, as it is seen from the instrument. The only calculations required in the operation are those for the tangential angles, which are readily formed by dividing constant 1719 by the radius of the curve in chains, which gives the angle in minutes for the first chain in the curve. The angle for any other point is found by multiplying this angle by the distance in chains or by $D \cdot 1719/R$; D being the distance in chains, but the use of tables that give the angle for every chain up to 20 for any curve obviates any calculation whatever. Thus the widths on a curve may be almost as readily set out as those on straight line, as the manipulation of the instrument is easily performed during the time the assistant is driving the pegs, and walking from one point to another. In curves of large radii, the variation of the vertical angle for some distance from the instrument is so slight that the depression of the telescope is practically immaterial; but in curves of smaller radii, and in those of larger radii when the widths to be set out are at a considerable distance from the instrument, it is necessary that the variation be attended to.

Application of the Instrument on the Ground:—When the Line is straight.—In proceeding to set out a series of widths when the line is straight, first set out in the ordinary way (using the instrument as a level) a pair of widths as far apart as possible, or at any convenient distance according to the nature of the ground or where practicable, put out the formation width at the commencement of cutting or bank, and make this one of the points established. Then clamp the telescope by means of the vertical arc to the angle or inclination of the slope, and set it at one of the points already determined, so that the cross-wire shall cut the top of the peg, or coincide with the plane of the slope at that point. Then direct the telescope to the distant peg, or other point determined, adjusting it by means of the screws on the horizontal arc, when it will now revolve in the plane of the slope, the cross-web coinciding with the surface line of that plane throughout the whole distance to the next change of gradient, which may now be “nickod out” or the batter peg be put in on either side of the instrument, as far as the range of the telescope will admit. The operation is then repeated for the other side of the cutting or bank, the two points for a basis being already founded by the former operation.

When the Line is on a Curve.—In setting out widths on a curve, first accurately lay off in the usual way, by means of a cross staff or optical square a “square” line, or line at right angles to the tangent of the curve at each chainage peg in the centre line. Then set out the pair of widths as a basis, and fix the instrument at one of them, as described in case of straight line. Having done this, calculate the tangential angle for the distant peg, $D \cdot 1719/R$, D being the distance in chains, and R the radius of the curve in chains (or refer to the tables). Clamp the horizontal arc to this angle, and the vertical arc to the same angle on the graduated scale, and direct the telescope to the peg; clamp to the lower peg, unclamp the top plate, and the telescope when set at zero is now on the tangent line and the method of procedure is the same as setting out an ordinary curve by tangential angles *i.e.*, doubling

the angle for the first chain, for the second, and adding it again for each chain forward (or by using the tables) the vertical are being clamped to the same angle for each chain respectively. The intersection of the cross-wire of the telescope with the square line set off at each peg, determines the square width at that point.

800. The Levelling Staves are graduated staves used to measure the vertical distance from the horizontal line established by the collimation line of the telescope down to the station on the ground. Their name is legion, each with its own peculiarities in graduation which allows the height to be taken to a very small fraction of a foot.

- There are two types, Self-reading or 'Speaking' staves and Target Rods. In America the levelling staff is called a *rod*.

800a. A Self-speaking or Reading staff is usually marked by lines across at every hundredth of a foot the spaces being alternately black and white, and is so graduated as to enable the observer to read the point cut by the line of sight, this reading being in all cases the distance to the bottom of the staff. The observer reads and records the reading, the staff-man only holds the staff vertical.

800b. A Target-rod or Staff is one furnished with a sliding vane or target, painted black and white, so as to obtain very precise intersection with the cross-hairs. The level being directed towards the staff, the staff-man moves the target in response to signals from the leveller until a line on it is bisected by the line of sight. The staff-man then reads the staff, i.e., determine the position of the target upon it, by means of a vernier scale. It can be read at a greater distance than the self-reading rod, but requires an able assistant to book the readings correctly. This is probably the reason why these rods were not introduced into this country. With target-rod to increase the length, the staff is made up of two or more pieces sliding on each other and held in any desired position by a clamp. The target rods, known as Philadelphia, New York, Troy, and Boston, are in common use in the United States of America.

The Philadelphia rod, Fig. 686, can also be used as a self-reading rod. This rod is made in two strips, each about $\frac{3}{4}$ " thick by $1\frac{1}{2}$ " wide and $7\frac{3}{10}$ feet long connected by two metal sleeves, the upper one has a clamp-screw for fastening the two parts together when the rod is extended for a higher reading than 7'. Both sides of the back strip and one side of the front are recessed, painted white, graduated into feet, tenths and hundredths of a foot, and the feet and tenths figured. The front piece reads from the bottom upwards to 7ft., the foot marks being red and the tenths black. When the rod is extended to full length the front surface of the rear half reads from 7 to 13 ft. When the target is used some makers add to it a scale reading to half-hundredths. Others attach a vernier. In fig. 686 the scale or vernier is shewn at the opening in the target. When a level of less than 7ft. is desired, the target is moved up and down the front face, the rod being closed and clamped; but when a greater height is required the target is fixed at 7 feet and the rear half extended, the vernier scale on the back giving the readings like those of the target to thousandths of a foot.



Fig. 686.—Philadelphia Rod.

The back face of the rear half is figured from 7 to 13 ft., reading from the top down; and it has a vernier scale by which the rod is read to thousandths of a foot as it is extended.

The Philadelphia Rod, in three parts, $5\frac{3}{10}$ ' long when closed and 13' when extended is also introduced. In reading, about 5' of the rear part is extended, the readings being

made on the graduated edges of the rod by vernier to $\frac{1}{1000}$ of a foot. When fully extended the front surface becomes a self-speaking rod to 13', the graduations being to $\frac{1}{100}$ ths. of a foot.

The New York Rod, Fig. 687, is similar to the Philadelphia rod except that the graduations are made on the wood in its natural color rendering it unfit for use as a self-reading rod. The cut B shows the rod in 2 parts, which is in common use in America. Fig. 688 shows the rod to which a third or fourth piece is added giving it a greater length. The three-part rod allows a reading of 12 $\frac{1}{2}$ ft., and when closed is 5 ft. long. The four-part rod when closed is 5 ft. long, but it can be extended to 16 ft.

The Boston Rod, (Fig. 689), is formed of two strips each about 6 ft. long, sliding easily by each other in either direction. One side is furnished with a clamping piece and screw, with a small vernier at each end; the other or front piece carries the target and has on each side an inlaid strip on which graduations of feet are marked. The target is a brass disc and is fixed permanently with its centre line just $\frac{1}{10}$ of a foot from the end of the rod. Each tenth graduation is figured decimally to hundredths of a foot, and by the verniers is read to thousandths. When any height is taken above six feet, the rod is changed end

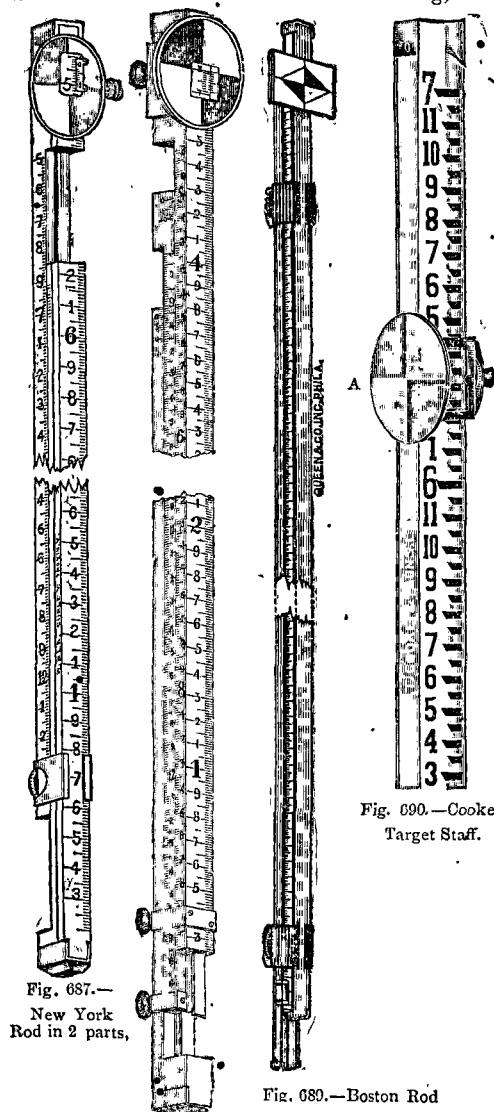


Fig. 687.—
New York
Rod in 2 parts,

Fig. 688.—New York Rod
in 3 or 4 parts,

Fig. 689.—Boston Rod

Fig. 690.—Cooke's
Target Staff.

for end and the graduations read by the other vernier, the height to which the rod can be extended being a little over 11 ft.

Cooke's Target Staff, Fig. 690, used with the Farm Irrigation Level (Fig. 625) is 7' long, divided on edge to $\frac{1}{4}$ ". It is provided with a target, A, which is



Fig. 694.



Fig. 693.



Fig. 691.

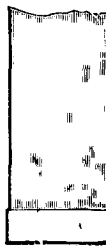
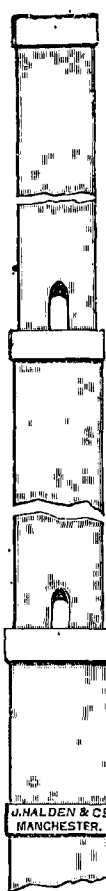
Fig. 69²

Fig. 695.—Full Staff.

free to slide up or down the staff until clamped in any desired position by the milled-head screw, B.

Various patterns of self-reading rods are also used in America nowadays.

800c. English Levelling Staves.—There are various patterns of self-reading levelling staves; the usual pattern now used in this country, is the telescopic staff invented by Thomas Sopwith, F.R.S. Fig. 691 shows Cooke's ordinary Sopwith telescopic staff divided into feet and $\frac{1}{10}$ th parts of a foot. Fig. 692 shows the same staff graduated to feet and $\frac{1}{50}$ parts of a foot only. Fig. 693 is another staff of the same kind graduated by figures and dots only. Fig. 694 is a metric staff divided to metres and half centimetres. Fig. 695 shows the full Sopwith staff as closed and is of about the same length as the tripod, 5'6". For ordinary field-work this is made 14, 16, or 18 feet, in its extended length, *i.e.*, when drawn out

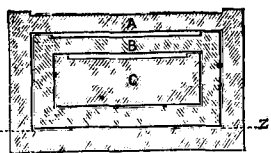


Fig. 696.—Section of Staff.

A, which is in the 14' staff, is 5' long, and is made of mahogany

$\frac{5}{16}$ " thick, the front being $\frac{1}{4}$ ". The outer dimensions are $3\frac{1}{8}$ " $\times$ 2" to 4 " $\times$ $2\frac{1}{2}$ ". The second tube, B, 5' 1" long, of outer dimensions $2\frac{3}{8}$ " $\times$ $1\frac{1}{4}$ ", made so that it will slide easily within the first one.

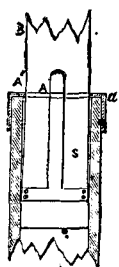


Fig. 697.

The inner solid slide, C, is 5' 2" long, $1\frac{3}{4}$ " by $\frac{3}{4}$ ". All the slides are sunk on the face about $\frac{1}{16}$ " to prevent the divisions being rubbed by exposure in sliding together. The slides have each a brass shoe and cap. They are held in position when extended by a spring catch, S, of $\perp$ form at the back (Fig. 697) which is section on y to z of Fig. 696. The catch is made at A over the edge of brass cap a. The spring should be of very hard rolled brass.

Sopwith staff is liable to be shaky when extended, besides which it often gets jammed if put away damp from rain. Now the square parallel tubes are made slightly conical, so that they form a fair fit when opened out, but are free when closed which remedies the defects just men-

tioned. The telescopic work should fit well and the boxes should be glued up square, and when the glue is set, be screwed up with brass screws 6" apart, to secure the joints which may afterwards in use be exposed to long-continued rain. The interior of the slides should be finished with raw linseed oil, and the outer surfaces be well-soaked in shellac dissolved in spirit, and then French polished. The brass work should be well-lacquered. These should be attended to after use from time to time.

Thornton's Improved Levelling Staves.—The improvement consists in the repetition at every 3 inches of the number of feet in small red figures on the left of the staff (Fig. 698). By this means with short lines of sight exact readings can be taken on any part of the staff where the hair-line falls, without the necessity of lifting or lowering the staff. The large feet figures and the small feet repetitions are painted on left in bright red. The divisional lines in centre (excepting those indicating feet and 6 inches, which are also in red) and figures on right are painted in black. *Thumb clamping screws are fitted, if desired, on first two lengths so as to keep the slides when extended perfectly secure and independent of the spring catches.* These staves are supplied with the figures either upright (Fig. 698) or reverse (Fig. 699) in sizes of 6', 8', 12', 14', 16', 18', 20' and 22' in length.

Fig. 698. —
Upright.Fig. 699. —
(Reverse.)

The Rurki College pattern staff is a simple straight forward staff. Figures indicating full feet are in red and might with advantage be larger and the upper edges of the bar graduations are even numbers. 3'00 is the top of the red bar opposite the red figure 3.

The Solid Scotch Pattern Level Staves are made in 3 lengths of solid hard-wood with strong brass bands, slots, and nut screws; one part fits closely in the next, the two being secured by a clamp. The face is hollowed and the divisions painted either upright or reverse in lengths of 14' or 16'.

800d. Reading with the Sopwith Staff.—In the staff divided into hundredths of a foot, the hundredths are colored black and white alternately. The feet and tenths are numbered, the figures for the tenths being always painted black, while those for the feet are distinguished by their large size, and often also by being painted red. The foot readings are taken from the tops of the red figures. The '1 foot figures are in black, as also all other odd numbers 3, 5, 7, 9. These read also from the top. The height of the figure is exactly '1 foot, so that the bottom of each figure reads the lower even number—thus the bottom of 3 reads '2, of 5 reads 4, of 7 reads 6, and of 9 reads 8. The 6 and 9 foot figures from the effect of telescopic inversion may cause error and should be carefully read. Thus by reference to Fig. 695, it will be seen that there are in each foot five figures 1, 3, 5, 7, 9, representing $\frac{1}{100}$, $\frac{3}{100}$, $\frac{5}{100}$, $\frac{7}{100}$, $\frac{9}{100}$, respectively. Consequently the bottoms of 3, 5, 7, and 9 of the black figures and the top of each red figure represent $\frac{1}{100}$, $\frac{4}{100}$, $\frac{6}{100}$, $\frac{8}{100}$ and $\frac{100}{100}$, the 0 of the scale being at the bottom of the staff. Each $\frac{1}{100}$ or tenth is delineated by a black stroke across the left side and each $\frac{1}{20}$ or $\frac{5}{100}$ is delineated by a longer black stroke. The readings will be as under:—0 bottom of white stroke at bottom of staff '01 top of white or bottom of black stroke, '02 top of black or bottom of white stroke, '03 top of white stroke, '04 top of black stroke, '05 top of white or bottom of black stroke, '06 top of black, '07 top of white, '08 top of black, '09 top of white, '1 top of black '11 top of white, and so on. It will be seen that, when sighted through the telescope, the staff appears inverted, as well as the figures upon it, the telescope having an astronomical eye-piece, to avoid the loss of light that would be incurred by the use of an erecting eye-piece. This produces no inconvenience the eye soon becoming familiar with their appearance in this form, and readily reading the figures downwards, as in fact in upward measurement from the ground. Fig. 699 shows a staff in which the figures are written reversed, but in which they appear erect through the telescope.

800e. The Metre Self-reading Staves are graduated into metres, decimeters and centimeters, 3 metres long.

800f. Feet and Inches Staff.—For building and drainage works, the staff graduated into feet and inches, and sub-divided again into quarter-inches may be conveniently used (*c.f.*, Fig. 690, p. 632).

800g. Mine or Sewer Levelling Staves.—The mining staff is exactly similar to the Sopwith staff except in its length. 2' 3" and 3' 3" are the lengths commonly used when closed, to open out respectively to 5 and 8 feet. The staff is lighted by means of a miner's candle or Safety lamp. Mr Stanley has made a portable staff in lengths of 18' like a French folding rule. The staff may be formed of 3, 4, 5, or 6 lengths opening out respectively 4' 6", 6' 0", 7' 6", and 9' 0". A spring at the end of each piece with a catch and notch keeps the length opened or closed with rigidity. The whole length of the staff when closed is 20½". Fig. 700 shows a mining telescopic staff similar to the Sopwith in total lengths of 6, 8, or 9 ft., but the top length reads downwards being taken off top edge of second joint reading $\frac{1}{16}$ ths, $\frac{1}{8}$ ths, $\frac{1}{10}$ ths of a foot.



Fig. 700.



Fig. 701.

800h. Jee's Pit Levelling-Staff (Fig. 701).—It consists of 3 lengths sliding one into the other. The bottom length is graduated upwards. At the top of the first division of the staff is attached a 2-inch band, which is graduated upwards, and forms an accurate continuation of the scale on the lower division of the staff. The band passes over a brass roller attached to the top division of the staff, and thence is wound round a brass drum, fixed just below the roller, to the top of the same length of the staff, the band being kept in tension by means of a box-spring attached on one side to the axis of the cylinder. It is thus evident that, when the second length of the staff is drawn out, the band unwinds and gives a continuous reading to any distance that the roof will allow. In the same way, when the third length is drawn out, a continuous reading may be obtained up to its full length, or to any intermediate distance required. They are in 6' and 9' lengths closing down to 2' 6" and 3' 6" respectively.

800l. Papering and Repapering a Sopwith Staff.—The staff, if new, is painted, with 3 coats of thin white-lead paint on the face, and left till the paint is quite hard. It is then washed thoroughly with a sponge dipped in Guinness's or other stout, until, this adheres without beading; and is again left to dry. For repapering an old staff, it is soaked with hot water mixed with washing soda and rubbed until the old paper is removed. The staff, in either of the above conditions, has to be made warm and coated with one or two coats of size, made of a piece of glue soaked in water for a night, and then melted in a jam-pot placed in a pan of water over a slow fire. After the staff is sized and dry, it has to be graduated into foot-lengths, by means of a set square, a scale and a pencil, to be marked across the face of the staff. The papers, which are printed short, are then pasted over and laid aside, pasted side upon pasted side, to thoroughly absorb the paste for a few minutes. The paste should be made of starch with boiling water, but not afterwards boiled. While still wet, the upper paper of the

two is lifted up and cut with scissors at the same time as the lifting to the boundary lines. The foot length of pasted paper is then laid by setting the upper edge exact to the upper foot line; and gradually bringing the paper down from this by dabbing with a clean cloth. If the paper does not reach the foot mark when laid, it is again lifted, and a little more pressure used in laying it the second time, which will lengthen it out as required. Other lengths are laid in the same way. There should be no lap to the joints of the papers but should be brought edge to edge. When the papers are dry they should be coated twice with isinglass size, and then a coat or two of varnish. Paper varnish can be brought; but in defect a varnish may be made of Canada balsam dissolved in oil of turpentine. This should be laid on with a flat bristle varnish-brush and set in a warm room to dry for a day or two.

800j. Rubber Cloth Levelling Tape, with foot-plate and hand-ring (Fig. 702) can be had in lengths of 6'.



Fig. 702.

800k. Pocket Levelling Rod, Fig. 703, made of woven linen tape 2" wide, can be coiled up in the pocket and when used tacked on to a board or lath in lengths of 6', 9', 10', 12', 14' reading on one side only in feet, 10ths, and 100ths.



Fig. 703.

800l. Care Necessary in Using a Levelling Staff.—The staff-man should be well-up in his duties. He should see that each length of the staff is drawn out to its proper length, and that the spring clips are in their places, and in carrying it he should not allow it to strike the branches of trees and buildings. In open country the lowest portion may be placed over the shoulder when carrying, but in woods, orchards, etc., it would be better to carry it with the top in front. In crossing a ditch or stream the staff-man should either get some one to hold his staff whilst he gets over, or should lay it gently across with the bottom in the direction in which he is going and upon no account to use it as a jumping pole. Non-verticality of the staff is a common error. If the staff leans towards the observer or away from him the readings are greater than if the staff were held erect. If the lean were equal for fore-sight and back-sight the error would be compensating, but this would seldom occur. In levelling up-hill the error for verticality would be little or nil, as the fore-sight would be taken near the bottom of the staff, but for back-sight the staff-station being lower than the position of the instrument, the staff would be sighted higher up, and the error for verticality would probably be great. These errors would be compensating if the leveller check level down-hill and closed on the starting point, and

though the closing error may be nil, yet the height of the hill would be higher than it really is. **The staff must always be held plumb.** The staff-man must hold it by standing behind it with his fingers at the sides of the first joint, the staff standing between the toes or balls of his feet and in front of his nose. To help the staff-man to do this, a small plummet may be suspended in a groove cut in the side of the staff, by means of which its verticality can be established in one direction. The leveller can then detect its verticality in the other direction by means of the two vertical wires in the diaphragm, or a staff-level, Figs. 709-710, may be fitted to the staff.

The staff-man should only draw out one length of the staff, unless signalled to do otherwise. When the staff is once on a point it should never be moved unless signalled to do so. The staff should be examined before setting it down to see that no dirt is sticking to the bottom of it. The staff-man should always stand erect behind the staff, so that the figures face the level, and no part of his hand should come before the face of the staff, nor any downward pressure be put on it. If the grass be long, tread it down, so that the staff shall have a firm footing.

In using the staff the leveller must see carefully where the horizontal wire of the diaphragm cuts the staff, counting the red figures from the bottom and the nearest subdivision—in other words, supposing the hair were to cut the staff between the red figure 6 and 7 and the black figure 3 of the sub-division, and intersect the second of the black strokes, it would represent that the line of collimation cuts the staff at 6.34.

800m. Readings near Top of Staff.—When the horizontal hair, i.e., collimation line, cuts the staff near the top, so that the staff-reading is large, great care should be taken to hold the staff perfectly plumb, otherwise true reading will not be obtained. Thus, in Fig. 704, if AB be the collimation line, and the staff be held perfectly plumb, the true reading is BD . If, however, the staff be held slanting as DC or DE , the reading DC or DE being greater than DB in the proportion of the hypotenuse of a right-angled $\triangle$ to its longest

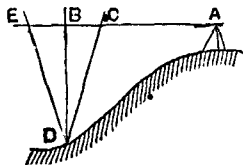


Fig. 704.

side, the error may amount to several hundredths of a foot. Without the use of a plummet or round spirit-level (see p. 640) this error may be avoided by signalling to the staff-man to slowly "*wave the staff*" backwards and forwards from the position DC to the position DE on each side of the vertical until the *smallest reading* DB has been obtained. With this method there is liability of a gradual settlement of the staff, and the base of the staff taking up mud or dirt, hence the ground below should be hard and firm or a Δr foot-plate (Fig. 708) may be used.

The error caused by the staff being slanting is sometimes considerable. Let r be the reading on the staff when it is held

vertical, and r_1 the reading when the staff is slanting at an angle a from the vertical. Then $r/r_1 = \cos a$, and $r_1 = r/\cos a$. Thus, if $a = 2^\circ 30'$, and $r = 13.5$, then $r_1 = 13.512$.

800n. Preservation of the Levelling Staff in Use.—When two staves are used, they may be placed face to face and be strapped together for carrying. When only one is used it is generally strapped to the tripod. Then there is a great amount of risk of the cylindrical tripod pressing against the front of the staff and splitting it. To avoid this German silver spring pads covered with leather are used between the staff and the tripod. The section of this spring is shown, Fig. 705, at *E*, with the general arrangement of the staff *S* and the tripod *D* fixed upon it by the strap *F*. Fig. 706 shows how the tripod and staff are strapped by another pattern of protector, shown in Fig. 707. The advantages of using this protector will be at once evident. The brass flanges fit over the sides of the staff, and the tripod is placed in the hollow on the opposite side; the whole is then strapped together. The face of the staff is thereby protected from wear and injury. For the entire protection of the staff a leather-bound sail-cloth bag is generally used. This may be divided into 2 compartments for the staff and tripod with pads between. This forms a protection from slight bruises and dirt, either in travelling or when set up slanting in an office corner for future use.

800o. Repairing Figures and Divisions.—It will be

convenient for the surveyor when going abroad to take a few tubes of artists' oil colors—white, black, and vermillion, with one or two sable hair brushes to touch up any divisions or figures upon the staves that have become accidentally injured and worn off by friction. A tube of *medium* is also useful, which will dry the color quickly and leave it bright. The brushes after use should be washed with soap and hot water,

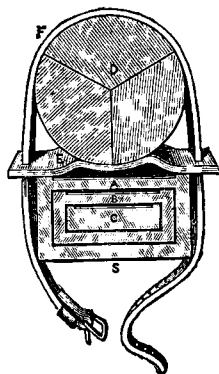


Fig. 705.—Spring pad for staff and tripod when carrying.

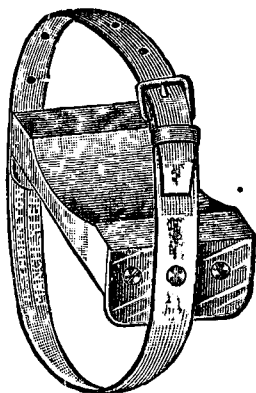


Fig. 707.—Staff Protector.



Fig. 706.

800p. An Iron Triangular Foot Plate of sheet iron, about $\frac{1}{10}$ " in thickness, having the corners turned down, is used to rest the staff on in soft ground (Fig. 708). The corners are trodden down firmly into the ground by the staff-man before he places the staff upon it. It forms a base to turn the staff upon from fore to back-sight, and thus forms a convenient implement where there are no solid stones or hard ground to hold the staff on for the *change points*. A chain and ring are attached to the plate for lifting and carrying from place to place.



Fig. 708.

800q. When the Staff is too near to read the Division.
—When no figure is visible, direct the staff-man to raise or lower the

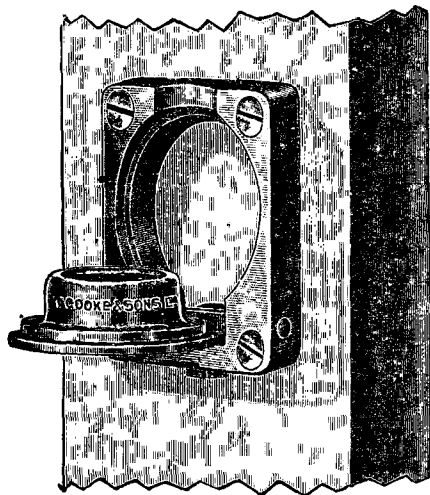
Fig. 709.—Section of Staff Level, $\frac{1}{2}$ Scale.

Fig. 710.—Staff-Level opened for Use.

with studs which fit in two holes with bayonet slots in the holding plate, which is attached to the back of the staff. When the bubble is in the centre the staff is vertical.

800s. The Staff-Holder (Fig. 711) striding a staff is in general use in Europe. The staff is sunk into one side of a hard wood block with a handle at one end and a nut at

the other. The staff is slowly raised or lowered until a figure can be seen. If the staff is too near to be focussed have a finger or a field-book placed against it, and raise or lower the finger or the field-book until its edge is in the collimation line. The staff may then be read where the edge of the field-book or the finger cuts it. This method is not sufficiently accurate for back and fore-sights, but it may be used for intermediate ones.

800r. Staff-Level.—This is a small level shown in section, Fig. 709, and Fig. 710 a perspective view of another folding pattern, having a slightly concave glass top fixed over a short cylindrical box filled with spirit. The level (Fig. 709) is mounted on a plate

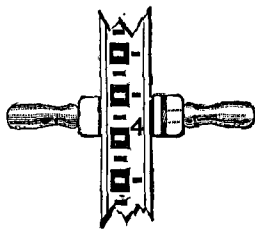


Fig. 711.—Staff-holder.

the other into which another handle is screwed. With these handles the staff is held more truly vertical.

800t. Cooke's Sight Detector.

—This simple *Tell-Tale* attachment (Fig. 712) enables the observer to ascertain whether the staff is held at right angles to the line of sight when so required.

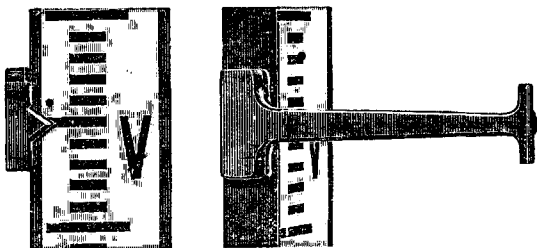


Fig. 712.—Sight Detector.

800u. The Folding Self-Reading Staff or Stadia Rod, used in connection with the stadia to ascertain distances, is formed of 2 pieces of pine, 6' long by $2\frac{1}{2}$ " width, connected by brass hinges and are folded for transportation. When in use they are laid flat and held firmly in line by a clip on the back of the rod. The inner surfaces of the rod are recessed and painted white with graduations in black to feet, tenths, and hundredths, the feet figured in red, tenths in black (Fig. 713).

800v. Messrs Thornton of Manchester make stadia rods divided to feet, $\frac{1}{10}$ ths, $\frac{1}{20}$ ths, $\frac{1}{60}$ ths (N for 9), (Fig. 714); reading 1 to 100 (Fig. 715); reading 1 to 50 (Fig. 716); divided to metres, decimetres and centimetres (reverse reading) (Fig. 717); Metres and centimetres (Fig. 718).

800w. Messrs. John Davis & Son make **Melville Stadia and Levelling Staff**, (Figs. 719-723) which meets the objections to the Sopwith pattern and facilitates the reading of a staff in hot climates, when the flickering effect of the mirage makes accurate reading difficult. The advantages claimed are as follows:—(1) The solid stepped tenth divisions are clearly recognisable as far as it is practicable to read distances by the stadia method. (2) Each figure intersected by a horizontal wire in the telescope is the one noted in the field-book, and not the one less as in the Sopwith pattern. More errors have, it is said, been traced to the latter method of numbering than to any other cause. (3) When the distance is so great that the figures are no longer visible, the foot divisions can be clearly distinguished, and the feet are readily counted by noting that the odd feet slope to the left, and the even feet to the right. The foot numbers are larger than those of the Sopwith staff.

The division of the foot into half also facilitates the finding of the tenths. The "8" and the "9" figures of the tenths being indistinguishable from each other at a distance, an "N" is substituted for the latter. (4) The tenths are sub-divided so that the horizontal edges of the black divisions denote the even hundredths (2, 4, 6, and 8), and the points denote the odd hundredths (1, 3, 5, 7, and 9). The sub-divisions are easily read at a distance

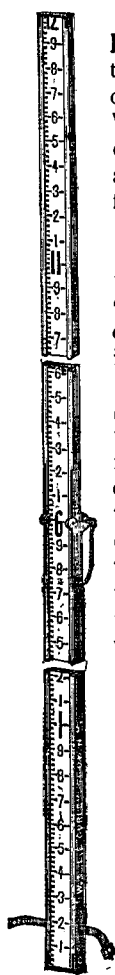


Fig. 713.—Folding Staff.

of 1,000 feet by a telescope of 20 magnifying power. (5) The top section is as wide as the middle section of the Sopwith, thereby allowing a broader design to be used. The divisions of the lower and middle sections are uniform. (6) The 15 feet is a serviceable average convenient length, not unwieldy, and enables it to be used as a fixed rod, 10 feet long, by reading the 0.5 and 10.5 divisions without drawing out the top section.

Fig. 714.



Fig. 715.

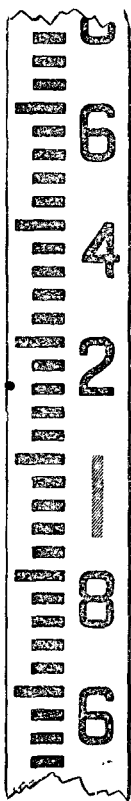


Fig. 716.



Fig 717.

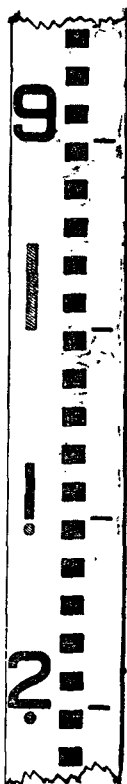


Fig. 718.



Figs. 714—718.—Thornton's Stadia Staves.

800x. High Precision Levelling Staff with Invar Tape. (Fig. 724-726).—For use with the Zeiss Level No. III with Micrometer Attachment a suitably devised levelling staff is used. It consists of a wooden staff of U section 3 metres long and fitted at either end with two steel shoes, the lower one of which is hardened and ground flat to right angles to the longitudinal axis of the staff. The latter has attached at it one of the ends of a tape of "Invar" nickel steel 26 mm wide. The tape lies in two channels in the staff in such a manner that it is nowhere permanently attached to the wood, the upper end of the Invar tape is attached to one end of a two-armed lever having its fulcrum in the steel shoe, whilst the other end is under the tension of a long spring exerting a pull of about

45 lbs. Any change in the length of the staff occasioned by moisture is accordingly taken up by the spring, whilst the tape remains unaffected.

On the back the staff is fitted with two handles which fold into the channel of the U section when the staff is to be carried about.

The Invar tape is divided on two sides into half-centimetres by plain lines, one set of divisions being $2\frac{1}{2}$ cm in advance of the other. The dividing lines are 1 mm thick and have very sharply defined edges. The lines are drawn with such precision that the mean error of a line does not exceed 0.015 mm. The marking and figuring are on the wood at the side of the graduated tape, the figuring proceeding at intervals of 5 cm. The numbering of one scale extends from 0 to 59, that of the other from 60 to 119. In either scale the zero point is at the bottom and, accordingly, readings as furnished by the two scales will differ by a constant amount. The drum of the No. III Level being figured to read intervals of 0.5 mm, it follows that the readings furnished by the drum may be simply added to the staff readings in the form of a decimal fraction, which saves a considerable amount of writing and reckoning.

In addition, the staff is fitted with a detachable circular bubble. Heavy foot-plates provided with hardened spikes for resting the staves upon are likewise supplied. To prevent the staff from slipping off the cusp of the foot-plate a loop fitting may be attached to the staff-shoe. As these staves are entirely unaffected by changes in the humidity of the air, in view of their design, whilst the thermal coefficient of expansion of the tape of Invar is extremely small, being about ten times less than that of steel, there is no necessity, as in the case of wooden staves, to verify the accuracy of the divisions several times in the course of a day. This saves much time and ensures confidence in working. Occasionally, the accuracy of the graduation may be verified with the aid of a comparator. Any deviation in the correct position of the zero point may be ascertained by taking with

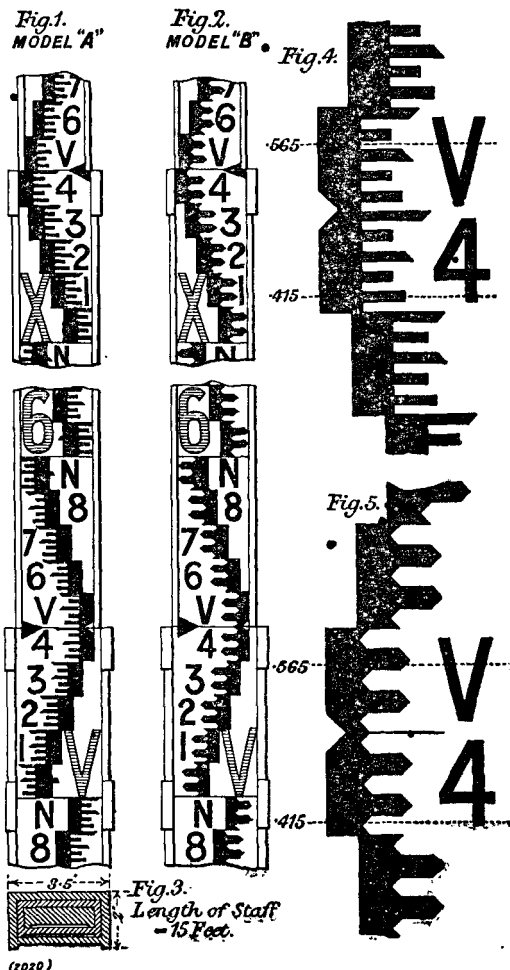


Fig. 719—723.—Davis' Stadia Staves.

a level with Slight Line Displacer readings from a given station point, both from a bar of standard length between the end faces and from the staff. A few repeated readings furnish an accuracy of 0.01 mm.

Fig. 724.



Fig. 725.

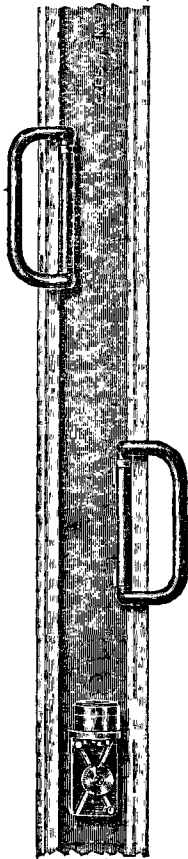


Fig. 726.

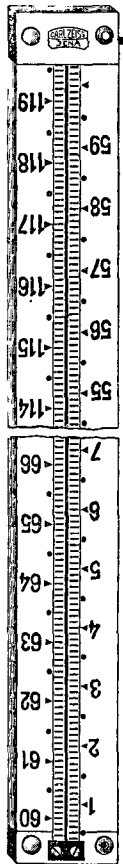


Fig. 724—726.—Precision Levelling Staff with Invar Tape (Carl Zeiss of Jena.).

convenient form of instrument for levelling in close towns. It consists of a pair of standards connected with rubber-hose with cast-iron bases and uprights of glass tubes 1" in diameter, 3' long, with wood back or put in a casing tube for protection. The casing tube is graduated into feet and subdivided into $\frac{1}{4}$ " or the 3' scale is a separate piece. A cock at the bottom allows the water to flow to level in the pair of tubes (Fig. 729). There is a handle at the top which unscrews to fill the level, and a small air lock. It is clear that the water finds its level, and the difference of reading of the two level standards is the difference of level of the surface upon

801. Water Level.

—This form (Fig. 727) consists of two phials fixed vertically on the ends of a horizontal tube $\frac{1}{2}$ " in diameter, and 3' long, made of tin-plate or brass, and partly filled with colored water, by which a level line is established in looking over the surface of the water. By means of this line, the vane of a levelling staff is adjusted to the proper height. The horizontal tube is made so as to revolve on a light stand. This instrument is well-adapted for contouring where a rapid delineation of a country is an object. Its best recommendation is that it can be made by any intelligent workman at a slight expense, and that it requires no adjustment. Levels of this kind but with long tube (Fig. 728) are found convenient in measuring through close buildings.

801a. The Brown's Water Level

is a

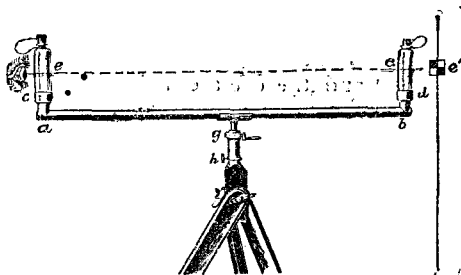


Fig. 727.—Water Level.

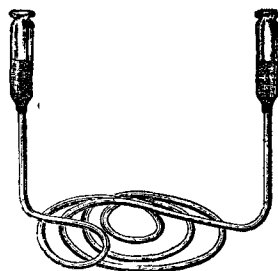


Fig. 728.—Long Tube Water Level.

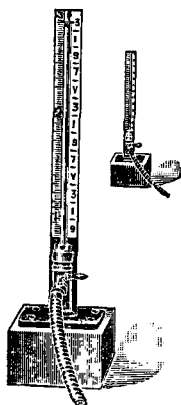
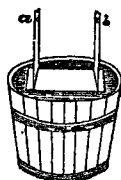


Fig. 729.—Davis' Water Level.

Fig. 730.—
Water Bath Level.

which they are set up. It is made portable when the cocks are closed. It is a very exact level for laying drain pipes in clear weather and for setting out foundations of buildings and bridges at dead level.

801b Water-bath Level.—If a float (Fig. 730) made up of a piece of plank with two pin-hole sights, *a* and *b*, of the same height erected upon the plank, is placed in a water bath, the line of sight established by the two pin-holes will be a level line. This small device can be successfully used in levelling off a small area such as a play ground, etc.

802. The Reflecting Instruments for giving a Level Line of Sight are the following :—

802a. Reflecting Sighted Pocket Level (Fig. 731).—It consists of a square tube with a pin-hole sight at the closed end, *E*. The object-sight, *F*, of the tube is half-closed. A bubble-tube, on the top of the tube at *B*, has its metal casing cut away *F* on the upper and under sides to make the bubble visible from the interior of the tube by means of a mirror, *R*, which occupies half the vertical section of the interior of the tube. The mirror is fixed at an angle of 45° to the axis of the tube upon an inner tube which can be removed for cleaning. The sight is taken by looking through the pin-hole across the edge of the window, *F*. When the level is set

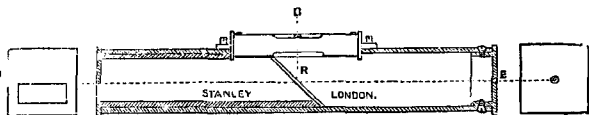


Fig. 731.—Reflecting Sighted Pocket Level.

horizontally, a distant object in the direct sight line is seen through half the tube; and at the same time the reflection of the bubble in the other half appears, as shown above, R bisecting the edge of the screen, F , when the distant object and the eye are in a level. This level is about 4" long and weighs about 8 oz. with case.

Use.—This only gives a level line of sight, and may be used for fixing contours. The leveller stands over a peg on the ground whose reduced-level is known and directs a staff-man, furnished with a $\top$ rod of same height as the observer's eye, up or down-hill till the top of the rod corresponds with the line of sight.

802b. The Locke Hand Level (Fig. 732) consists of a brass tube about 6" long, having a bubble-tube on top near the object-end

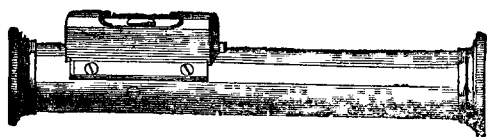


Fig. 732.—Locke Hand Level.
(By Lietz of San Francisco.)

There is also an opening in the bubble-tube beneath through which the bubble can be seen, as reflected by a prism fixed at an angle of 45° with the vertical and immediately under the bubble-tube. There is at the inner end of the sliding or eye-tube a semi-circular convex lens, which serves to magnify the level-bubble and the cross-wire beneath, while it allows the object to be clearly seen through the open half of the tube, both ends of which are closed by plain glass discs to prevent dust. The cross-wire is fixed to a little frame moving under the level-tube and adjusted to its place by the screw shown on the end of the level-case. The level of any object in line with the eye of the observer is determined by sighting upon it through the tube and bringing the bubble into a position where it is bisected by the cross-wire. **To adjust the instrument** obtain two points of the same height and hold the level at one so that on looking through it the cross-wire covers the other. If then the hair does not bisect the bubble, move the level vial until it does. The movement is effected by the screw shown at the end of the bubble-tube. A serious objection to the Locke Level is that any movement of the observer's eye will cause a considerable movement of the cross-wire on a rod observed. It is used in rough levelling.

802c. Pocket Telescopic Reflecting Level.—In this instrument (Fig. 733), the bubble is reflected by a silver reflector to the diaphragm where the image from the object-glass is formed and when the bubble is adjusted by the milled-head under the stage to the centre of its run, the object which coincides with the bubble is in

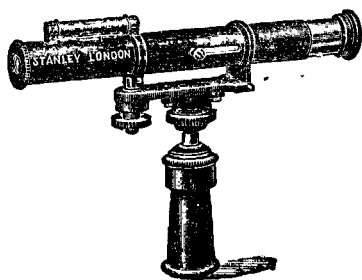


Fig. 733.—Pocket Telescopic Reflecting Level.

the centre of its run, the object which coincides with the bubble is in

a level line with the line of collimation of the instrument. The socket is made to fit a stick or Jacob's staff.

802d. The Reflecting Level with Prismatic Reader (Fig. 734) is constructed on the principle of the Precise Level (*q. v.*, p. 612-614). The two main features of the Precise Level, are maintained. The accuracy of the instrument is equal that of a 10" dumpy. With the prismatic reader attached to the compass horizontal angles can be taken without

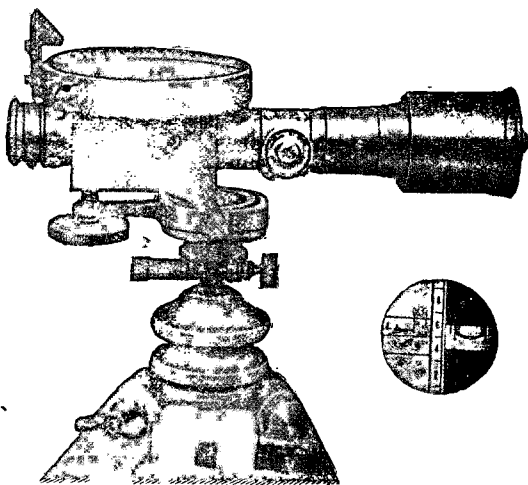


Fig. 734.—Watts & Sons' Reflecting Level, with Prismatic Reader.

changing the position of the observer. A clamp and slow-motion screw is fitted to the horizontal movement.

802e. The Abney Hand Level and Clinometer (Fig. 735) is constructed on the same principle as the sighted level with reflect-

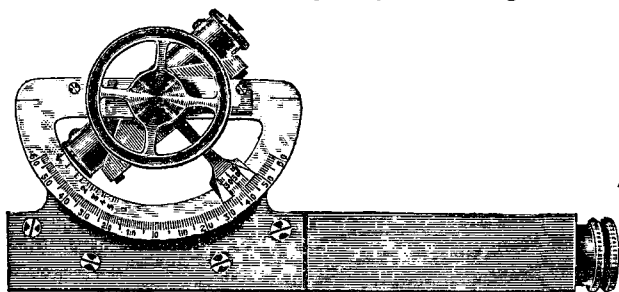


Fig. 735.—Cook's Ordinary Abney Level.

er, R (Fig. 731), and the Locke level, but the level instead of being fixed on the top of the tube is placed above it upon an axis which is the centre of a graduated arc, the middle point of which is marked zero. The graduations on both sides increase to 60°. A light milled wheel placed in front of the arc turns the axis of the arc to any angle. The axis carries the pointer provided with a vernier to read the degree graduations of the arc showing the inclination of the level, while the outer edge of the vernier-plate reads the proportion of slopes of ground, which are marked at the inner rim of the semi-circle. If the observer looks through the instrument with its tube on any slope, at the same time turning the level vial until the reflected image of the bubble

is seen in the reflection, the line of sight is horizontal.

appears bisected by the hair, the reading of the vertical arc will give the tube's angle of inclination to the horizontal. This instrument is used to determine the slope of the ground. The observer looking through it makes the cross-hair cover a point on a rod at the same distance as his eye above the ground. The vertical circle

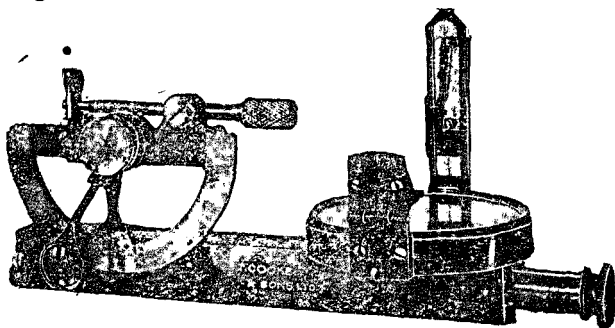


Fig. 736.—Cooke's Abney with Tangent Screw & Prismatic Compass.

will then give the slope of the ground between the observer and the rod. **Abney Level with Tangent Screw adjustment** and a prismatic compass is an excellent instrument for mapping on a small scale (Fig. 736).

802f. Telescopic Abney-Troughton Hand Clinometer, Fig. 737.—In this instrument the arc is toothed, and it is moved by a

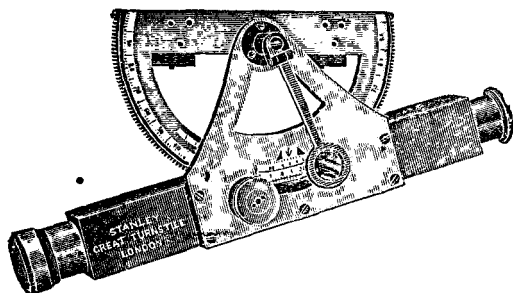


Fig. 737.—Abney-Troughton Hand Clinometer.

pinion similar to the movement of the box-sextant so that the bubble moves slowly in relation to the motion of the fingers in adjustment. This enables it to be set with greater facility and accuracy. A telescope is added to the clinometer as shown.

802g. Steward's Improved Abney Level—In this (Fig. 738) instrument a larger arc has been provided than in the original form, and a long reading Vernier makes it possible to get open divisions of ten minutes each. The bubble is moved by a worm-wheel adjustment and milled horizontal head, which not only is a more delicate movement than the rack or ordinary vertical wheel adjustment, but moving horizontally, any slight movement of the instrument in turning must naturally be from left to right, and so the level is preserved, whereas, no matter how carefully done by the old plan, the instrument was apt to be tilted. The scale shows degrees of inclination or declination and reads by the Vernier to 10 minutes of arc. Supplemental scales give rise in inches per yard and the "one in" scale, with clear readings to each. The divisions of the arc are

continued to 90° , so as to take side slopes, batters of wall and dips of strata, and for plumbing. The bubble is seen by reflection on a half-mirror placed at an angle of 45° in a tube, and the telescopic tube extends so as to enable the bubble and line to be focussed and secure portability when not in use, the size then being $3\frac{1}{2}$ inches long by $1\frac{0}{8}$ inches by $1\frac{1}{8}$ inches deep.

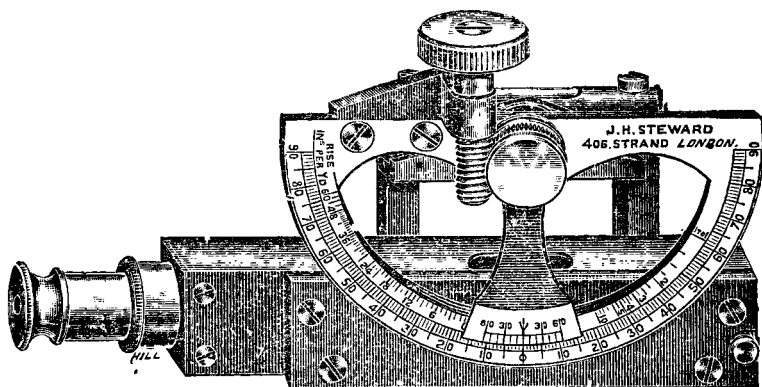


Fig. 738.—Steward's Improved Abney Level.

802h Burel's Reflecting Levels (Fig. 739) are constructed on the principle that a ray of light which strikes a reflecting plane at right angles is reflected back in the same direction. When the eye is reflected in a plain mirror the line joining the eye and its image is at right angles to the mirror, and if the mirror is vertical, this line is horizontal, and may therefore be used for finding points at the same level as the eye. It is made up of a parallel glass which has the lower half silvered to form a reflector. The mirror is placed in a metal frame which is suspended in such a manner that the glass hangs vertically by gravitation. The ring at the top of the level is placed over the thumb or finger to support it when in use. A stout bar is screwed through a prolongation of the lower part of the frame which permit adjustment by filing so as to put the mirror in a vertical plane when suspended. The instrument, fitted into a neat case, weighs from 5 oz. to 9 oz. **In Using the Reflecting Level,** it is held upon the thumb at about arm's length, and adjusted by raising or lowering the arm until the reflection of the pupil of the eye seen in the mirror is exactly bisected by the diameter which divides the mirror from the plane glass. The distant object seen in front that cuts the sight-line passing through the diameter, and the image of the pupil of the eye will then be on a level with the eye of the observer, provided the air is still and the mirror is vertical. As it is not possible to keep the hand steady so it is difficult to keep the level

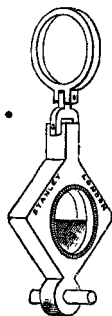


Fig. 739.—
Burel's
Reflecting
Level.

free from oscillation. This may be avoided by resting the hand against a tree or by clutching a staff by the hand and suspending the level from the thumb projected out for that purpose. The best way to hold the instrument is to put the handle of a stick in the waistcoat pocket, to clutch the stick with the hand which holds the instrument, and to steady it with the other hand, the two arms and the stick forming a steady tripod.

802 i. Reflecting Level in Case.—When the pendulous level is fitted in a case, (Fig. 740) much greater accuracy may be obtained when used in the presence of wind. The case consists of double tubes, so that the opening on one side may by a half turn of the outer tube close and protect the instrument when not in use. In this instrument the edge of the mirror is placed vertically as shown at R and the weight, W, keeps the mirror vertical. When this form of instrument is used, it may be, if required, made to fit on to the top of a light staff by means of the screw, s. The eye is then brought with much greater certainty to the point of bisection on the edge of the mirror and much greater accuracy is thus obtained in levelling with it.

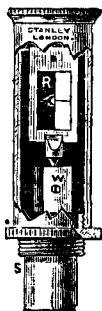
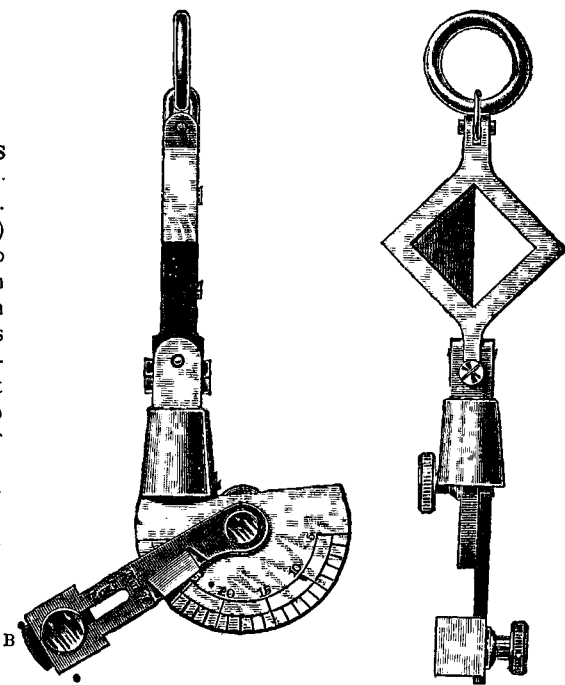


Fig. 740.—
Reflecting
Level in
Case.

802 j. De Lisle's Reflecting Pendant Clinometer. (Figs. . 741-742) (Length 6," weight 10 ozs.) is a modification of the Burel level in which a heavy arc is jointed upon a vertical axis at C so that it may be revolved to bring the arc either forward or backward, to take inclines upwards or downwards, or to rest at an intermediate position to make the instrument flat and portable in its case. A radial arm, the edge of which forms the index, with a sliding



Figs. 741-742.—Cooke's De Lisle's Clinometer

weight, is fitted at the centre of the arc. The sliding weight, B, when slid out to the end of the arm will counterbalance the weight of the arc in a horizontal position to make the mirror vertical. In this position it is a Burel level. The radial bar index set to one of the graduations of the arc, 1 : 50 to 1 : 5, gives the amount of inclination that will result from the coincidence of the reflection of the centre of the pupil of the eye cutting the object observed. This Instrument is intended for setting out slopes and gradients in the construction of roads, paths, railways, water-courses and the like.

The method of using it is as follows :— First, twist the semi-circular weight forming its lower part one-half round, so that it lies at right angles to the face of the mirror. Then procure a couple of poles of the nature of short ranging poles, and into one of them screw a hook at such a height that when the Clinometer is hung upon it by its ring, the mirror is some 5 ft. or other convenient distance from its foot. On the second pole fix a short cross-piece at a similar height, so that when the two poles are held vertically side by side, the centre of the mirror and the centre of the cross-piece are on the same level. The Clinometer may now be set by the movable arm to any gradient required between the limits of 1 in 5 and 1 in 50; care being taken that the square sliding weight on the arm is kept well up to the inner stop. Now look into the mirror the Clinometer being suspended from its hook and the pole held at a short arm's length, and raise or lower the eye until its reflection is clearly seen. The inclination of the line of sight now exactly corresponds to the grade indicated, and it naturally follows that if it be extended through the opening at the right-hand side of the mirror, it will continue unchanged for an indefinite distance. It is easy, therefore, for an assistant to fix height-pegs at all desired points by holding the second pole with cross-piece on the pegs in turn, and driving them until the cross-piece appears exactly opposite the centre of the eye as seen in the mirror. By turning the arm of the Clinometer back to its fullest extent, and then drawing the sliding counter-piece up to the outer stop at the end of the arm, the word "level" will appear, and when set thus the line of sight is horizontal. By twisting the semi-circular weight through an angle of 180 degrees, rising gradients will be changed to falling ones, and *vice-versa*.

802k. Watkin's Mirror Clinometer.

—Fig. 743 shows the instrument with the cover removed. It consists of a brass circle pivoted at the centre, counterbalanced, and caused to swing as a pendulum by means of a weight shown like a screw-head in the diagram. The inner rim of the left-hand half circle is fitted with an ivory scale reading degrees. A concave mirror, R, reflects a magnified image of this scale to the eye placed at E. The eye can also at the same time see the reflection of a line drawn on a fixed slip of ivory, I, placed just beside the eye-hole, E, and distant objects through the opening, O. A spring shown above the reflector, R, clamps the graduated arc for travelling. By pressing a knob at the side of the clinometer acting on the spring the arc is freed and swinging freely brings the bar forming the diameter of the arc into the vertical position. A screw, C, adjusts the clinometer if required.

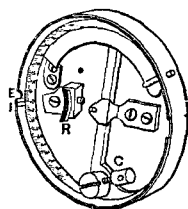


Fig. 743.—Watkin's Mirror Clinometer.

To find objects on a level with the Eye of the observer.— Hold the instrument in the left-hand, free the swinging scale by pressing the

Knob at the side of the Clinometer, bring the reflection of the line on I to 0° on the scale, and note the object which is bisected by the line.

To obtain the Inclination of any Object.—Having freed the scale, bring the reflection of the line on I to bisect the object, and read the scale opposite the line. The red scale denotes angles of elevation, the black scales of depression. In observing a slope, the sight line must be directed parallel to the surface of the ground.

802l. Surveying Compass-Clinometer (Figs. 744-746).—It consists of a $2\frac{1}{2}$ to 4 ins. circular box about $\frac{1}{2}$ in. deep, which contains

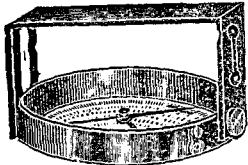


Fig 1

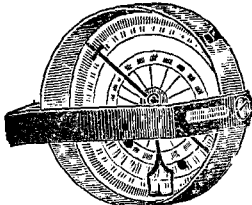


Fig 2

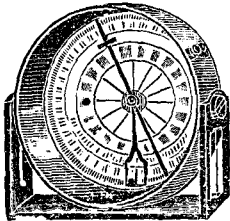


Fig 3

Figs. 744-746.

a card or metal dial, a clinometer pendulum, a magnetic needle (which is balanced on a selected agate stone and steel pivot), and the mechanism for throwing the needle and pendulum out of action when not in use. The sights and foot are in one piece, and are attached to the box with two screws, which permit the sights to be raised over the box (for taking horizontal and vertical angles) or folded down (for obtaining bearings of walls, etc.) A milled-headed screw and leg-of-mutton piece is attached to the side of the box for throwing the instrument out of action. A spring bezel secures a stout glass in position. For simplicity in reading bearings the Compass points are reversed in the same manner as for a theodolite, etc.

For taking vertical angles hold the instrument vertically (see Fig. 2), and the index on the pendulum will indicate, on the degree scale, the angle between objective and horizontal plane. The rise in inches per yard will also be registered by the same index on the rise scale. For instance, if the index registers 15 inches, this means that the line of slope rises 15 inches to every yard of horizontal. The height may be found by measuring the horizontal distance, in yards, between objective and observer and multiplying by the number of inches in the rise.

For example:—Distance in 250 yds. $\times$ 15"
Rise in ins. = 312.5 Height in feet.

(This is not a Reflecting Instrument).

802m. Combined Prismatic Compass and Clinometer.—

This form of the Altitude and Prismatic Compass is of the same shape, size and depth as an ordinary Hutchinson Prismatic, and exceeds it but very trifling in weight. The diameters of the Compass Dial and Altitude Circle are both $2\frac{1}{2}$ inches. Prismatic Compass Dial being divided to half degrees, and Altitude Circle in the same manner, which circle has also a scale of rise and fall in inches per yard for use with the foot E, Fig. 747. The clinometer card is, mounted over the Compass Card on a pin-axis and is an well-balanced disc a part of which is cut away so as to permit the Compass Card to be read beneath,

To Use the Instrument.—For taking vertical angles or levels, hold the Instrument as in Fig. 747. pulling stop B well out, which will release the weighted disc A, after which focus prism C so as to read the divided circle distinctly, raise sight vane D, and bisect with cross-hair the object you wish to observe. The reading then shown through the prism will be correct reading, or if zero be shown the object will be on a level with the observer.

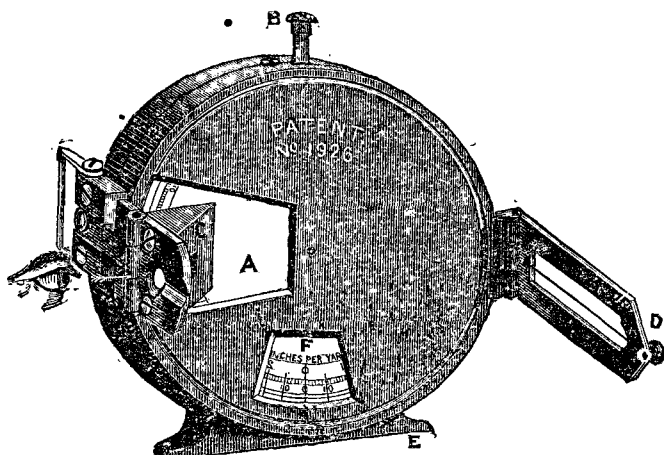


Fig. 747. —Hutchinson Type, Prismatic Compass Clinometer.

For use as a Prismatic Compass for horizontal angles, change Instrument from a vertical to a horizontal position; in changing, let the cut away part of the disc fall opposite the prism box C, then push the stop-piece, B, in so as to keep disc from moving, and proceed as with ordinary prismatic compass. When Instrument is not in use always keep stop-piece pushed in.

Hunter Form Altitude Instrument and Prismatic is shown in Figs. 28 and 748. It is like Fig. 747 above and is used in the same manner.

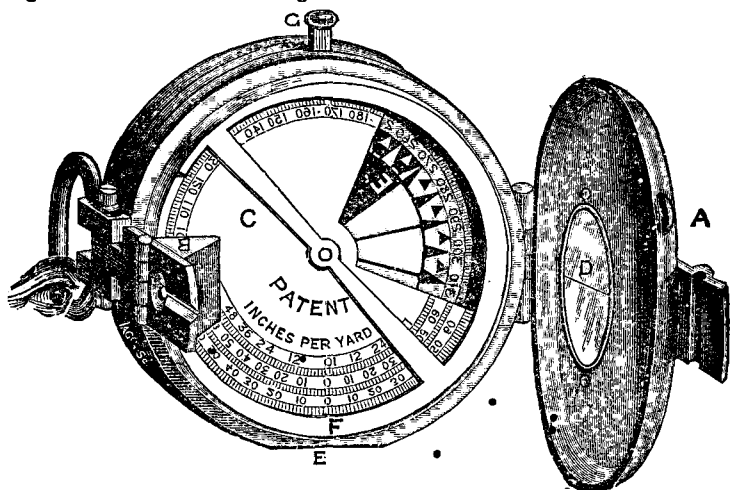


Fig. 748. —Hunter Form, Prismatic Compass Clinometer.

802n. Telescopic Prismatic Clinometer.—By it both vertical and horizontal angles can be taken. Inside the compass Box is a silver ring divided to $30'$; this gives the magnetic bearing. On the tripod (Fig 749) with the spirit-level and telescope, horizontal angles can be taken independently of the magnetic needle. This is important in iron districts. To measure vertical angles, the compass is used side-ways (Fig. 750). The level then shows the horizontal line, and all angles can be taken from the Zenith round. All observations can be verified by reversing the compass in azimuth and altitude.

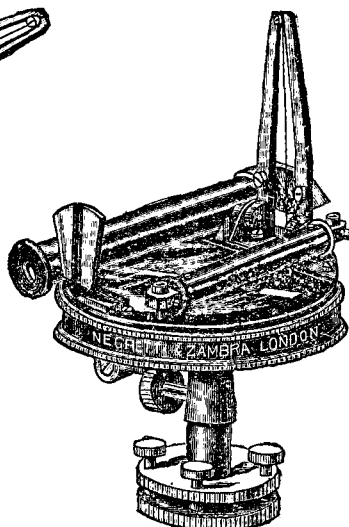
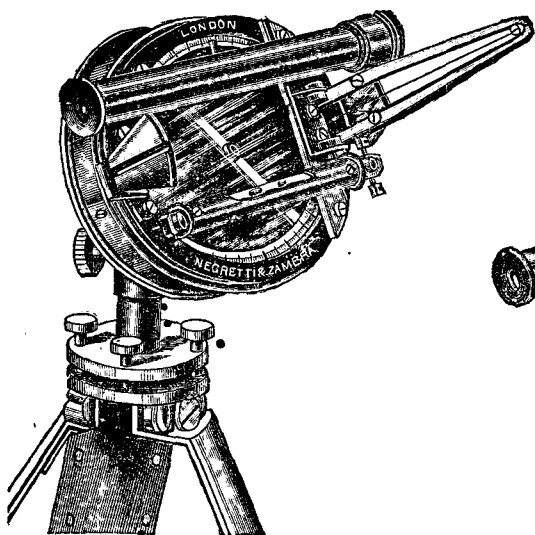


Fig. 750. Figs. 749-750.—Telescopic Prismatic Clinometer. Fig. 749.

802o. Pocket Alt-azimuth.—The Altitudes, Azimuths, Compass Bearings, Clinometric degrees and Levels are

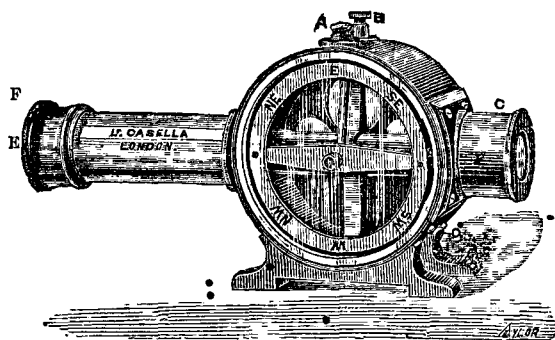


Fig. 751.—Pocket Alt-azimuth.

all obtainable by this little Instrument, (Fig. 751). Diameter $2\frac{1}{2}$ ins., thickness $1\frac{1}{8}$ in., and weight 8 oz. The Altazimuth contains: 1. An unusually good Azimuth Compass, with aluminium disc. 2. A weighted Disc for Altitudes: both these are graduated on their

edges; and read off through a lens, in a far more simple manner than the ordinary prismatic compass. The instrument is ready for use immediately the catches, that hold the discs, are released. 3. One face of the aluminium disc is plainly engraved to serve as an ordinary compass. 4. One face of the weighted disc is divided to serve as an ordinary clinometer, the box of the instrument being furnished with a graduated edge. **A** stops or liberates the compass on being turned back-wards or forwards. **B** acts similarly on the clinometer on being pushed in or drawn out. **C** is an adjustable eye-piece to compass and clinometer circles. **E** is a dark glass fitted in cap so as to enable the observer to take sun's azimuth. **G**, brass foot, by means of which the rise or fall of any surface can be obtained. **F** is a slide by which to adjust focus of Telescope. **D** is to be pushed in as a steady pin before reading the compass.

Accuracy of Performance.—The performance of an Azimuth Compass is well known and appreciated. That of the weighted disc must clearly be more delicate, inasmuch as the directive force of gravity is enormously greater than that of terrestrial magnetism. Both discs are graduated to degrees. A careful observer will read off to tenths.

To use the Instrument.—Adjust the lens C so as to read either the Compass ring or Clinometer disc, after which focus Telescope to object that you may wish to observe: the degree then intersected by the cross-hair is the correct reading; hold it vertically for altitudes, horizontally for azimuths.

Index and other Errors.—The principle of the weighted disc admits of these errors being discovered, checked, and eliminated. Two different readings may be obtained from the vertical circle by observing with the Clinometer face, first to the left and then to the right. The mean of these is clearly independent of index error. Again, by similar observations of the image of the object, as reflected in the water, two additional readings may be obtained, making four in all, or one on each quadrant of the disc. The mean of these four must be almost wholly independent of instrumental error. A surveyor furnished with the Altazimuth may by its use alone map his country, take the height of mountains, etc., the dip of strata, and correct his position by very fair astronomical observations as being correct to one-tenth of a degree; his latitude will be right to six miles, and his longitude, by occultations or (if he has a telescope) by Jupiter's satellites to within thirty miles.

802p. The Clinometer Rule, Fig. 752, is an useful instrument for taking heights and distances of objects at any known distance from the point of observation, it being founded on the well-known principle that the base and one angle of a right-angled triangle being given, the height of the $\perp$ is obtained; thus, the base being 100' and the angle subtended at the vertex at one extremity of the base being $45^{\circ} = 1$, the height will be 100 as $\tan. 45^{\circ} = 1$.

In form, it is similar to an ordinary rule, having two arms which turn on the joint, E. On each arm is placed a spirit-level, so as to enable either arm to be placed perfectly horizontal; the upper leg, M, is provided with a pair of sights in each of which there are pin-pole and cross-wires as H-F and I-G. to bisect any object.

When the arms of the instrument are open the brass divided scales at joint are: one scale, B, containing the ordinary degrees, and the other, A, being divided for rise and fall per yard. The edge of rule is divided to eighths and tenths of an inch, the compass fitted into

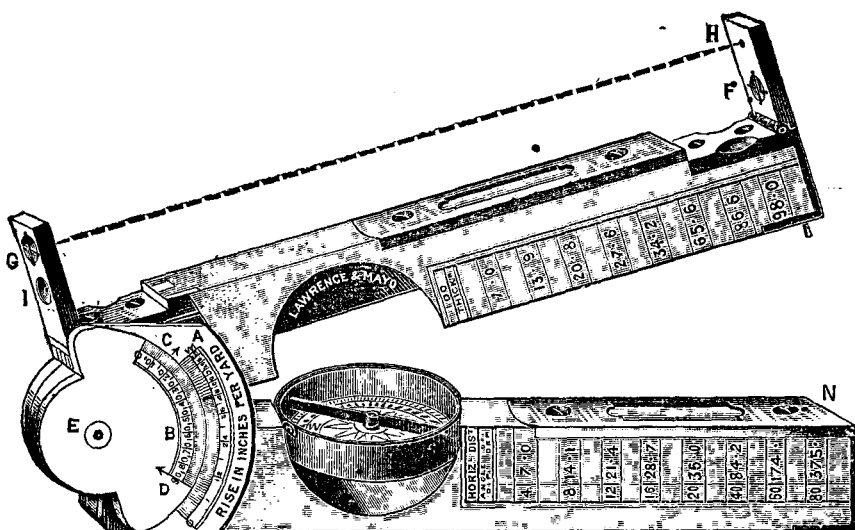


Fig. 752.—The Clinometer Rule.

surface of rule is useful for taking bearings of walls or any other objects, and is in some cases furnished with a swing action to compass to facilitate its use with the pair of sights.

To Obtain the Vertical Height of a Distant object.—1st.—When above the eye.—Place the instrument so that leg, N, is horizontal, arrange the sights and open the instrument until the object viewed through the small hole (as in figure) is opposite to the centre of the cross-wire of the other sight. Then the arrow, C, upon the head will point to the number of degrees upon scale, B, which measures the angle subtended by the object, and the arrow, D, will mark upon scale, A, the number of inches of rise per yard of run horizontal, which corresponds to that angle of elevation. Thus, if scale, B, registers that the angle between the two arrows is 23° , scale A will denote that the height of the object viewed above the horizontal of the eye can be obtained by allowing 15 inches vertical to 36 inches of horizontal. Further, the table on front of leg N will record for certain angles up to 85° , the amount of rise per 100 of run. Thus for 1° the rise would be 1.7 per 100 horizontal, and an object 400 feet from the observer (measured) horizontal, and subtending an angle of 1° , would be $1.7 \times 4 = 6.8$ feet in vertical height.

On the back of the instrument a table shows certain angles of elevation or depression which give unity of rise per amount of run.

Thus 11° , of elevation gives 1 vertical to 5 horizontal, etc.

2nd.—When the object is below the Eye.—The instrument is used with the sights reversed, the observer's eye being nearest to that sight which is farthest from the head, the small hole is opposite to the corresponding glass disc in the other sight. The angle of depression together with all the measurements are then registered in the same way as described for objects above the eye.

802q. Ceylon Ghat Tracer (Fig. 753).—It consists of a brass sighting tube, AB, hung from a pivot C by two rods, with square sides, and having a very small aperture at one end and a larger opening, with cross-wires at the other. Rigidly attached to the tube by

two vertical bars, is a horizontal rack up and down which a suspended weight, W, can be moved by means of a milled-head-screw, working a pinion on the rack. The upper part of the weight forms an index by which the scale of gradients of road slopes from 1 in 6 to 1 in 120 marked on the flat side of the sight-tube can be read. By the movement of the weight along the rack by turning the milled-head, the tube and consequently the line of sight can be set to any gradient required. This is a very useful instrument for carrying out the preliminary survey of a hill route of any required gradient.

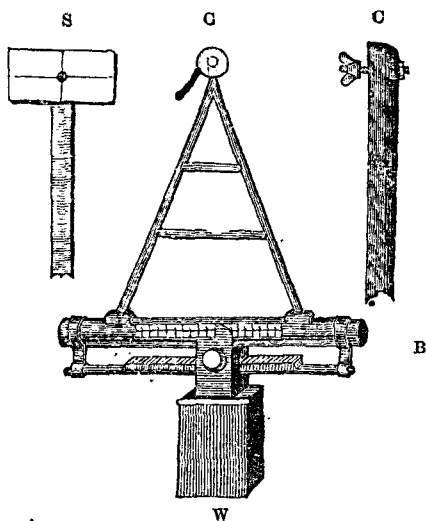


Fig. 753.—Ceylon Ghat Tracer.

Use of the Ghat Tracer.—Suppose a gradient of 1 in 37 has to be laid out on a hill slope. Move the weight, W, along the rack until the index points to 37 on the scale of gradients marked on the side of the tube. Then send a man with a sight vane S, (fig. 754) set at the height of the axis of the sight-tube, along the hill-side to a convenient distance, usually from 150 to 200 feet, until the place centre of the sight-vane. The foot of the vane is then resting on a point, from which the slope down to the observing point is one of 1 in 37. The assistant drives a peg in here to which the surveyor then proceeds and repeats the operation.

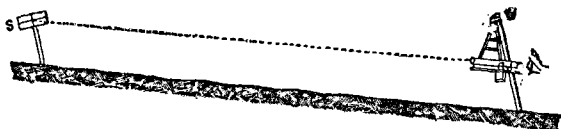


Fig. 754.—Use of the Ghat Tracer.

sight-tube, along the hill-side to a convenient distance, usually from 150 to 200 feet, until the place centre of the sight-vane. The foot of the vane is then resting on a point, from which the slope down to the observing point is one of 1 in 37. The assistant drives a peg in here to which the surveyor then proceeds and repeats the operation.

802r. The Bellamy Road-Tracer (Fig. 755).

—The difference between the above and Bellamy pattern lies in the adoption of an arc instead of a straight scale. The weight for ensuring the horizontality is attached to a pendulum swinging from the same fulcrum as the segmental frame and moving in a vertical plane parallel with that of

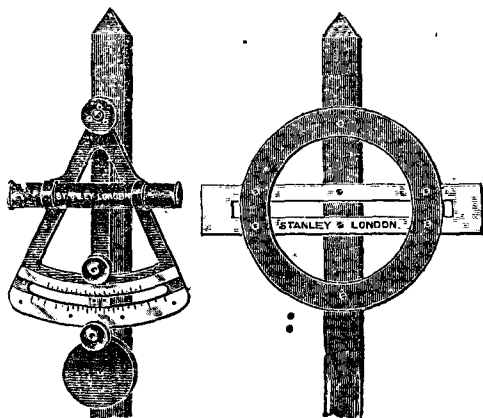


Fig. 755-756.—Bellamy Road-Tracer & Sight.

in a vertical plane parallel with that of

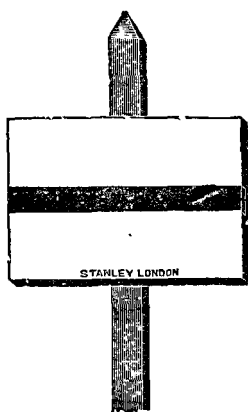


Fig. 757.—Target Rod.

the frame to which the sight is fixed. When the sight-tube is fixed at right angles to the plane the line of sight must be horizontal. The point of intersection of the centre line of the pendulum with the vernier, when the pendulum and the sight-tube are at right angles to each other, is taken to be zero, and the vernier is marked off on either side of this point in grades or degrees, every such division of the vernier representing a variation in the sight-plane from the horizontal, one way or the other. The weight is moved along the arc of the frame by means of a milled-head and pinion fixed on the pendulum, which engages a rack on the outer edge of the segmental frame. A clamp is also provided, whereby, in case a long distance of the same gradient has to be run, the instrument may be fixed at this gradient, and the risk of a movement of the scale while changing the point of observation may be avoided. Further improvements over previous types will be found in the fittings of the sight-tube, which is furnished with adjusting-screws, as in the telescope of a level, so that the tube may be adjusted from time to time should it get out of order from long or rough usage. The eye-pieces also are made reversible, being fitted with sliding shutters, so that in case a back-reading requires to be taken, this may be done by merely changing the shutters, and without unclamping the instrument or altering the vernier. The sight-staffs are shown by Figs. 756 and 757, the last being much more visible in woods and jungles than the plain cross, commonly used.

802s. The Bald's Self-Registering Clinometer (Figs. 758-760) consists of an aluminium or brass dial graduated to show 60° on either side. Sight vanes are attached to the dial, which and the swinging weight are pivoted from the staff at the centre of the line of sight marked on the dial. The top point of the pendulum forms the index and coincides with the zero of the graduations when the line of sight is horizontal. By rotating the dial by means of a milled-head the sight vane can be inclined to any gradient required. This is a cheap instrument for drainage operations, road tracing, measuring heights, levelling sites for buildings, etc.

803. Instruments for Measuring Altitudes by Differences of Atmospheric Pressure:—The practical instruments constructed for measuring altitudes by the differences of pressure due to the weight of the superincumbent atmosphere are the portable barometers, the Syphon or Cistern Mountain Barometers, the Aneroid, and the Hypsometer.

The pressure of the atmosphere decreasing with the elevation, the barometer is used to measure this pressure directly, and hence

indirectly the elevation of the point at which the instrument is held. Barometric hypsometry furnishes a means of determining

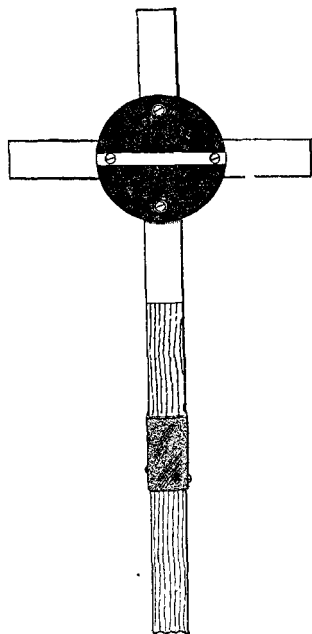


Fig. 760.—Target Rod.

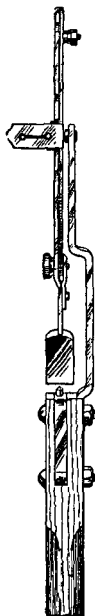


Fig. 759.—Side View showing sight vanes.

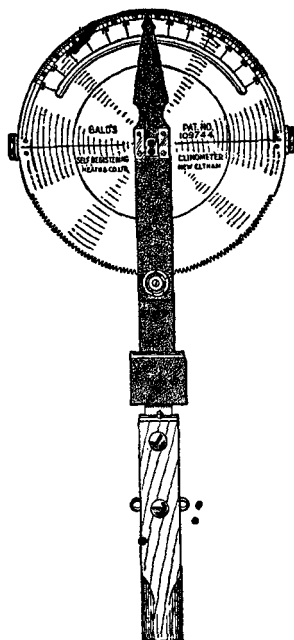


Fig. 758.—Face View showing dial.

Figs. 758—760.—Bald's Self-Registering Clinometer & Sight.

quickly the difference in Elevation of widely separated points, but its results are only approximations. These become useful in the *reconnaissance* of a new country. Special methods of observation eliminate in part the errors to which barometric results are liable. In point of precision the mercurial barometer is a better hypsometric instrument than the aneroid. The aneroid on the other hand is more easily portable, and hence more serviceable. It is also more sensitive. The aneroid gives earlier indications than the more sluggish mercurial.

803a. The Syphon Barometer (Fig. 760) has no cistern the tube is turned up at its lower end. A small hole admits the air. The height of the mercurial column is equal to the difference of the height mercury in the two branches of the tube. The whole is protected by a brass case, is fitted with verniers, thermometers, etc.

803b. The Collie Portable Mercurial Barometer (Fig. 761—763).—The cistern and the vacuum tube at the top are of equal diameter, and are connected by a flexible tube, and the difference in level of the mercury may be measured directly by means of a graduated rod, or as in Deasy's mounting by means of a vernier. There is

no attached thermometer, but if the instrument is used in the open air, and is exposed for 10 or 15 minutes before using, it will be

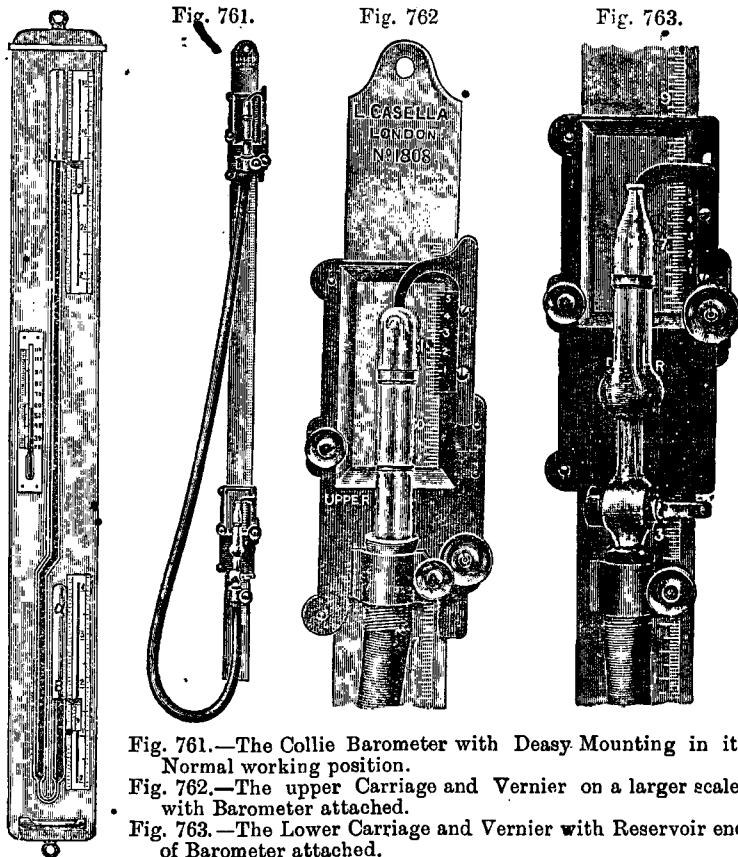


Fig. 761.—The Collie Barometer with Deasy Mounting in its Normal working position.

Fig. 762.—The upper Carriage and Vernier on a larger scale, with Barometer attached.

Fig. 763.—The Lower Carriage and Vernier with Reservoir end of Barometer attached.

Fig. 760.—Hick's Syphon Barometer.

sufficient to note the temperature of the air in the usual way. The upper end (Fig. 762) is about 2.5" long, and contains an air-trap, into which all the air that may accidentally enter the barometer, either by tap leaking, through the rubber-tubing, or through either of the joints, must find its way. The lower or reservoir end (Fig. 763) is about 4.5" long and has an air-tight glass tap about .1" below the broad part. These ends are forced into the rubber-tubing, and as a precaution against leakage, copper wire is bound round the joints. The scale is cut on an aluminium bar, along which two carriages, to which the barometer is attached, move up and down, and they can be clamped to the bar at any place (Fig. 761). By means of the verniers attached to the carriages, which are divided to 0.002 of an inch, it is easy to estimate the height of the mercury to 0.001.

To use the Barometer, the carriages are put on the Scale bar, the lower one is clamped at the bottom of the bar, the upper one some inches higher up; the barometer is attached to the carriages by clamps which fit over the joints; the rubber cap is removed from the reservoir end, the tap opened, the verniers put in the middle of their runs, and the upper carriage moved up the bar until there is a vacuum. By means of the screws on the right of the mercury, the verniers are moved up or down until the top of the mercury at each end is in line with the edges of the rings attached to the verniers, which fit round the glass ends. Both verniers are then read, and the difference gives the height of the barometer. The rubber cap on the reservoir end is merely to prevent the small quantity of mercury, which should be left above the tap when it is closed, from being shaken out when travelling.

To pack up the barometer, lower the upper carriage *very slowly* until the mercury has touched the top of the glass; then detach the barometer from this carriage, and either let the upper end hang vertically below the reservoir, or detach the reservoir end from its carriage and raise it till the barometer hangs vertically. By this means the barometer is completely filled with mercury, and then the tap must be closed. The tube is then to be coiled away in its padded box. When too much air is found in the trap, it must be extracted by means of the air-pump.

803c. Fortin's Mountain Barometer.—In measuring altitudes when portability is required, the construction devised by Fortin is employed. Fig. 764 shows the arrangement of the Barometer tube, A, which is placed in a brass case, B, Fig. 765, and is mounted upon a tripod made of 3 light tubes with steel points.

The Cistern.—Fig. 765 shows the cistern upon Fortin's plan, made of glass, and box-wood tubes, GH, with flexible kid leather bottom, J, and the box-wood top, D, to which is inserted a small ivory-point, E; the lower end of which is the zero of the barometer scale; and in order that the readings should possess uniform value, it is necessary at each observation that the level of the mercury in the cistern should be adjusted by the screw, L, by screwing it up, until the ivory-point appears to touch its own reflection in the mercury surface. The reading is then taken. In the top part the glass sighting-tube, $1\frac{1}{2}$ " long and 1" to $1\frac{1}{2}$ " internal diameter, the glass being of $\frac{1}{8}$ " to $\frac{1}{4}$ " in. thickness, ground square at its ends, is fitted between the box-wood collars D and G with the intervals of India-rubber washers to make the joints air-tight. To the lower end of the collar, G, another box-wood collar, H, is screwed to which the leather bag, J, is fixed by silk and glue. The bag is made of fine close-grained leather, a stout kid being the best. The pores of the leather must be sufficiently mercury-tight, and at the same time the leather should be soft and flexible to transmit the exterior pressure of the air. Fortin's plan makes the cistern secure from leakage of the mercury in whatever position the barometer may be placed, as there is fixed also the *top leather*, C, which not only makes the joint between the box-wood socket, D, and the glass tube, A, mercury-tight, but permits the air changes to have their full effect on the mercury in the cistern. The mercury is represented by the part shaded black, above the leather bottom, J, the flexibility of which provides a means of adjusting the mercury level in the cistern to the ivory-point, E, which is done by turning the screw, L. The screw, L, is made of a length just sufficient to force the mercury to fill the tube, so that when it is screwed home there is no jar or percussion of the mercury in

carrying the barometer. On the lower end of the bag is glued a leather button which rests on the box-wood socket, K, which again takes the end of the screw, L.

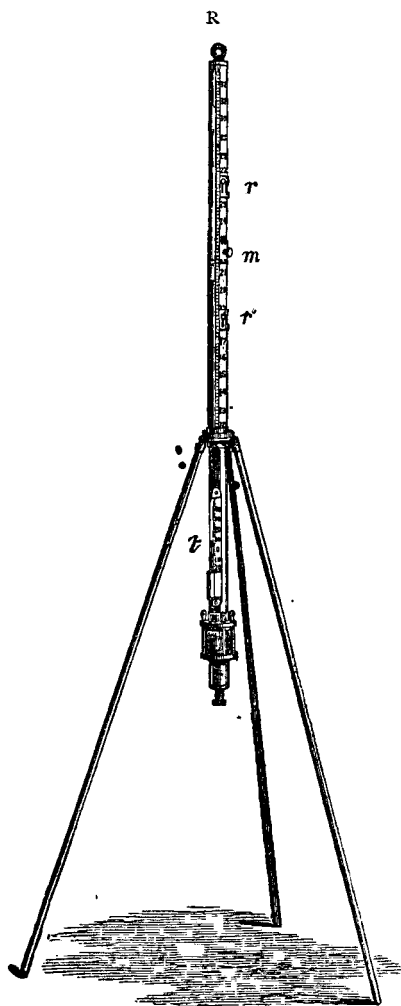


Fig. 764.—Fortin's Mountain Barometer erected for use.

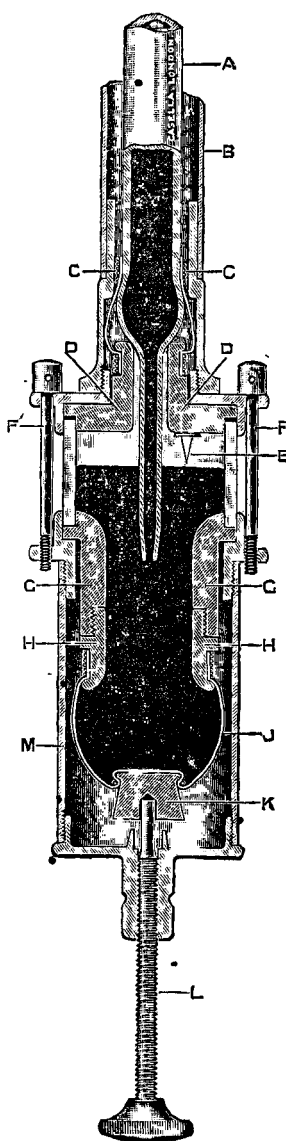


Fig. 765.—Section through Cistern of Fortin's Mountain Barometer

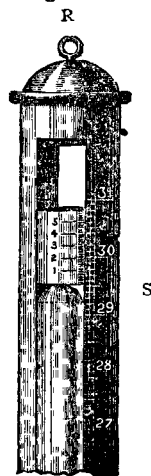


Fig. 766.—Top Part, Vernier Reading showing Gauge point, S.

The Barometer Glass tube, A, is made of mild flint-glass sufficiently stout and thoroughly annealed to resist all wear and tear with fair usage. The top end of the tube is hermetically sealed by the blow-pipe and the lower open end dips into the mercury in the cistern.

Mercury.
—Filling the Barometer Tube.—The

mercury must be distilled before filling the barometer tube which should be perfectly clean. 36" of the tube is then filled with distilled mercury and boiled in a circular charcoal stove with a vertical iron tube 2½" in diameter in the centre. The barometer tube is inserted from the bottom of the stove, to heat about 4" of the top of mercury only. The tube is left in this position till the mercury boils. It is then raised another 4" and again kept till the mercury boils. The process is repeated till the lower end of the tube is reached. By this process the air and impurities rise to the surface of the mercury. The end of the tube is then cut off to its proper length and inserted in the cistern. The lower part of the barometer tube, after it is filled with distilled mercury, is attached to the box-wood socket, D, about 1" in depth, with hot glue, and dips into the mercury. The socket and the tube is then covered with a piece of silk or buckskin, C, which is again covered with glue and varnish. This mode of closing prevents the mercury from escaping when the barometer is inverted, while the pores of the leather transmit the atmospheric pressure.

The Cistern Casing (Fig. 765) consists of socket-pieces made of brass. The 4 brass capstan-head screws, F, F, bring these pieces together. The lower socket-piece has a brass tube, M, screwed upon it, covering the bag and the upper part of the screw, L, which again passes through a nut formed in the under cover of the tube, M, and in the axis of the tube. The upper socket-piece is screwed into the socket of the brass tube, B. The upper and lower socket-pieces connect the box-wood fittings, D, and G, with a glass cylinder held firmly between them so that the ivory-point shall be visible through it.

The Stem or Barometer Casing Tube, B, is made of brass, about ¾" in diameter. At the top part of this case there are two longitudinal slots on opposite sides, of about ¼" in width. The casing tube is graduated along one open edge in inches and tenths, these being again subdivided to twentieths, and marked to read from 13" to 32" of mercury, the zero point from which they are reckoned being the lower extremity of the ivory-point, E, (Fig. 765). If metrical measure be adopted then the same space is divided into centimetres and millimetres. An inner tube about 1' long fits telescopically within the outer tube to move smoothly. This inner tube carries one vernier at top and one at bottom, *r*, *r'*, (Fig. 764). The top vernier (Fig. 766) is placed above a slot in this tube which corresponds with outer tube, so that the level of the mercury can be seen below the top vernier at S. The inner tube has a rack about 11" long which moves by a pinion fixed upon a *cock-piece*, *m* (Fig. 764). A ring, R, is fitted at the top of the barometer to suspend it in a room, to be used, if required as an ordinary meteorological barometer (Fig. 766).

Mounting of the Barometer.—In making observations in the field, a barometer is usually suspended from a tripod stand (Fig. 764) by gimbals, so that it always takes a vertical position by the superior gravity of its cistern and lower parts. The light tubes forming the tripod screw into a collar which is packed in the cap of the leather case.

The Thermometer, *t*, (Fig. 764) is placed with its bulb in contact with the glass tube enclosed in the casing tube to ensure its indicating the same temperature as the contained mercury. It is divided with both centigrade and Fahrenheit scales.

The Packing Case is made of leather lined with felt to fit the barometer. The tripod is placed outside the case. In packing for carriage the screw, *L* (Fig. 765), is screwed tight home, so that the mercury can not move. The instrument should be carried in an inverted position as in this position air can not enter it. A strap is attached to the case for holding it over the shoulder when travelling.

The vernier is a movable scale for sub-dividing parts of a fixed scale (q.v. pp. 445-446).

The new Temperature Scale adopted is the Centigrade scale starting from the Absolute Zero. This is based on the law which states that the volume of a gas under constant pressure increases by $\frac{1}{273}$ rd of its volume at 0° centigrade for every rise of 1° C. The Absolute Temperatures are those on the Centigrade Scale plus 273, thus, 0° C. = 273° A, -20° C. = 25° A, 30° C. = 303° A.

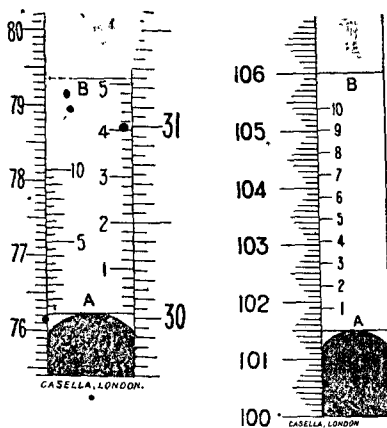


Fig. 767.—Vernier Reading Inches & Millimetres. On the right-hand (inch) scale the height of the mercury is shown as 30.022 inches. On the left-hand millimetre scale the height is 762.2 mm.

Fig. 768.—Vernier Reading to 0.1 Millibar. In this diagram the height of the mercury is shown as 1015.0 mb.

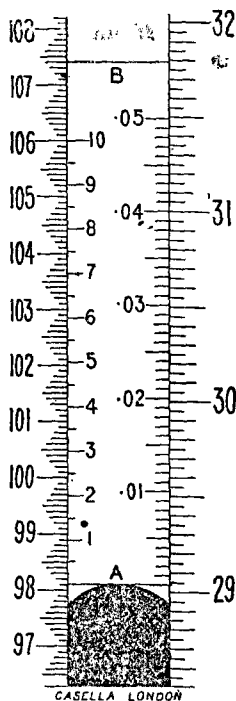


Fig. 769.—Vernier Reading 0.001 Inch & 0.05 Millibar.

New Meteorological Units.—The introduction of an international Meteorological service for the use of ships at sea, the increased interest in the study of the upper atmosphere, the progress of aviation, etc., have lead the authorities to adopt New units of measurement based on the centimetre-gramme-second System. The unit of pressure in the C, G, S

system is one Dyne per sq. centimetre; the C. G. S. unit of Atmospheric pressure is one million dynes (called a **Megadyne**) per Sq. Cm. This may be looked upon as the "C. G. S. Atmosphere", and has been given the name of **bar**, the practical Meteorological units used for recording observations being the **Millibar** $\frac{1}{1000}$ of the bar = 0.029 inches of mercury, and the **Centibar** = 10 millibars. The millibar may be regarded as the thousandth part of the ordinary atmospheric pressure at sea-level.

The Barometers are now often divided in inches and millibars, the attached thermometer having Fahrenheit and Absolute scales, or in millimetres and millibars (thermometer Cent. and Abs.).

Ex. 1. Fig. 667 shows the Vernier of a Double Scale Fortin Barometer reading on one side to $\frac{1}{100}$ inch, and on the other to $\frac{1}{10}$ millimetre. Taking the Right-hand scale.—Here 25 divisions on the vernier are equal to 24 on the fixed scale, which is divided to $\frac{1}{100}$; Hence the distance between 2 divisions on the vernier is shorter than the distance between 2 divisions on the scale by $\frac{1}{25}$ of $\frac{1}{100}$ of an inch, i.e. by $\frac{1}{2500} = 0.0002$ ". Here the height of the mercury is shown as 30.022" (the figures on the inch vernier represent $\frac{1}{100}$ ", but for clearness they are put as 1, 2, etc., instead of .01, .02). On the left-hand millimetre scale the height is 762.2 mm.

Ex. 2. Fig. 766 shows the vernier for a Millibar Scale of a Fortin Barometer reading to 0.1 millibar. Here 39 divisions on the fixed scale are made to correspond to 40 divisions on the vernier giving readings to $\frac{1}{40}$ millibar. The fixed scale is divided in centibars and millibars, the millibar graduations being of unequal length for the sake of clearness in reading. The figures on the Vernier represent tenths of a millibar. The height of the mercury is shown as 1015.0 mb.

Ex. 3. Fig. 769 shows the Verniers of the most accurate types of an observatory standard Barometer. On the right-hand side the vernier is divided to read to 0.001", and on the left to 0.05 millibar. The readings are 29.040 and 981.10 on the inch and millibar sides respectively.

The Mercury is represented by the parts shaded black in Figs. 765, 767-769. In Fig. 767-769 the lower edge of the vernier is set to the top of the column of mercury.

Reading the Barometer.—1. Note the temperature from the attached thermometer. 2. Raise or lower the mercury in the cistern by turning the screw, L, underneath so that its surface just touches the ivory-point, E, and the reflected image of the ivory-point on the mercury will appear as a double cone. Adjust the vernier by the milled-head until its lower edge just touches the top of the mercurial column when the scale and vernier will indicate the height of the barometer. The inches, tenths and half-tenths, are read on the scale, and the thousandths on the vernier. For altitude, readings at upper and lower stations simultaneously should be taken with a barometer at each, and the difference obtained of the barometrical scale.

803d. The Boylean Mariotte Mountain Barometer (Fig. 770), whose action depends on the laws of Mariotte and Boyle, consists of a short centre glass tube, a lower open air tube, or bulb, with

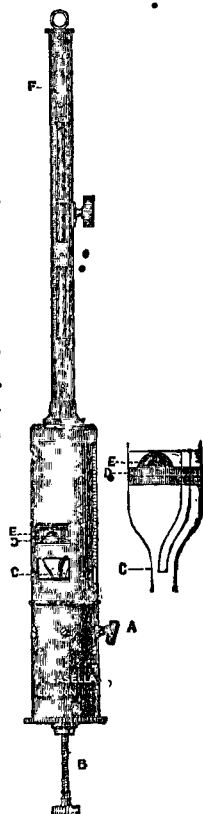


Fig. 770.

diaphragm joined to it, with a vulcanite covering for insulation, and a brass tube, or shield, which covers the glass tube, and on which the graduations and figures are engraved. Attached to this is a cistern filled with mercury, which has a stop-cock and a coarse screw adjustment.

The weight and pressure of the atmosphere is determined accurately, at each observation, by an uniform volume of air being admitted to the cistern, and compressed by the advance of the mercury to a fixed point, or zero, so that no accumulation of error is possible, and the reading will be equally accurate for all times and for all places.

It is not necessary that the same quantity of mercury be always used, so that the loss of a little is of no moment, and it need not be boiled.

The readings are taken from two points, as in Fortin's barometer, and the scale is read to hundredths of an inch. The total length of the Mariotte barometer is from 12 to 15 inches; but, for convenience of carriage the cistern may be detached at pleasure and carried separately.

Instructions for using the Boylean Mariotte Barometer.—

(1) It is very important that the barometer shall be strictly vertical while readings are being taken. If possible it should be suspended from a fixed hook rather than held in the hand. (2) Turn the brass stopcock, A, (projecting from the cistern) into a vertical position. This connects the tube C with the cistern. (3) Carefully screw up the brass screw below the cistern, B, until the mercury, rising in the closed tube in the lower part of the instrument appears through the diaphragm, D, in the form of a drop at E, which drop or convex surface must be raised so as barely to intercept the light and form a tangent to the top opening and which is the zero of the instrument, just cuts off the light passing through the slot at the top of the opening in the lower part of the barometer. (4) *Immediately* read the height of the mercury column in the tube, F, at the top of the instrument by the help of the vernier, one division of which is .01 inch. It is important that this

should be done as soon as possible after screwing up the mercury. (5) Release the stopcock, A, and lower the mercury into the cistern after each observation. In the case of the Mariotte barometer, unlike ordinary mercury barometers, the instrument is not always in action. The tube must be emptied after each observation and only refilled shortly before a reading is required.

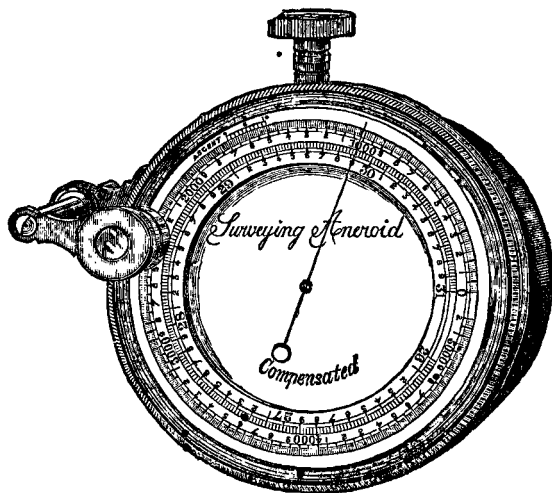


Fig. 771.—Surveying Aneroid.

803e. The Aneroid Barometer consists of a circular metallic box, hermetically

sealed, both sides being covered by corrugated plates, mostly exhausted of air, enough only being left in to compensate the diminished stiffness of the corrugated covers at higher temperatures. The bottom cover being fixed to the base-plate the top cover rises or falls as the outer atmospheric pressure is less or greater, and this slight motion is greatly multiplied and transmitted to an index pointer moving over the scale on the outer face. The scale is graduated arbitrarily by comparison of the motion of the index with a standard mercurial barometer. Inasmuch as all barometric tables are prepared for mercurial barometers, wherein the atmospheric pressure is recorded in inches of mercury, the aneroid is graduated so that its readings are identical with those of the mercurial column. Two forms of aneroids in use are the Vidi or Naudet aneroid (Fig. 771), and the Goldschimid (Fig. 777).

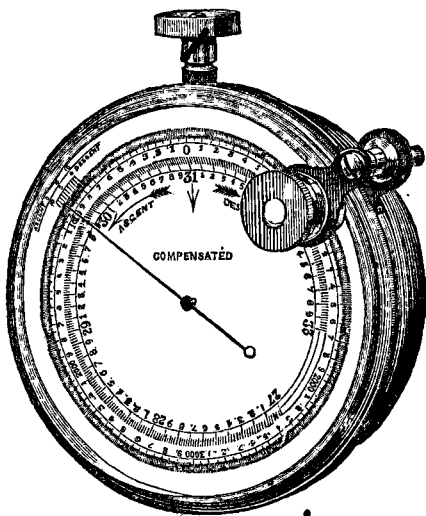


Fig. 772.—Surveying and Mining Aneroid.

The Vidi Surveying and Mining Aneroids (Fig. 771-772) are either 3" or 5" in size with Altitude scales from 5000 to 20,000 feet. The altitude scale is graduated to hundredths and subdivided so as to be read with a vernier to single feet, in addition to the barometric scale of inches. There is a rackwork adjustment attached to the vernier scale and a rotating magnifier for taking minute readings. The elevations cannot be determined with anything like this degree of accuracy by any kind of aneroid. The aneroid simply indicates the pressure at the given time and place, but for the same place the pressure varies for various reasons. Hence all barometric changes cannot be attributed to a change in elevation when the barometer is taken from one place to another.

The Altitude Scale is preferably a fixed one, but in some aneroids it is made to revolve. The object of a revolving scale is to enable the observer to set its zero opposite to the pointer before beginning to take readings, so that no farther calculations are necessary. It is not quite so exact as a fixed scale.

The Surveying and Mining aneroids have been found of great utility in Surveying and Mining Work, from their extreme sensitiveness, for ascertaining slight variations in gradients, levels, etc. **For Mine Surveying** the entire circle of the dial is graduated to represent 6 ins. of the mercurial column, i. e., from 27" to 33" (Fig. 772). This scale affords observations from about 2000' below sea-level to 4000' above.

For Surface Surveying, the instrument (Fig. 771) is made with the

scale divided from 26° to 31° thus giving an altitude of 5000' above sea-level only. The instruments are compensated for temperatures.

803f. Field's Engineering Aneroid (Fig. 773).—In this instrument, besides taking into account the varying ratio between the pressure and altitude in computing the altitude scale, an adjustable scale has been provided for the variable temperature of the air (not the temperature of the instrument) which will affect the result by affecting the density of air, and hence the ratio of pressure to altitude.

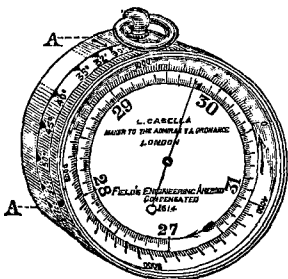


Fig. 773.—Field's Engineering Aneroid.

The adjustment for the temperature of the air is applied by shifting the scale in accordance with the figures engraved on the outside of the instrument marked, A A. The rim which holds the glass should be slightly raised, so as to be free from the locking-pin, and then turned until the figures corresponding to the air temperature are opposite to the pin when the glass should be depressed so as to relock it.

In the process of observation, first determine, either by observation or estimation, the air temperature likely to prevail during the series of observations (if this is done within 5° Fah. it will be correct within 1% nearly). The scale must then be set to this temperature. Then the readings must be taken from the outer scale of feet, and the difference will give the difference of elevation. For example, if the temperature of air be 40°F. for observations between two places A and B, set the scale accordingly:

For Journey from B to A :	Journey from A to B :
Reading at B = 1440 feet	Reading at A = 1018 feet
• „ „ A = 1000 „	„ „ B = 1440 „
Differences = 440 „	422 „
Mean = 431 feet.	

803g. The Watch Shape Pocket Aneroid with Metallic Thermometer (Fig. 774) is 1 $\frac{3}{4}$ " in size, with keyless action and Revolving altitude scale to 10,000 feet.

To use it with revolving altitude scale for ascertaining heights turn the keyless knob, until the 0 (zero) on the altitude scale coincides with the hand, and on ascending a mountain or other altitude the hand will be seen to travel backwards until your destination is reached, when the height can be read off, and, if necessary corrected for temperature. For very accurate readings, especially for high altitudes use an instrument with a fixed altitude scale, i.e., zero being fixed at 31, and by making a note of the reading before commencing, this amount can be deducted from the aggregate result being the height ascended.

803h. The Mechanism of the Vidi Aneroid.—The internal mechanism of the Aneroid (Fig. 775) consists of the *vacuum chamber*, A, made of two discs of corrugated German silver, soldered together, forming a box, exhausted, and to each side of which is

soldered a brass centre—one with a thread on it, to screw the chamber to the $\odot$ brass base-plate, C, $\frac{1}{8}$ " thick; the other plain with a hole drilled across it, to receive a *knife-edge* which suspends the vacuum chamber from a *powerful spring*, B. As the vacuum chamber is compressed with a weight of the atmosphere equal to about 15 lbs. per sq. in. when the air is pumped out it is kept from collapsing, by the spring, B, which rising upwards keeps the upper disc away from the lower one bolted to the base-plate, C. the varying pressure of the atmosphere produces tiny pulsations in the vacuum which are greatly multiplied by levers D and E, and chain F. The strong *iron-carriage*, H, H, fixed across

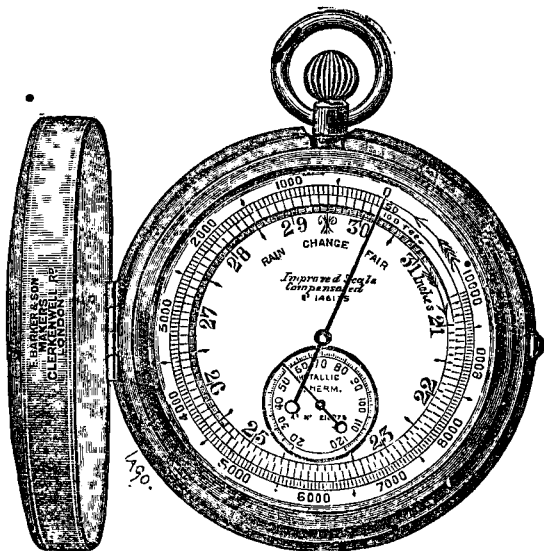
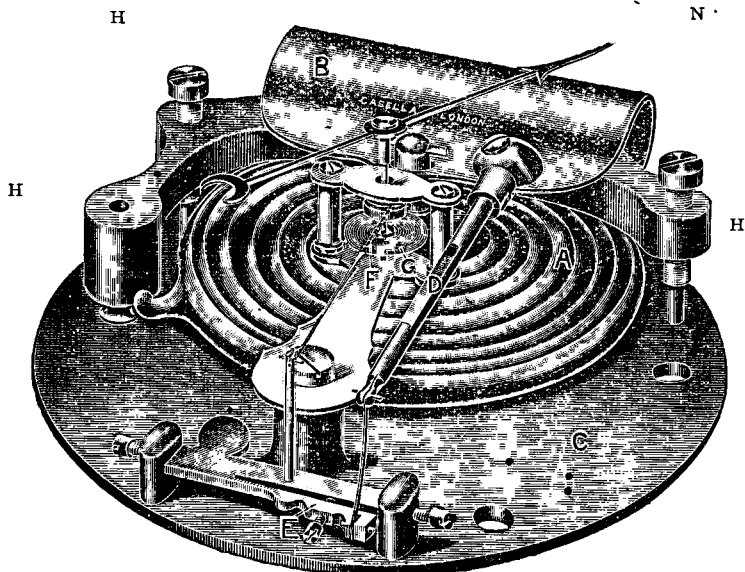


Fig. 774.—Pocket Aneroid (Barker's).

Fig. 775.—Mechanism of the Vidi Aneroid. Scale About $\frac{3}{4}$ ths.

the chamber, supports the *main spring* B, which, acting in direct opposition to the undulations of the *vacuum chamber*, A, gives rise to the variations of the *needle* N, right and left, on the *dial*. To the *main spring* B, is attached the *main lever* D, a bar of steel $\frac{1}{4}$ in. long the long brass lever about 2" long which serves the double purpose of connecting the action of the vacuum chamber and spring to the indicator and of correcting the effect on the instrument of changes of temperature. Brass expands more than $1\frac{1}{2}$ times as much as steel for a given rise in temperature; hence, in the case of a compound bar such as this, the effect of the different expansions of the two metals is to cause the bar to *curve*, and its effective length to be slightly reduced, thus neutralizing the expansion due to the rise of temperature. This, called the compensation for temperature, is applied to all good aneroids. To the end of this arm, D, is attached a small *rod of steel* to connect it to the regulator, E, which is an arrangement for increasing or diminishing the leverage at that point, and is a workshop adjustment for altering the range of the aneroid so as to make an inch on the scale of the aneroid exactly equivalent to an inch of the mercury barometer scale. It is again furnished at its centre with a *vertical arm* of brass from which the fine chain, F, extends to communicate with the movement. To the brass *base-plate*, C, a *short stout brass pillar* is screwed, bearing a flat brass *projecting arm*, F-G, at the end of which are two *smaller brass pillars*, supporting a *thin double convex plate* of brass, and between this and the flat arm works the *arbor*, on which is fixed friction tight the hand. The *fine watch chain*, F, is attached to and wound round the arbor and causes it to rotate, the hand rotating with the arbor on the rise or fall of the lever and a fine *hair*

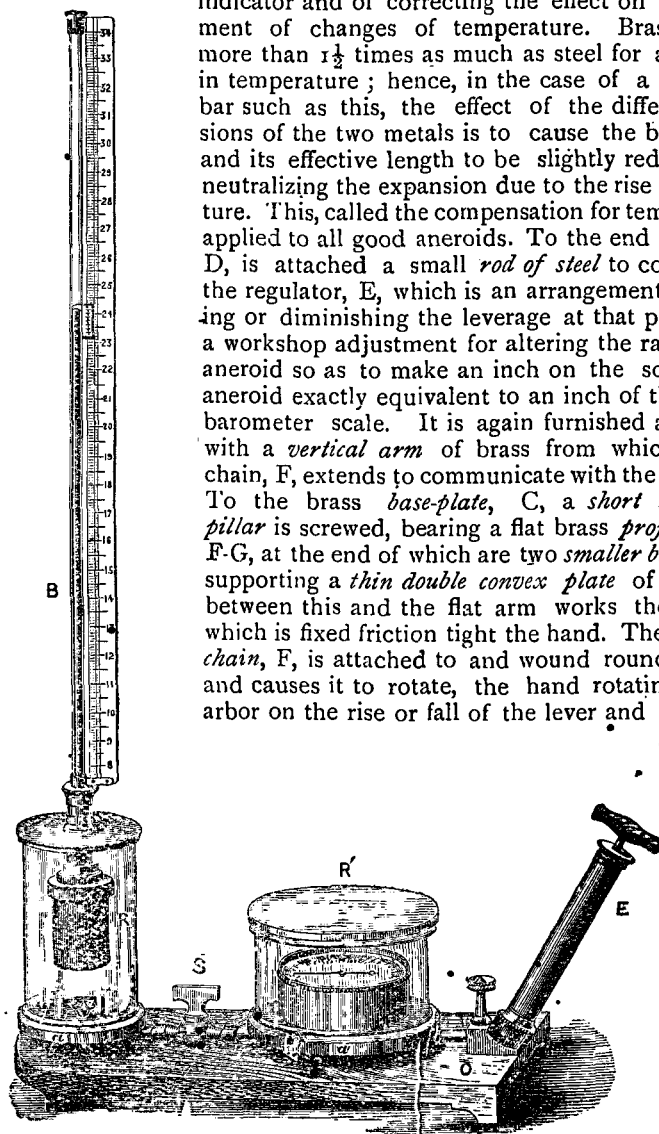


Fig. 776.—Apparatus for testing the Aneroid.

Scale about $\frac{1}{6}$ ths.

spring of coiled steel, G, the end of which is attached to one of the pillars keeps the hand in its proper position. G, the *hair spring*, takes up the watch chain and move the hand to the

left when the pressure is low. A movement of the vacuum box of $\frac{1}{200}$ inch may cause the hand to move through more than 3" on the dial. The connection having been made between these various parts the entire movement is screwed to the iron foundation-plate C, and connected with the main lever, by means of a fine steel rod at the end, and the communication thus rendered complete between the various parts of the instrument, it is ready for the final adjustment of the dial and hand. This done, it only remains to graduate the scale, which is effected by placing the instrument under the glass receiver of an air-pump with a Standard Mercurial Barometer attached. The air is then exhausted, and, as the mercury falls inch by inch, the Aneroid Scale is pointed off and graduated from 31 inches to any required range.

803h. Apparatus for Testing the Aneroid (Fig. 776).—

This consists of a Standard Mercurial Barometer, B, with an engine-divided metal scale, each inch of which is compensated for capacity; the cistern being enclosed in the receiver R, which communicates by the Tube and stop-cock, S, with the larger receiver R', containing the Aneroid, while undergoing the process of testing. It will thus be seen that the two receivers virtually form one chamber, the compartments of which are simultaneously exhausted of air by the powerful exhausting syringe, E. The base-board, O, and the circular plates, *a a'*, are the same as in an ordinary air-pump. In using the apparatus, all joints should be made absolutely air-tight in the usual manner by the application of a little lard: the Aneroid, to be tested, is placed in the receiver, R', and the exhaustion commenced. If the scale of the Aneroid be rightly divided and works in accord with those divisions, the pressure in inches which they indicate should coincide tenth for tenth with the divisions on the mercurial scale as the mercury falls or rises.

803j. In using the Aneroid, see that the pointer has come to a position of equilibrium, and tap it gently before reading. The eye must be brought over the end of the pointer and the reading made, the aneroid being held in a horizontal position. As the aneroid is mechanically compensated for temperature, the correction for temperature of the instrument itself (correction for temperature of mercury) is not required. The correction for temperature of the air must, however, always be made, as also other corrections when any great degree of accuracy is aimed at. As a rule, when several trips are made between two stations, it will be found that the mean of the *up-hill* journeys is more correct than the mean of the *downhill* journeys as the pressure of the atmosphere puts a strain on the spring inside the vacuum box of the aneroid, and in going *up-hill* the pressure of the atmosphere decreases and this strain is relieved. In going *downhill*, reverse is the case, and the instrument is apt to be slower in responding to the changes of atmospheric pressure. The instrument should always be held in the *same position* when taking readings, i. e., either always horizontally or always vertically, as a mere change of position usually affects the readings.

In most cases aneroids, even when they have been in the first instance correctly graduated, do not read accurately against the mercurial barometer at low pressures, and will be found almost always to possess more or less considerable + or - errors. These index-errors are fairly constant in good instruments, but will always be considerably increased when low pressures have been experienced for a length of time. Aneroids should be treated with

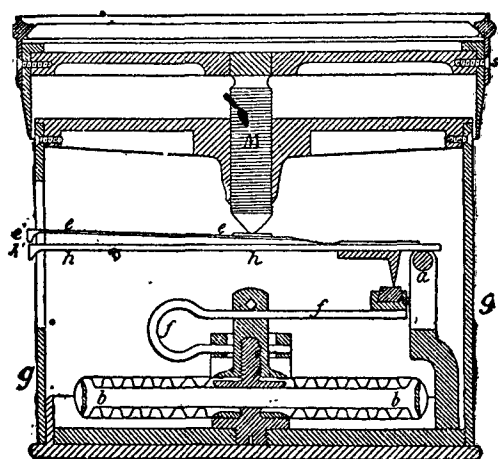


Fig. 1.

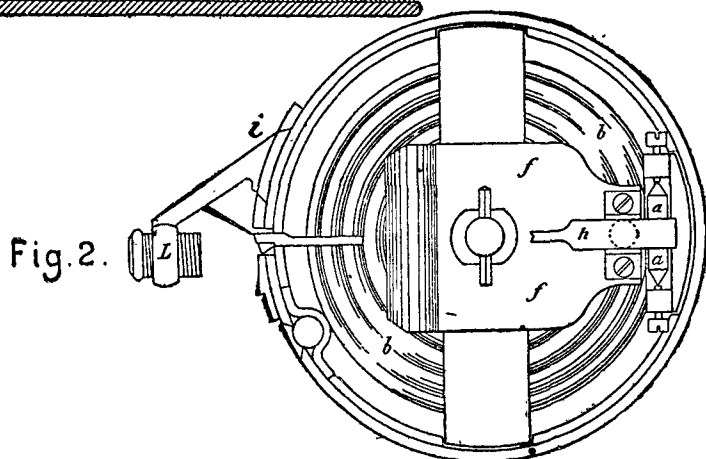


Fig. 2.

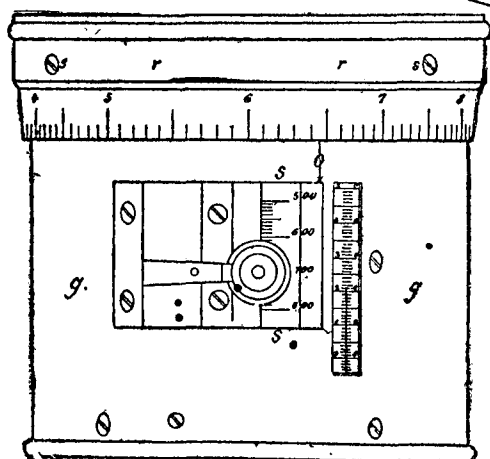


Fig. 3.

almost as much care as chronometers, and should not be allowed to dangle about the person or to be shaken up in pockets. If watch size is used they can be carried in extra watch pockets. It should always be carried and used in a leather case, and protected from sudden changes of temperature, and when observations are made it should have the temperature of the surrounding air. It should not be carried so as to be affected by the heat of the body, and should be read out of doors. The index should be adjusted by means of a screw at its back to agree with a standard mercurial barometer and then this adjustment left untouched.

803k. The Goldschmid-Hottinger Aneroid.—It need hardly be stated that all aneroid barometers make use of the elasticity of a hermetically sealed box, or other closed compartment in which a partial vacuum has been established, to measure changes in atmospheric pressure. The movements of the box, always small, are magnified by means of a somewhat complex system of delicate levers in the ordinary aneroid such as that of Naudet's make. The extreme delicacy of the intermediate transmission has always proved the objectionable feature and the main source of inaccuracy, and it was with the object of totally suppressing this transmission that Goldschmid successively designed the series of instruments which bear his name.

Hottinger's Goldschmid Aneroid No. 1, numbered 1, 2, 3, in (Fig. 777).—The vacuum-box, *b b* is kept in tension by the steel spring, *f f*, terminating under the knife-edge, this transfers the movement of the box-center to the lever, *h h*, turning on the fulcrum *a*. To the top of this lever, *h h*, is attached a hair-spring, *e e*. The movements of the box are measured by means of a micrometer screw, *M*, whose contact with the hair-spring, *e e*, must be ascertained with greater accuracy than is possible by the mere sense of touch, and is accordingly indicated by the coincidence of two black lines on the heads of the lever, *h h*, and the hair-spring, *e e*, observed by means of a magnifying glass, *Z*. The scale, *S S*, (Fig. 3), gives the full revolution of the micrometer screw. The first instruments of this class were provided with a scale on which the readings increased with the altitude, the value of the scale-unit being arbitrary. The scale of the more recent instruments is divided to correspond with the mercurial barometer. Each revolution of the micrometer screw corresponds to 10 mm. of the mercury column, and the head of the micrometer screw is decimally graduated, into 100 equal parts, admitting of accurate reading to the one-hundredth part of a millimeter.

The usual size of this instrument is $3\frac{1}{8}$ inches in diameter and $2\frac{1}{2}$ inches high. Fig. 1 is a vertical section; Fig. 2, a top view, the cover being removed; Fig. 3, a side view; *g g* is the outer shell; *r r* the revolving and graduated head; *N*, Fig. 1, the end of the main lever, works in a slit in the side of the case, and *e'* that of the hair-spring lever; each provided with an index-line; *S S*, Fig. 3, the scale by which the number of full revolutions of the micrometer screw, *M*, Fig. 1, are measured. At the side of the slit, in which the outer end of the lever, *N*, works, is a vertical scale (Fig. 3) graduated downwards to agree with the even inches of the mercurial barometer, and is read by a horizontal line on the lever, *h h*. To read the $\frac{1}{100}$ ths and $\frac{1}{1000}$ ths the spring, *e e*, is used in connection with the micrometer screw, *M*. *M* is moved by turning the top of the outside case, its graduations being on the side of this top (scale *SS*), and

the zero on the side of the box just below. With the micrometer reading zero, the spring passes upwards against the point of the screw. As h' rises, its horizontal line reads less on the scale. A complete revolution of the micrometer corresponds to one division of the vertical scale. To read the instrument the micrometer is turned down until the horizontal line on e' coincides with that on h' . When the line on e' coincides with the division just above h' , the micrometer reads zero. Hence its reading, when the two lines are made to coincide, will give the $\frac{1}{10}$ ths and $\frac{1}{100}$ ths between h' and the scale division next above, the combined scale and micrometer readings giving the total reading of the instrument.

New aneroids give elevations at the ordinary atmospheric pressure (25 to 30 inches) correctly within one-half of one per cent, the difference of elevation not being more than 750 to 1,000 feet. For the determination of greater differences of elevation old aneroids should be used, in which the box has attained its equilibrium. Indeed, it may be said that these instruments improve with age.

The influence of temperature upon 44 of these aneroids, tested for a period of six months, at temperatures of 10° to 30° C. (50° to 86° Fahr.), was only 0.4 mm. per 1° C.

Each instrument is provided with a table of reduction, giving the value of its unit at different atmospheric pressures in millimetres, or inches of the mercurial barometer, and its value in inches or feet, and the corrections for temperature. Each is rated individually and tested daily for four months at the factory before being sent out to the users.

804. Boiling-Point Thermometer or Hypsometer.—Apparatus for Determining Elevations by the Temperature of the Boiling-point of Water.

The Barometrical Thermometer is constructed to meet the requirements of travellers in circumstances where the mercurial barometer cannot be conveniently employed. The instrument is portable, and affords an accurate means of measuring heights by observations of the temperature of boiling water, as the temperature at which water boils differs with the pressure of the atmosphere and decreases as we ascend. The apparatus (Fig. 778)

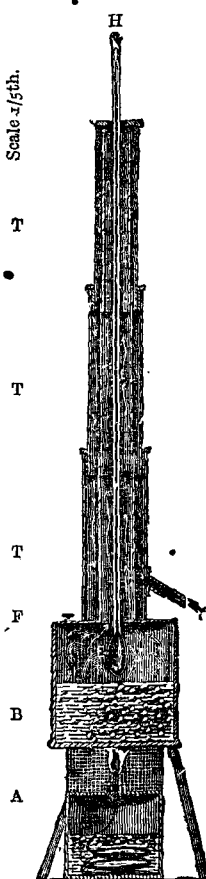


Fig. 778.—Boiling Point Thermometer. consists of a delicate thermometer, H, almost 12" long, the scale ranging from 180° to 215° having each degree subdivided to show 0.1 ; (2) A metal boiler, B, mounted on a small tripod stand; from the boiler, B, pro-

ceeds 3 to 5 double tubes, T, T, open at top and screwed on the top of the boiler; the outer tube has two openings, one at the top, through which the thermometer is inserted, passing down to within an inch of the water, which it should never touch, in the boiler, supported by means of an India-rubber washer at the top; the second opening forming an outlet for the steam at G. These double tubes are now constructed to separate at the joints by a simple slide fitting, so that any length of the Thermometer Stem can be made visible varying with the elevation at which the tubes are adjusted. The object of the double tube is to ensure a steady boiling-point, which it would be impossible to obtain in open-air experiments, were only a single tube employed. A, is a metallic spirit-lamp, surrounded with wire gauze to prevent the flame being extinguished when experimenting in the open air. The whole instrument is packed in a leather case for travelling. **To pack this instrument** for travelling, withdraw the thermometer, and put it in a brass tube lined with India-rubber, having a pad of cotton-wool at each end; take off the tube T, shut it up, and put the small end into the boiler, which it fits, then withdraw the spirit-lamp, screw the cover over the wick and replace it. The whole of this apparatus fits into a circular tin-case, 6" long and 2" in diameter. Each instrument is furnished with a carefully computed set of tables, from which may be obtained, by an easy calculation, the elevation corresponding to any observed boiling-point between the temperatures of 180° and 212° .

To use the Boiling-Point Apparatus, pour into the boiler, through the small opening, F, on its surface, a sufficient quantity of water to fill it about one-third, afterwards close it by means of the screw; the lighted spirit-lamp to hold a proper quantity of spirits is then applied. When the water is made to boil the steam rises, surrounding the bulb and tube, and descending between the two tubes issues from the opening at G. After a few seconds, the mercury in the thermometer will rise and become stationary, the degree indicated by it must then be noted, when, by reference to the tables, the elevation of the spot where the experiment has been performed may be obtained. The temperature of the air in the *Shade* should be observed by a reliable thermometer at the same time. A screen of some sort is required on the windward side of the thermometer, and at low temperatures is indispensable. The heat of the lamp will otherwise be carried off too rapidly for the water to boil properly.

To determine a height with accuracy, pure water should be used, distilled water if possible, and a similar observation should be made at the same time at a lower station, not very remote laterally from the upper, and both should be many times repeated. When such observations have been very carefully conducted the height of the upper station above the lower may be ascertained with great precision, as has been repeatedly verified by subsequent trigonometrical measurement of elevations determined. If the lower station be at the sea-level absolute height of the upper is at once obtained.

CHAPTER XIV.—PRACTICE OF SPIRIT-LEVELLING.

805. The Science of Levelling comprises two distinct operations. First, the determination of the difference in elevation between any two points near together ; and 2nd, the elevations, with respect to some datum, of any number of points, a point's elevation being plus or minus as the point is above or below the given datum. The datum is always some surface parallel with the surface which the earth would assume were it entirely liquid, *i. e.*, some level surface (Art. 765). It is customary to speak of it as a plane. In the case of small areas this is nearly true.

The 2nd operation depends on the first. Having found the difference in elevation between two points, we can then determine the difference in elevation between the 2nd point and a 3rd, and so on ; from the data thus obtained referring all the elevations to that of one of the points. The whole process of levelling, then, depends on the determination of the difference in elevation between two points. Such a determination may be effected by means of (1) Spirit-Levelling, (2) Physical (Barometric and Thermometric) Levelling, (3) Trigonometrical Levelling.

Different Classes of Spirit-Levelling are : (1) **Differential Levelling** to find the difference of level between two points.

(2) **Compound, Profile or Longitudinal, Levelling** in which elevation of several points are found with the object of finding the exact slope or section of the ground lengthwise, from which the grades for a Road or Railway, or the fall of a Canal may be established.

(3) **Cross-section Levelling.**—In which the object is to supply an additional series of sections at right-angles to the longitudinal levelling for finding the general cross-slope of the ground and thereby estimating quantities of earthwork of *cutting* and *filling* accurately. Scotch staves 16' or 19' will expedite work of this kind.

(4) **Check levelling** is running a 2nd set of levels to check the first set between bench-marks left when taking the first set.

(5) **Flying levels** for approximate elevations (q. v., p. 684).

(6) **Reciprocal levelling** to obtain the difference of elevation between two points which are separated by a natural obstacle like a river where it is impossible to have an intermediate set up of the level (q. v., p. 689).

(7) **Precise levels** for finding the difference of elevations of two points a considerable distance apart for the establishment of Bench-marks over a large tract of country, under the ægis of a Government, *e. g.*, the G. T. S. Levels in India, Ordnance Levels in the United Kingdom, U. S. A. Coast and Geodetic Levelling, etc.

805a. Differential Levelling with the Spirit-Level.—The simplest case of levelling is to find the difference of level

between any two points, not very far apart, which can be seen without shifting the instrument, *e. g.*, to find the difference of level between two points *A* and *B* (Fig. 779) by **simple levelling**, set up the level at *C*, any where between *A* and *B*, either on the line *AB*, or on either side of it, and from where both *A* and *B* are visible.



Fig. 779.—Differential Levelling.

Now perform the temporary adjustments and direct the staff-man to hold the staff, vertical at *A*. The telescope is then turned on to the staff at *A*, focussed, reading taken, and recorded in the level-book, care being taken to see that the bubble retains the centre in the tube while the reading is being taken. The staff-man is then signalled to hold the staff at *B* vertical, and the telescope is turned round on its axis and directed on to the staff there, and the reading recorded. Suppose the reading *Bb* at *B* is 5.26 ft., and the reading *Aa* at *A* is 2.72 ft. Then the collimation line, *ab*, being horizontal, and the horizontal line *AA*, being drawn through the point *A*, it is clear that the difference of level *BA*, is equal to the staff-reading *Bb* less the staff-reading *Aa*, or $5.26 - 2.72 = 2.54$ ft., *i. e.* the point *A* is 2.54 ft. higher than the point *B*. When levelling in this way between any two points the sight *CA*, *i. e.*, the reading at *A* is called the *back-sight*, and the sight *CB*, *i. e.*, the reading at *B* is called the *fore-sight*. When the instrument is once set up and levelled, its range of action in that one position is limited; *e. g.*, when the instrument is levelled at *C*, it can not read a staff placed on any point of ground above the line *ab*. If it were therefore required to ascertain the difference of level between the points *A* and *E*, it could not be done with the instrument placed at *C*. The instrument would either have to be shifted between the points *B* and *E* after taking the first readings at *a* and *b*, or placed upon some higher ground, which would enable readings to be taken directly upon the staff placed first at one point and then at the other.

805b. Practical Hints on Setting up the Level.—Excepting when it is impossible, set up the level midway between the back-sight and fore-sight points, as then, if the instrument is out of adjustment, the errors of the two readings being exactly alike eliminate each other, as also errors due to curvature and refraction, *e. g.*, if the level is out of adjustment, and the reading on the back-sight is 0.23 too small, the reading on the fore-sight will also be 0.23 too small, if the level be half-way. The difference of the readings will however be correct, as the errors in both are the same in amount, the correct difference of level is therefore obtained. Let the level *C* be set up at *c* so that $Ca = Cb$, *c* may not necessarily be on the line *AB*, but may be on either side of it, and let *ab* be a horizontal

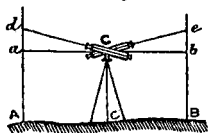


Fig. 780—Setting up Level Midway between two Points.

line established by the level when in perfect adjustment (Fig. 780). But if the level be out of adjustment, and then if all the sources of error combine in giving the axis of the telescope the direction Cd when the telescope is directed towards the staff at A , making an angle, aCd , with the horizontal line, the cross-wires of the diaphragm will not coincide with the point a , but with the point d , and on turning the telescope towards the staff at B , the cross-wire will not coincide with the point b , but with e . As the sources of error are the same in both positions of the telescope, the angles aCd and bCe must be equal. $\therefore Ca = Cb, ad = be$. The difference of level equals $(Aa + ad) - (Bb + be)$. It is here assumed that the refraction is the same both for back and fore-sights, the condition of the atmosphere being same at the time observations were made. **The limits under which a difference of level may be taken by one pair of sights are :** (a) that the powers of the telescope should be such that the greatest distance sighted will give clear reading of the graduations of the staff, (b) that the sum of the distances at the ends of which sights are taken should be such that the line of sight may not pass below the zero of one staff or above the top of the other, *i.e.*, the maximum difference of level, either rise or fall, that can be observed at one setting-up of the instrument should not exceed the total length of the staff, (c) that the distance to be sighted should not exceed 660 ft., and hence the two points A and B should not be more than 1320 feet apart. This limit is so fixed that in levelling operations the effect of curvature and refraction may be neglected.

805c. To set out a Level with the Farm Level (Fig. 625, p. 559).—Having set up the instrument (see p. 559) instruct the staff-holder whereabouts to take up his position. Direct the telescope towards the staff, and when same is in the centre of the field of view, clamp screw, G , (Fig. 625) to lock the level securely in position. Bring long bubble to the centre of its run by means of screw, E , and then signal the staff-holder to raise or lower the sliding-target on the staff (Fig. 690, p. 632) until the pointer of the telescope intersects the centre of the target which will now be on the same level as the telescope itself. Similarly if this same level from the same standpoint be required in another direction, send the staff-holder to the new position, unclamp screw, G , of level and direct telescope in the direction of staff (readjusting eye-piece to give clear definition, if necessary,) and then bring long bubble to centre again by screw E . Signal the staff-holder as before to raise or lower target until pointer of telescope intercepts centre of target which will then be on the same level as telescope and the target at first position of staff: Take care not to interfere with the legs of the tripod. If by accident the tripod is knocked out of position the levelling must begin again from the beginning.

805d. To find the Difference in Level between two places with the Farm Level.—If the difference in level between 2 places A and B is required (Fig. 781) first set up level between the two points, preferably midway, and erect the staff at A , then bring the target into the centre of the field of view and record its position on the staff. Next send the staff-holder to the second point, B , and instruct him to slide target up or down as necessary, until the leveller signalled that the centre of the target coincided with the cross-lines or pointers in the telescope. Then the target is clamped and a note made of its position by the staff-holder. The difference between the two readings would represent the difference in level between the two places. Say the first reading had been 3', and the second reading 4', we should know that the second place was 1' lower than the first, and if the distance between the two points be 400', there is a difference in level of 1' in 400', or the grade is 1

in 400. Should the distance between the 2 points be too great to allow the difference in level being found at one setting up of the instrument, then of course, the operation is to be extended 2 or 3 times, adding together the differences in level as shown by the staff-readings.

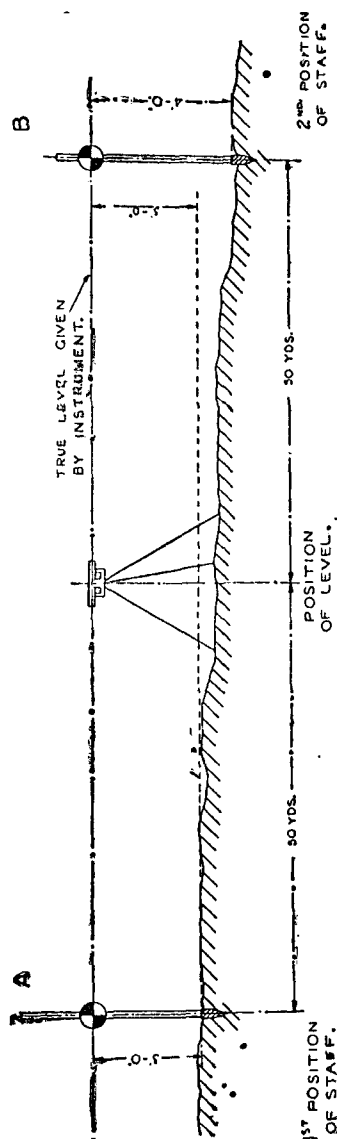


Fig. 781.—Levelling with the Cooke's Farm Level.

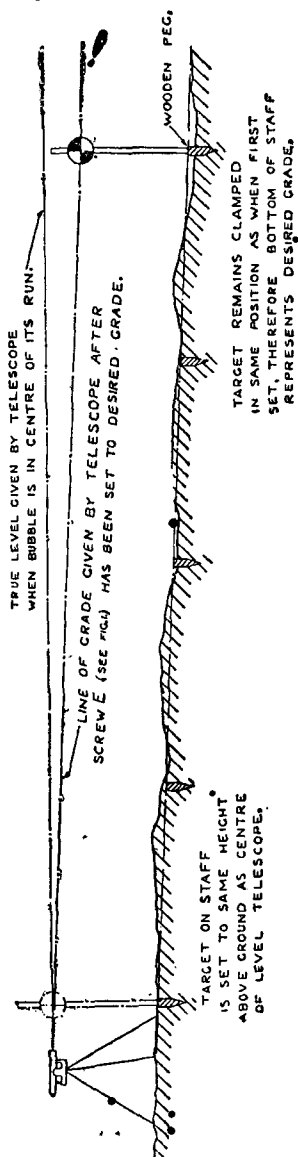


Fig. 782.—Grading with the Cooke's Farm Level.

805e. To Lay down a Grade for an irrigation trench or bed, (Fig. 782) set up the level (see p. 559) and then hold the staff along side the

level and clamp target at the same height above the ground as the centre of the telescope. Now if the desired grade or tilt be given to the telescope by rotating the screw *B* (see p. 580), and the staff be erected at any desired point and the target slid up or down such a height as to bring the centre of target on to the cross-lines in the telescope then the bottom of the staff (*e. g.*, an wooden peg) will indicate the desired grade.

806. Correction for Curvature.—The two errors incidental to levelling are those arising from the curvature of the surface of the earth, and from the peculiar nature of the atmosphere by which it is surrounded. The error due to the curvature arises from the fact that horizontal lines are not level lines. A strictly level line is one in which all the points are equidistant from the centre of the earth, and a horizontal line can not be a level line, since the point at which it is a tangent to the earth's surface must be nearer to the centre of the earth than any other point in its direction. In Fig. 783, let *BD* be any portion of the surface of the earth, *ad* a horizontal line representing the line of sight of the telescope of the level and parallel to the line tangential to the surface of the earth at *D*; and *C* is the centre of the earth. The points *D* and *B* and consequently *d* and *b* are clearly on the same level, whereas *b* and *a* are not.

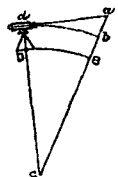


Fig. 783.—
Correction for
Curvature.

But *a* appears to be the level of the point *b* as seen by the observer at *d*. Thus *db* is the true horizontal line, *da* is the apparent horizontal line, and *ab* the depression of the true below the apparent horizontal line. Then the correction for curvature is $-ab$. This distance is hence the measure of the error due to the sphericity of the earth's surface, commonly called the *error due to curvature* or the *correction for curvature*. The effect of curvature on the level of the point observed is that it makes the point appear lower than it really is, *e. g.*, *b* which is really on the same level with *d* appear to be depressed below it.

Now to find *ab*, we have:—*ad* the distance or length of sight practically equal to the arc *bd* and chord *bd*; *Cb* and *Cd* are radii of the earth plus the height of level. Therefore $aC^2 = ad^2 + Cd^2$ (Euc. I. 47); but $aC = ab + bC$. $\therefore (ab + bC)^2 = ad^2 + Cd^2$. But $bC = dC$; $\therefore (ab + dC)^2 = ad^2 + dC^2$ or $ab^2 + 2ab \times dC + dC^2 = ad^2 + dC^2$, and equating, the formula becomes $ab^2 + 2ab \times dC = ad^2$. But *ab* must always be relatively very small compared with the radius of the earth plus the height of the instrument *bC*. The square of *ab* will be still smaller and it may be neglected in calculation, and the equation becomes: $ab = ad^2 / 2dC$; or putting $ad = d$ = distance in miles, and $dC = r$, the correction $= d^2 / 2r$, $2dC$ being practically the mean diameter of the earth = 7913 miles = 41,781,184 ft., and is therefore a constant quantity.

$$\text{If } ad = 1 \text{ mile or } 5280', \quad ab = \frac{(5280)^2}{41,781,184} = .6673' = 8.004'' = \frac{2}{3} \text{ ft. say.}$$

$$\text{If } ad = 2 \text{ miles.} \quad \dots \quad ab = \frac{2^2 \times (5280)^2}{41,781,184} = 2^2 \times \frac{2}{3} \text{ ft.}$$

$$\text{If } ad = n \text{ miles} \quad \dots \quad ab = \frac{n^2 \times (5280)^2}{41,781,184} = n^2 \times \frac{2}{3} \text{ ft.}$$

A rule for the difference of level is that "*correction for the curvature in feet is equal to $\frac{2}{3}$ rds of the square of the distance in miles.*"

807. Correction for Refraction.—The second cause of error is known as that of refraction, and is due to the varying density of the atmosphere surrounding the earth. Rays of light coming through the atmosphere are *refracted* or curved downwards. The effect of refraction is to raise the object above its true level than it really is; and its correction will therefore be of an opposite character to that for curvature. This is explained by Fig 784 where ac representing the correction for curvature equals ab in Fig. 783; ab (Fig. 784) equals that for refraction. Calling the former negative and latter positive, the total correction $= ab - ac = -bc$. Owing to the uncertain conditions attending the atmosphere, the error due to



Fig. 784.—Correction for Refraction.

refraction is not capable of being so accurately determined as that due to the sphericity of the earth, and can not be expressed by any formula. It may be fairly considered to have an average value of about 0.1348 of the former error. **The correction for curvature** in feet has been found to be $.6673d^2$, where d = distance in miles. That **for refraction** is equal to $.6673 \times .1348d^2 = .0899d^2$. But since the former is negative and the latter positive, or, for the sake of simplicity and to avoid a negative result, these signs may be reversed. The total corrections represented by bc (Fig. 784) will be equal to the difference of these two fractions. Putting C to represent the total correction for both curvature and refraction, $C = (.6673 - .0899)d^2 = .5774d^2$. From this the rule is—"To ascertain the *total correction*, *square the distance in miles, multiply the result by .5774 and the product will be the total correction in feet and decimals of a foot.*"

It will be troublesome if calculations for curvature and refraction are to be allowed for all points on which staff-readings are taken, so in ordinary levelling these corrections are very seldom applied. The distance at which graduations of a staff are visible is very limited. It is with ordinary levels about 300', and with levels of the best type, like those used by the tidal levelling party of the G. T. Survey of India, about 500'. The error due to a distance of 500' is $.00513$, which may be neglected in all but the very most exact and scientific observations, as it is very small, and not readable on the ordinary levelling-staff reading to $\frac{1}{100}$ th of a foot. At distances up to 660 feet the combined error being $.009 = (\frac{1}{8} \times \frac{1}{8} \times .5774)$ feet may be neglected. This extreme case occurs only in the very unusual condition when one staff is to be read close to the instrument and the other at 660' off. The errors, due to the instrument not being in perfect order, are much greater than those caused by curvature and refraction, but these also are neutralized when back and fore-sights are taken on staves at equal distances from the level, for even if the telescope sights upwards or downwards instead of horizontally it makes the same error at both readings which cancel and give a *correct difference of level*. The uncertainty of the correction

for refraction makes it advisable to avoid, in exact levelling, all sights at distances exceeding about an eighth of a mile.

Effect of Curvature.—The curvature of the earth causes an object on its surface to appear lower than it really is with reference to the point of observation. The correction for curvature is therefore *additive*.

Effect of Refraction.—As refraction causes all objects to appear higher than they really are, the correction for it is *subtractive*.

The following are examples on curvature and refraction:—

Ex. 1. If the distance be 5 miles, then

$$\text{Correction for curvature} = \frac{1}{2} \times 5^2 = +16.66 \text{ feet.}$$

$$\text{„ refraction} = 0.1348 \times 16.66 = -2.24 \text{ „}$$

$$\text{Total correction} = +14.42 \text{ „}$$

Ex. 2. A light is 150' above mean sea-level. At what distance out at sea will it be visible from the deck of a ship 15 feet above mean sea-level?

The line joining the deck of the ship and the light will be a tangent to the earth's surface at a certain point. We shall have to find the distance of the tangent point from the light and the deck of the ship in order to get the required distance.

$$0.5774 \times d^2 = \text{Combined correction for curvature and refraction.}$$

$\therefore 0.5774 d^2 = 150$; $\therefore d^2 = \frac{150}{0.5774} = 259.7851$. $\therefore d = 16.12$ Miles = distance of light from the tangent point. Again $0.5774 d^2 = 15$ ft., $\therefore d = 5.09$ miles = distance of deck of ship from the tangent point. $\therefore$ Total the distance between light and deck of ship = $16.12 + 5.09 = 21.21$ miles nearly.

Ex. 3. It is required to construct a drain between *A* and *B* (Fig. 785), a distance of 1320', let a level be set up at *C*, and directed on to a staff held vertical at *B*. The horizontal line, *CD*, represents the line of sight which gives the staff-reading at *B* to be 17.88 with an 18 ft. staff; the height of the level above the ground was $4\frac{1}{4}'$, and the depth of the point *A* below ground at *C* where the level was set up 10.55; therefore the difference of level between *A* and *B* is obtained as under:—

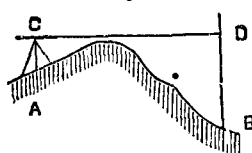


Fig. 785.

Reading of staff	...	...	17.88 ft.
Height of Instrument	...	4.25	
Depth <i>CA</i>	...	10.55	
Curvature and refraction for 20 chains	...	0.03	14.83 „
Difference of level = 3.05 „			

Ex. 4. A level is set up midway between *A* and *B*, from which the staff at *A* reads 7.27 feet and the staff at *B* 11.72 feet, how much is *B* higher or lower than *A*? ... Ans.: 4.45 feet lower.

808. Time available for Work.—Refraction is greatest early in the morning, diminishes till 10 A. M., is nearly constant till 4 P. M., and then begins to increase. Great variation in refraction sometimes occurs over ground which is passing from light to shade, or *vice versa*. For exact work in clear weather not more than .3 or 4 hrs. a day are available.

809. Continued Series of Levels.—**Compound Levelling.**—In all long lines *Compound levelling* has to be resorted to. It consists of a continued series of levels taken between every two consecutive

points on the whole line. For example, the difference of level between two points, *A* and *B*, (Fig. 786) is required, and that owing

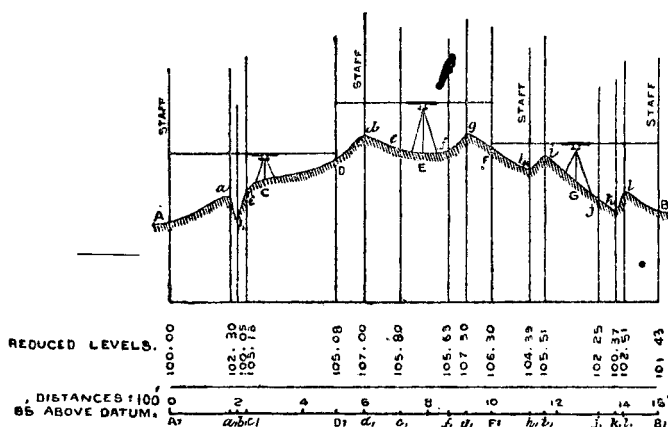


Fig. 786.—Example of Continued Series of Levels.

either to the inordinate inclination of the ground, or to the great distance between *A* and *B*, it is not feasible to observe the staff at these points from any intermediate point, the method of procedure then is as follows:—Set up the level at *C* between *A* and *D*, level it up, and direct the telescope on to the staff at *A*. Carefully observe the reading where the first collimation line cuts the staff, and of which the height above the datum line is known. Suppose the reading on the staff as shown by the horizontal cross-wire at *A* is 5'45". Enter this in the level-book, and then direct the staff-man to hold the staff at *D* which point should be as high up as it is possible to sight. The point *C* should be about *half-way* between *A* and the highest visible point, *D*. In holding the staff at *D*, a well-defined hard point should be chosen, or in defect an iron Δ (Fig. 708, p. 640) should be used by the staff-man to hold his staff on, otherwise the foot of the staff may be moved while the reading is being taken. Now turn the telescope to *D* and record the reading, which is say, 0'37". Having done this turn the staff simply half-round so as to face the point, *E*, midway between *D* and *F*, and carry the level to that point, and set up there. Now direct the telescope on to *D*. The reading on the staff still held vertical on the same point *D* is now observed where the 2nd collimation line cuts it, say it is 4'67". The staff is then taken to *F*, and the reading is observed, say 3'45". Here again a firm point should be chosen by the staff-man at *F*. The level is now taken to *G*, and set up there, and the staff still at *F* is turned half round and read where the 3rd collimation line cuts it, say the reading is 0'21". Lastly direct the staff-man to hold his staff at *B*, and turn the telescope to *B*, and take the reading, say 5'08", which finishes the work. Now the booked reading at *A* being 5'45", and that at *D*, 0'37", it follows, as before explained for Fig. 779, that the point *D* is $5'45'' - 0'37'' = 5'08''$ higher than *A*. Similarly the reading at *D* being

4.67 and that at F 3.45, the point F is $4.67 - 3.45 = 1.22'$ higher than D ; and again the readings at F and B being 0.21 and 5.08, the point B is therefore $5.08 - 0.21 = 4.87'$ lower than F . Thus there is a rise of 5.08' from A to D , and a rise of 1.22 from D to F , i.e., a total rise of $5.08 + 1.22 = 6.30$ ft. from A to F . Deducting from this the fall from F to B which is 4.87', we find that the point B is $6.30 - 4.87 = 1.43$ ft. higher than A . It shows a rise when the sum of the back-sights is the greater, and a fall when that of the fore-sights is the greater. In a long line of levels between any two points the sights, CA , ED , and GF , i.e., the readings on the staff at A , D and F , are called *Back-sights*, and the sights CD , EF and GB , i.e., the readings on the staff at D , F and B are called *Fore-sights*. The points D and F on which the staff is held and turned half-round while the level is shifted forward and to which both a fore-sight and a back-sight are taken, are called "**Change or Turning Points.**" This is the method of conducting a **Flying Level**, i.e., a levelling operation merely to determine how much one point is above or below another.

809a. Flying Levels are preparatory levelling operations for ascertaining the elevations of detached points of primary importance, and are carried on to give an approximate idea of the nature of the ground, of the practicability and cost of work for Engineering Projects, and to enable the Engineer to fix upon the line along which he would take detailed levels to prepare the project. The operation is the same as that before described, only that it is much more rapid, and less detailed and correct. The heights of detached points are most conveniently recorded on the plans. Detached levels may be taken to determine the elevations of important points on existing works, such as bridges, roads, railways, canals, etc., or of objects suitable for Bench-marks. The stadia-hairs will be found as a means of measuring distance quickly when *flying levels* are taken.

809b. Section Levelling.—When a profile or section of the ground is to be drawn the distances between the several staff-stations must be carefully measured as well as the bearings observed of each staff. The level should be set up as nearly as possible half-way between the two stations and the levelling operation is then carried on in the way described above. The readings are booked as follows:—

Level-Book No. 1.—For Section Levelling.

1	2	3	4	5	6	7	8
Level Station.	Back-sight.	Fore-sight.	Rise.	Fall.	Height above Datum, 100.	Distance, feet.	REMARKS.
C	5.45	0.87	5.08		105.08	500	A to D
E	4.67	3.45	1.22		106.30	500	D to F
G	0.21	5.08		4.87	101.43	500	F to B
	10.33	8.90	6.30	4.87	—100.00		
		1.43		1.43	1.43		

809c. Intermediate Sights.—When the ground is undulating, and there are elevations and depressions between any two back and

and eminences at *a, c, d, g, i, j, l*, and their levels are also required. Before taking the fore-sight at *D* (Fig. 786) from *C*, the level is to be directed on to the staff at *a, b, c*, successively. Similarly from *E* and *G* the sights are to be taken to the staff held at *d, e, f, g*, and *h, i, j, k, l*, respectively. The readings at *a, b, c, d, e, f, g, h, i, j, k, l*, are say, 3'15, 5'40, 2'32, 2'75, 3'95, 4'12, 2'45, 2'12, 1'00, 4'26, 6'14, 4'00 respectively. Remember that the *first sight* after planting the level is always a *back-sight*, and the last before removing the instrument is always a *fore-sight*, and all others are *intermediate sights*.

In order to draw the correct section of a continuous surface, the levels of the intermediate points as well as that of the back-sight and fore-sight points must be determined with the level at one station and the operation repeated by moving the instrument from station to station and the readings recorded in the level-book as described above.

809d. Level-Book : Rise-and-Fall System.—The ordinary form of level-book for booking the levels is as under :—

Level-Book No. 2.—For a Correct Section of a Continuous Surface.

1	2	3	4	5	6	7	8
Back-Sight.	Intermediate Sight	Fore-Sight.	Rise.	Fall.	Reduced Level.	Distance.	REMARKS.
5-45	3-15		2-30		100-00	0	On A Bench-mark.
	5-40		0-05		102-30	175	" a
	2-32		3-18		100-05	200	" b
4-67		0-37	(5-08)		108-13	225	" c
	2-75		1-92		(105-08)	500	" d, Level moved.
	3-95		0-72		107-00	600	" e
	4-12		0-55		105-80	700	" f
	2-45		2-22		105-63	850	" g
0-21		3-45	(1-22)		107-30	900	" h
	2-12			1-91	(106-30)	1000	" i, Level moved.
	1-00			0-79	104-39	1100	" j
	4-26			4-05	105-51	1150	" k
	6-14			5-93	102-25	1329	" l
	4-00			3-79	100-37	1375	" m
		5-08		(4-87)	102-51	1400	" n
					(101-43)	1500	" B
10-33			6-30	4-87	101-43	= last reduced level.	
8-90		8-90	4-87		100-90	= first " "	
1-43			1-43		1-43		

The first seven columns are usually on the left page and the 8th column is on the right page of the level-book. The first column contains the *back-sights*, the second the *intermediate sights*, the third the *fore-sights*, the fourth the *rises*, the fifth the *falls*, the sixth the *reduced levels*, the seventh the *distances*, and the eighth the *remarks*. The first three columns, viz., of back-sight, intermediate and fore-sight, taken from the staff observations, and seventh and eighth columns from chain measurements of distances and remarks, constitute the field operations, whilst the fourth, fifth, and sixth columns, for *rise*,

from chain measurements of distances and remarks, constitute the field operations, whilst the fourth, fifth, and sixth columns, for *rise*, *fall*, and *reduced level* need not necessarily be worked out in the field, but it is always as well to do so if time and circumstances permit. -

809e. Booking the Levels.—The procedure in booking the levels in Fig. 786 is as under :—The first sight of 5'45 on *A* is a back-sight, and is recorded in the first column as shown, and noted as *A* on the same line in the column for *remarks*. The next three sights on *a*, *b*, *c*, viz., 3'15, 5'40 and 2'32 are intermediate sights, and are entered on the *next* three lines below and noted as *a*, *b*, *c*, in the column of remarks. The next sight of 0'37 on *D* is a fore-sight, and as this is to another *new* point, and is the last that can be sighted from the level at *C* is recorded in the *next* line below in the third column and is described as *D* in the column for remarks. We have now done with the first collimation line, and will record the sights after establishing another. The second line of collimation having been established by planting the level at *E*, the reading on the staff still held at *D* but turned to face the instrument, viz., 4'67 being a back-sight to the same point *D* is taken down on the *same line* as 0'37 in the first column. Similarly the next four sights being intermediate ones on four *new* points *d*, *e*, *f*, *g*, viz., 2'75, 3'95, 4'12, 2'45 are booked on the *next* four lines one by one in the second column and are described in the column of remarks as *d*, *e*, *f*, *g*, respectively. The next and the last sight of the 2nd collimation line being a fore-sight to a *new* point, *F*, is booked on the fore-sight column on the *next line*, viz., 3'45 and is noted, on the space for remarks as *F* in the same line. Now the third collimation line being established by planting the level at *G*, the back-sight 0'21 to the same point is entered in the first column on the *same line* as 3'45. The next five intermediate sights on *h*, *i*, *j*, *k*, *l*, viz., 2'12, 1'00, 4'26, 6'14, 4'00 are entered in the next five lines in the second column and are described in the space for remarks as *h*, *i*, *j*, *k*, *l*. The last sight being to the *new* point, *B*, and being a fore-sight it is booked on the next line in the third column, viz., 5'08, and is described in the remarks as *B*. The last sight of a series of levels is *always* a fore-sight.

809f. Chain and Staff Men.—In all levelling operations besides the level-man to carry the level, two chain-men are required to measure the distances with the chain, and one staff-man to hold the staff at the several staff-stations which represent the angles of the ground. For rapid levelling 2 staves may be used if they are tested and found to give the same readings at different parts of the staves. In common staves there are errors in graduation, hence one staff is generally used in practice. The leveller is to guide and control the men generally. With stadia-hairs in the diaphragm the services of the chain-men may be dispensed with.

809g. Reducing the Levels.—**Level Book No. 2.**—It is now to be borne in mind that whenever the intermediate or the fore-sight is *less* than the back-sight the difference of level between the two points is a *rise*, and when the intermediate or fore-sight is *greater*

is a *fall*. Working diagonally downwards from left to right (vide, level-book No. 2) each of 3'15, 5'40, 2'32, 0'37 being less than 5'45, their differences are *risés* of 2'30, 0'05, 3'13, 5'08, respectively. Again each of 2'75, 3'95, 4'12, 2'45, 3'45 being less than 4'67 their differences are also *risés* of 1'92, 0'72, 0'55, 2'22, 1'22, respectively. Now 0'21 being less than each of 2'12, 1'00, 4'26, 6'14, 4'00, 5'08 their differences are *falls* of 1'91, 0'79, 4'05, 5'93, 3'79, 4'87. The *risés* and *falls* are recorded in columns 4 and 5 respectively.

809h. Level-Book, No. 3.—For a Correct Section of a Continuous Surface.

1	2	3	4	5	6	7
Back-sight.	Fore-sight.	Rise.	Fall.	Height above Datum 100.	Distance.	REMARKS.
5'45	3'15	2'30		100 00	0	On A. B. M.
3' 5	5'40		2 25	102'30	175	a
5'40	2'32	3 08		100 05	200	b
2'32	0'37	(1'95)		103'13	225	c
4'67	2'75	1'92		(105'08)	500	D, Level moved
2'75	3'95		1 20	107'00	600	d
3'95	4'12		0 17	105'80	700	e
4'12	2'45	1'67		105'63	850	f
2'45	3'45		(1 00)	107'30	900	g
0'21	2'12		1'91	(106'30)	1000	H, Level moved.
2'12	1'00	1'12		104'39	1100	h
1'00	4'26		3 26	105'51	1150	i
4'26	6'14		1'88	102'25	1329	j
6'14	4'00	2'14		100 37	1375	k
4'00	5'08		(1 08)	102 51	1400	l
				(101'43)	1509	B
51'99	50'56	14'18	12'75	100 00		
50'56		12'75		1'43		
1'43	Rise	1'43	Rise	Rise		

Here with the level at *C* the first back-sight gives 5'45'. The first intermediate-sight taken with the staff at the point marked *a*, is at the same time a fore-sight to *A*, and a back-sight to the point marked *b*; *a* is a fore-sight to *b* and a back-sight to *D*, and so on for other points. In this form column for intermediate sight is dispensed with. The first back-sight and last fore-sight of each line of sight are the most important in point of accuracy. Any error made in taking an intermediate sight affects that line only, whilst any error in the back-sight or fore-sight is carried on throughout the section.

809i. Level-Book No. 4.—Some surveyors, instead of using the back-sight and its subsequent intermediate and fore-sights to find out the *risés* and *falls*, adopt the following method (vide, Level-book No. 4): 3'15 being less than 5'45 is a *rise* of 2'30; 5'40 being greater

than 3'15 is a fall of 2'25; 2'32 being less than 5'40 is a rise of 3'08; 0'37 being less than 2'32 is also a rise of 1'95. We have now completely done with the first series. Now take up the next series:—2'75 being less than 4'67, the difference is a rise of 1'92; but 2'75 being less than 3'95 shows a fall of 1'20, and 3'95 less than 4'12 a fall of 0'17, 2'44 being less than 4'12 is again a rise of 1'67, 3'45 being greater than 2'45 shows also a fall of 1'00. This finishes the second series. Now with a new back-sight of 0'21, we have a fall of 1'91, a rise of 1'12, the two intermediates showing falls of 3'26, 1'88 respectively, a rise of 2'14, whilst the fore-sight gives a fall of 1'08. The total difference of level between *A* and *B* is now the difference of the sums of the rises and falls in columns 4 and 5, or 14'18—12'75 = 1'43; and as the sum of the rises is the greater, the result is a rise of 1'43, *i. e.*, there is a rise of 1'43 from *A* to *B*.

**Level-Book, No. 4.—For a Correct Section of a
Continuous Surface.**

1	2	3	4	5	6	7	8
Back-sight.	Inter-mediate sight.	Fore-sight.	Rise.	Fall.	Reduced Level.	Distance.	REMARKS.
5'45					100'00	0	On A. B. M.
	3'15		2 30		102'30	175	„ a
	5 40			2'25	100'05	200	„ b
	2'32		3'08		103'13	225	„ c
4 67		0'37	1 95		105'08	500	„ D, Level moved.
	2'75		1'92		107'00	600	„ d
	39'5			1'20	105'80	700	„ e
	4'12			0'17	105'63	850	„ f
	2'45		1'67		107'30	900	„ g
0'21		3'45		1'00	106'30	1000	„ F, Level moved.
	2'12			1'91	104'39	1100	„ h
	1'00		1'12		105'51	1150	„ i
	4'26			3'26	102'25	1329	„ j
	6'14			1'88	100'37	1375	„ k
	4'00		2'14		102'51	1400	„ l
		5'08		1'08	101'43	1500	„ B
10'33		8'90	14'18	12'75	101'43		
8'90			12'75		100'00		
1'43 rise			1'43 rise		1'43 rise		

809j. Check on the Reduction of Levels.—It is manifest that the same result will be arrived at by adding up all the back-sights and all the fore-sights in columns 1 and 3. When the sum of the back-sights exceeds the sum of the fore-sights, the result is a *rise*, when it is less their difference is a *fall*. In this case the sum of the back-sights = 10'33, and the sum of the fore-sights = 8'90, the difference

of level is therefore $10.33 - 8.90 = 1.43$ as before. This is a valuable check on the reduction of levels, and should always be adopted.

809k. Comparison on the Reductions by the Rise-and-Fall Methods.—In the first method (Level-book No. 2) the rises and falls are obtained from the back-sight of a series, and each subsequent intermediate and fore-sights, but by the second and third methods (Levels book Nos. 3 and 4) the rises and falls are obtained from each intermediate and its previous back-sight, or the fore-sight and its previous intermediate sight. **The first method** of reduction is **preferable** as any mistake occurred in reducing an intermediate reduced-level will not be carried through

809l. Levelling across a Hill.—Instead of setting up the instrument on the top of the hill, it will save time to set it up on one side at *C* and to sight the staff as shown by *BD* in Fig. 785. This also applies to levelling across a hollow.

809m. Reciprocal Levelling with One Instrument.—In crossing over a deep valley, a marsh, or at the crossing of a wide river

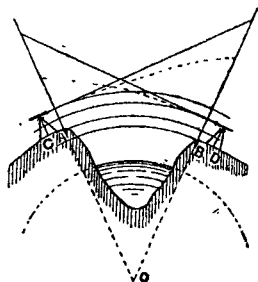


FIG. 787.—Reciprocal Levelling.

this method is useful, when it is not feasible to set up the level midway between two points. In such cases two bench-marks *A* and *B* should be established, one on each side of the river (Fig. 787.)

First Method : Set up the level at *A*, and sight on the staff held on *B*. The difference between the height of instrument above the peg at *A* and the staff-reading is a difference of level. Then set up the instrument at *B* and sight on the staff held on *A*. This gives another difference of level. The mean, of these two differences of level, is the correct difference.

Ex : H. I. at *A* = 4.46, staff-reading *B* = 5.51 ;

$$\therefore B - A = 5.51 - 4.46 = 1.05 ; -$$

H. I. at *B* = 4.94, staff-reading *A* = 3.75 ;

$$\therefore B - A = 4.94 - 3.75 = 1.19 ;$$

Hence the mean difference of level = $\frac{1}{2}(1.05 + 1.19) = 1.12$. If the reduced level of *A* be 500 ft. the reduced level of *B* is $500 - 1.12 = 498.88$ ft. The instrument at *A* and *B* may be raised or lowered and reset to obtain another set of readings ; the more sets of readings the greater the accuracy of the mean. **2nd Method :** With the level at *C* near *A*, and staves on *A* and *B*, so that *CAB* is a right line, ascertain their difference of elevation. Then repeat the operation with the level at *D*, a point on the opposite bank of the river, such that *BD = AC*, and, therefore, *AD = BC*. The mean of the two differences is the correct difference of elevation. If E_1 and E_2 are the absolute elevations of *A* and *B*, their true difference of elevation is

$$X = E_1 - E_2 \quad \dots \quad (1)$$

Letting r_1 represent the correction for curvature and refraction for the distance $AC=BD$, and r_2 the corresponding correction for $CB=DA$, the difference determined with the instrument at C is

$$X^1 = (E_1 - r_1) - (E_2 - r_2) \quad \dots \quad (2)$$

With the Level at D , the difference found is $\dots$

$$X^2 = (E_1 - r_2) - (E_2 - r_1) \quad \dots \quad (3)$$

Adding (2) and (3) and dividing by 2

$$\frac{1}{2}(X^1 + X^2) = E_1 - E_2 \quad \dots \quad (4)$$

See Instructions for *Precise Levelling* levelling across a river, Post.

809n. Bench-Marks.—A bench-mark is a mark of reference the level of which mark above a certain assumed datum is determined and recorded for future reference. Standard bench-marks carefully selected and distributed according to the needs of the country, are obtained by *precise levelling* by Government, and are found along trunk roads, railways, or on buildings, where going is easy, the grades flatter and clear sights are obtainable. In England Ordnance bench-marks are



Fig. 788.—Ordnance Bench-Mark.

placed at numerous points all over the country, and are marked by an arrow (Fig. 788) chiselled on the stone of various buildings and walls, etc. The positions of these bench-marks and their heights above Ordnance datum are marked on the Ordnance maps, and they may be easily obtained by consulting the maps. In India

also the Great Trigonometrical Survey bench-marks of masonry, with a square unplastered stone slab in the middle, are distributed all over the country and are shown as in Fig. 789. Bench-marks along railways and roads are not altogether satisfactory owing to the continual vibration set up by traffic, and although the leveller might work along the railway or road, bench-marks are set up at a distance. In the practice of levelling

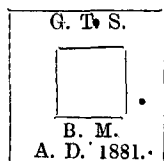


Fig. 789.

it is usual to leave at convenient intervals bench-marks which consist of permanent objects on which a distinguishing mark is cut so that it may be recognised afterwards. In a city bench-marks should be at closer intervals than in the country.

809o. Object of the Bench-Marks.—Their use is chiefly for future reference, either to check levels by repetition, to change the direction of the line of levels from any point, or to take up and continue the levels at the commencement of a day's work a bench-mark having been left at the close of the previous day's work. Whenever a temporary bench-mark is set, a peg driven into the ground, or a spike in the root of a tree will do. A good bench-mark shall not change its elevation during the time it is to be used. If this period is not more than 2 or 3 years, a nail driven in the spreading root of a tree near the trunk and above ground will do. The wood should be cut away from it so as to mark a projecting spur that will not grow. Bench-marks are sometimes engraved and fixed. This is not suitable in a semi-civilised country where a brass disc or a stout nail is of a local value and may be removed and the mark destroyed. In such localities an engraved

DIAGRAMS OF BENCH MARKS.

CHAP. XIV.

PLATE VII.

SURVEY OF INDIA DEPARTMENT.

EMBEDDED BENCH MARK

TYPE A SECTIONAL DIAGRAM

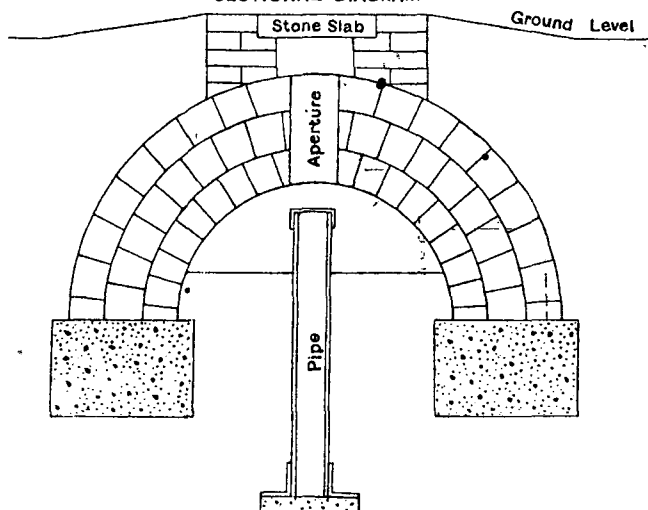


Fig. 790, a

EMBEDDED BENCH MARK

TYPE B PERSPECTIVE DIAGRAM

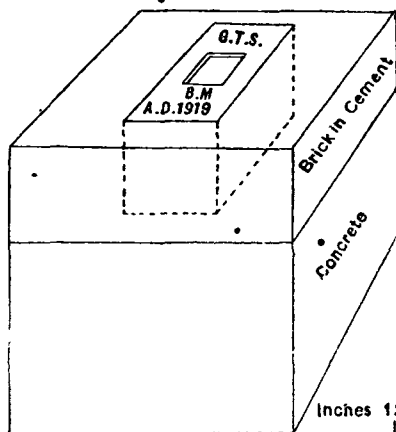


Fig. 790, b,

ROCK CUT BENCH MARK

TYPE C SECTIONAL DIAGRAM

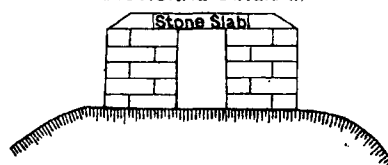
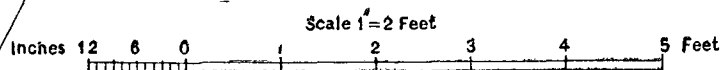


Fig. 790 c

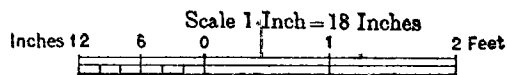
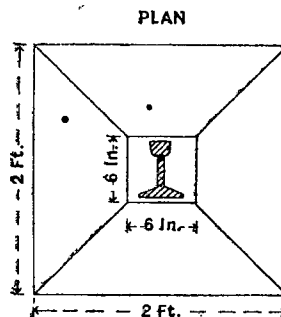
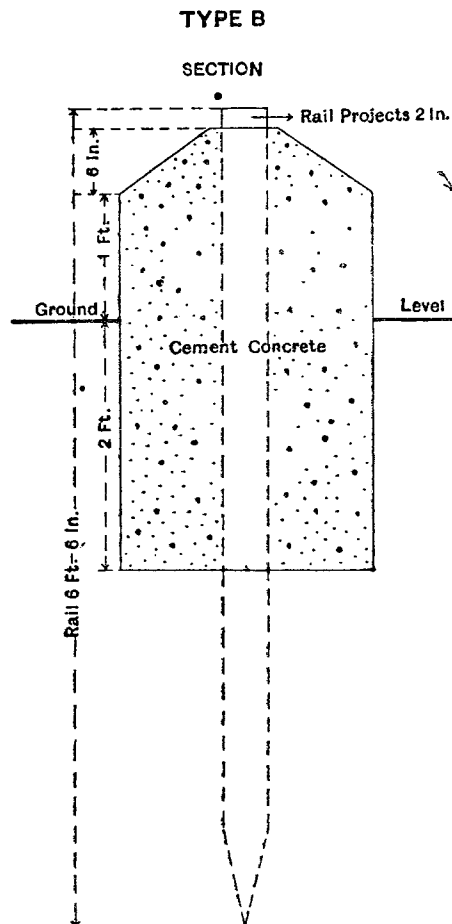
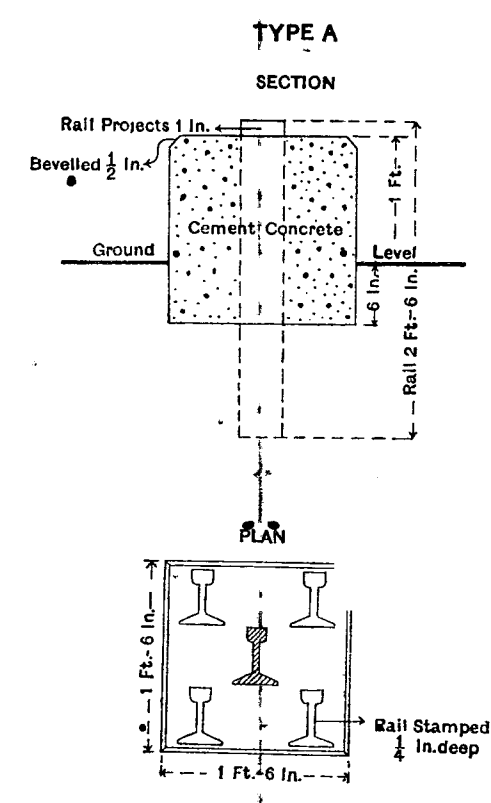


DIAGRAMS OF BENCH MARKS.

CHAP. XIV.

PLATE VIII.

USED IN MESOPOTAMIA.



Figs. 790 d

Fig. 790 e.

unplastered stone slab away from the traffic is suitable. The selection of suitable places for bench-marks should be done with great care. The place should be one that can be easily recognised and the exact position should be marked in some way or other : a broad arrow and bar across the points like the Ordnance bench-marks, being the one usually adopted and the staff to be held on the bar. B. Ms. need not necessarily be at or near ground level, a cornice may be chosen and measured to by means of staff held upside down. The plinth, and corner of door-steps of a house, copings of retaining walls, piers and abutments, parapets and floors of bridges and culverts, gate-posts, mile stones, tops of wells, or copper bolts leaded in natural rock and other permanent objects, Figs. 790a-d, the levels of which are not likely to be disturbed should be selected for bench-marks if the mark is to serve for from 5 to 50 years. In running a continuous line of levels a bench-mark should be left about every $\frac{1}{4}$ to 2 miles ; if there is no permanent object near the line; either one should be made, or else an auxiliary line of levels should be run to the nearest suitable object. The best method of recording bench-marks is to make careful sketches and descriptions of them in the level-book indicating their positions and to note the measurements relative to near well-recognised objects ; the north point should always be shown on the sketch.

809p. Datum Plane or Datum.—In all important levelling operations it is usual to refer all levels to some fixed plane of reference or datum, *i. e.*, to find the heights of the various points above an assumed level-line taking it as zero level, which is called the “Datum” line. The mean surface level of the sea is the most convenient datum to assume. The datum of the Ordnance Survey of the United Kingdom is the mean Sea-level (M. S. L.) at Liverpool. In India all important levels should be reduced to the Great Trigonometrical Survey datum, *i. e.*, at Karachi and 8 other stations. In Calcutta, the Sill of the stone on the Tide Gauge* at Kidderpur old Dock has been accepted by the River surveyors as equivalent to Mean Low-water ordinary springs. At Kidderpur the same Datum is 10·80' below the mean level of the river or identical with the Zero of the Gauge, or 23·93 feet below a Bench-mark, engraved as in the margin on the coping of the West Wall of the Main Entrance of the Kidderpur New Dock 10½ feet North of the Swing Bridge.

The heights of some chief Bench-marks in Madras* above Mean Sea-level are as follows :—

G.T.S.	B.M. at Astronomical Observatory—20·35	Height in ft. above Mean Sea-Level.
Do	„ Harbour Memorial Stone—15·76	
Do	„ Main Entrance to C. Ry. Stn.—14·57	
Do	„ Government House lowest Step off Entrance to Banqueting Hall—9·82	

The False-point Datum* for Tide Tables is 5·06 feet below

* Taken from “the Tide Tables for the Indian Ports for the year 1906.”

mean Level of the Sea or 2'50' above the Zero of the tide gauge or 15'45' below a Marine Survey B.M. $\bigcirc$ cut on the S. W. pile of the Refuge house at Hookey Tolah. The Chittagong Datum* for tables is 6'87' below the mean level of the water or 1'17' above the zero of the gauge or 21'54' below the B.M.A. which is near the S. E. corner of the Port and Customs Office.

809q. The Reduced Level of any point is the depth or height of that point above or below any datum. It is usual for the sake of convenience to take any imaginary datum line below the earth's surface some even number of feet, such as 100, below the *first station*, and to reduce all levels to this, if the relative heights are required for a small detached area or on a short length of line as in level-books: 2 to 4. In choosing such a datum, to avoid contrary signs (+ and —) in the calculation and for facility in plotting, it is necessary to have it sufficiently below the first point to underlie all other points.

809r. Fifth Form of Level-Book.—Collimation Level or Height-of-Instrument Method.—The system of booking and reducing the levels by the *rise-and-fall system* entails more labor, than the *collimation level* or *height-of-instrument* method. It saves one column of figures as the columns for rises and falls are not required. All surveyors know how easy it is to make a mistake and to enter a rise in the fall column or *vice versa*, and it will be well to replace these columns by a single column known as that of the *collimation level* or *height-of-instrument*. Taking the same example as in Fig. 786 the level-book under the *Height-of-Instrument method* is kept as on p. 693.

The method of reduction consists in *adding* all the back-sights to the reduced levels of the points on which they are taken to find the collimation level, and *subtracting* each intermediate and fore-sight from the previous collimation level to obtain the reduced levels of the points to which the intermediate and fore-sights are taken. Hence the simple rule:—"Add the back-sights to the last reduced level to find the height-of-instrument and subtract from it the intermediate and fore-sights to get the reduced levels." Referring to Fig. 786, we find that if the reduced level of the point *A* is 100, and the back-sight on *A* is 5'45, the reduced level of the collimation line of the instrument at *C* is $100 + 5'45 = 105'45$, or in other words, the height of the collimation line, ordinarily called the "height of instrument" is 105'45. This is booked on the same line as the reduced level 100 of *A* in the fourth column of "height of instrument." To find the reduced levels of the points *a*, *b*, *c*, and *D*, the staff readings on *a*, *b*, *c*, and *D* being 3'15, 5'40, 2'32 and 0'37, i.e., these being the lengths of the staff between the collimation line of the instrument and the ground, it is only necessary to subtract these from the "height of instrument" in order to give the new positions of these points above datum. Thus, it is evident from Fig. 786 that *a*, *b*, *c*, and *D* are 3'15, 5'40, 2'32, 0'37 below the collimation line, and we get

* Tide tables for the Indian Ports for 1906.

Level-Book No. 5.—For a Correct Section of a Continuous Surface: Height-of-Instrument or Collimation Plane Method.

1.	2	3	4	5	6	7
Back-sight.	Int. Sight.	Fore. Sight.	Height of Instrument.	Reduced. Level.	Distance.	REMARKS.
5'45			105'45	100'00	0	On A, B. M.
	3'15			102'30	175	a
	5'40			100'05	200	b
	2'32			103'13	225	c
4'67		0'37	109'75	105'08	500	D, Level moved.
	2'75			107'00	600	d
	3'95			105'80	700	e
	4'12			105'63	850	f
	2'45			107'30	900	g
0'21		3'45	106'51	106'30	1000	F, Level moved.
	2'12			104'39	1100	h
	1'00			105'51	1150	i
	4'26			102'25	1329	j
	6'14			100'37	1375	k
	4'00			102'51	1400	l
		5'08		101'43	1500	B
10'33		8'99		101'43		
8'90				100'00		
1'43 rise.				1'43 rise.		

105'45—3'15=102'30 for the reduced level of *a*, 105'45—5'40=100'05 for the reduced level of *b*, 105'45—2'32=103'13 for *c*, 105'45—0'37=105'08 for *D* where fore-sight was taken. The level is then moved and the process repeated, the back-sight as ascertained from the new position being added to the reduced-level at the point upon which levels are changed. Thus, the height-of-instrument when placed at *E*=reduced-level of *D*+the back-sight on *D*=105'08+4'67=109'75. To find the reduced-levels of the points *d*, *e*, *f*, *g*, *F* subtract the staff-readings of these points, 2'75, 3'95, 4'12, 2'45, 3'45, from the new collimation level 109'75, *viz.*, 109'75—2'75=107'00, 109'75—3'95=105'80, 109'75—4'12=105'63, 109'75—2'45=107'30, 109'75—3'45=106'30 respectively. Similarly for the third height-of-instrument at *G*, add the reduced-level at *F* to the back-sight on *F*, *i.e.*, 106'30+0'21=106'51, and for the reduced-levels of *h*, *i*, *k*, *l*, and *B* subtract the staff-readings at these points from the third height-of-instrument=106'51.

A modification of this form may also be adopted. Only two columns may be used for recording the staff-readings. The intermediate and fore-sights are booked in one column, the latter being distinguished by being underlined.

809s. Comparison of "Rise-and-Fall" and "Height-of-Instrument" Systems.—On comparison of both these systems, it will be evident that by the collimation or height-of-instrument method

in the case of the intermediate sights there is a considerable saving of labour in the reducing as this method dispenses with finding rises and falls as required in the "rise-and-fall" method. By the rise-and-fall 2nd and 3rd methods as given in Level-book Nos. 3 and 4, there is, however, a check on the reduction of the intermediate sights, as any error in the reducing of an intermediate sight, affects the reducing of the next fore-sight, and therefore, of all the levels after it. On the other hand, by the height-of-instrument method and the rise-and-fall first method any intermediate sight may be wrongly reduced without affecting any other levels. For example, the reduced levels of the intermediate sights, a, b, c, d , etc., might have any value in the reduced level column without affecting any of the other reduced levels. By these two methods levels may be checked correctly on to a bench-mark, and still have the intermediate sights wrongly reduced. Important points and not intermediates should therefore be made *change or turning points*.

Some surveyors object to the collimation method, saying that it deprives them of a check. Under the rise-and-fall methods, the difference between the total back-sights and total fore-sights should equal the difference between total rises and total falls, and between the first and last reduced-levels, and necessarily if there are no rises and falls one of these checks is absent. It is, however, only a check upon arithmetic and upon the entry being made in the proper column, may well be omitted. Mistaking the column is then impossible and one arithmetical computation is avoided. If the collimation method be used, the check is quite as perfect, the difference between total back-sights and total fore-sights being mere tested against the difference between the first and last reduced-levels. With such simplification the collimation method should be universally used.

809t. Distances.—Usually levelling operation is carried out along lines previously ranged out and measured and in which pegs are driven at even distances apart. Staff-readings are taken on these pegs as well as on points showing the change of slope of the ground. In the **distance column** the respective distances are entered from the commencement. With Stadia measurements a "Total Distance" column should be added.

809u. Scales.—In plotting longitudinal sections of levels, a larger scale is adopted for the vertical than for the horizontal distances. An exaggerated representation of the section is thus obtained. The space is thus economised in the length of the section, and the slopes of the ground, especially in roads, canals, etc., the depth of cuttings, height of embankments, etc., are shown with much greater clearness than if the two scales were equal. In cross-sections equal scales are adopted both for horizontal and vertical distances. The scales for distances on longitudinal sections are identical with, or bear some convenient ratio to, those used on the corresponding plans. The scales for heights on sections are generally made to be 10 times the scale for distances, as this is often the most convenient; another

mode is to use 10 times the number of chains in horizontal distances per foot in vertical; thus for 8 chains horizontal, 80 feet vertical, and so on. Records of height or depth made on plans are generally in feet and decimals, metres and decimals.

809v. The Bengal Public Works Department has prescribed the following Scales for Plans and Longitudinal Sections:—

	1 inch = 1 mile.		R 8 inches = 1 mile.
R, C	2 inches = 1 "	R	10 " = 1 "
3	" = 1 "	R	20 " = 1 "
R, C	4 " = 1 "	40	" = 1 "
R	5 " = 1 "		

The two scales marked C are most useful for canals while they could also be used for roads, or if preferred any of those marked R according to the character of road. The vertical scale should always be $\frac{1}{10}$ th or multiple of this fraction.

809w. For Indian Railways the Scales used are as under :

For Index plans and Sections : a scale of 1 mile to 1 inch horizontal and 100' to 1 inch vertical. For detailed plans and sections : Scale of 400' = 1" horizontal and 40' = 1" vertical.

809x. Reduced Levels or Surface Elevations.—Having now explained the meaning of the term, *datum*, the use of column 6 in level-book Nos. 2 and 4 may be explained. For instance, suppose the height of the point *A* (Fig. 786) above the Great Trigonometrical Survey datum, or any other chosen datum, to be known, column 6 is used to obtain the heights of the other points *a, b, c, d*, etc., above this datum. If the point *A* is supposed to be 100 ft. above a certain datum, it is called the *reduced-level* of the point *A*, and is booked in the same line as the back-sight of *A* in column 6 (pp. 685 and 688). From the rise-and-fall columns in Level book No. 2, p. 685; *a, b, c, D* are 2'30, 0'05, 3'13, 5'08 feet above *A*. To find the reduced-levels of *a, b, c, D*, add each of 2'30, 0'05, 3'13, 5'08, to the reduced-level of *A*, viz., $100 + 2'30 = 102'30$, $100 + 0'05 = 100'05$, $100 + 3'13 = 103'13$, $100 + 5'08 = 105'08$ which are noted in column 6 as the reduced-levels of *a, b, c, D*. Similarly the reduced-levels of *d, e, f, g, F* are $105'08 + 1'92 = 107'00$, $105'08 + 0'72 = 105'80$, $105'08 + 0'55 = 105'63$, $105'08 + 2'22 = 107'30$, $105'08 + 1'22 = 106'30$, while *h, i, j, k, l* and *B* being 1'91, 0'79, 4'05, 5'93, 3'79, 4'87 ft. lower than *F*, their reduced-levels are $106'30 - 1'91 = 104'39$, $106'30 - 0'79 = 105'51$, $106'30 - 4'05 = 102'25$, $106'30 - 5'93 = 100'37$, $106'30 - 3'79 = 102'51$, $106'30 - 4'87 = 101'43$. It is clear that when all the levels are reduced, the difference of level between any two points is equal to the difference of their reduced-levels. For example, the difference of level between *A* and *B* is $101'43 - 100 = 1'43$ as previously obtained.

809y. Plotting Longitudinal Section.—If the distances of the points, *a, b, c, D, e*, etc., from *A* are measured and noted in the distance column (pp. 685 and 685) in order to plot the section of levels between *A* and *B*, rule a line to represent the datum line

from which the heights are calculated ; and on it lay off the distances $A_1a_1=175$, $A_1b_1=200$, $A_1c_1=225$, $A_1D_1=500$, etc., Then erect perpendiculars A_1A , a_1a , b_1b , c_1c , D_1D , etc., at the points A_1 , a_1 , b_1 , etc., and scale on them lengths equal to the reduced-levels, 100, 102'30, 100'05, 103'13, 105'08, etc., when the points A , a , b , c , D , etc., are found ; and on joining these points the "longitudinal section" $A a b c D d e \dots B$ of the ground between A and B is obtained (Fig. 786, p. 683). When the perpendiculars A_1A , a_1a , etc., become inconveniently long the datum-line A_1B_1 is ruled at a certain distance above the datum of the levels. In the present case (Fig. 786) the datum A_1B_1 is assumed to be 85 ft. above the datum of the levels, and the perpendiculars A_1A , a_1a , b_1b , c_1c , D_1D , etc., are plotted equal to 15, 17'30, 15'05, 18'13, 20'08, etc., instead of equal to 100, 102'30, 100'05, 103'13, 105'08, etc. The assumed level of the datum-line A_1B_1 is to be noted below the line A_1B_1 thus : 85 feet above datum, or as the case may be.

809z. Profile Paper — For plotting longitudinal sections this

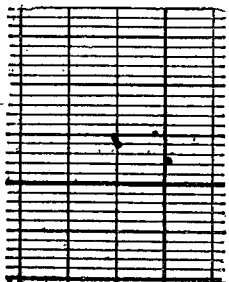


Fig. 791.—Profile Paper. are printed in green or red.

Profile Tracing Cloth and Bond Paper (Semi-transparent) are manufactured for photographic reproduction, both profiles and the drawing being reproduced on the blue print. Available in rolls of 20 yds. $\times 22'$.

698. Marquois Scales.—Level Sections may be plotted with great rapidity by the aid of the Marquois scales (q.v., p. 421.)

810. Check Levels.—In levelling there is no check on the accuracy of the work unless it is checked by running a second set of levels called *check levels*, between *bench-marks* left during the first operation. In the running of check-levels it is unnecessary to chain the distances or read the bearings. The object of check-levels is only to obtain the difference of level between certain intermediate, and the extreme points of the section previously made. It is immaterial by what route these levels are carried, the most convenient being chosen for the purpose, but they must always commence and end in a bench-mark. If the reduced-levels of the bench-marks—which are the terminal points—obtained by the second operation agree with those obtained at first it is inferred that the intermediate operations must be correct. The field-book required for check-levels is merely a simple entry of back and fore-sights, the difference of the sums of which will be the difference of level between the extreme points of the section line.

The following is a Specimen Check-level Field-book :—

Stations.	Back-sights.	Fore-sights.	REMARKS.
1	5 27	7 36	B. M. on top of wall ∇
2	3 41	0 50	$\square$
3	4 32	4 49	
4	5 83	4 91	
5	4 86	4 78	B. M. on 13th Mile-stone.
	23 69	22 04	
Sums	22 04		
	1 65 = Difference.		

811. The Principal Sources of Error in Spirit-Levelling are : 1. *Errors of observation* ; 2, *Errors from unstable supports* ; 3, *Instrumental errors* ; 4, *Atmospheric errors*. The corresponding remedies are : (1) errors of observations are mostly unavoidable, and arise chiefly from (a) the bubble not being properly centred, (b) from the inaccurate staff-reading caused by the assistant not holding the staff precisely vertical, (c) from the distance error caused by the curvature of the earth affecting the result, when the back and fore-sights are of unequal length. These may be reduced on noticing the cause of error. (2) Errors from unstable supports can be avoided by using a heavy firm stand, permanently attached to the lower part of the instrument. Errors can also be eliminated by repeating the levelling in the opposite direction, and by taking the mean of the results. (3) Errors due to want of instrumental adjustment can only be avoided by setting up the instrument midway between the back-sight and fore-sight points. Errors due to curvature and refraction are also thereby eliminated. (4) Atmospheric errors arise from wind, a heated atmosphere which causes the image of the levelling-staff to tremble, and variable refraction due to sudden bursts of sunshine on the line. When the velocity of the wind is moderate the level may be protected in a tent or by a screen, but when it exceeds 8 to 10 miles per hour work must be kept in in abeyance. During a heated atmosphere work may be proceeded with, with short sights, but if the length of sights has to be reduced to less than about $2\frac{1}{2}$ chains to read the staff distinctly, it is better to stop, as the errors of observation due to the large number of stations used for the level and staff become excessive.

812. Admissible Error in Levelling.—The error or difference in any circuit of levels, ought not to exceed *one foot per 100 miles traversed*. The result of four series of test-levels in France, of from 45 to 140 miles, averaged a difference of 1.10 of a foot in 43 miles, and the greatest error was 1.3 of a foot in 56 miles ; another series of test-levels in Scotland of two sets of levels of 26 miles was .02 of a foot. Mathematical analysis shows that for the same grade of work throughout, the error should be expected to

accumulate as the square-root of the distance increases. The limiting errors permitted on high grade levelling work on various Government Surveys in the United States are as given below :—

	Formula (or its equivalent).	Permissible errors in	
		10 miles.	100 miles.
U.S. Coast & Geod. Survey	$E = 0.029 \sqrt{\text{distance in miles}}$	0.092 foot	0.29 foot
U. S. Geol. Survey ...	$E = 0.05 \sqrt{\text{distance in miles}}$	0.158 foot	0.50 foot
New York State Survey ...	$E = 0.020 \sqrt{\text{distance in miles}}$	0.063 foot	0.20 foot
" " " " ...	$E = 0.016 \sqrt{\text{distance in miles}}$	0.051 foot	0.16 foot

It should be made a custom, at the end of each day's work, to level back and check on the starting point. No level-work can be depended upon without a check of this kind. In ordinary work, after a day's labor the original and check reduced-levels should agree within $\frac{1}{10}$ of a foot. With Zeiss Levels the mean error in the case of Model III with optical variation Plate and Invar staff is ± 0.3 mm. per kilometre (± 0.019 in. per mile.), in that of Model I it is 2 mm. per kilometre (± 0.127 in. per mile.).

813. Precautions to be taken in Section Levelling.—Equal back and fore-sights as far as practicable should be insisted upon as by this means any inaccuracy in the level, even if the run of the bubble is kept in the centre, is thereby compensated ; but it is not always convenient, and when it is not, the precision necessary must depend upon the adjustment of the level. Owing to obstructions occurring on the line, of woods, marshes, rivers, etc., it is not always practicable to take equal back and fore-sights. In these cases the level should be set up at some lateral position and equal angular back and fore-sights from the apex of an equilateral triangle may be taken thus : *e. g.*, an obstacle occurs on the chain line by a pond or marsh, but that both points, of which the levels are required, are visible at some lateral position. The level should be taken to this place and sights taken, and if the intermediate point is equal from both staff stations there will be no instrumental error, or if it is found impossible to take equal sights at any time, the inequality should be balanced off at the next one or two settings, by making them unequal in the opposite direction by the same amount. The equality of sights can be ascertained by pacing with sufficient accuracy (*q. v.*, Arts. 805). In levelling over hilly country it is not practicable to take equal back and fore-sights except at a great loss of time, the best course then to be taken is to make as much use of the length of the staff as possible. Here staves longer than 14 ft. can be advantageously used. In descending a hill back-sight is to be taken to the lowest visible part of the staff which will be sometimes as much as 4 or 5 ft., if there be grass or other obstructions, whereas for the fore-sight

* When the old method of observing was used on this service the limit was : $E = 5\text{mm} \sqrt{2K}$, $K = \text{distance in Kilometres}$.

reading may be taken to the top of the staff to the height of the collimation line. The distance apart, the levels and staves may be set up, depends thus on the length of the staff and other conditions present. Through valleys the levels may be checked from hill to hill by reciprocal levelling. In wide valleys of 5 to 10 miles check-levels may be taken from hill to hill with a 14" level in clear weather in dark nights, by the aid of an oil lamp, allowance being made for curvature and refraction. **In traverses generally**, it is usual to mark the distance run over into parts of 100 ft. each by means of pegs. The numbering is kept up continuously without regard to angles either horizontal or vertical in the line. In levelling for section it is necessary to take a reading at every change of slope. It is customary, further, to take the level of all even and 100 ft. stations and all horizontal angles in the traverse.

814. Obstacles and Difficulties.—To level across a wall take a sight to the wall, and mark where the cross-hair cuts it. Find height of the top of the wall from this point and record this, as a fore-sight with plus sign. Move the level to the other side of the wall and take another sight to it, and mark where the cross-hair cuts it. Again find the height of the top of the wall from this point and record this as a back-sight with a minus sign. The same result will be attained by holding the staff upside down with the foot of the staff on a level with the top of the wall, and the back and fore-sights being booked with the contrary signs as above stated. **To level through a House**, plant the level in such a position that you can see the staff by opening doors and windows through the house, or hold the staff in such a position inside the house that it may be visible from both sides of the house by opening the doors and windows.

Sun.—The sunshine on the object-glass sometimes gives trouble. If the level has a ray-shade draw it out. If not shade the objective with the hat or hand or the level-book, or shift the level on one side of the line so as to obviate the sunshine on the objective while sighting.

815. Cross-Sections.—These are taken at right angles to the longitudinal section in order to enable an Engineer to get an idea of the configuration of the ground on each side of the line of the longitudinal section. The points to be fixed for taking cross-sections should be those where the surface of the ground is irregular, and uneven, or where the ground has a steep slope in a direction transverse to that of the line of longitudinal section, *i.e.*, the centre line. The cross-sectional lines may be squared off by the eye with sufficient accuracy, but if more correctness is demanded, an optical-square, box-sextant, or a theodolite may be employed. They are taken to accompany the longitudinal section, for the purpose of finding and checking the quantities of cuts and fills. For very irregular ground it may be necessary to take levels and distances of other points in the cross-section in order to better find its area. These are taken by simply reading on the staff of the critical points on the outline and measuring the distances out from the centre. The points can then be plotted on cross-section paper, and joined by

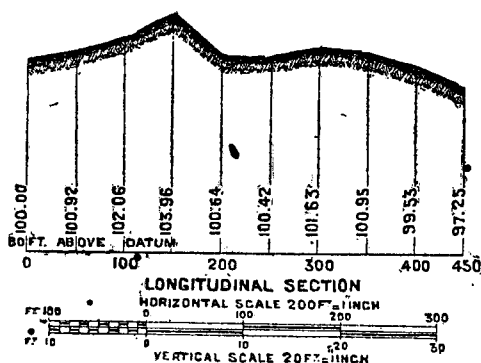


Fig. 793.

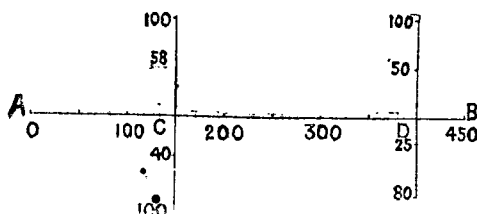


Fig. 792.—Plan of Longitudinal Level Line.

section is over. In the latter case the method of booking the levels will be as in pp. 685-688. In the former case the collimation method of recording will much simplify matters.

Level-Book No. 6.—Combined Level-Book for Longitudinal and Cross-Sections.

Back-sight.	Inter-sight.	Fore-sight.	Collimation Plane.	Reduced Level.	Distance.			Remarks.
					Left.	Centre.	Right.	
8-26	7-34 6-20 4-30 7-43 8-01 7-52 9-16 7-62 7-82		108-26	100 00		0		
				100 92		50		
				102 06		100		
				103 96		150		
				100 83	58	150		
				100 25	100	150		
				100 74		150		
				99 10		150	40	
				100 64		200	100	
				100 42		250		
8-23	8-91 10-33 11-00 9-90 10-43 8-20	6-63	109-86	101 63		300		
				101 63		300		
				100 95		350		
				99 53		400		
				98 86	50	400		
				99 96	100	400		
				99 43		400	25	
				101 66		400	80	
				97 25		450		
								Level Moved.

straight or curved lines. The area in this case can be expeditiously determined by the aid of a planimeter. Thus in Fig. 792, *AB* is the plan, and Fig. 793 is the longitudinal section taken on *AB*, and at the points *C* and *D* cross-sections are taken, the cross-sectional lines being ranged at right angles to the centre line, *AB*, and marked by flags at the ends of each of them. The levels of the cross-sections may either be taken along with the longitudinal section, or they may be taken after the work of the longitudinal

This form of book is in some respects more valuable when used for complicated levels than for a simple section. The most ordinary complication met with is that of entering cross-sections as well as a longitudinal section. An example of this is given above. It is quite possible to use the same level-book with remarks for distances made in the "Remarks column" at each of which levels are taken on either side of the centre line on the cross-sections, but it is better to add special columns in the way shown under the head of "*Distance*," with levels taken on the left entered in the left-hand column and those taken on the right entered on the right-hand side. The longitudinal section is shown with the addition of cross-sections at right angles to the centre line at distances of 150 and 400 feet along it. The distances along the centre line are entered exactly the same way as in the previous books (*q. v.*, pp. 685–693); but when the cross-section is required at 150 (Fig. 794) the chain is set out at right angles to the centre line and sights are taken to the prominent points on the cross-section. It has already been ascertained that at 150 ft. along the centre line the staff reading is 4.30, and on subtracting this from the collimation level of 108.26 the reduced-level is found to be 103.96. At 58 ft. to the left of this point another reading is taken of 7.43, which, by subtraction, gives a reduced level of 100.83. The entries are made in the book exactly as they would be for running levels,

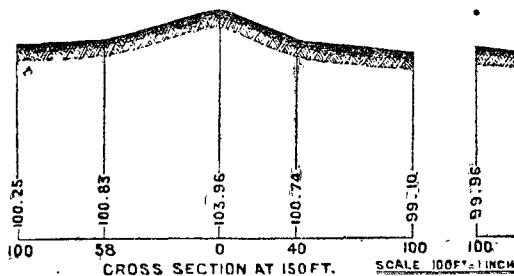


Fig. 794.

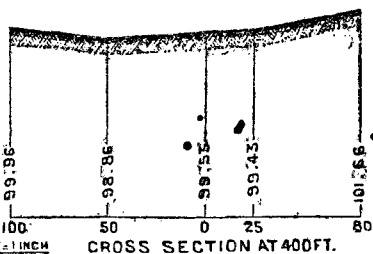
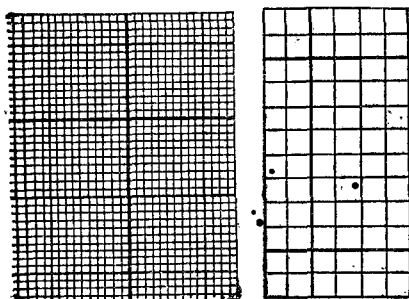


Fig. 795.

except that the distances are entered in the way indicated, 150 appearing under the word "centre," and 58 under the word "left." The plan shows where the cross-level occurs, and a distance 58 is indicated on it, while the distance and reduced-level are shown on a cross-section projected on side of the longitudinal section. In the same way levels are taken at 100 ft. on the left, and at both 40 and 100 ft. on the right, of 150 along the centre line, there being thus 5 readings on the cross-section, two being on either side of the centre line and one at the point of crossing. All these sights are taken from the level when in its first position, and after they have been entered, the next sight is taken on to the staff held at 200 along the centre line, and so the work proceeds. Another cross-section is also shown at 400 (Fig. 795) on the centre line, the entries being of a similar character and it is plotted by the first cross-section. It is better to plot each cross-section separately with an indication in its title as to where it crosses. It will be seen that such an indication is made below the cross-section itself as "at 150" and "at 400."



Figs. 796, 797.—Watt's Cross-Section Papers.

815a. Cross-Section Papers and Cloths.—These are very useful in plotting cross-sections

as no scaling and ruling are required. Figs. 796, 797 give cuts of cross-section papers, the lines being printed green.

They are printed in inches and eighths, inches and tenths, and inches and twentieths, in sheets of $22\frac{1}{2} \times 17\frac{1}{2}$, or in rolls of 50 yds. or 10 yds. $\times 26$. Mounted paper in rolls of 10 yds. $\times 26$. Tracing Cloth or paper in rolls of 10 yds. $\times 26$.

816. American Method of Cross-sectioning with the Hand-Level.—To estimate the quantities of earthwork the American method of cross-sectioning will be found useful. Let Fig. 798 illustrate the cross-section of a bank having four warped surfaces

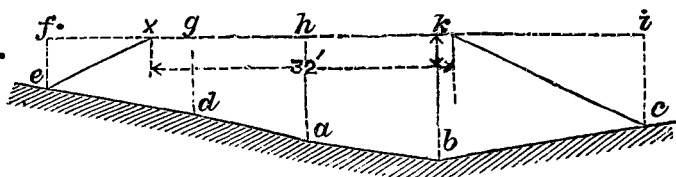


FIG. 798.—American Method of Cross-sectioning.

of which the field-notes are the following, the formation width of bank being 32 ft., and the side slopes 2 to 1. The field-notes are recorded in the form of fractions. The numerators are the distances out from the centre line, and the denominators are the heights

Field-notes for Cross-section :

$26\frac{4}{5}$ —5.5, $11\frac{75}{8}$ —8, 0/—9.94, 14 —13, $34\frac{4}{9}$ —9,

of bank or depths of cutting at the ends of these distances, thus in the above note, the middle fraction indicates a height of 9.94 ft. bank or 9.94 ft. "fill", as it is usually called at the centre line at right angles to which the cross-section is taken. The hand-level is used to determine the height of bank or depth of cutting in the following way:—Standing at the centre peg, *a*, hold up the level placed in a notch cut in a wooden board 4.5 ft. above the ground. Now direct the level to a staff held at *b* and take the reading, which is say 7.56 ft. This gives a fall of 3.06 ft. The bank on the centre line at *a* being 9.94 ft., the bank at *b* will, therefore, be 13 ft. The distance from the centre line as taped or staffed being 14 ft., there is entered in the field-notes 14 —13 showing the depth of bank to be 13 ft. at a distance out of 14 ft. to the *right*, the record being to the right-hand of the centre fraction. + is used to indicate "cutting," and — to show bank or "fill." The points *c* and *e* where the bottom of slope comes are marked, and the distances out of height of bank are entered in the field-notes. These are $34\frac{4}{9}$ —9 and $26\frac{4}{5}$ —5.5. Similarly other intermediate readings where the ground-slope changes as at *d* are noted as shown. This method of cross-sectioning is very rapid, and one advantage in its favour is that the areas of the cross-sections and the cubic contents of the earthwork may be calculated from the field-notes *without plotting the cross-sections* thus :

Area of the trapezoid $abhc = (ah + bk)/2 \times kh = (9.94 + 13)/2 \times 14 = 160.58$ s. ft.
 " " " $bkc = (bk + ci)/2 \times ki = (13 + 9)/2 \times 20.4 = 224.40$ s. ft.
 " triangle $kci = \frac{1}{2} ki \times ic = \frac{1}{2} (9 \times 18.4) = 82.8$ s. ft.
 $\therefore$ The area of the right half of the cross-section $= 160.58 + 224.4 - 82.8 = 302.18$ s. ft.
 Again the area of the trapezoid $hadg = (ha + dg)/2 \times hd = (9.94 + 8)/2 \times 11.75 = 105.40$ s. ft.
 " " " $dffe = (gd + fe)/2 \times gd = (8 + 5.5)/2 \times 14.65 = 98.89$ s. ft.
 " " " triangle $x fe = \frac{1}{2} fx \times fe = (10.4 \times 5.5)/2 = 28.60$ s. ft.
 $\therefore$ The area of the left-half of the cross-section $= 105.40 + 98.19 - 28.60 = 175.69$ s. ft.
 $\therefore$ Total area of cross-section $ex k c b a d = 302.18 + 175.69 = 477.87$ s. ft.

The cubic contents of banks or cuttings are then found by adding up the areas of two contiguous cross-sections and multiplying by half the common distance in the usual manner. This method of finding volumes of earthwork obviates the necessity of plotting the cross-sections and as the quantities can be computed from the field-notes, there is no chance of errors from incorrect plotting and scaling.

817. Slope and Batter Protractor.—For correct plotting of the side slopes this instrument (Fig. 799) facilitates very greatly the

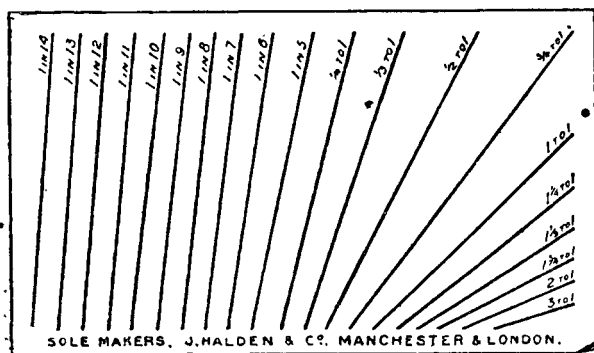


FIG. 799.—Slope and Batter Protractor.

production of Slopes and Batters upon the drawing when designing Retaining Walls, Breakwaters, Railway Embankments, Roofs, etc. It consists of a rectangular piece of pear-wood, celluloid, or Brass, measuring 12" by 7". It is used in conjunction with the T-square in the same way as an ordinary set-square. There are 20 slots cut into it, each having a different angle and marked with its slope or batter, of such a width as to enable a line to be drawn through it, thus immediately transferring the required angle or batter to the drawing. There are 40 angles in all, each slot indicating one angle to the horizontal and one also to the vertical. The outside edges can be used if required as a set-square.

818. In Using the Abney Level or Clinometer the point to be sighted must be at the same height above the ground as the eye. Referring to Fig. 800, to obtain angle of slope dca , the clinometer must be held up to the eye at a and the point, b , to be sighted should be at such a height that $bd = ac$. The line of collimation of the instrument is then parallel to cd , the inclination of the ground, and the reading on the graduated arc is the angle of slope;

cd. When greater steadiness is required in holding it the clinometer may be held in a notch cut in a ranging rod at a height convenient for the eye and a mark made at the same height on another rod may then be sighted.

818a. Cross-Sectioning with the Abney Level.—The slope of the ground is easily obtained by the aid of this instrument and thereby saves the trouble of carrying heavy levels for cross-sections. Referring to Fig. 800, the $\angle deg$ being known the slope cd may be plotted, and if the distance $cd = ab$ along the slope is also known the point d may be plotted. Similarly the $\angle hde$ being read and the distance de measured, the point e is located. In this way all the salient points of a cross-section may be fixed by the aid of the Abney Level much more

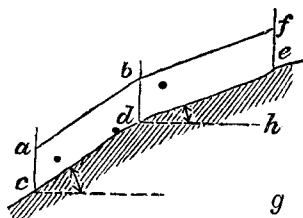


FIG. 800.—Use of Abney Clinometer.

quickly than by the ordinary level. The readings of the Abney level are not so accurate as that of levels, but for ordinary works, such as cross-sections for locating a road or railway this is quite useful. This instrument is much used in railway surveys for taking cross-sections for obtaining contours for locating purposes. For final earthwork quantities the level should be used..

819. Cross-Sectioning with the Theodolite.—The application of the theodolite affords the most rapid method of taking cross-sections on steeply inclined ground or hill sides. The instrument is set up at one end of the line previously ranged out by flags or ranging rods at every change of the general inclination of the ground ; and the height of the axis of the telescope found by taping from the peg over which the instrument is set up and levelled. The collimation line is then set nearly parallel to the general inclination of the ground, and the vertical circle clamped at that angle which is now recorded. The staff-man then holds the staff at the points where the slope of the ground changes, and the assistant at the theodolite taking readings as if with a spirit-level. The distances along the slope may either be measured before the angles are taken, marks being left at every irregularity on the surface where the staff is required to be held ; or both operations may be performed at the same

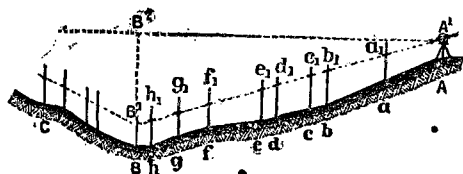


FIG. 801.—Cross sectioning with Theodolite.

FIG. 801.—Cross sectioning with Theodolite.

required, the mean of this and the reciprocal angle of elevation from B to A is taken, and the vertical arc being clamped to this angle, the telescope is again made to bisect the mark at B . On arriving at B , after the staff-readings are taken at a, b, c , etc., and measuring distances Aa, ab, bc , etc., the instrument must be set up at B , and the angle of elevation taken to C , the same process repeated to obtain the outline of the ground between B and C , and onward.

819a. In Plotting the Cross-Section, first plot the height of the axis of the instrument AA' , then draw a horizontal line $A'B'$ through A' , and protract the angle of depression at A' , which will give the collimation line, $A'B'$. The distances Aa, ab, bc , etc., are then scaled off along the collimation line as $A'a_1, a_1b_1, b_1c_1$, etc.; the respective staff readings, then scaled down the vertical lines a_1a, b_1b, c_1c , etc., from the collimation line, will give the points a, b, c , etc., on the surface of the ground where the staff was successively held. Upon connecting these points a, b, c , etc., the required cross-section is obtained. **With stadia-hairs in the diaphragm** the distances along the collimation line may be simultaneously found without chain or tape measurement. By holding the staff at right angles to the collimation line the distances along the collimation line may be calculated easily, and the staff readings are then to be plotted at right angles to the collimation line. By holding the staff vertical the calculation becomes tedious and will have to be made according to the principles to be described in the Chapter on Tacheometry in part II of this Work.

Where a number of cross-sections are required, the theodolite is very useful, as so many can be taken without moving the instrument. It is also well-adapted for *trial sections* where economy of time and money is an object. The theodolite can also be used in running check-levels, to test the accuracy of those *taken in detail with a spirit-level*. Reciprocal angles of elevation and depression, taken between bench-marks, whose distances from each other are known, afford a proof of the general accuracy of the work. The effect of curvature and refraction may be nearly neutralized by taking reciprocal angles; *i. e.*, if A and B be two stations, A being higher; take altitude of A as seen from B , and the depression of B as seen from A , half the difference of these angles will be the combined correction, and the *tangent of half their sum*, being multiplied by the distance, will give the difference of level nearly. The reciprocal angles should be taken as nearly as possible at the same time, lest the refracting power of the air should change in the interval. The altitude of an object can best be taken with the sextant (see, pp. 89-101.)

820. Levelling by the Theodolite may be effected by placing the collimation line horizontal and using the theodolite like a level when it is not at hand.

821. Cross-sectioning with Deacon's Optical-square Attachment (Fig. 644, p. 581).—When a number of Cross-sections at right

angles to a main longitudinal sectional line along the middle of a valley is required, having ranged out the main line, and driven pegs into the ground along this line at the points at which cross-sections are to be taken, the level is set up over one of these pegs, say at B (Fig. 801), and the *line of sight* directed along the main line; keeping the telescope still in the same position, the shutter at the end of the cap is then closed, the line of sight will no longer be directed along the main line, but at right angles thereto, by the prism, and as the cap can be revolved in either direction about the longitudinal axis of the telescope, points may be sighted on either side of the valley, either in, above, or below the horizontal plane of the axis of the telescope, the points may be brought to focus with the focussing-screw. Having ranged out the cross-section, the level may be moved to the second station, and so on. With the scale of degrees engraved at one end of the cap and a pointer or vernier attached to the telescope, angles of elevation or depression of the sights taken on the staff may be taken and from these the heights of the various points may be determined or with the stadia-hairs in the diaphragm tacheometric readings of the staff may be taken.

822. Levelling for Fixing a Grade.—In order to fix a grade the longitudinal section may be plotted and then the grade-lines may be marked on it. The difference of level between the ground-line and the formation or grade line, *i.e.*, the vertical distance between them, at any point is the depth of cut or fill at that point, and this may be marked on the centre pegs at once, without the aid of a level and staff, if only centre depths are required, as in the case of a trench for laying down a water-main. If a given slope is required for the side the use of the level and staff becomes imperative to fix the horizontal distance of the slope pegs. These slope lines may be marked on the cross-sections beforehand. If the grade and the reduced-level of the starting point be known before the longitudinal section is drawn as is often the case with trenches for water-mains or sewers, then the depth of cut (or fill) may be at once ascertained and marked on the centre line pegs when the longitudinal section is taken. The field-book might be the same as given before for longitudinal sectioning with the addition of a few columns as given in the specimens above. The formation level above datum would be found for each longitudinal section point by adding if an up, and subtracting if a down grade, the differences of elevation corresponding to the successive distances in the longitudinal section. The difference between the corresponding "ground level" and "formation level" would be cut or fill at each point, which could be at once ascertained and marked to the centre line peg.

CHAP. XIV.]

SPECIMENS OF LEVEL-BLOCK.

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CHAPTER XV.—PRACTICE OF BAROMETRIC AND HYPOSOMETRIC LEVELLING.

824. Measurements of Heights with the Aneroid.—In taking altitudes of mountains and hills, etc., where close accuracy is necessary, two instruments should be employed. An observer should be left at the lower station noting at agreed-up intervals, preferably at 9.30 a.m. and 3.30 p.m. changes both in temperature and pressure that take place. The observer making the ascent should note at the summit of the mountain the barometric reading, the temperature of the air and the time. On returning to lower station he should compare the records, and make the necessary corrections for difference in temperature between base and summit.

It is an excellent instrument for laying down contour lines; but for absolute heights it should be checked by the boiling-point thermometer, because its index-error is apt to change; when thus checked it is a reliable instrument for measuring heights up to 8000'. On most aneroids the zero of the altitude scale is 31 inches of the scale corresponding to the mercury column. The absolute height of the barometer varies systematically throughout the year, and in India there is a large diurnal variation having two maxima and minima in the 24 hours at 9.30 a.m. and p.m. and 3.30 a.m. and p.m. This is the reason why it is always better to read the barometer at 9.30 or 3.30 when it is changing but slowly.

824 a. Barometric Formula and its Constants.*—The fundamental formula for reducing the barometer to sea-level and for determining heights by the barometer is the original formula of Laplace, amplified into the following form:—

$$Z = K (1 + \alpha \theta) \left(\frac{1}{1 - 0.378 \frac{e}{b}} \right) (1 + k \cos 2\phi) \left(1 + \frac{h + h_0}{R} \right) \log \frac{p_0}{p} \quad (1)$$

In which h and h_0 = Heights of the upper and lower stations respectively.
 p and p_0 = Atmospheric pressures at the upper and lower stations respectively.

θ = Mean temperature of the air column between altitudes h and h_0 .

e = Mean pressure of aqueous vapor in the air column.

b = Mean barometric pressure of the air column.

α = Co-efficient of the expansion of air = 0.002039.

k = Constant depending on the figure of the earth = 0.002662.

$Z = h - h_0$.

R = Mean Radius of the earth
= 20899600 ft. = 6367324 metres.

ϕ = Latitude of the Stations.

K = Barometric constant.

The pressures p_0 and p are computed from the height of the column of mercury at the two stations; the ratio B_0/B of the barometric heights may be substituted for the ratio p_0/p , if B_0 and B are

* Smithsonian Meteorological Tables, 3rd Edition, p. xx.

reduced to the values that would be measured at the same temperature and under the same relative value of gravity. The correction of the observed barometric heights for instrumental temperature is always separately made, but the correction for the variation of gravity with altitude is generally introduced into the formula itself. If B_0 , and B represent the barometric heights corrected for temperature

only, we have the equation:—
$$\frac{p_0}{p} = \frac{B_0}{B} \left(1 + \frac{\mu Z}{R} \right),$$

μ being a constant depending on the variation of gravity with altitude.

$$\log \frac{p_0}{p} = \log \frac{B_0}{B} + \log \left(1 + \frac{\mu Z}{R} \right)$$

Since $\frac{\mu Z}{R}$ is a very small fraction, we may write

$$\text{Nap. log} \left(1 + \frac{\mu Z}{R} \right) = \frac{\mu Z}{R}; \text{ and } \log \left(1 + \frac{\mu Z}{R} \right) = \frac{\mu Z}{R} M,$$

M = modulus of common logarithms = 0.4342945.

By substituting for Z its approximate value $Z = K \log \frac{B_0}{B}$,

$$\text{We have } \log \left(1 + \frac{\mu Z}{R} \right) = \frac{\mu K}{R} M \log \frac{B_0}{B}.$$

With these substitutions, and putting $\beta = 0.378 e/b$, $r = k \cos 2\phi$, $\chi = \frac{\mu K}{R} M = 0.002396$, the barometric formula becomes

$$Z = K (1 + a \theta) \left(\frac{1}{1 - \beta} \right) (1 + r) \left(1 + \frac{h + h_0}{R} \right) (1 + \lambda) \log \frac{B_0}{B}. \quad (2)$$

824b. Value of the Constants :—The Barometric Constant $K = \Delta B_n / \delta M = 60367.7$ ft ; where B_n = normal barometric height of Laplace = 29.921 inches, Δ = Density of Mercury at the temperature of melting ice = 13.5958. δ = Density of dry air at standard temperature (62°F.) and under the pressure of a column of mercury B_n at the sea-level and at latitude $45^\circ = 0.001293052$; $a = 0.00367$ for 1°C ; $r = k \cos 2\phi = 0.002662 \cos 2\phi$.

For determining heights by the barometer Eqn. (2) is arranged as under in English measure :—

$$Z = K (\log B_0 - \log B) \left[\begin{array}{l} (1 + a \theta) \\ (1 + \beta) \\ (1 + k \cos 2\phi) (1 + \lambda) \\ \left(1 + \frac{Z + 2h_0}{R} \right) \end{array} \right] \dots \quad (3)$$

In which $K (\log B_0 - \log B)$ is an approximate value of Z , and the factors in the brackets are correction factors depending respectively (1) on the air temperature, (2) the humidity, (3) the variation of gravity with latitude, (4) the variation of gravity with altitude in its effect on the weight

of mercury in the barometer; (5) and the variation of gravity with altitude in its effect on the weight of the air. With the constants already given the formula becomes, in English measures :

$$Z \text{ (feet)} = 60368 (\log B_0 - \log B) \left[\frac{[1 + 0.002039 (\theta - 32^\circ)] (1 + \beta)}{(1 + 0.002669 \cos. 2 \phi) (1 + 0.00239)} \left(1 + \frac{Z + 2 h_0}{R} \right) \right] \quad (4)$$

In order to make the temperature correction as small as possible for average air temperatures, 50°F. will be taken as the temperature at which the correction factor is zero. This is accomplished by the following transformation : $1 + 0.002039 (\theta - 32^\circ) = [1 + 0.002039 (\theta - 50^\circ)] [1 + 0.0010195 \times 36^\circ]$.

The second factor of this expression combines with the constant, and gives $60368 (1 + 0.0010195 \times 36^\circ) = 62583.6$.

$$\therefore \text{The first approximate value of } Z = 62583.6 (\log B_0 - \log B) \\ = 62583.6 \left(\log \frac{29.9}{B} - \log \frac{29.9}{B_0} \right).$$

Tables XI to XV in Part III of this work gives Tables to facilitate calculations.

Table XI contains Values of the Expression $62583.6 \times \log \frac{29.9}{B}$ for values of B varying by intervals of 0.01 inch from 12 to 30.90 inches. The first approximate value of Z is then obtained by subtracting the Tabular value corresponding to B_0 from the Tabular value corresponding to B (B and B_0 being the barometric readings observed and corrected for temperature at the upper and lower stations respectively.)

Table XII gives the Temperature correction, $Z \times 0.002039 (\theta - 50^\circ)$. The side argument is the mean temperature of the air column, θ , given for intervals of 1° from 1° to 100°F. The top argument is the approximate difference of altitude Z obtained from Table XI. For temperatures above 50°F. , the correction is to be added, and for temperatures below 50°F. , the correction is to be subtracted. It will be observed that the correction is a linear function of Z , and hence, e.g., the value for $Z = 1740$ is the sum of the corrections in the columns headed 1000 , 700 and 40 . In general, accurate altitudes can not be obtained unless the temperature used is freed from diurnal variation.

Table XIII gives the correction for latitude, and for the variation of gravity with altitude in its effect on the weight of the Mercury. When altitudes are determined with aneroids the second factor does not enter the formula. In case the effect of the latitude factor can be obtained by taking the difference between the tubular value for the given latitude and the tabular value for latitude 45° . The side argument is the latitude of the station given for intervals of 2° . The top argument is the approximate difference of height Z .

Table XIV gives the Correction for the Average Humidity of the air at different temperatures; the values of the factor of $(1 + \beta)$ have been used. This correction could have been incorporated with the temperature factor in Table XII, but it is given separately in order that the magnitude of the correction may be apparent, and in order that, when the actual humidity is observed, the correction may be computed if desired, by the expression $Z (0.378e/b)$ where e is the mean pressure of vapor in the air column, and b the mean barometric pressure. The side argument is the mean temperature of the air column, varying by intervals of 2° from -20°F. to 96°F. , except near the extremities of the Table where the interval is 4° . The top argument is the approximate difference of altitude Z .

Table XV gives the correction for the variation of gravity with altitude in its effect on the weight of the air. The side argument is the approximate difference of altitude Z , and the top argument is the elevation of the lower station h_0 . The corrections given by Tables XIII, XIV, XV are all additive,

Example 1. Let the barometric pressure observed, and corrected for temperature, at the upper and lower stations be, respectively, $B=23.61$ and $B_0=29.97$. Let the mean temperature of the air column be 35°F. , and the latitude $44^\circ 16'$. To determine the difference of height.

Table XI, Argument 23.61 , gives ... 6420 feet
Table XI, " 29.97 , " ... -64 "

Approximate difference of Height $Z=6484$ "
Table XII, with $Z=6484$ and $\theta=35^\circ\text{F.}$, gives -198 "
Table XIII, with $Z=6300$ and $\phi=44^\circ$, gives +16 "
Table XIV, with $Z=6300$ and $\theta=35^\circ\text{F.}$, gives +17 "
Table XV, with $Z=6300$ and $h_0=0$, gives +2 "

Final Difference of Height, $Z=6321$ "

If in this example the barometric readings be observed with aneroid barometers, the correction to be obtained from Table XIII will be simply the portion due to latitude factor, and this will be obtained by subtracting the tabular value for 45° from that for 44° , the top argument being $Z=6300$. This gives $16-15=1$.

824 c. Loomis's Formula for Determining Differences of Height with the Mercurial Barometer (English measures) is:—

$$Z=60158.6 \text{ ft. } \left\{ \log \frac{B_0}{B} - M\theta (T_0 - T) \right\} \times \left\{ 1 + \frac{t_0 + t - 64}{900} \right\} \\ \times \left\{ 1 + 0.00265 \cos. 2\phi \right\} \times \left\{ 1 \times \frac{Z+52251}{20888629} + \frac{S}{10444315} \right\} \dots\dots\dots (4)$$

taking the mean radius of the Earth = 20888629 feet.

Where B_0 and B are heights of the barometer in inches at the lower and upper stations respectively. T_0 and T are the temperatures of barometer in degrees (Fah.) at the lower and upper stations. t_0 and t = temperatures of the air in degrees (Fah.) at lower and upper stations. ϕ = Mean Latitude.

S = Height of the lower station above sea-level in feet.

Z = Difference between the heights of the two stations.

M = Modulus of the common system of logarithms = 0.4342945.

θ = Difference of co-efficient expansion between mercury and brass scale. for 1°Fah. = 0.0001 - 0.0000104 = 0.0000896; and $60158.6 \times M\theta = 60158.6 \times 0.4342945 \times 0.0000896 = 2.3409$.

$\therefore$ The first two factors become, as given by Loomis,

$$60158.6 \log \frac{B_0}{B} - 2.3409 (T_0 - T).$$

In Table XVIII, Part I (in Part III of this Work) gives the value in feet of the expression $60158.6 \log B$ for heights of the barometer from 11 to 31 ins.

In Table XVIII, Part II (in Part III of this Work) gives the corrections for $-2.3439 (T_0 - T)$ depending on the difference of temperatures of the barometers at the two stations. This correction is to be omitted in the case of Aneroids.

In Table XVIII, Part III (in Part III of this Work) gives Correction due to the term $(0.00265 \cos. 2\phi)$.

In Table XVIII, Part IV (in Part III of this Work) gives Correction due to the term $(Z+52251)/20888629$.

In Table XVIII, Part V (in Part III of this Work) gives Correction due to the term $S/10444315$.

Example 2 of use of Table XVIII in Part III of this work:—

Dehra (Lower Station) : $B_0=27.563$ inches, $T_0=79^\circ.0\text{F.}$, $t_0=79^\circ.6\text{F.}$,

Mussoori (Upper Station) : $B=23.408$ inches, $T=70^\circ.1\text{F.}$, $t=65^\circ.1\text{F.}$

Mean latitude = 30° .

Calculation by Table XVIII.

Part I gives for 27.563 inches = B_0 , ...	25396.1 Feet.
„ „ „ 23.408 „ = B , ...	21127.2 „
Difference	4268.9 „
Part II gives for $T_0 - T = 79^\circ - 70^\circ.1 = 8^\circ.9$.	20.9 „
α = First Approximate Difference of Height =	4248.0 „
$\frac{4248}{900} (t^\circ + t - 64^\circ)$	= + 380.9
b = 2nd Approximate difference of Height	4628.9
Part III gives for $b = 4629$ feet and lat 30°	+ 6.1
Part IV „ „ $b = 4629$ „	+ 12.6
Part V „ „ $b = 4629$ „ and $B_0 = 27.56$ inches + 1.0	
$\therefore$ Height of Mussoori above Dehra	= 4648.6

824 d. Dr. Richard Rühlmann's Formula (Leipsic, 1870) is given as the simplest and best, being based on the most recent results :—

$$Z = 18400.2 \left(1.00157 + 0.003675 \frac{t_0 + t}{2} \right) \left[1 + \frac{0.378}{2} \left(\frac{a_0}{B_0} + \frac{a}{B} \right) \right] \\ (1 + 0.002623 \cos. 2\phi) \left(1 + \frac{2S + Z}{6378150} \right) \log \frac{B_0}{B}; \quad \dots (5)$$

in which Z is the difference in metres of level between the two stations ; t_0 and t , the temperature centigrade of the air at the two stations ; B_0 and B the heights of barometer in millimetres, corrected for temperatures, and for all instrument errors ; a_0 and a the elastic force of vapour ; ϕ the mean of the latitudes of two stations ; and S the height of the lower station above the sea. Making :

$$A = \log. \left\{ 18400.2 \left(1.00157 + 0.003675 \frac{t_0 + t}{2} \right) \right\};$$

$$C = \log. \left\{ 1 + \frac{0.378}{2} \left(\frac{a_0}{B_0} + \frac{a}{B} \right) \right\}; D = \log. \{ 1 + 0.002623 \cos. 2\phi \}$$

$$E = \log. \left\{ 1 + \frac{2S + Z}{6378150} \right\}; \text{ Rühlmann has calculated the values}$$

A, C, D, E for the different values of the respective arguments, which are given in the tables appended to his work.

From formula (5) we get :

$$\log. Z = \log. \{ \log. B_0 - \log. B \} + (A + C + D + E). \quad \dots (6)$$

It is assumed that the whole stratum of air between the two heights is in a state of rest, and that the means of the temperature and humidity observed at the two stations are the means respectively of the stratum of air between them.

824 e. Babinet's Formula for Travellers and Engineers

for the Determination of Heights by the Barometer in English Measures :

$$Z \text{ feet} = 52494 \left[1 + \frac{t_o + t - 64^\circ}{900} \right] \frac{B_o - B}{B_o + B}; \quad \dots \quad (7)$$

in which Z = the difference of elevation between a lower and upper station at which the barometric pressures corrected for all sources of instrumental error are B_o and B , and the observed air temperatures are t_o and t respectively at the lower and upper stations.

Calculations can be facilitated by the aid of Table XVII in Part III of this work.

Ex. 3. Let the barometric pressures, observed and corrected for temperature, at the upper and lower stations be respectively $B = 23.61$ and $B_o = 29.97$ inches. Let the air temperatures be respectively, $t = 30^\circ \text{F.}$ and $t_o = 40^\circ \text{F.}$ To find the approximate difference of height.

With $\frac{1}{2}(t_o + t) = \frac{1}{2}(30^\circ + 40^\circ) = 35^\circ \text{F.}$ the Table XVII in English measures gives C , the first two factors in Eqn. (7), = 52844 ; $(B_o - B) / (B_o + B) = 6.36/53.58$. The approximate difference of height = $52844 \times 6.36/53.58 = 6272.6 \text{ ft.}$

824 f. Belville's Rule for Rough Work, which is correct between temperatures of 50°Fah. and 60°Fah. , is as follows :—

$$Z = 55000 (B_o - B) / (B_o + B) \quad (8)$$

For engineering purposes, however, barometer observations are practically worthless without temperature corrections.

824 g. Calculation of Heights for Ordinary Engineering purposes with the Aneroid.—For the ordinary purposes of the Engineer it is sufficient to correct the index-error of the barometer and then to make the correction for the temperature of the air. As the aneroid is mechanically compensated for temperature, no correction of the barometer readings themselves is required. The Eqn. 4 will then be : $Z (\text{feet}) = 60158.6 (\log B_o - \log B) \left(1 + \frac{t + t_o - 64^\circ}{900} \right)$ (9)

Ex. 4 : Let the barometric pressure observed and corrected for temperature at the upper and lower stations be respectively $B = 23.61$ and $B_o = 29.97$ ins. Let the air temperatures be respectively $t_o = 30^\circ \text{F.}$ and $t = 40^\circ \text{F.}$ Find the difference of height.

Part I of Table XVIII gives for 29.97 ...	...	27583.54 ft.
" " " " 23.61 ...	...	21351.65 "
Difference	=	6231.89 "
$\frac{6231.89}{900} \times (40^\circ + 30^\circ - 64^\circ) = 6.924 \times 6$...	...	= + 41.54 "
		6273.33 "

If Table XX. Part II had been used, we have $\frac{1}{2}(t + t_o) = 35^\circ$. The corresponding multiplier is 1.0066. $\therefore 6231.89 \times 1.0066 = 6273.02$. For greater accuracy corrections in Parts III, IV, V of Table XVIII must also be made.

824 h. Airey's Formula, in which the Difference in two altitude readings is corrected for temperature by $\frac{1}{1000}$ part for every degree the sum of the temperatures when the sum is more or less than 100°F. , is :—

$$Z = 62759 - (\log B_o - \log B) \left(1 + \frac{t_o + t - 100^\circ}{1000} \right) \quad \dots \quad (10)$$

Prof. Airey's Table supposes the mean temperature of the air to be 50° F., or 10° C. For other temperatures the following correction must be applied : Add together the temperatures at the upper and lower stations. If this sum, *in degrees Fahrenheit*, is greater than 100° , *increase* the height by $\frac{1}{1000}$ part for every degree of the excess above 100° ; if the sum is less than 100° , *diminish* the height by $\frac{1}{1000}$ part for every degree of the defect from 100° . Or if the sum *in centigrade degrees* is greater than 20° *increase* the height by $\frac{1}{550}$ part for every degree of the excess above 20° ; if the sum is less than 20° *diminish* the height by $\frac{1}{550}$ part for every degree of the defect from 20° .

Taking $Hx = 1$, we get Z feet $= 62759 (\log 29.97 - \log 23.61)^{\circ} (1 + (-.03)$

$$= 62759 (1.47669 - 1.37310) (0.97),$$

$$\therefore \log Z = 4.797750 + 1.015318 + 1.986772 = 3.79984,$$

$$\therefore Z = 6307 \text{ ft.}$$

Prof. Airey's Tables facilitate Calculation of work done by aneroids :—using (Prof. Airey's) Table XIX and the above Example, we get

23.61 gives	...	...	...	7422.10 feet
29.97 „	...	...	...	920.91 „
				6501.19 „

$$6501.19 \left(1 + \frac{70 - 100}{1000} \right) = 6306.15 \text{ „}$$

825. Determination of Heights by the Boiling-Point Thermometer Tables.—Table XX in Part III of this work will facilitate computations of heights by the Boiling-Point Thermometer.

Ex : At upper station, A, boiling point $187^{\circ}.3$, temp. of air, 26° .

At lower station, B, „ „ $210^{\circ}.4$, temp. of air, 68° .

Boiling point, $187^{\circ}.3$, gives height from Table XX, Part I = 13394 ft.

„ „ $210^{\circ}.4$, „ „ „ „ „ $\frac{835}{\text{ „}}$

Approximate Difference of Height = 12,559 „

Mean temperature = $\frac{1}{2} (26 + 68) = 47^{\circ}$.

From Table XX, Part II, multiplier for $47^{\circ} = 1.0333$.

$\therefore$ Corrected difference of height = $12559 \times 1.0333 = 12977.2 \text{ ft.}$

CHAPTER XVI.—TRIGONOMETRICAL AND PRECISE LEVELLING.

826. Trigonometric Levelling neglecting Curvature and Refraction.—(1) *To find the Height of a Tower, BE, the surface of the ground being Level.*—For short distances if the length AB and $\angle ECD$ are measured then the height, $BE = DB + AB \tan DCE$ (q.v., Fig. 169, p. 152). (1)

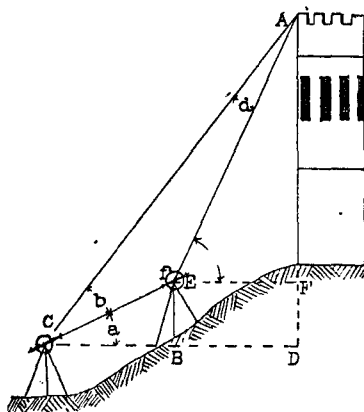


FIG. 802.—To find the Height of a Tower on an Inaccessible Hill.

out the line CE in the same Vertical Plane as C and A; take $\angle s$, a , b , and f , and measure CE. AD may be measured from a plotted plan, or computed as follows:—

$$\begin{aligned} AD &= \{CE \times \sin. f \times \sin. (a+b)\} / \sin. d \\ FD &= CE \sin. a; \text{ and } CD = AD \cot. (a+b) \end{aligned} \quad (2)$$

$\angle d$ being $= 180^\circ - (b+f)$.

Or measure the angle of elevation FEA at E, and in the $\triangle CEA$ calculate AE and then $AF = AE \sin. AEF$; and $EF = AE \cos. AEF$.

(4) *To find the Height of an Inaccessible Hill, when a distance CE in the same vertical plane, as C and A, cannot be measured,* run out a line CD (Fig. 803) in any convenient direction and measure $CD = 500'$, and the $\angle$ of elevation $ACB = 42^\circ$; and the horizontal angles $BCD = 95^\circ$ and $BDC = 62^\circ$, i. e., angles subtended by A at each end of the base, CD. Then in the $\triangle BCD$ we have $CB = (CD \times \sin. BDC) / \sin. CBD$; and $AB = BC \tan. ACB$. Substituting for BC, $AB = (\sin. BDC \times \tan. ACB \times CD) / \sin. CBD$ (3)

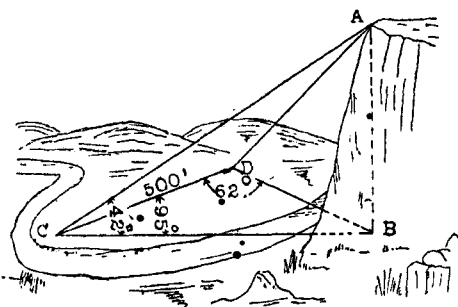


FIG. 803.—To find the Height of a Inaccessible Hill.

(5) *To find the Inaccessible Distance between two points, A and D (Fig. 804), when the Height of the Hill, BC, is known.*—Measure the $\angle$ s of depression at B of A and D which are equal to the $\angle$ s of elevation CDB and CAB, also measure at B the horizontal angle DCA. Compute CD and CA from known height BC and $\angle$ s of elevation at D and A. Then the distance, AD, may be found from the sides, CA and CD, and the included angle ACD, A and D being in the same plane and BC the known height above this plane. In each of the above cases the height of the instrument is to be added as in Fig. 169.

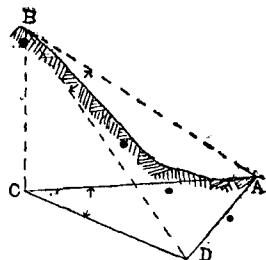


FIG. 804.—To find Inaccessible Distances.

827. To Find the Difference of Level when Curvature only is taken into Consideration.—(1) For long

distances the effect of curvature must be allowed for. In Fig. 805 the horizontal line given by the instrument is PD, the tangent to the earth's surface at P. The observed angle of elevation of the object, Q, is $\angle QPD$, and the computed height will be QD. The true level line being the arc PB, the true height of Q is QB.

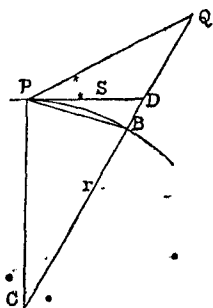


FIG. 805.—Difference of Level when Curvature is allowed for.

For moderately long distances $\angle PDQ$ may be taken equal to 90° , and we have $DB = PB^2/2CB = S^2/2r$, where $S =$ distance PB, and $r =$ earth's radius CB or PC. If the earth's mean radius be taken as 20,902,358.4 ft. (q.v., p. 350) then $DB = 0.0000000239207S^2$, and $QB = S \tan. QPD + 0.0000000239207S^2$... (4)

The arc PB and the lines PD and PB are taken nearly equal.

Another method is to correct the observed angle, QPD, by adding the $\angle DPB$ which is equal to half the central angle PCB; with the above value of earth's radius, $\angle DPB$ in minutes $= 0.000822S$; adding this to the observed angle, QPD, and assuming $PBQ = 90^\circ$, we have $QB = S \tan. (QPD + 0.000822S)$ (5)

(2) For very long distances $\angle PDQ$ cannot be taken equal to 90° , and here in the $\triangle PQD$, $QD = S \sin. QPD / \sin. PQD$.

$$\text{Also } \angle PQD = 180^\circ - (PCB + QPC) = 180^\circ - (PCB + 90^\circ + QPD) = 90^\circ - (PCB + QPD).$$

$$\therefore \sin PQD = \cos. (PCB + QPD);$$

$$\therefore QD = (S \sin. QPD) / \cos. (PCB + QPD);$$

$$\therefore QB = QD + DB = (S \sin. QPD) / \cos. (PCB + QPD) + 0.0000000239207S^2.$$

Substituting for PQ its value in minutes = $\cdot 00016446S$.

$$QB = \frac{S \sin QPD}{\cos (0\cdot 00016446S + QPD)} + 0\cdot 0000000239207S^2 \quad \dots (6)$$

For the other method correcting the $\angle QPD$ to $\angle QPB$ in $\triangle QPB$, we get $QB = S \sin (QPD + \frac{1}{2}C) / \cos (QPD + C)$; substituting the values of C we get,

$$QB = S \sin (QPD + 0\cdot 00008223S) / \cos (QPD + 0\cdot 0001644S). \quad (7)$$

828. Geodetic Levelling is of two kinds: (1) *Trigonometrical Levelling*, and (2) *Precise Spirit-Levelling*. In trigonometric levelling the relative elevations of triangulation stations are found by reading the vertical angles between the stations. When these are corrected for curvature of the earth's surface and for refraction the actual difference of elevation is found.

Refraction.—In trigonometric levelling correction only for curvature of the surface of the earth would have been necessary if the rays of light passed through the atmosphere in straight lines, but it is found that rays of light near the surface of the earth curves downwards, *i.e.*, their paths are convex upwards. This curve is variable, sometimes it is convex downwards in some places. It has its greatest curvature at daybreak, diminishes quickly till 8 A.M. and is nearly constant from 10 A.M. to till 4 P.M. when it begins to increase again. The curve may be taken as a circle having a variable radius the mean value of which is 7 times the radius of the earth.

829. To find the Difference of Level when both Curvature and Refraction are taken into Consideration.—The refraction increases the observed angle QPD , *i.e.*, its effect is to make objects higher than they really are. The increase varies with the state of the atmosphere, temperature, wind, etc., but on an average is ordinarily taken as $\frac{1}{7}$ of the curvature DB (Fig. 805). We must, hence, deduct $\frac{1}{7}$ DB from the Computed height or deduct $\frac{1}{7}$ of the $\angle DPB$ from the observed $\angle QPD$. Here

$$\text{Eqn. 4 (p. 717) becomes, } QB = S \tan QPD + 0\cdot 000000019947S^2. \quad (8)$$

$$\text{Eqn. 5 „ becomes, } QB = S \tan (QPD + 0\cdot 00007049S). \quad (9)$$

$$\begin{aligned} \text{„ 6 „ , } QB &= \frac{S \sin (QPD - 0\cdot 00001175S)}{\cos (QPD + 0\cdot 0001527S)} + \\ &\quad 0\cdot 0000000239207S^2. \quad (10) \end{aligned}$$

$$\text{Eqn. 7 „ , } QB = S \left(\frac{\sin (QPD + 0\cdot 00007049S)}{\cos (QPD + 0\cdot 0001527S)} \right). \quad (11)$$

Ex. Taking example on p. 719, the angle of elevation $\angle QPD$ being equal to $2^\circ 2' 30''$ and neglecting the height of instrument and signal we get

$$QB = \left(\frac{\sin (2^\circ 2' 30'' + 33'' \cdot 8352)}{\cos (2^\circ 2' 30'' + 1' 13'' \cdot 296)} \right) 8000' = 287' \cdot 845 \text{ feet,}$$

Note.—By *Computation of Heights* is meant the computing of the height of an unknown station above the mean height of the sea (*mean sea-level*), having already given the height of a *known station* above mean sea-level, the vertical angle from it to the *unknown station*, and the horizontal distance between the two stations, e. g., if P and Q are the known and unknown stations respectively, then the relative height of Q with reference to P would be thus found; multiply the horizontal distance between P and Q by the tangent of the vertical angle at P of Q, to which add the correction for curvature and refraction. When the vertical angle is an elevation add this height to, and if a depression subtract it from the given height of P, above mean sea-level for a similar height of Q. Again (1) the height of instrument (eye) at P, and (2) the height of, if any, signal or object at Q must be taken into consideration. The rule for obviating this is that in depression the H. I. is —, in elevation the H. I. is +; in depression the height of the object is —, in elevation the height of object is +.

Ex. 1. At P, a station 450' 28 above mean sea-level, the vertical angle of elevation of another station Q is $2^{\circ}2'30''$ and the horizontal distance between them is 8000 ft., H. I. = 4' 80" at P, and 21' = Ht. of signal at Q. Find the height of B above mean sea-level. Ans. 720' 56 feet.

Ex. 2. In the above if $2^{\circ}2'30''$ were a depression, find the height of Q above mean sea-level. Ans. 181' 86 feet.

Stations.		Horizontal Distances, Feet.	Vertical Angles.	Calculations	Feet.	Relative height, Feet.	Reduced Level, Feet.	Remarks
From	To							
P	Q	8000		$\log. = 3.903090$ $E + 2^{\circ}2'30''$ L. tan. = 8.552046 $\log. \text{ sum. } 2.455136$ Cor: Curvature & refraction + 1.29 H. I. or $i'p$ + 4.80 H. O. or $h'q$ - 21.00 	+ 285.19 + 1.29 + 4.80 - 21.00 	+ 270.28	450' 28 720.56	
P	Q	8000		$\log. = 3.903090$ $D - 2^{\circ}2'30''$ L. tan. = 8.552056 $\log. \text{ sum. } 2.455136$ Cor. Curvature & refraction + 1.29 H. I. or $i'p$ - 4.80 H. O. or $h'q$ + 21.00 	- 285.19 + 1.29 - 4.80 + 21.00 	- 268.70	450.28 181.86	

830. Computation of Heights from Reciprocally Observed Altitudes. 1st Method.

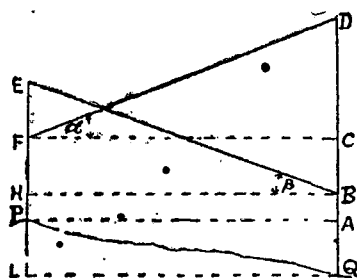


FIG. 806.—Reciprocally observed Altitudes.

At stations P and Q (Fig. 806) reciprocal and simultaneous observations have been made on both faces of the theodolite, and the heights of signals E and D and the heights of the instrument at F and B above P and Q have been booked; and FC, HB, PA, LQ are all equal horizontal lines, and PA = horizontal distance between P and Q at level of P = d . Let PE and QD heights of signals E and D from ground level be h_p and h_q respectively, and let PF and QB heights of instruments at P

and Q be i_p and i_q respectively; the observed mean altitude of D at P = $\angle DFC = \alpha$, and of E at Q = $\angle EBH = \beta$. Then $DC = d \tan \alpha$, and $EH = d \tan \beta$, $AQ = PL$ = difference of level between P and Q. But $AQ = DQ - (CD + AC) = DQ - CD - PF = h_q - d \tan \alpha - i_p$, and $PL = EL - PE = EH + HL - PE = EH + BQ - PE = d \tan \beta + i_q - h_p$. Adding the last two equations the difference of level between P and Q = $AQ = PL = \frac{1}{2} \{d \tan \beta + i_q + h_q - (d \tan \alpha + i_p + h_p)\}$.

Whether P or Q is higher can be determined by the observer when he is at any station. Then the quantities belonging to the higher station are deducted from the quantities of the lower station and $\frac{1}{2}$ the difference is the true difference of level between the two stations.

2nd Method.—Computation of Heights.—Let P and Q (Fig. 807) be two points on the surface of the earth, C being its centre.

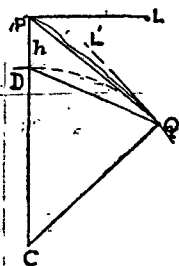


FIG. 807.

PL and QL' $\perp$ s to PC and QC indicate the directions of a level surface at P and Q respectively, and $\angle QPL$ will sensibly equal to $\angle PQL'$ when the reciprocal angles LPQ and L'QP had been taken at the same time and the refraction would have been equal or constant. Assume $CD = QC$, then $DP = h$ = difference of height. Let the observed depressions at P and Q be Dp and Dq . Now $\angle PQD = \angle CDQ - \angle QPC$, and $\angle PQD = \angle PQC - \angle DQC$; and since $\angle CDQ = \angle DQC$, then $\angle PQD = \frac{1}{2} (\angle PQC - \angle QPC)$; and since $\angle QPL = \angle PQL'$, then $\angle PQD$, the subtended angle, $As = \frac{1}{2} (\angle L'QC - \angle LPC = \frac{1}{2} (Dq - Dp)$. Now DQ is an insignificant part of the surface of the earth, hence h , the difference of heights between the ground levels of P and Q, may be taken equal to $DQ \tan. PQD$ or $h = DQ \tan. \frac{1}{2} (Dq - Dp)$, or if one angle is an elevation $h = DQ \tan. \frac{1}{2} (E_p + E_q)$, or $DQ \tan. \frac{1}{2} (E_q + Dp)$, i.e., the subten-

taking the staff away from it without any jar on ground. Errors due to settling of supports are therefore evidently *cumulative*, and the only way in which these errors can be eliminated is by *re-levelling* over the same ground in the opposite direction and the mean result of this and that of the first one is nearer the truth.

Atmospheric Errors are due to *wind*, *heated air-currents* causing the image of the staff to *dance*, or form variable *refraction*. During a moderate wind the level may be shielded by a screen or in a tent, but when the velocity exceeds 8 or 10 miles an hour, work must be suspended, as the instrument and staff vibrate. During a heated atmosphere the length of sights is shortened, but if the length of sights has to be shortened to less than about 150 or 50 metres to enable the staff to be read distinctly, it is better to stop, as the errors arising from the large number stations occupied by the instrument and staff will be too much. Refraction is practically constant between 11 A. M. and 3 P. M., and is greatest in the early mornings and late evenings. Great variations in refraction sometimes occur on lines over ground which is passing from light to shade, and *vice versa*. Then the observer should take readings between staves when they are both in the shade or in the sun-light. In clear weather not more than 3 or 4 hours a day are available for best work, and sometimes when the days are very hot and nights very cool it is impossible to get through an hour's good work in the day.

Observations.—In taking observations the bubble is brought to the centre of its run; the observer being able to effect this by the aid of the thumb-screw under one Y and the mirror which reflects the image of the bubble to the observer at the eye-piece. When no mirror is fitted to the bubble, the recorder reads the bubble brought approximately to the centre, while the observer reads the three horizontal cross-wires. In any case the bubble-reading is recorded in the level-book, and if it was not in the centre a correction is made for the eccentric position by means a Table prepared for the purpose. The mean of the 3 horizontal cross-wire readings which should agree with the central hair reading is taken as the staff-reading, the observer estimates the tenths of the centimetre-spaces, thereby reading each wire reading to the nearest millimetre. The stadia-hairs being spaced at as nearly as possible equal distances from the central hair, the two differences between the central and the extreme hair-readings give also the distance, and should agree very nearly, otherwise one or more of the 3 hair-readings is in error. *This is an important and essential check* on any serious error in the readings, and should be made in the field for each reading before the level or staff is moved to the next station. Two staff-holders to each level are employed in order not to lose time between the back and fore sight readings from a level station. As only a very small part of the day can ordinarily be available, rapid progress is a necessity when the weather is favourable. When two staff-holders are employed in favorable weather with air so steady that 100 metre length of

sights can be taken, an experienced leveller can level about a mile an hour or 4 miles day of precise levels.

The instrument is always shaded from the sun, both while set up on a station and while being carried between stations. It is essential to do this in order to keep the adjustments almost constant and the bubble from continually changing.

843. Limit of Error.—On the U. S. Coast and Geodetic Survey the limit of difference between duplicate check lines is $4\text{ mm } \sqrt{K}$, where K is the distance in kilometres. On the U. S. Lake Survey it was $10\text{ mm } \sqrt{K}$, and on the Mississippi River Survey it was $5\text{ mm } \sqrt{K}$; these limits are respectively 0.017, 0.041, 0.013 ft. multiplied by the sq. root of the distance in miles. When greater differences occurred the line was re-levelled.

The Probable Error of the mean of several observations may be found as follows:—If $d_1, d_2, d_3, \text{ etc.}$, be the discrepancies of the various results from the mean and if $zd^2 = \text{sum of the squares of these differences}$, $n = \text{number of observations}$, then the **probable error of the mean** is: $P. E. = \pm 0.6745 [zd^2/n(n-1)]^{\frac{1}{2}}$. Where there are only two observations, the equation becomes $P. E. = \pm \frac{1}{2}d$. The European International Geodetic Association fixed upon the following limits of probable errors per Kilometre in the mean or adopted result:

± 1 millimetres per kilometre is very precise;	On the U.S.C. and Geod. line of precise levels, from Sandy Hook to St. Louis, 1109 miles, the probable error per kilometre was ± 1.2 millimetres. On a line of precise levels on the Mississippi River Survey, 670 miles, run by Prof. J. B. Johnson with a probable error in the mean result of 23.5 mm, for the whole distance, which is less than 1" while the probable error per kilometre was $\pm .7\text{ mm}$.
± 2 " " " is fair;	
± 3 " " " Tolerable;	
± 5 " " " Too large	

844. Adjustment of Errors in Closed Circuits of Precise Levels.—If a line of levels closes upon itself the actual error is of course known, as the total difference of elevations should be zero. The corrections are applied in two ways. In one method, the error is distributed among the lines of the polygon proportionally to the length of the lines or to the square root of the lines. In the other, the corrections are made proportional to the computed *probable error* of each line as found from the discrepancies between each separate result and the mean result. The corrections are calculated by the method of *least squares*, as if they are the most probable ones, i.e. the sum of the squares of the corrections is a minimum. The last method is very laborious; and if the closing error is simply distributed so that the error on each line is taken as proportional to the square root of the distance of the line, the result is practically the same as obtained by the second method. The errors in levelling being compensating in their nature are more nearly proportional to the square root of the distance than to the distance itself.

Bavarian System.—When the operation of levelling comprises a number of closed polygons (Fig. 231, *q.v.*) the closing error of polygon *df hg* is first distributed on the sides *dg*, *gh*, *hf*, *fd*. In adjusting for the polygon *feh*, the corrections are distributed on the sides *he* and *ef*, the error in side *hf* having already been corrected. In the same way in adjusting *efc*, the error is distributed between *ec* and *cf* only, and for the circuit *cfa*, its closing error is distributed in side *ca* only. The polygon whose closing-error is the greatest should be adjusted first, then that with the next greatest error, and so on.

845. To Determine Correctly the Level of Mean Tide at any point on the coast continuous observations are required by the aid of an automatic self-registering gauge for a period of several years. A float enclosed in a perforated box rises and falls with the tide, and this movement reduced in scale by mechanism is recorded by a pencil on a continuous roll of paper which is rolled over a drum at a uniform rate by means of a clockwork (Fig. 812).

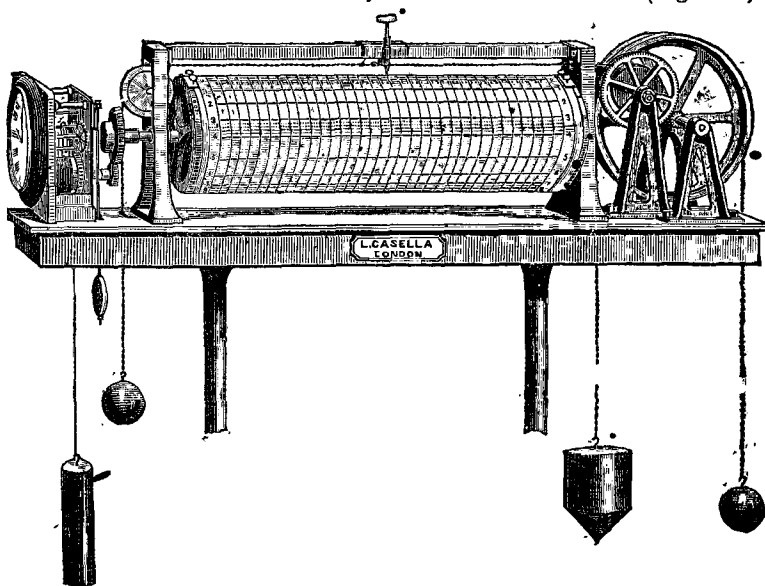


FIG. 812.—Automatic Self-Registering Tide Gauge.

An attendant reads, once or several times in a day, an outer staff gauge, and book the height of water and the time of observation on the paper roll. This staff is connected with bench-marks in the locality by careful levelling and this connection is tested at intervals. To determine from this automatic record the height of mean tide, ordinates are measured from the datum line of the sheet to the tide-curve for each hour of the day throughout the entire period which should be a certain number of entire lunar months. The mean of all the hourly readings for the period may be taken as

mean tide. All readings in stormy weather should be rejected, in that case the entire lunar month ought to be rejected. There is another type of **Automatic Water Gauge** (Fig. 813) in which a printed record of the height and time is made.

846. Instructions for Precise Level Work by the Mississippi River Commission obtained through the courtesy of the Secretary to the Commission and Messrs. W. L. E. Gurley Surveying and Engineering Instrument Makers of the United States of America :—

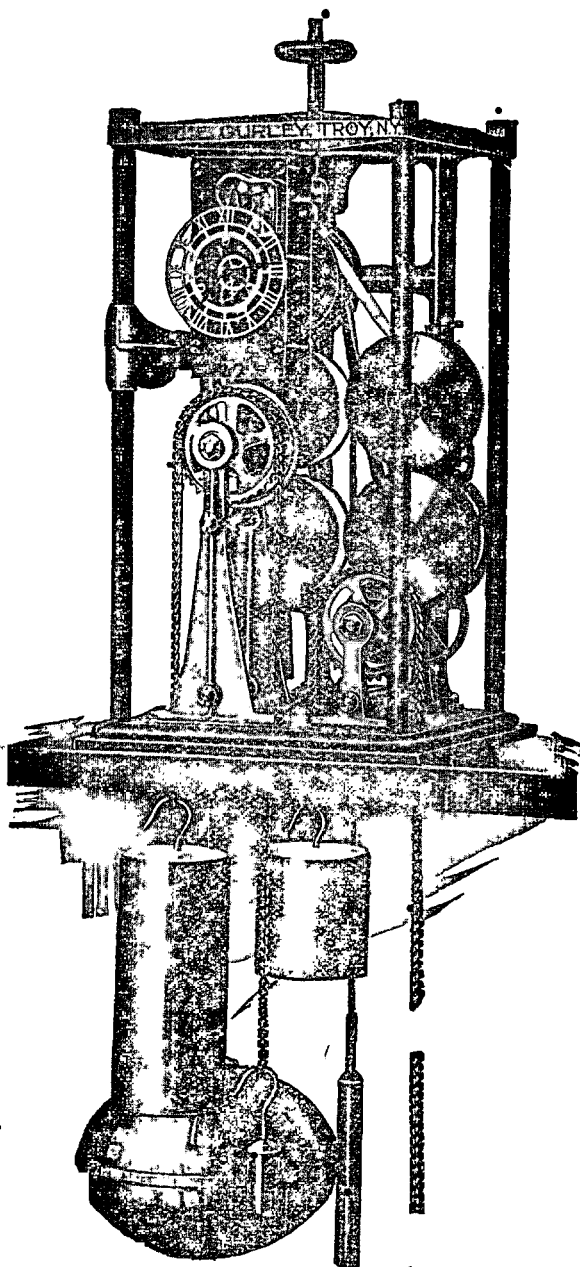


FIG. 813.—Automatic and Autographic Tide Gauge.

APPENDIX 1 L: INSTRUCTIONS FOR PRECISE LEVEL WORK.

OFFICE OF THE SECRETARY, MISSISSIPPI RIVER COMMISSION,
St. Louis, Mo., February 28, 1899.

The following general instructions, amended to date, are issued for the guidance of assistant engineers and others under the orders of this office.

MASON M. PATRICK,
*Captain, Corps of Engineers,
Secretary Mississippi River Commission.*

Instruments.—The instruments used on this work are the Kern levels and rods, similar to those used in the precise levels of Switzerland, and were made by J. Kern, of Aarau. The instrument, with tripod, weighs 22 pounds. It has three steel leveling screws, with enlarged spherical ends at bottom, which are held to the tripod head by clamps fitting over the spherical enlargements. The wye adjustment is by means of a thumb-screw of very fine thread, so that the last final centering of the bubble is made with this screw. The bubble-tube is fixed in a striding metal case covered with wood, with brass ends fitting on the collars of the telescope. The bubble case has a glass cover and a mirror that may be raised to the angle of 45° and so reflect the bubble image to the eye of the observer at the eye-piece. The telescope clamps also hold the bubble case in position. This is removed, however, when the instrument is carried. The telescope gives an inverted image. The telescope has three horizontal wires, all of which are read and the mean taken as the reading of that sight. This also gives a stadia measurement of the distance.

The rods are 3 meters long, made of two pine boards with a T-shaped cross-section, provided with an iron spur at bottom with a flat end, and a watch level by which to hold it vertical. They are graduated to centimeters, and are read by estimation to millimeters. No target is used except at river crossings.

2. Value of Level Divisions.—Before commencing operations the constants of the instruments will be determined. The most important of these is the value of one division of the level-tube. This can best be determined by means of a *level-trier*. It can also be determined in the field as follows: Set up the instrument firmly, mounting it on a wooden post, if possible, or better still, on a stone pier. Set up a rod in its tripod at such a distance that it can be distinctly read through the telescope. The distance should be at least 50m, or, if the air is very still, 100m, and should be carefully measured. Adjust the instrument carefully, taking such length of bubble in the level tube that its ends will be about the middle or tenth graduated line on each side. Direct the telescope to the rod, and by means of the elevation screw cause the bubble to run to near one end of the level. Carefully note the position of the three wires on the rod and the reading of the level. Now, by means of the elevation screw, cause the bubble to run to near the other end of the tube and note the reading of the wires and bubbles as before. One result for value of one division of level can then be obtained. This operation should be repeated ten times. The elevation of the rod should be changed occasionally between sets in order to avoid estimating the same part of the same centimeter on the rod. It will be sufficient to run the bubble five divisions each side of its central position.

If k =distance from instrument to rod, in millimeters, d, d' =distance through which eye and object ends of bubble move when run from near eye-end to object-end, $(d+d')/2$ =amount of displacement of bubble between two readings, r, r' =corresponding means of three thread readings on rod, and v =value of one division of level in seconds of arc, v'' =value of one division of level in millimeters per meter. Then

$$v = \frac{2(r' - r)}{k \sin 1'' (d + d')} ; \text{ and } v' = \frac{2,000(r' - r)}{k(d + d')} .$$

3. **Tables.**—With the value of one division of the level-Tables will be constructed showing the correction to be applied to a rod-reading for an observed inclination of the level and for a distance determined by interval between extreme threads.

If the level-bubble is well-ground equal displacements of the bubble, say of two divisions, will correspond to equal displacements on the rod.

4. **Level fastenings.**—Before using the level or determining its value the fastening of the tube in its case should be examined. One end should be clamped down just tight enough to prevent the tube from moving easily but not tight enough to strain the glass. The other end should be lightly clamped so that the tube may be free to expand and contract with temperature changes. The cotton packing at the ends should not exert a lateral strain on the tube. All level tubes will be numbered and have their numbers marked upon them.

5. **Inequality of Telescope Rings.**—In order to determine the inequality in the telescope rings the instrument should be mounted on a stone pier or other firm support and carefully leveled. The ends of the horizontal bar directly under the wyes should be firmly supported by blocks or large nails to prevent motion of the instrument in a vertical plane. The level should be carefully adjusted, and the instrument clamped to prevent its moving in azimuth. Now, with the eye-piece of the telescope over the elevating screw, note the reading of the bubble when level is set on telescope, both in direct and reversed position. Now reverse the telescope in the wyes and read the level as before. Several sets of observations should be made.

Let b, b' = inclination of telescope as denoted by means of level readings with telescope direct and reversed, then the inequality of rings, $p = \frac{1}{4}(b - b')$ = value of the inequality of rings in millimeters per meter.

• Sixteen determinations of the value of p of two instruments in use on the Lake Survey gave probable errors of $\pm 0''.046$ and $\pm 0''.041$. The inequality may be expressed in seconds of arc if desired, but for purposes of computation is best expressed in millimeters per meter, as it can then be combined directly with the errors of adjustment of level and collimation.

6. **The Centering of the Object-glass** will be examined as follows: Draw out the eye-piece until the threads are no longer visible. Direct the telescope upon some well-defined object, and while looking at it rotate the telescope in its wyes; if the object remains steady the object-glass is sufficiently well centered. Should the object appear unsteady the fault can only be remedied by a maker. The objective should be firmly screwed into the telescope.

7. **The Values of the Wire intervals** will be determined as follows: Set up a rod at carefully measured distances of 10m, 20m, 30m, 100m from the instrument. Read the rod ten times at each distance. The rod may be altered in elevation, the level may be caused to change and the telescope may be rotated 180° (inverted) in order to change the position of the wires, or thread, on the rod. Taking the mean of the ten observed differences of readings of the extreme threads at each station occupied by the rod, a Table will be constructed giving in meters the distance of the rod from the instrument for any observed difference of reading between extreme wires.

8. **Length of Rods.**—Unless the rods used have been previously compared with some known standard they will be compared with each other and their relative lengths determined. This may be done by establishing two fixed points, or two foot-plates, at equal distances from the instrument and differing in elevation about 2.7m. The distance should be about 10m. Determine the difference of elevation of the points by reading each rod on each point. A comparison of the resulting differences of elevation will give relative lengths of meters on rods. Ten measurements with each rod will be determined. The elevation of the instrument will be slightly changed between each set in order to eliminate errors in estimating the millimeters. Each rod will be numbered and have its number marked on it. The rods should also be kept dry and provided with canvas-covers to protect them while being carried to

and from work. The distance, which will be called A , between the zero of graduation of the rod and the bottom of the steel spur on which the rod stands will be carefully determined; this distance should be made exactly the same for each pair of rods by filing off the steel spur if necessary. This may be determined by measuring the distance between bottom of spur and zero or first graduation on rod by the use of a rightangled triangle and scale. It may also be determined by means of another leveling rod, the graduations of which commence at the foot of the rod, by determining the height of the instrument above some fixed point and subtracting it from the reading of the rod to be determined. The relative lengths of the rods must be known. Whenever a bench-mark is connected with in such a way that the rod is not placed directly on the B. M. this quantity, A , enters into the computation of difference of elevation.

9. **Adjustments.**—Before commencing work at any time all adjustments will be carefully made. The telescope will be collimated by having a rod set up at a distance of 50m, and noting the position of the wires on the rod when the telescope is normal, and the bubble centered, and when inverted or rotated 180° about its axis and the bubble centered, then repeat with instrument in first position. Let r = mean rod reading when the telescope is normal. Let r' = mean rod reading when the telescope is inverted.

Let D = distance in meters from instrument to rod. Then collimation error $C = (r' - r)/2D$ in millimeters per meter. The collimation error of the mean of the horizontal threads should not exceed 0.025mm per meter.

The horizontality of the horizontal wires will be examined by moving the telescope in azimuth so that the rod shall appear to move through the field of the telescope. If the threads are horizontal the reading on the rod will be the same, the position of the level, which should be closely watched, remaining the same. If the threads are found to be not horizontal they will be made so by turning the telescope a small amount in the wyes. When the thread wires have once been made horizontal, small screws which abut against projection of wye above elevating screw should be so adjusted that when they press against this projection the wires are horizontal. If the vertical thread is then inclined, as shown by the plumb-line attached to the rod, it must remain so.

To Adjust the striding level, i. e., to make the tangent of the level-tube parallel to the upper surface of the telescope rings, it is necessary to make a lateral adjustment and a vertical adjustment as follows: *

To make the lateral adjustment, raise the clips fastening the level to the telescope and revolve the level about the telescope a short distance each side of the vertical. If the bubble runs in opposite directions when on opposite sides of the vertical, the level is to be adjusted by means of the opposing horizontal screws at one end of the level until such is not the case.

To make the vertical adjustment, set up a rod at a distance of about 50 meters and read the wires with the telescope normal, level direct, and the bubble in the center; reverse the level, center the bubble, and read the rod; then read the rod with the level in the first position, to ascertain if any changes have occurred during the observations. Be careful to eliminate parallax when reading the bubble in the mirror. Let r = mean of rod readings with level direct. Let r' = mean rod readings with level reversed. Let D = distance in meters from instrument to rod. Then error of inclination $I = (r' - r)/2D$ in millimeters per meter.

The error is to be adjusted by the opposing vertical screws at one end of the level case. The error of adjustment must not be allowed to exceed two divisions of the level. * Care must be taken that the telescope rings are free from dust while adjusting the level. After having made the vertical adjustment it will be necessary to examine the lateral adjustment again, since making one of these adjustments affects the other.

To make the tangent of the level-tube and vertical axis of

revolution perpendicular to each other, loosen the small clamp screw at one end of the horizontal bar fastened to the vertical axis and by means of the elevating-screw raise or lower that end of the upper horizontal bar until the telescope can be rotated 180° from any position and have the level reading the same in both positions.

To adjust the level attached to the rod, set up the rod in its tripod in such a position that when a plumb-line is attached to the small hook near the top of the rod the point of the plumb-bob shall coincide with the point of a small cone attached to the rod near its foot. Now bring the level bubble to the center by means of the levelling screws. In making this adjustment the rod should not be exposed to the wind, as the plumb-line is influenced thereby. This adjustment will be made at least once each day.

10. Setting up the Instrument.—Each time that the instrument is set up at a station its axis will first be made vertical by means of the levelling screws in such manner that the telescope may be turned around the horizon without the bubble of the level running a great number of divisions. The telescope is finally made horizontal by means of the elevating-screw. The bubble must be in the center at the moment of observing.

11. Sheltering the Instrument.—The instrument when in use ought always to be sheltered from the sun and wind. It is carried from station to station without being dismounted, but the level should be taken off and carried in the hand. The small clamp screw at the end of the horizontal bar and the large screw which fastens the instrument immovably to the tripod should both be turned tight before moving the instrument.

12. Holding Rods.—The rods must be placed on the plates or pins which accompany them and held in a vertical position as indicated by the spherical level attached. It is advisable to always use the same rod with the same foot-plate. In placing the foot-plates, great care should be taken that they be horizontal, on firm ground, and not liable to change. The surface of the ground, if not firm or level, should be removed. When pins are used in hard ground they should not be driven down hard, but only far enough to be stable; they will rise during observations if they are driven hard.

13. Errors of Adjustment.—The errors of adjustment of collimation and inclination will be determined at beginning and end of each series of observations—that is to say, after having mounted the instrument and before dismounting it, and in all cases at least once each day. If the instrument has been deranged by a jar the corrections must be determined anew.

The algebraic sum of the errors of adjustment must not exceed two level divisions, and commonly should not exceed one. All the details of the determination of the errors of judgment must be entered in the note-book in their proper places. It is always advisable to have the errors of adjustment as small as possible, and necessary that they be well-determined. The time of making these determinations will be recorded in the note-book.

14. Duplicating Work.—In all work along the main line of levels each observer will duplicate his own work by running over the line in opposite directions, preferably under similar conditions as to illuminations, etc.

15. Order of using Rods.—While connecting two bench-marks the order of using the rods will be as follows:—In Fig. 814, let I, I^1, I^2 , etc., represent the successive stations occupied by the

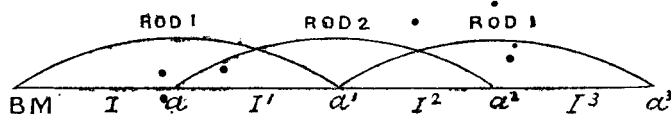


FIG. 814.—Order of using Rods in Precise Levelling.

instrument. B.M., a^1, a^2 , etc., occupied by rod 1, and a, a^2 , etc., the positions

occupied by rod 2. The instrument being set up at I, rod 1 is placed on B. M. and rod 2 at a , making the distance $1-a$ equal to $I-B.M.$ Rod 1 is then read, and immediately afterwards rod 2. The time elapsing between these readings should not exceed five minutes and commonly will not exceed one minute. The instrument is then carried to I^1 , and rod 1 to a^1 , the distance $a - I^1$ and $I^1 - a^1$ being equal. Rod 1 will then be read and immediately afterwards rod 2. The instrument will then be taken to I^2 and the rods read in the order 1, 2. Work will be continued in this manner until the other bench-mark is reached. Rod 1 must be placed upon this bench-mark, which will be the regular order if there have been an even number of instrument stations. If there have been an odd number of instrument stations, at the last station use rod 1 for both back-sight and fore-sight.

If the lengths of meters on rods are the same, and the values of A of the rods are equal, the stretch may be closed on either rod. While levelling, the rate of progress in favorable weather will be about 1 kilometer per hour.

In order to reduce the discrepancies due to settling of the instrument, the order of reading the rods will alternate. That is, at I the back-sight will be read first and then the foresight; at I^1 the fore-sight will be read first and then the back-sight, and so on throughout the length of the line.

16. Order of Observing.—After having properly leveled the instrument at any station, always making the vertical wire occupy the same relative position with regard to the center line of the rod, the observations will be made and recorded in the following order: The bubble of the level being kept in the center while threads are being read. First read the level, then the position of the threads on the rod will be read, the millimeters being estimated, and finally the level will be read again. The observer will then read the rod a second time to make sure that no error has been made. The recorder will then take the differences between the readings of the middle and extreme wires to guard against errors, and if these differences denote any error the observations must be repeated. If an error exists, it will be shown by too great a difference between the differences. This is a most important check and must not be neglected. These differences will also serve as a check upon the distances between the instrument and rods. *The recorder should also check the level readings, when bubble is not centered, to make sure that errors of whole divisions have not been made. This may be done by summing up the readings and noticing the length of the bubble.*

17. In Reading the Level by means of the mirror, care should be taken that the position of the eye is such that there will be parallax. Such positions can be determined once for all when the mirror is at its greatest angle of elevation, by a second person reading the level directly while the observer finds the position from which the reading of the level in the mirror is the same.

18. Form of Field-notes.—The notes will be kept in the form given in note-books, as shown in the sample below. When once a number has been written down it must not be erased or made illegible. If wrong, a line will be drawn through it and the correct number written above. (Left-hand page.) Date, October 22, 1898.

Back-sight.

Thread readings.	Means.	Difference of threads.	Level.		Rod.	Remarks.
			Eye.	Object.		
1196.....		128				T. B. M. 249.
1324.....	1320.3	117	15.5	15.5	18	
1441.....		245				

Observer, W. S. Williams. Recorder, E. E. Whitehead. (Right-hand page.) Level No. 4. Tube No. 214 Fauth.

Fore-sight.

Thread readings.	Means.	Difference of threads.	Level.		Rod.	Remarks.
			Eye.	Object.		
1068.....		127				
1195 - ...	1192 0	118	15 5	15 5	19	
1313.....		245				

19. **The Length of Sights** taken will depend upon the condition of the atmosphere, but the rod should always be near enough to be seen distinctly. It will be seldom that lengths of sights greater than 100 meters can be taken. The back-sight and fore-sight corresponding to any instrument station must not differ in length by more than 4 meters and the sum of the lengths of the back-sights and fore-sights between any two bench-marks should be equal. If the observer has the slightest cause to think the instrument has been in the least disturbed, or that the refraction has changed between the times of reading the two rods, he should repeat the observation until satisfactory results are obtained.

20. **Crossing Streams.**—Whenever it is necessary that the line of levels should cross a river or other wide obstruction, a narrow place should be chosen. Firm points should be set upon the two banks, levels in good adjustment are set up on opposite sides of the stream about 10 meters from each bench-mark, and both levels go through the same operation. The distant rod will require a target. Each observer first reads on the bench-mark nearest him, once with the telescope normal and level direct, and once with the telescope inverted and level reversed; then takes simultaneous readings on the rod across the river with the instrument in the following positions:

1. Telescope normal, level direct.
2. Telescope normal, level reversed.
3. Telescope inverted, level direct.
4. Telescope inverted, level reversed.
5. Telescope inverted, level reversed.
6. Telescope inverted, level direct.
7. Telescope normal, level direct.
8. Telescope normal, level reversed.

These eight readings constitute one set for the direct run. Then take eight more readings in the same order for the reverse run, then each observer turns to the nearest bench-mark and reads as he did at the beginning. Observers and instruments change places and repeat the observations at these new stations. The simultaneous levels eliminate refraction; the change of station eliminates curvature and small instrumental errors for each individual instrument. Unless good results are obtained the observations should be repeated until the results obtained come within the prescribed limits of error. If but one level can be used, the operation will be performed in the same order, but the time occupied in crossing must be as small as possible.

21. **Permanent Bench-marks.**—Permanent bench-marks will be established at intervals of 3 miles along the river and 5 miles on lines connecting the river line proper with the other levels or bench-marks.

These bench-marks will consist of a thoroughly vitrified tile 4 by 18 by 18 inches, placed 3 feet below the surface of the ground and surmounted by a 4-inch wrought iron pipe as a surface mark. The tile should have time to settle before leveling to it. Both tile and pipe will be suitably marked to designate the character of the point. In the center of the upper surface of the tile a copper bolt will be leaded, the upper surface of which will be the point of reference. These bench-marks will be placed where they can be easily found and where they will not be disturbed. Property corners should be utilized where practicable. In addition to the above, benches should be established on permanent brick or stone structures by leading into them a horizontal or vertical copper bolt with the letters U. S. P. B. M. cut near it. A small hole in the center of the bolt will be the point of reference.

22. In Connecting with a Bench mark if the bolt is vertical the foot of the rod is placed directly upon it. If the bolt is horizontal in the wall of a building or other structure, it will be connected with in the following manner: Set up the instrument, in such a position and at such an elevation that the small hole in the bolt may be bisected by the middle thread without displacing the level by more than five divisions, using the elevating screw for making this bisection. Since the instrument can be raised or lowered about 2 centimeters by means of the levelling screws, the instrument can be placed in such a position by two or three trials. Now bisect the bench-mark with the telescope normal and also inverted, noting the reading of the level. Read the rod on the plate with the telescope in both positions and the bubble in the center. It is necessary to eliminate collimation of the middle wire by inverting the telescope, since the collimation of the middle wire is not the same as that of the three wires. The quantity A (distance between the zero of the rod and the bottom of the spur) must be taken into account when a bench-mark is connected with in this manner. The distance of bench-mark from instrument must be determined and recorded.

23. Closing Work for the Day.—Whenever work is stopped at least two temporary bench-marks should be established. These will consist of large nails or spikes driven their entire length vertically into the base of trees or in the tops of sound stakes. The mean elevation of these two benches will be used in continuing the work.

24. Describing Bench-marks.—Every bench-mark will be fully described in the note-book and a careful sketch will be made of its surroundings. Its position with reference to the most prominent objects near it should be given by distance and direction. Public buildings, such as depots, court-houses, churches, etc., are the best positions for permanent bench-marks. In a village or town several permanent bench-marks should be established to insure at least one being always available.

25. Determine Elevation of Base of Rail.—If a railroad is crossed the elevation of the base of the rail will be determined, and if levelling along a railroad the elevation of the base of the rail at depots will be determined.

26. Gauge Zeros.—The elevation of the zeros of all water-gauges and also the gauge bench-marks will be determined.

27. Datum-planes of Cities.—The main line of levels will be connected with the datum-planes of cities along the line and the elevations of these planes will be deduced.

28. United States Engineer Bench-marks.—The main line of levels will be connected with all accessible important United States Engineer bench-marks.

In reducing the observations the nearest tenth of a millimeter will be retained. The distance will be taken out from the Table to the nearest meter.

29. The Limit of Discrepancy in closing a Polygon will be: $3\text{mm} \sqrt{\text{distance in kilometers}}$. The distance referred to is the entire length of the polygon from B. M. 1 to B. M. 2 and back to B. M. 1, and the limit of discrepancy refers to the polygons between successive bench-marks. If the discrepancy exceeds the prescribed limit then the entire line must be re-run one or more times, or until the difference of the direct and reverse results is within the limit. In re-running lines the third line run should alternate between direct and reverse directions so as to keep the total number of direct and reverse lines equal.

Field reductions of all precise level notes will be made and checked at the end of each day's work, so as to ascertain before leaving the ground whether the results come within the prescribed limits.

CHAPTER XVII.—TOPOGRAPHY BY MEANS OF CONTOURS.

847. Definitions.—A *topographical survey* is one which gives not only the positions of the points and objects on the surface of the ground but also furnishes the data from which the nature of the surface may be delineated with respect to the relative elevations and depressions, and a *topographical map* is a graphic delineation of the results obtained.

848. The best practical method of representing topography is by means of "contour-maps." A *contour line* or *contour* is the representation on the plane of the paper of the intersection of a horizontal plane with the surface of the ground surveyed. If a series of horizontal planes be assumed at equal distances vertically apart, they will intersect an irregular or inclined surface in lines; or if a water level be supposed to be raised or lowered by equal stages, the successive water-lines or edges thus formed will represent the contour lines of the surface. These contours are shown by projecting them upon a horizontal plane, i.e., the positions of these horizontal lines are surveyed and plotted on a map in the same way as any other boundary lines. Let the plan in Fig. 815 represent the contoured map of

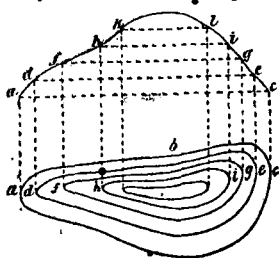


FIG 815.—Contour Map of a Hill.

of a hill, the figure above is a sectional elevation parallel to the line *ac*. Then each of the curved lines in plan would be seen in elevation as follows:—*abc* as *ac*; *de* as *de*; *fg* as *fg*, etc., the vertical heights between *a* and *d*, *d* and *f*, *f* and *h*, etc., being all equal, say, 1, 3, 5, 10, 20, 50, 100ft, apart beginning with the datum plane which is usually taken the lowest point in the surface of the tract surveyed. Mean Sea-level should always be used when an accurate connection with the sea can be obtained, since the mean of the rise and fall of the tides is approximately the mean level of the sea. The curved lines represented on the map are called contour lines. It is evident that for a given vertical distance apart, the steeper the slope the nearer the contour lines will be on the map, while the flatter the slope the further apart the contour lines will be. In the case of a vertical cliff the lines would be shown directly superimposed. A few main principles may be stated that will assist the student in becoming expert in this art.

1. All points in one contour line have the same reduced level above the datum.
2. The contours must be equally spaced when ground is uniformly sloping, and where the ground is plane they must also be straight and parallel.
3. Contours never split, as shown in

Fig. 816; nor do two contours run into one, as shown in Fig. 817; nor cross each other, except in the rare instance of an overhanging cliff



FIG. 816.—Contour that never splits.

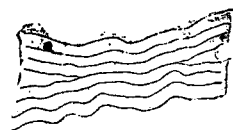


FIG. 817.—Two contours that never run into one.

as shown in Fig. 818. When an auxiliary contour is introduced, no more of it is drawn than is sufficient to delineate the special feature which makes it necessary. A principal contour cannot have an end within the map, if it commences at one edge it must terminate at another.



FIG. 818.—Contours for overhanging cliff.

4. A closed contour encircled by one or more closed contours is either a hill, as shown in Fig. 819; or a depression or hollow as shown in Fig. 820, the arrows showing the direction along which water would run. The summits of all hills of importance should have their elevations determined and marked on the map. All depressions without an outlet and which do not contain a pond or lake should be marked with a D at their lowest point. 5. A series of contours is either a croupe (the end of a ridge or shoulder or promontory), as shown in Fig. 821, or a valley as shown in Fig. 822. If a *croupe*, the contours will have their concave sides towards the higher ground; if a *valley*, the contours will have their concave sides towards the lower ground. 6. A combination of four sets, like Fig. 823, with convex sides turned towards each other, represents a dip in a ridge, or the junction of two ridges, and is called

FIG. 819.



HILL

FIG. 820.



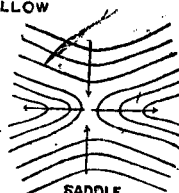
HOLLOW



SHOULDER



VALLEY



SADDLE

FIG. 821.

FIG. 822.

FIG. 823.

a *saddle*, and in case of a mountain range it takes the form of a *pass*.

849. The Contour Intervals, i.e., the vertical distances between contour lines are taken 3, 5, 10, 20, 50, or 100 ft, apart beginning with the datum. They are fixed by the object of the survey. the

amount of irregularity of the surface, and the scale of the map. When, however, the contour runs very near to some remarkable feature of ground, as a prominent spur or indentation, a slight deviation above or below its true plane is admissible to include this feature although it is preferable to avoid doing so, if possible, by the introduction of an auxiliary curve. In abruptly mountainous and comparatively inaccessible regions where sketching must be relied upon 100-foot contours may suffice to develop all necessary features.

For Engineering purposes, such as, for the plan of a railway, the contour interval may be 5 ft., small reservoirs 3 ft., and for large ones as much as 25 ft., according to the scale employed. By diminishing the vertical interval, *i. e.*, equidistance, the delineation of the undulations become more accurate. It will therefore vary with the scale of the plan, and the nature of the country surveyed; and the larger the scale, the smaller the vertical interval. In France, as a rule, the ratio between the vertical interval and the denominator of the scale is constant and $= \frac{1}{2000}$, a great advantage is therefore gained, since at whatever scale a plan is made, the same inclination will always be obtained by contours equally distant. At the scale of $\frac{1}{10000}$ the sections are 5 metres apart; at $\frac{1}{20000}$, 10 metres, and so on.

850. Determination of Contours.—There are several methods of fixing contours or lines of equal elevation:—

The first method consists in determining the contours on the ground at once, each contour being located on the ground with the level reading equal elevations only, driving in pegs where the contour bends. The contours thus set out on the ground are then surveyed by any of the survey methods, traversing or planetabling. On long strip of ground, such as that required for the location of a road or railway, cross-sections in the line of the steepest slope are taken at every $\frac{1}{2}$ or $\frac{3}{4}$ mile as shown at CD and EB (Fig. 824). Pegs of the desired vertical intervals are then set by means of the level on these lines. The contours are then located and marked by pegs, the level giving equal reading for each contour between the corresponding points on CD and EB. Thus contour 30 is marked by levelling from 30 on EB to 30 on CD. Having thus located the pegs of equal elevation for each contour marked $\times$ in Fig. 824 these points are then surveyed and plotted on the plan.

Contouring by Water-level.—When the equal vertical intervals have been fixed by means of the dumpy or Y-level the contours can be located by means of the Water level (Fig. 727, p. 645). The water in both the phials of the instrument is in free communication, and the water surfaces on sighting along them to a staff with a vane set to the height of the eye of the observer give a horizontal line in whichever direction the tube is turned by which the vane of the levelling-staff is adjusted.

Second Method.—By Interpolating Contours.—This method consists in fixing the highest and lowest contours only on the ground and interpolating for the intermediate ones. The ordinary

method of doing this is from cross-sections which having been taken and plotted, the contours are plotted on the plan by their aid as

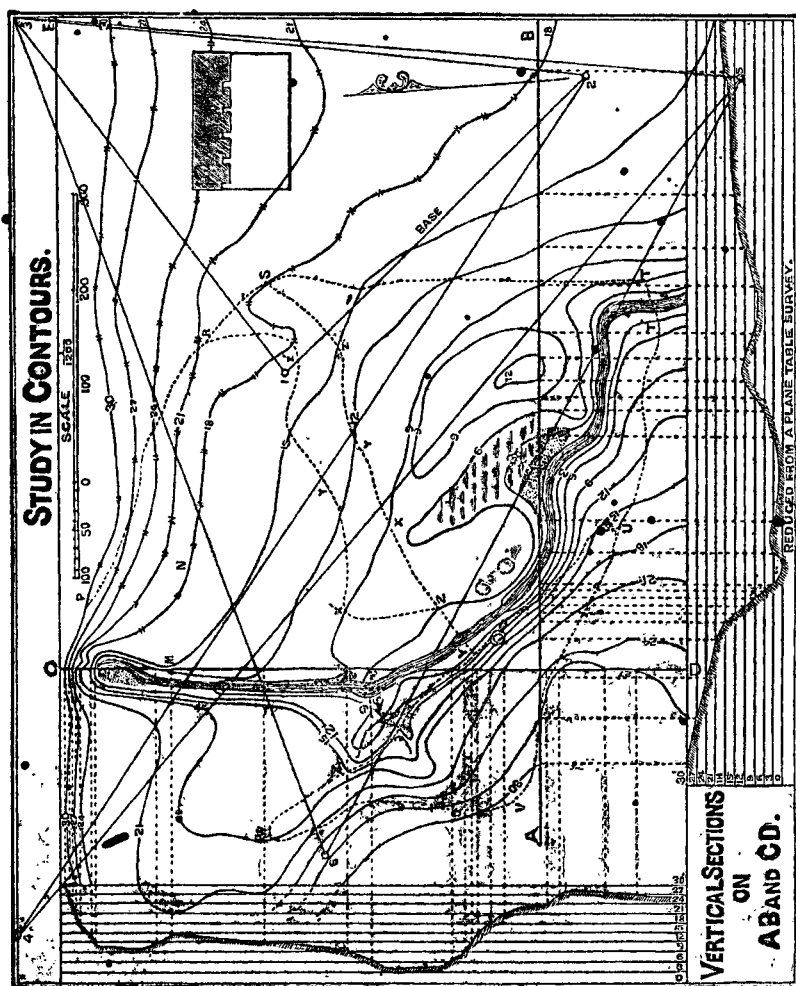


FIG. 824.—Determination of and Study in Contours.

under:—Fig. 825 shows, for example, a cross-section across the centre line of a railway. Draw a vertical line AB on the cross-section through C, the point on the centre line of the railway, as shown on the plan, and of which the reduced level is 172.60, and that contours are required to be shown with vertical intervals of 5 ft. It is plain that the first contour above the centre line of the railway will be 175.0 ft. contour, and plot therefore $Ce = 175.00 - 172.60 =$

2'40 ft., and ed, dc, cb, ba each equalling 5 ft. Similarly the 170 ft. contour will form the first contour below the centre line, therefore plot $Cf = 172'60 - 170'00 = 2'60$, and fg, gh, hi, ij, jk, each equaling 5 ft. **The reduced levels** of the points a, b, c, d, e, f, g, h, i, j, k, will then be 195'00, 190'00, 185'00, 180'00, 175'00, 170'00, 165'00, 160'00,

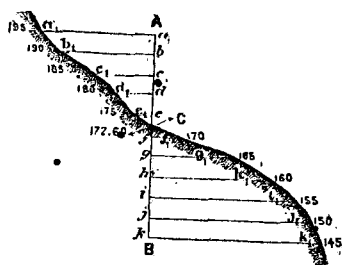


FIG. 825.—Interpolating Contours.

155'00, 150'00, 145'00 respectively; and the horizontal lines aa', bb', cc', dd', ee', etc., will give the horizontal distances from the centre line at C of the railway to the 195, 190, 185, 180, 175, etc. contours. These distances when plotted square off the centre line on the plan will fix the positions of the contours at the point C. Each cross-section is similarly dealt with and the positions of the contours marked on the plan

Plotting the Work.—The outline is first plotted by the ordinary methods and the parallel lines with the contour points drawn, applying at the same time the check described in the previous article. Contours are then drawn in black by joining points of equal elevation on the successive parallel lines. The elevation of each contour should be written directly on it and for this purpose sufficient number of breaks perpendicular to the contours should be left. The lines can be drawn conveniently with a curve pen (Fig. 435) or an ordinary drawing pen (Fig. 429). They are best put in free hand.

Third Method—The position and heights of a number of characteristic points of the terrene are determined, and with these as guides tracing the contour lines. It has the merit that the development of the terrene proceeds with the survey of the map, and does not necessitate a return to a station when once occupied. In connection with the determination of position by **re-section** (*q. v.*, p. 283) it works harmoniously and economically, since points that would be selected for position as having the best outlook are likely to be the characteristic ones of the terrene. In order to obtain data for a topographic map, the first step will be to survey the boundary of the tract to be mapped before the proceedings to be taken as above stated.

Fourth Method.—This consists in surveying and levelling the base of each level-section by the aid of the plane table with telescopic alidade. To determine the base of each level-section, the table is set up in position where this level intersects the profile, and using the alidade as a levelling instrument, with a target fixed on the staff at the height of the optic axis of the telescope, the line is traced by locating the staff in successive positions at characteristic points of the terrene, when the target comes in the horizontal plane of the optic axis, direction and distance of the staff being determined and

drawn in each case. A line drawn through these points locates the curve. In this operation allowance should be made for curvature and refraction, when the distance becomes sufficiently great to make it a factor.

Fifth Method.—Surveying and levelling the parts of several level sections from one station.—When parts of several level sections are run from one station, set up the table at a point on a contour, and observe on a staff the height of the optic axis of the alidade. Set a target on the staff above this height as many contour intervals as its length will include. The aid carries the staff below the instrument and is signalled to stop when the target comes in the horizontal plane of the optic axis, at successive steps traverses the lower curve. The target is then lowered on the staff one contour interval, and the next curve above is traced in the same manner, continuing the proceeding until the level of the instrument is reached, when the table is moved to an upper station, and the proceeding continued until the summit is reached. (Applies only to very small contour intervals).

Sixth Method.—This consists in dividing the ground into squares, triangles, or parallelograms. By this method pegs are driven at regular intervals, and heights determined of the rectangle corner and intermediate changes of slope by level in the way that may be most convenient, a spirit-level instrument being the most accurate.

851. Contour Sketching.—The topographer will be assisted in sketching contours, where the modelling of the ground is intricate, by lightly drawing a skeleton composed of the ridge lines and lowest lines of valleys in their proper positions around the station. On the ridge lines will be found the extreme convex bends of the contours, and on the lowest valley lines the extreme concave bends. It can be readily imagined that if each spur and each small depression was represented by its approximate line, and on each of them were located, either by observation or estimation, points having elevations equal to some multiple of the contour interval, it would be only necessary to connect those points having the same elevation with a smooth curve to have a correct plan of the contours. It will simplify the sketching at a station to draw the highest, lowest, and middle contours first, as they will then serve as guides for estimating the position of the others. In a contoured map the levels of the contours are to be marked against them to show which are the lowest and highest and also to ascertain their elevations.

852. Method of Topographical Survey for Small Areas.—If a topographic survey is required for a small area, perhaps a half a mile wide, the first step to be taken will be to divide one or more sides into 50 or 100 ft. stations, a peg driven at each with the appropriate number. A *Rough sketch* in the note-book should be made indicating the locations and numbers of the pegs. Next level over these pegs with an Engineer's level obtaining reduced levels over each, and continuing the levelling operation round the entire boundary of

the cheapest road that can be built upon the given conditions. Crossing the stream at TT^1 , upon a level 6 ft. above the water to reach the top of the hill at V the grade must be increased to 5—100 or 1—20 or 3 in 60; and the centre line is located in the same manner; but with a radius of 60'. Descending from V to W , the grade is the same; but from W to X it is reduced to 1—40 to prevent cutting at X . From Z back to P two routes are shown, which are the shortest for the given grade 3—100, although a third might be located following the stream to a point M , where it must be diverted to N , and thence zig-zag up the hill, but it would be longer than the others. The two distances $Zxy zSR$ and $Zx'y'z'R$ are equal; but the first gives a more graceful curve, and will require less construction, the part SR being supposed to have been already built for the descending grade.

854. Computation of Contents from Contours.—The cubic contents of a hill or mound to be excavated, or a hollow to be filled in, or the cubic capacity of a reservoir, etc., may be obtained from contours, the contents between successive contours being practically equal to the product of the mean of the areas of the upper and lower contour surfaces and the vertical interval. See Author's "Practical Earthwork Tables and Calculations of Banks and Cuts made Easy", p. 149, for "Estimate by Means of Contours of a Rectangular Reservoir on a Hill side, etc.

855. Profile or Longitudinal Section.—As each curve has equal vertical ordinates at all points, the elevation or profile of a hill, as well as a model in relief can be constructed from the map, when it is accurately executed on a large scale, without further field measurements. A profile or section of a hill is the outline or trace formed with its surface by a vertical plane cutting the hill in any direction. Similarly, the profile of a hollow can be drawn without much difficulty from a contour map. Fig. 824 shows the profiles through the lines AB and CD of a hollow as represented on a topographic map. The full parallel horizontal lines upon the profile represent the successive heights or sections of the hollow of 3'. A reference to the letters of the diagram is all that is necessary to a full understanding of the subject: the 30' lines on the sections show the high-water lines on the map and 3' ones the lowest points of the watercourse shown. A mound is shown by the closed curve marked 12. It will be plain that if we were to suppose the water to go down by 3' below the high-water line, the 27' curve upon the map would be on the water-line; and if the water were to go down by another 3', the 24' curve would form the shore line, and so on. It will now be clear that if horizontal lines through the points of equal vertical intervals of 3, 6, 9, 12, 15, 18, 21, 24, 27 and 30 feet be drawn parallel to the sectional lines AB and CD , as shown in the sections, and if lines be drawn from the points of intersection of the contours with the sectional lines on plan at right angles to the corresponding horizontal lines through the points of equal vertical intervals and if these points are joined sections of the hollow will be obtained as shown in the figure.

856. Interpolation of Contours by Proportion.—When reduced levels and positions of principal points are plotted on a plan, points of any intermediate level corresponding to any required contour may be obtained by proportion making the horizontal distances equal between succeeding contours. Thus in Fig. 828 the position of the points A, B, C, D, E, F, G, H is shown. Suppose the position of the 65 contour between E and A is required to be plotted, the distance AE being 120 ft. The total amount of fall between A and E being 20 ft., the distance in which the fall is 10 ft. equals $120 \times \frac{1}{2} = 60$ ft., or the 65 ft. contour lies at a distance of 60 ft. from A towards E. Similarly any number of contour points can be interpolated when the levels of a certain number of points are on the plan.

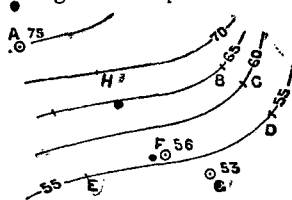


FIG. 828.—Interpolation of Contours by Proportion.

857. Interpolation of Contours with the Sector.—A sector (Fig. 421, p. 410) may be employed to interpolate the contours between two points (calling them X and Y, the latter being the higher) of which the reduced levels are known. The horizontal distance between the two points is to be taken off on a pair of dividers and the sector opened until the distance between the points on its arms numbered to correspond with the difference in elevation of the two points X and Y is just spanned by the dividers. The sector is now set for interpolation. For example, to obtain the horizontal distance from X to a point 13 feet higher it is only necessary to spread the dividers between the points marked 13 on the arms of the sector and take off that distance on the map from X towards Y.

858. Representation of Ground: Relief.—There are two methods of representing it—by hill-shading and contours.

Hill-shading is generally effected by a system of lines, called **hachures**, drawn in the direction of the slope, i. e., perpendicular to the contour lines. When it is steep, the hachures are thick and closely spaced. On the other hand, a gentle incline will be indicated by fine lines widely separated.

859. Advantages and Disadvantages of Hill-shading and Contours.—In a mountainous country the method of hill-shading presents a picture which expresses more forcibly to the eye the configuration of the country than a system of contours. But the objection to its sole use arises from the fact that although one ridge is perceived to be higher than another, there is no guide for stating in terms of some linear unit this difference in elevation. It also obscures the symbols representing other details on the surface. A system of contours furnishes a convenient means for obtaining the heights on any part of a map, but does not adapt itself to the representation of the small but important accidents of the ground, such as gullies, ledges, rocks, etc.; nor does it satisfactorily delineate

such features as cliffs, bluffs, quarries, railroad cuts and embankments. For these reasons both methods should be adopted, employing hachures for the smaller features, and where the steepness of the slope would make the contour lines approach together so closely that individual lines would become indistinguishable, and relying on the contours to delineate less precipitous ground. The two systems can be seen combined when it is necessary to indicate a rocky and broken mountain face.

860. Order of Development of Contours.—As the progress of topographic work is usually from the shore line inward, this affords

the most favourable direction for drawing the curves of equal elevation, and as it is desirable that all work at a station shall be completed when it is first occupied to avoid the necessity of returning to it, the curves should be drawn by estimation from the shore line to the points sighted and determined for position and height, to be checked by drawing from those points when in turn occupied. The heights of a sufficient number of points must be determined to guard against any wide range of estimate of height by the eye. In abrupt slopes of considerable extent the use of pocket clinometer is valuable in determining the degree of slope, and in order to draw curves by the width of their zones (cosines of angles of slope) from a paper scale prepared for the purpose (Fig. 829).

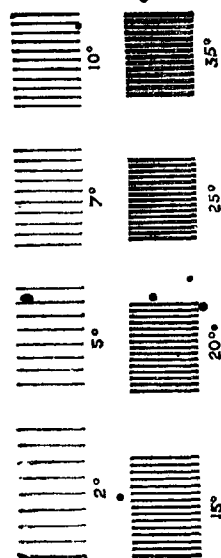


FIG. 829—Example of Scale of Shades for Slopes.

Filling in.—Having completed the work at a given station, the topographer proceeds with his party and instruments to an adjoining locality, where he selects a new station from which he can gather the details of an area bordering upon the one last surveyed. In this manner the skeleton map is filled in by successively occupying stations over the whole expanse of the sheet.

861. Theory of Shading Ground from Light Falling Vertically.—The theory of shade supposes the light to fall upon the ground vertically in parallel rays, and the steeper slopes, receiving the rays at a more oblique angle than gentler ones do, are lighted less than the latter and therefore must be indicated on plan by a darker shade; while horizontal grounds which receive most light falling perpendicularly are left unshaded in the plan. Practically shading never can give the true height of a hill as it cannot be exactly delineated. From Fig. 830, it will be seen that the slopes ac , ad , ae , being at 30° , 45° , 60° , $ac \cos. a_1 = ad \cos. a_2 = ae \cos. a_3$. Now, referring to Fig. 830, as the steeper and longer the slope is, the less lighted it will be, for it is seen that the same amount of

vertical light falls on each slope, and therefore the amount of light is directly proportional to the cosine of the angle of slope.

The different shades may be put in with Sepia or Indian ink, the colours being made light for flatter slopes and dark for steeper ones. The shade may also be effected by horizontal contours making them numerous on steep slopes and less on flat ones. The most usual method of shading is to make the thickness and distance between the hatchings proportional to the angle of the slope as above described.

French Method.—By this method the degree of slope is indicated by varying the spaces between the centres of hatchings. The rule is:—"The distance between the centres of the hatchings equals $\frac{1}{100}$ in. $+\frac{1}{4}$ of the denominator of the fraction denoting the slope (i.e., the tangent of the angle of slope) in $\frac{1}{100}$ in." For flat slopes the lines are made fine and for steeper ones thicker lines. Only slopes from $\frac{1}{100}$ to $\frac{1}{25}$ are indicated by this method.

Lehmann's Method.—This method consists of nine shades for slopes from 0° to 45° , the first being white and the last absolute black. The ratio of black to white is equal to the ratio of the angle of a slope to its supplement to 45° .

$$\text{Thus, for a slope of } 35^\circ, \frac{\text{White}}{\text{Black}} = \frac{45^\circ - 35^\circ}{35^\circ} = \frac{10}{35} = \frac{2}{7}.$$

In this method the features of the ground are strongly marked but the tints are too dark, and it is often difficult to read the small writing and see the details.

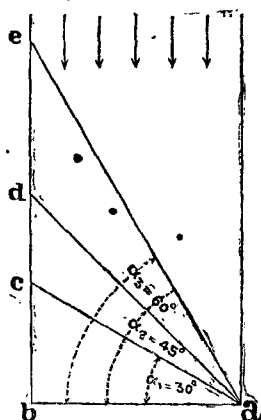


FIG. 830.—Theory of Shading slopes.

Only slopes from $\frac{1}{100}$ to $\frac{1}{25}$

CHAPTER XVIII.—CURVE-RANGING.

862. Curve-Ranging may be defined as deflecting by degrees, and is effected either with the chain only, or with the transit and chain as being *par excellence* curve-ranging. To take all round, curve-ranging with the chain only is, as compared with the work by the transit and chain, or the tachometer, poor and unreliable.

863. Definitions :—Railway curves are mostly circular, though parabolic curves are sometimes used. A circular railway curve, A I J K L M B (Fig. 831), is an arc of a circle joining the two straight lines or intersecting tangents XAE and YBE, in the survey.

The Point of Curvature, or P. C., is the starting point, A, of the curve, or that end of it first reached by the survey in its progress.

Apex Distance.—The line EN joining, E, the point of intersection and the crown, or summit, or apex, N, of the curve.

Circular Curves may be *simple* or *Compound*. A *Simple Curve* may be defined, as a single circular curve uniting two tangents meeting at an angle. A *Compound Curve* is one in which a curve of one radius runs into another of different radius but of similar flexure.

A *Serpentine* or *Reverse* or *Ogee Curve* is one in which a curve of one flexure runs into another of different flexure. A serpentine curve is also a compound curve. A *Curve of deviation* or *Diversion curve* is a double compound curve and is one which starts from the straight and returns to the same straight again.

It is always preferable to have a piece of straight between the P. T. of one curve and the P. C. of another to allow for the change in super-elevation of the outer over the inner rail.

The *Transition curve* is a means of toning down the abrupt passage from a tangent to a circular curve.

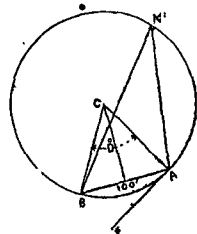
The advantages of transition curves are: (1) less wear upon the running gear; (2) Less discomfort to the passengers; (3) danger of derailment obviated.

A Curve as laid down forms the centre line between the rails.

864. Nomenclature of Curves.—A curve is designated (1) by the length of its radius in feet, links or metres as in England; or (2) by its degree of curvature in degrees, minutes and seconds (in ordinary practice 15 seconds will be found accurate enough), as in India, the U. S. A., and South America. It is found convenient to classify them according to the angles at the centre subtended by a chord of 100 feet on the curve, or 100 links, or 20 metres. The degree of a curve, D° , is the angle subtended at the centre of the curve by a chord of 100', or 100 links, or 20 metres = 2 dekameters (where the metric system is in use). Thus a **circular curve** is known as a one, two, or a three-and-a-half degree curve as its 100-foot, or 100-links, or 20-metre chord subtends at the centre an angle of 1° , 2° or $3\frac{1}{2}^\circ$.

The Tangential angle for 100'-chain chord under the *degree system* is therefore half the *degree* of the curve. The reason for this is clear. When the *total number of degrees* contained in the *whole deflection angle* have been turned the point of tangency will have been reached and the number of chain lengths contained in the curve will be obtained by dividing the *whole angle of deflection* by the degree of the curve.

865. Connection between the Degree of a Curve with its Radius.—Let D represent the degree of the curve (Fig. 832), R its radius, CA



or CB , then

$$R \sin. \frac{1}{2} D^\circ = 50' \text{ or links; } \dots (1)$$

$$\text{or } R \sin. \frac{1}{2} D^\circ = 10 \text{ metres } \dots (2)$$

Ex. 1. Find the radius for a 2° curve with a 100' or 20 metre chord.

FIG. 832.

$$R = 50 / \sin 1^\circ = 50 / 0.01745 = 2865.32 \text{ feet.} = 2865 \text{ ft. say ;}$$

$$\text{or } R = 10 / \sin 1^\circ = 10 / 0.01745 = 773 \text{ metres.}$$

The radius of a 1° curve having a 100' chord is 5729.56'

$$\therefore \frac{1}{360^\circ} = \frac{1^\circ}{2\pi r} ; \therefore r = \frac{50 \times 360}{3.1416} = 5729.56 = 5730' \text{ say.}$$

The radius of a 2° curve is half of that of 1° curve = 2865' ;

„ 3° „ one-third „ „ „ = 1910' ;

„ 4° „ one-fourth „ „ „ = 1432.5' ;
and so on.

866. Geometrical Theorems.—The methods of curve-ranging with the theodolite depend on the two following theorems :

(1) The angle formed by a tangent and a chord equals half of the central angle subtended by the chord ;

(2) The angle at the centre of a circle is double the angle at the circumference standing on the same arc (Euc. III, Prop. 20) ; thus, in Fig. 832, $\angle BAF = \frac{1}{2} \angle ACB$; and $\angle ANB = \frac{1}{2} \angle ACB$.

867. Properties of Circular Curves.—The following are the most useful problems occurring in practice (Fig. 831). Let the whole deflection $\angle \Delta$ be denoted by $\angle FEY$, the radius by AC or CB, the tangent distance or sub-tangent by EA or EB, and the apex distance by EN.

1. Prove that $\angle FEY = \angle ACB$.

Proof.—In the quadrilateral, EACB, $\angle$ s EAC and EBC are right angles, but the internal angles of any quadrilateral are together equal to four right angles. Therefore $\angle AEB + \angle ACB$ are equal to two right angles. But $\angle FEY + \angle AEB$ are also equal to two right angles ; hence equating and eliminating $\angle AEB$, the $\angle FEY = \angle ACB$.

2. Prove that the $\angle EAB = \frac{1}{2} \angle FEY = \frac{1}{2} \angle ACB = \angle ACE$.

Proof.— $\angle ACE = \angle BCE$, because N is the crown of the curve. $\angle EAB = \angle EBA$, because EAB is an isosceles triangle. The three angles of any Δ being equal to two right angles, and $\angle AEB + \angle ACB =$ two right angles ; therefore in the Δ AEB, angles $AEB + EAB + ABE$ are equal to $\angle AEB + \angle ACB$. Equating, and eliminating, $\angle AEB$, and setting $\angle EAB + \angle ABE = 2 \angle EAB$; $2 \angle EAB = \angle ACB$, or, $\angle EAB = \frac{1}{2} \angle FEY$, or $\angle EAB = \frac{1}{2} \angle ACB = \angle ACE$.

3. Prove that the tangent distance (sub-tangent), EA = Radius, AC, multiplied by the tab. tan. of the whole tangential angle, EAB.

It is evident that $T = EA = AC \times \tan ACE = AC \tan EAB = R \tan EAB = R \tan \frac{1}{2} \Delta$ (3)

4. Prove that the apex distance (A. i.e., EN (N being the crowning point or Apex) = radius (sec. EAB - 1).

Proof.— $EN = EC - NC$, but $\frac{EC}{AC} = \frac{1}{\cos. ACE} = \sec. ACE$,
 $\sec. EAB$. $\therefore EN = AC \cdot \sec. EAB - AC$. Because $AC = NC = \text{Radius}$,
 $\therefore EN = \text{Radius} (\sec. EAB - 1)$, i.e., $Ad = R (\sec. \frac{1}{2} \Delta - 1)$. (4)

Ex. 2. Find the length of the tangent for a 2° -curve containing an angle of 32° . Here $R = 50 / \sin. 1^\circ = 2865.32$ feet.

$$\therefore T = 2865.32 \times \tan. 16^\circ = 2865.32 \times .28675 = 821.51 \text{ ft.}$$

Ex. 3. Find the apex distance of a 2° -curve with a central angle of 32° .

$$EN = R (\sec. 16^\circ - 1) = 2865.32 \left(\frac{1}{\cos. 16^\circ} - 1 \right) = 2865.32 \left(\frac{1}{.96126} - 1 \right) = 115.47 \text{ ft.}$$

5. To find the Length of the curve, L, measured along the chords. If $\Delta^\circ =$ central angle, and $D^\circ =$ degree of curve, then,
 $D^\circ : \Delta^\circ :: 100 : L$. $\therefore L = 100 \Delta^\circ / D^\circ$... (5)

Ex. 4. Find the length of a 2° -curve having the central angle equal to 32° .
 $\therefore L = 100 \times 32/2 = 1600$ feet.

6. To find the Length of the Long chord, C, $= AB$

$$\cos. \frac{1}{2} \Delta = \frac{1}{2} AB / \text{Sub-tangent} = \frac{1}{2} AB / R \tan. \frac{1}{2} \Delta.$$

$$\therefore AB = 2R \cos. \frac{1}{2} \Delta \times \tan. \frac{1}{2} \Delta = 2R \sin. \frac{1}{2} \Delta = 2R \sin. y. \dots (6)$$

Ex. 5. Find the length of the Long Chord $AB = C$ for a 2° -curve having a central angle of 32° . Here $R = 2865.32$ feet.

$$\therefore C = 2R \sin. \frac{1}{2} \Delta = 2 \times 2865.32 \sin. 16^\circ = 2865.32 \times 2 \times .27564 = 1579.593 \text{ ft.}$$

7. The Middle Ordinate, V, is the $\perp$ from N on the Long Chord $AB = R (1 - \cos. \frac{1}{2} \Delta) = R \text{ Versin. } \frac{1}{2} \Delta$... (7)

Ex. 6. Find the Middle Ordinate in a 2° -curve with a whole angle of deflection of 32° .

$$\therefore V = 2865.32 (1 - \cos. 16^\circ) = 2865.32 (1 - .96126) = 111.002 \text{ ft.}$$

868. To Lay out a Simple Curve without an Angular Instrument, but with a Chain, Ranging-Rod, offset-staff, or tape.—For short curves the useful methods are :—

Method 1 (a).—By Chords and offsets outside the Curve (Fig. 833).— so and s_1o are two radii, ss_1 a chord, and AI the tangent line of the curve which commences at the springing point s . Set off $sd = ss_1$, and $pd = s_1p$, the tangential distance. ss_1o and ss_1d are similar Δ s, for they are both isosceles, and the $\angle oss_1 = \angle ss_1d$.

$$\therefore os : ss_1 = ss_1 : s_1d.$$

$$\therefore s_1d \text{ the deflection distance} = ss_1^2 / os = \text{chord}^2 / \text{radius}$$

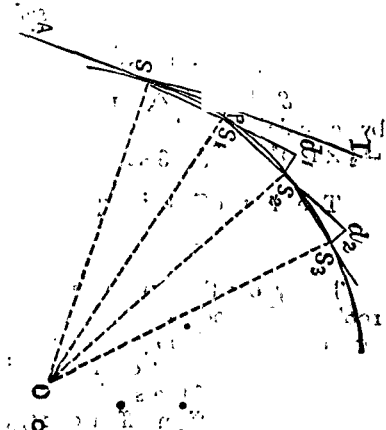


FIG. 833.—Curve-ranging with the Chain.

Similarly, if the chord $s_1s_2 = ss_1$, and $s_1d = s_2d_1$; s_2d_1 the deflection distance = chord² / radius. In order to lay out the curve the straight portion As is first pegged out. The curve $ss_1s_2s_3\ldots$ commences at s , and points in it 100' apart are required to be shown. Make sp on the tangent, $AI = 100'$, and at p set off $ps_1 =$ the tangential distance $\perp sp$ and $= \frac{1}{2} s_1d$ as calculated before. This fixes the first point, s_1 , on the curve. Now range the line, ss_1d_1 , and make $s_1d_1 = ss_1 = 100'$. On s_1d_1 as base construct the $\triangle s_1d_1s_2$, having $s_1s_2 = 100'$ and $d_1s_2 = ds_1$ (the calculated offset). This determines s_2 . Similarly other points are fixed until the other springing point is reached. If the other springing point be at the full length of the chord 100', then set off, as at first, half the offset s_1d to revert to the other tangent line. If the springing point occurs at a fraction of the last chord, then set off a proportionate length of the half offset. It may here be explained that if $ss_1 = 100'$, sp will be necessarily less than 100'; but the difference is so small that it may be neglected during real work.

Ex. 7. If the radius of the curve is 1000', the offset

$$s_1d = \frac{100 \times 100}{1000} = 10', \text{ and } s_1p = 5';$$

$$\therefore sp = \sqrt{100^2 - 5^2} = \sqrt{9975} = 99.87 = 100' \text{ very nearly.}$$

The formula, deflection distance = chord² / radius, is not quite exact also for very small radii. On a curve of 100' radius the tangential distance for 100' chord would be 51.76' as against 50' by the formula. At 500 the formula would be but .013' too little. So that for curves of 10 chains and upwards, set out with 1 chain chords, the formula is practically correct.

Method 1 (b).—*With a sub-chord.*—If the P.T. occurs at a fraction of the last chord the offset is to be computed by proportion of squares and by direct proportion combined :

Ex. 8. $y = \frac{1}{2} \Delta = 16^\circ 40'$; $D = 2^\circ$; length of chord 100'. Here by calculation $R = 2865.32'$ and $L = 1666.5$. Tangential offset $= \frac{1}{2} \times 100^2 / 2865 = 3.49 / 2 = 1.75'$ say; offset from chord = 3.5'.

$$\text{The last offset is equal to } \frac{1}{2} \left(\frac{66.5}{100} \times 3.5 + \left(\frac{66.5}{100} \right)^2 \times 3.5 \right) \\ = 1.9' \text{ (Euc. III, 26).}$$

To lay out the curve put in the peg by offsets of 3.5' from the 100' chords produced. To get the last point on the curve or P. T. for the sub-chord 66.5' produce the last full chord (of 100) 66.5 ft. and set off an offset of 1.9' at right angles to last P. T. To find the direction of the straight measure 100' on the last chord produced and set off an offset of 1.75'.

Method 1 (c).—*Starting with a sub-chord.*—Let P. C. (Fig. 834) or A occur at a station 159+67; $D^\circ = 3^\circ$; $\therefore R = 1910$. Let XAE be the tangent to the curve at A (Fig. 834). At X, 67' back from A,

set off $Xa = 67'$ sin. $\angle AX = 1.169'$, and at E $33'$ from A set off $EB = 33$ sin. $\angle EAB = .288'$; since $\angle aAX + \angle EAB = 60^\circ + 30^\circ = 90^\circ = 1\frac{1}{2} D^\circ$.

Now a and B are two points on the curve, $100'$ apart and B is station 160 . Prolong aB to c and make $Bc = 100'$, also $BC = 100$, and Cc the offset from the chord = $(\text{chord})^2/R$; and find C and continue.

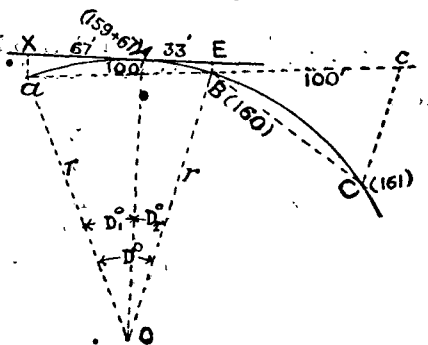


Fig. 834.—Starting with a Sub-Chord.

Method 2.—By Offsets inside the Curve. (Fig. 835).

—It often happens that the curve cannot be laid out with outside offsets, then inside offsets are laid out to range it out. Here calculate the versed sine of the angle which a chord of given length subtends at the centre of the curve; and at a the springing point, set off $aa_1 = \frac{1}{2}$ versed sine in a direction towards the centre of the curve; also set back as along the tangent equal to the length of the chord. Then range sa_1b , and measure $a_1b = sa$. This fixes the point b . Now set off bb_1 in the direction of the centre of the curve = the full versed sine, range ab_1c , and measure $b_1c = ab_1$, so that c is fixed, and so on. When the next tangent is to be resumed, set off $\frac{1}{2}$ the versed sine as in the method above.

Method 3.—Setting out Curves from the same Tangent

—There are two ways: (a) one is to obtain equidistant points upon a curve, by laying out unequal abscissæ BB' , $B'B'$, B^2B' , etc., along the sub-tangent of the curve and setting off ordinates B^1b' , B^2b' , B^3b' , etc., (Fig. 836) to the required points; (b) another is to set off equal abscissæ, thereby obtaining unequal distances on the curve (Fig. 837).

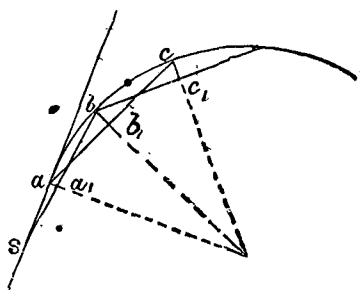


Fig. 835.—Curve-Ranging with the Chain with off-sets inside the Curve.

curve depends upon one of the properties of the circle, *i. e.*, equal chords subtend equal angles at the centre. It is easy to prove that in Fig. 836, $BB^1 = OB \sin a$, or $BB^2 = OB \sin(a + a')$; or that B^1b'

(a) Krohnke's Tangential System.

—The principle of laying off deflection distances to show how points may be set off from a tangent to any required equidistant points on a circular

$= OB (1 - \cos. a)$, or $B'b^1 = OB [1 - \cos. (a + a^1)]$; and that angles $a, a^1 + a^2$, etc., are each of them double the corresponding tangential angle.

Ex. 8a. Supposing the radius to be 2865', the tangential angle comes to 1° for a 100' chord, or the deflection or central angle comes to 2° .

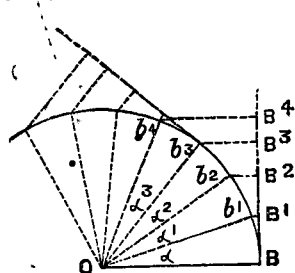


FIG. 836.—Curve-ranging with the Chain.

$$\therefore BB^1 = 2865 \times \sin. 2^\circ = 99.997 \text{ feet.}$$

$$BB^2 = 2865 \times \sin. 4^\circ = 199.852 \text{ feet.}$$

$$B^1b^1 = 2865 (1 - \cos. 2^\circ) = 1.745 \text{ feet.}$$

$$B^2b^2 = 2865 (1 - \cos. 4^\circ) = 6.978 \text{ feet.}$$

Kröhnke's tables are published for setting out curves of any radius.

When the offsets become too long, a new tangent is thrown off. This is effected by going back 2, 3 or 4 stations and measuring off from it the deflection distance due to its number

from the last peg. For instance, if we wished to set off a tangent at b^4 , we should go back to b^3 , and set off the deflection distance for 100'; or to b^2 and set off deflection distance for 200'. We should also set off from b^2 the abscissæ $= BB^1$ or BB^2 as the case may be, and this would be a true tangent at b^4 , which can be produced and treated in the same manner as BB^4 .

(b) **Jackson's Six-point Equidistant System.**—This is similar to the above method; but Jackson obtains a manageable curve, and much shorter tables by reducing the number of points to six. The tables are given in his 'Aids to Survey Practice.'

This is applicable to short curves when the ground outside the curve is only adapted for chaining, and when the curve does not exceed say $\frac{1}{4}$ of its radius. Lay off equal distances Ap_1, p_1p_2, p_2p_3 , etc., from A along AC , and $\perp$ off-sets p_1m_1, p_2m_2 , etc., the points m_1, m_2 , etc., fixing the curve (Fig. 837).

To find the offsets, $p_1m_1 = Ap_1 \tan. CA m_1$, or $= \text{Chord}^2 / 2 \text{ Radius}$.

$$p_2m_2 = 2^2 p_1m_1, \dots, p_m m = n^2 p_1m_1.$$

For draw m_1n_1, m_2n_2 , etc., $\perp$ to AO .

Then $m_1n_1^2 = Ap_1^2 = An_1 (2r - An_1) = An_1 \times 2r$, nearly; since An_1 is very small compared with $r = p_1m_1 \times 2r$.

And $m_2n_2^2 = Ap_2^2 = 2^2 Ap_1^2 = An_2 \times 2r$ nearly $= p_2m_2 \times 2r$;

$$\therefore p_2m_2 : p_1m_1 :: 2^2 Ap_1^2 : Ap_1^2, \text{ or } p_2m_2^2 = 2^2 p_1m_1^2;$$

Similarly $p_3m_3^2 = 3^2 p_1m_1^2$, etc., = etc.

These offsets are very easily and quickly laid off with a pocket sextant set to read 90° or by means of an optical square.

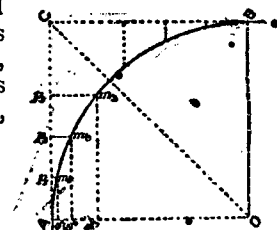


FIG. 837.—Curve-ranging with the Chain.

Ex. 9. For a curve of 20 chains radius and 66' chord,
 the first off-set = $\frac{66^2}{2 \times 20 \times 66} = 1.65$, the second off-set = $1.65 \times 4 = 6.60$,
 the third off-set = $1.65 \times 9 = 14.85$, the fourth off-set = $1.65 \times 16 = 26.40$,
 the fifth off-set = $25 \times 1.65 = 41.25$; etc., etc.

Method 4. The Versin Method.—Let A and B represent the P. C. and P. T. of the curve (Fig. 838), and as the directions of the Tangents are known, the $\angle EAB = \frac{1}{2}\Delta = y$ is known. Measure the long chord $C = AB$, and $AB = 2 \cdot R \sin y$; $\therefore R$ is known, and hence $DC = V = R \text{ versin. } y$ is obtained. To set out the curve, measure long chord AB, bisect it at D, and erect the middle ordinate $DC = V = R \text{ versin. } y$. C will represent the apex of the curve. Then join AC and BC, and bisect them, and erect ordinates over them FG and $HI = R \text{ versin. } \frac{y}{2}$, and proceed until a number of points are fixed to set down the curve.

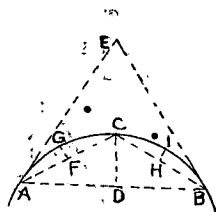


FIG. 838.—Versin Method.

Method 5.—Setting out Curves by means of Ordinates taken at right angles to a Long Chord.—This most accurate method for small curves consists in setting up upon a chord, ordinates, or lines at right angles thereto, to various points in the curve. It is extremely useful in jungle lands as it saves much needless cutting and clearing of jungle. The formulæ for finding the versed sine and ordinate (Fig. 839) are:—

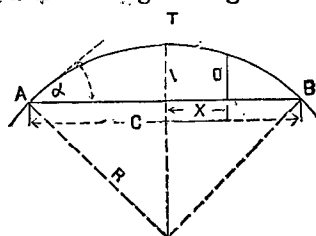


FIG. 839.—Setting out Curves by ordinates to a Long Chord.

$$V = R - \sqrt{R^2 - \frac{1}{4}C^2} = C^2/8R \text{ nearly;}$$

$$O = \sqrt{R^2 - X^2} - (R - V).$$

When C = Long chord, AB ; V = Versed sine, R = Radius, X = Distance of ordinate from centre of chord, O = length of ordinate.

Proof.—In Fig. 839, $C = 2R \sin \alpha$; and, $R^2 = (\frac{1}{2}C)^2 + (R - V)^2$; $\therefore V = R - \sqrt{R^2 - (\frac{1}{2}C)^2}$. Again $V(2R - V) = \frac{1}{2}C \times \frac{1}{2}C$ (Euclid III, 35). Now V^2 is very small in comparison with R , and hence may be neglected. $\therefore 2RV = \frac{1}{4}C^2$; or $V = C^2/8R$ nearly.

Take any abscissæ X and let O be the ordinate.

Then, $X^2 = (V - O)\{2R - (V - O)\}$ (Eucl. III, 35) = $R^2 - (O + R - V)^2$.

$$\therefore O = \sqrt{R^2 - X^2} - (R - V), \text{ and } X = \sqrt{R^2 - (O + R - V)^2};$$

$$\therefore R = (V^2 + \frac{1}{4}C^2)/2V.$$

If there be an obstruction, such as a mosque, which prevented certain ordinates being measured from the long chord, AB , then a tangent at T over V parallel to AB could be aligned and the ordinates from this tangent would be $V - O$.

Ex. 10. Having found 2 T. P.s on a curve for a tramway it was seen that the long chord is 125' and the whole tangential angle $\alpha = 16^\circ$. To find the abscissas and ordinates. Here $C = 2R \sin \alpha = 2R \sin 16^\circ$. $\therefore R = 125 / (2 \times .27564) = 226.7$; and $V = C^2 / 8R = 8.61$ ft.

Dividing the long chord into 10 equal parts each comes to 12.5' and the middle ordinate V will be at 62.5 from P.C. or P.T. Putting $x = 12.5, 25, 37.5, 50$, the values of the corresponding ordinates from the middle ordinate outwards are: 8.2, 7.2, 5.4, 3.0. feet respectively.

869. Theory of Simple Curves:—Curve-ranging with a Single Theodolite and a 100' chain.—First Method.—In Fig. 840, suppose it is necessary to link up the intersecting straight,

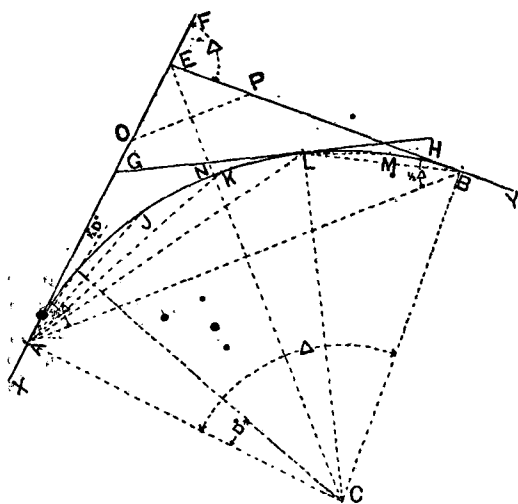


FIG. 840.—Curve-Ranging with a Single Theodolite and a 100'-chain.

XE and EY , by the curve, AB , of degree D° ; and radius $AC = BC = R$. The $\angle FEY$ is called the angle of *Total Deflection*, E being the point of intersection, $P. I.$, where the two tangents, XE and EY , meet. $\angle FEY$ equals the *central angle* $\angle ACB$. Both are designated by Δ . The distance $AE = BE$ is called the *Tangent Distance* or *Sub-tangent*, and is denoted by T . EN is called the *Apex distance* = Ad . L = Length of the curve in feet = AIJ —

NLMB. The curve will spring from the point A , called the *First Tangent point* or the *point of curvature*, $P.C.$ or $B.C.$, beginning of the curve; and end at the *Second Tangent point* or *point of Tangency*, $P.T.$ or $E.C.$, end of the curve, B .

The first step to be taken, then, is to locate the points, A and B . This is effected by computing by eqn. 3, (p. 756) the distance $AE = BE = T$; and chaining this distance back along each tangent, measuring the *Second Tangent* from the $P.I.$, i.e., E , and driving in pegs at A and B , which are then marked $P.C.$ or $B.C.$, and $P.T.$ or $E.C.$ After locating the points A and B , with the theodolite at A , vernier set to zero, and the cross-hairs directed to E , deflect to the right an $\angle EAI$ (Tangential angle) = $\frac{1}{2} D^\circ$. Next hold one end of the 100'-tape at A , and swing the other end with a flag, until it is cut by the cross-hairs. The point, I , on the curve will then be located and pegged. Similarly by deflecting further, the angle $IAJ = \frac{1}{2} D^\circ$ by directing the cross-hairs to J , and holding one end of the tape at I , and having swung the other end until it is cut by the cross-hairs, the next point

J on the curve will be fixed by a peg. So repeat the process around the curve until it is set out.

If it happens that L is the last point that is visible from A , having driven a peg at L move the theodolite to that station. Back-sighting to A , and deflecting the $\angle GLA = \angle FAL = \frac{1}{2} \angle ACL$, the line of collimation will again be in tangent by turning the telescope vertically, and further points can be located as at first (with the Everest theodolite 180° should be added to $\frac{1}{2} \angle ACL$ after back-sighting to A to be in tangent). Generally the curve will not make up of a number of full 100-foot chords. This can be ascertained by noting whether D° is contained in Δ an even number of times. Referring to Fig. 840, it is found that D° is contained in Δ $5\frac{1}{2}$ times. The theodolite having been moved to L , and the station M located by deflecting from the tangent LH the $\angle HLM = \frac{1}{2} D^\circ$, and measuring from L the distance $LM = 100'$, it is desirable to fix the point B , and see whether it checks closely enough with its location as fixed by measuring from E down the tangent EY . The $\angle MLB$ with equal $3/4 (\frac{1}{2} D^\circ) = 3/8 D^\circ$, but the length MB will have to be ascertained by the equation $MB = 2R \sin. 3/8 D^\circ$. Evidently it will be a little longer than $3/4 \times 100$ feet, i.e., 75 feet.

The above is a brief summary of the most important principles in the theory of circular curves. The following practical example will show the student all that is required for setting out an ordinary curve in the location of the centre line of a railway.

For curves of 0° to 5° set out with 100' chords.

„ „ 6° to 10° „ „ 50' „ „

„ „ 11° to 20° „ „ 25' „ „

Ex. 11. The centre line of a railway is required to be deflected to the right through an angle of $27^\circ 51'$, and it is decided to join the tangents by a " 2° curve" (Radius = 2865'.32). The chainage at E , the P.I., is found to be 142+95.5 ft. from the initial starting point. The P.I. of the straights is accessible. Required T = tangent length, Ad = apex distance, L = the length of curve, the chainage of P.C. and P.T., and of the crowning point, N .

$$T = R \tan. \frac{1}{2} \Delta = R \tan. \frac{1}{2} (27^\circ 51') = 2865'.32 \times .47964 = 710.5 \text{ ft.}$$

$$Ad = R (\sec. \frac{1}{2} \Delta - 1) = 2865'.32 (1.0302782 - 1) = 86.75 \text{ ft.}$$

$$L = \frac{100 \times 27^\circ 51'}{2^\circ} = 1392.5 \text{ ft.}$$

$\therefore$ The curve springs at 85 ft. beyond the 135th peg, ends at 77.5' beyond the 149th peg, the apex or crowning point of the curve is at 81.25' beyond 142th peg. The single tangential angle is 1° .

We have now sufficient data for ranging out the curve. Now lay off $\angle BEC$ (Fig. 840) = $\frac{1}{2} (180^\circ - \Delta) = 76^\circ 04' 30''$, and measure off $EN = 86.75'$, and drive a peg in marked "Apex, 142+81.25". Measure the sub-tangents bothways, and put in pegs with nails

aligning them from the theodolite at the P.I. Put in reference pegs marked P. C. :—135 + 85 ; and P. T. :—149 + 77.5.

Next move the theodolite to A, the P. C., and begin to set out the curve. The theodolite being set up over A, the P. C., and the vernier set to zero, direct the telescope on to the back flag at the beginning of the first straight, and reverse the telescope to see whether the point of intersection is bisected. Then set the vernier to the tangential angle for the whole curve, and see whether the cross-hairs also bisect B, the P. T., if so, the field-work so far is correct. The vernier is again set to zero, and the telescope directed to the P.I. The first odd distance is 15 feet from P. C., and the tangential angle for this distance will be $1^{\circ} \times \frac{15}{100} = 09'$. The instrument is set to read this angle, and the first peg put in at 15 feet marked 136. The next angle will be $1^{\circ} + 09' = 1^{\circ} 09'$, which is set off, and the 2nd peg aligned, and driven at full 100 feet, and marked 137, and so on. After peg 142 is put in at a tangential angle of $6^{\circ} 09'$, the odd 81.25 feet to the apex, N, is set off, the angle being $(81.25 \times 1^{\circ}) / 100 = 48' 45''$, or the angle from the beginning $6^{\circ} 57' 45''$. This will be $\frac{1}{2} \Delta$, or half the total tangential angle. Both the odd distances to the apex and to the P. T. are given to the chain-men, and when the tangential angle for the whole curve is set off, the cross-hairs should not be more than 1 or 2 in. at the outside off the P. T. In a long curve, in which the theodolite has to be shifted several times 1 or 2 ins. of error is permissible, and may be adjusted by altering slightly the last three or four pegs on the curve.

The foregoing constitute a brief summary of the operation when the whole curve was run in without having to move the theodolite from the P. C. For greater distances and where obstacles intervene so that P. T. cannot be seen from the P. C., the instrument must be shifted and re-set up somewhere on the curve. The operation at a turning point is this. Taking as an example, the first shift of the theodolite to station 141. A flagman is sent to the P. C. the instrument is planted at station 141 over a peg with a nail driven in it, and fixed by the theodolite from the P. C. and backsight taken to P. C. with one of the verniers clamped at $141^{\circ} 46'$. The lower plate is then clamped, the telescope reversed, (with Everest theodolite turned horizontally through 180°), the upper plate unclamped and the instrument directed to the tangent of the curve at station 141 by reading with the same vernier the angle $146^{\circ} 55'$. It will be clear that, as the tangential angle from the P. C. to station 141 is $5^{\circ} 09'$, if this is added to the bearing at station 141, i.e., $141^{\circ} 46'$ the angle of deflection is obtained. This laid off from sta. 141 throws the line of sight on to the tangent. Beyond this we have to increase the angle, by single tangential angle for each 100' chain for each peg in succession all of which is already calculated and recorded in the Field-book.

Forms I and II given below should also be prepared before curve-ranging is done in the field to facilitate operations.

FORM I.—DESCRIPTION OF THE CIRCULAR CURVE.

Total Angle of Deflection.		Bearing.				Degree of Curve.	Radius of Curve.	Tangent Distance.	Length of Curve.	Apex Distance.	Chainage of			
Right	Left	First straight	Crowning point.	Chord.	2nd. Straight						P. C.	Apex	P. T.	P. I.
27°51'	—	136°37'	148°43'	150°32'5"	164°28'	2°	2865	710.5	1392.5	86.75	135+85	142+81.25	149+77.5	142+95.5

FORM II.—FIELD-BOOK FOR SETTING OUT CIRCULAR CURVE.

Chainage Pegs.	Bearings.	Length of Chord.	Tangential Angles.	Total Tangential Angles.	Remarks.
135+85 ft	136° 37' 00"	0'	0	0	P. C. 3 pegs.
136	136 46 00	15	0° 09' 00"	0° 09' 00"	
137	137 46 00	100	1° 00 00	1° 09 00	
138	138 46 00	100	1° 00' 00	2° 09 00	
139	139 46 00	100	1° 00 00	3° 09 00	
140	140 46 00	100	1° 00 00	4° 09 00	
141	141 46 00	100	1° 00 00	5° 09 00	Turning Point
	5° 09 00				
141	146 55.00	000	0° 00 00	0° 00 00	On tangent at 141
142	147 55 00	100	1° 00 00	1° 00 00	Crowning point
142+81.25	148 43.45"	81.25	0° 48' 45"	1° 48 45	
143	148 55.00	18.75	0° 11.15	2° 00 00	
144	149 55 00	100	1° 00 00	3° 00 00	
145	150 55.00	100	1° 00 00	4° 00 00	
146	151 55 00	100	1° 00 00	5° 00 00	
147	152 55.00	100	1° 00 00	6° 00 00	
148	153 55 00	100	1° 00 00	7° 00 00	
149	154 55 00	100	1° 00 00	8° 00 00	P. T. 3 pegs.
149+77.5	155° 41' 30"	77.5	0° 46' 30"	8° 46' 30"	
	8 46 30				
	164 28 00				
				5° 09 00	On new Tangent.
				8° 46 30	
				13° 55 30	
				2	
				27° 51 00	Check.

The student is recommended to draw up the field-book for the same curve curved to the left instead of to the right as in the present case.

Practical Hints.—The P.C. and P.T. points are marked with two more reference pegs, one on either side of the centre peg, and close to it. Long pegs should not be driven with chainage chalked on them, as they would be welcomed for fire-wood by the villagers. Whenever the position of the theodolite is changed, the new position is marked by a peg with a nail on it, and a reference peg is also driven close to it. All other points are

marked by pegs having their distance chalked on them in stations of 100 feet and their excess. Thus 1257 would be marked 12+57.

Problem I.—To find the Position of the Point of Intersection when it is inaccessible.—When E (Fig. 840) is inaccessible, run any line OP, and measure it, O and P being intervisible; and observe the angles AOP and OPB. Then $\angle EOP = 180^\circ - \angle AOP$, and $\angle EPO = 180^\circ - \angle OPB$. $\therefore \angle AEB = 180^\circ - (\angle EOP + \angle EPO)$, and sides: $OE = OP \sin. EPO \times \text{cosec. } AEB$;

and, $PE = OP \sin. EOP \times \text{cosec. } AEB$;

$\therefore$ OE and PE are found. $\angle EAB = \text{whole tangential angle} = \frac{1}{2}(360^\circ - AOP - OPB) = \frac{1}{2}\Delta = y$. AO = AE - OE and PB = BE - EP are now measured as their directions are fixed; we get the tangent points. Then we get $EA = EB = R \tan. y$; as R is first decided upon.

Problem II.—Lay out a curve which shall pass through a given point, L, and shall join the given tangents, AE and BE, with an angle of intersection of 164° (Fig. 841).

Let AE and BE be the given tangents, and $\angle AEB = 164^\circ$. Measure the $\angle AEL$ with theodolite = 128° , and measure $EL = 50'$.

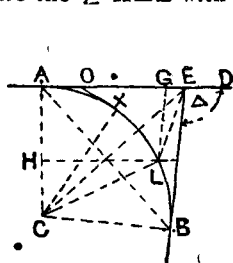


FIG. 841.—Curve-ranging through a Given Point.

Join CE and CL, and through L draw LG and LH perpendiculars to AE and AC. The $\angle AEB = 164^\circ$. $\therefore \angle BEC = \angle AEC = \frac{1}{2} \times 164^\circ = 82^\circ$; and $\angle CEL = 128^\circ - 82^\circ = 46^\circ$. Now $\sin. 82^\circ = CA/CE$; $\therefore CE = CA/\sin. 82^\circ = R/\sin. 82^\circ$. In the $\triangle ECL$, $EC : CL :: \sin. ELC : \sin. 46^\circ$. $\therefore \sin. ELC = EC \sin. 46^\circ / CL = EC \sin. 46^\circ / R = R/\sin. 82^\circ \times \sin. 46^\circ / R = \sin. 46^\circ / \sin. 82^\circ = .71934/.99027 = .72640$.

$\therefore \angle ELC = 133^\circ 25'$; and $\therefore \angle CEL = 46^\circ$, and $\angle ELC = 133^\circ 25'$; $\therefore \angle ECL = 35'$. Again $\angle ACL = \angle ACE + \angle ECL = 80^\circ 35'$; and $R \text{ vers. } ACL = LG = LE \sin. AEL = 50 \times \sin. 128^\circ$. $\therefore R = 50 \sin. 128^\circ / \text{versin } 80^\circ 35' = 50 \times .78801 / (1 - .98880) = 3517.9'$.

And $R = 50 / \sin. \frac{1}{2}D^\circ$; $\therefore \sin. \frac{1}{2}D^\circ = 50 / 3517.9 = 0.01421$ or $D^\circ = 1^\circ 38'$ very nearly. Now the whole tangential angle = $y = \frac{1}{2}\Delta = 8^\circ$; and Tangent Distance = $R \tan. 8^\circ = 3517.9 \times .14054 = 494.40$ ft. $\therefore AE$ and BE are 494.4 ft. from E.

Problem III.—To fix a point on the curve, say a 100 ft. peg at X (Fig. 841), or when an obstacle intervenes between A and X, and work up to X requires checking before proceeding on to B; the point, X can be found as follows:—Let $D^\circ = 2^\circ$, $\Delta = 42^\circ$; and the chainage at A, the P.C., is 9060. It is necessary to fix the peg at chainage 10000 at X. Now $L = AX = 10000 - 9060 = 940'$. The central

angles subtended by AX is $9'4 \times 2'' = 18''8 = 18''48'$. Let OX be a tangent at X , then $OX = OA = \frac{1}{2} \times 5729.58 \times \tan. 9''24' = 474.26'$. Therefore X can be fixed by setting off 474.26 from A to O along AE ; and at O an $\angle 18''48' =$ the $\angle EOX$, and OX is measured 474.26 to X . Now OX being a tangent to the curve at X , and if a theodolite is used the direction OX can be used for laying out the remainder of the curve.

870. Second Method.—Curve-Ranging with Two Theodolites.—If any two lines be set out from the starting points, $P.C.$ and $P.T.$, of the curve having the sum of their tangential angles $= \frac{1}{2} \Delta$; the intersection of these two lines will be a point in the curve. By a series of such points, the whole curve may be set out, if the country be sufficiently open to allow of it. Thus if two instruments were set up one at the $P.C.$, and the other at the $P.T.$ (vide field-book on p. 765) the first point at 136 would be fixed by the intersection of a tangential angle of $0''09'$ from the $P.C.$ with one of $13''46'30''$ (or rather $360'' - 13''46'30'' = 346''13'30''$) from the $P.T.$, etc. The points are fixed by a peg held by an assistant, first in line with one theodolite and then retreating and advancing along that line until in the line of the other theodolite. This method dispenses with chaining and is especially useful on a curve where chaining is impeded by undergrowth. The disadvantage of this method is that it takes up the time of two theodolite-men and is therefore rarely resorted to.

871. The compound curves are actually laid out as described before. A compound curve may consist of 2, 3 or more portions of arcs of different radii, and is adopted where the line is required to pass through given points to avoid obstructions, or where a principal station or terminus is required.

Case I.—Curve running Outside the Curve.—To find the radius of the second curve, the two tangents, the starting point, and one radius being given. In Fig. 842, from the given ($P.C.$) point A in the tangent, XE , draw the given radius $AC \perp$ to XE ; and draw the curve to some point D , where it is found convenient to alter the radius.

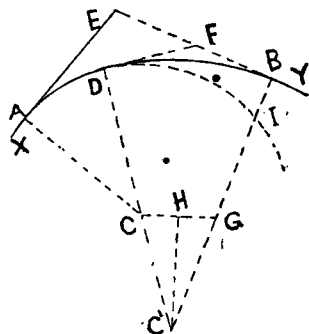


FIG. 842.—Compound Curve.

Draw the radius DC , and perpendicular thereto draw DF meeting the tangent YE in F . Measure off $BF = DF$, and from B , draw $BC^1 \perp$ to FY meeting DC prolonged, if necessary, in C^1 which is then the centre of the arc DB of the curve lying between the tangents.

Case II.—Curve Running outside the curve.—One of the two radii of the compound curve, and its starting and closing points being given, find the other radius.—Let XA

and YB be the tangents, A and B be the given $P.C.$ and $P.T.$ Draw AC and BG each = given radius and $\perp$ to the tangents; unite CG and bisect it in H , draw $C_1H \perp$ to GC meeting BG produced in C_1 ; join CC_1 and prolong it till $DC = BG$, then the points C and C_1 are the centres of the arcs AD and DB which completes the curve, $C'D = C_1B$ being the radius required.

Case III.—Curve Running inside the Curve, Fig. 843, R and r = radii of curves; G = Gauge of railway; $w = G + (R - r)$; $(y - z) = (r^2 - w^2)/R$; $y = \frac{1}{2}(R + (y - z))$; $x = \sqrt{r^2 - y^2}$; $L = xR/w$; $x = r \sin. \alpha^0$; α^0 = angle of crossing. If w be greater than r , then $(x - y) = (w^2 - r^2)/R$; $z = \frac{1}{2}(R - (z - y))$, $y = R - z$. $R = OG$ and $w = OC$.

872 Reverse Curves.—There should be no reverse curves in the main line of a railway, as they involve a sudden change of cant

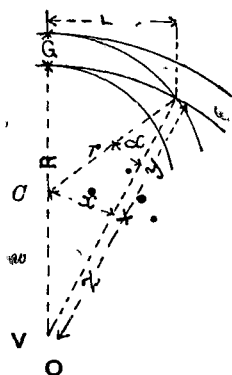


FIG. 843.—Compound Curve.

from one side to the other. It is always preferable, having regard to the running gear of the rolling stock, to put in a piece of tangent, at least 200' long, between the ends of reverse curves, and when this is done the problem is nothing different from ranging two distinct curves, one to the right and the other to the left, in the manner just described. When the main tangents are parallel it is impossible to unite them by any other than reverse curves; but when they are inclined to one another the only reason for preferring reverse to a plain connecting curve is to obtain a more rapid transit,

in which case the reverse curve will either intersect or lie wholly on the further side of one of the main tangents. A sub-tangent at the point of contrary flexure can be chosen, which will make any desired angle with the main tangents; and an endless number of cases can be formed with a pair of curves of equal deflection and unequal radii, or unequal deflection and equal radii. If curves are wanted which will meet without any intervening tangent, the lengths of sub-tangents are the fixed data, and from the formula on p. 756, transposed, we have $R = T \cdot \cot. \frac{1}{2} \Delta$, from which we can obtain the radius and other elements. The following problem is the commonest amongst true reverse curves, and suitable to turn-outs and cross-over roads between parallel tangents. Whatever the radii, the deflections Δ of the two curves (Fig. 844) will each of them be equal to twice the inclination θ of the common chord to either of the main tangents. When the radii are equal the point of contrary flexure I bisects the common chord. When unequal they will be proportional to the abscissæ of the common chord BI and IE .

Case I. Reverse Curves of Equal Central Angles and of Unequal Radii between Parallel Tangents.—If the distance D on the square between the tangents be given, and either L the straight reach, or BE the common chord be given, the other elements can be found from the following expressions; θ being $\frac{1}{2}\Delta$:

$L = D \cot. \theta$; $D = L \tan. \theta$; and $\tan. \theta = \frac{D}{L}$; $\cos. \theta = \frac{L}{BE}$; $R = \frac{BI}{2 \sin. \theta}$; $r = IE/2 \sin. \theta$; or if one of the radii R be given, and the distances L and D or BE be given, to find the other radius:

$$r = \frac{BE - BI}{2 \sin. \theta} = \frac{BE (BE - BI)}{2 D}.$$

Turn-outs are now generally curved from the commencement of the switch, when the switch is of the split type, such as is always used on passenger railways in England.

In America, the old-fashioned stub-switch, having its heel at the point of curvature, is still largely used. The switch, being common to both tracks, is straight, and the curvature to which it is a tangent commences from its further end. To lay out a turn-out for a stub-switch, instead of the main tangent, take the line of direction of the switch when directed towards the curve and which forms an angle i with the main tangent of which

$$\sin. i = \frac{\text{throw of switch}}{\text{length of switch}}.$$

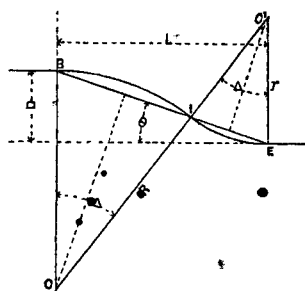


FIG. 844.—Reverse Curve.

Treat the end of the switch as the point of curvature, and calculate from it as if it were B . The two tangents will then not be quite parallel, but the curve can either be run out to a parallel with the switch, by foregoing formula, and produced to a parallel with the main tangent, or else a separate calculation can be made.

To Divert a Curve to a Parallel Tangent (Fig. 845).—Very often the end of a curve comes too near some fixed point, and it is desired to throw it to one side or other by advancing or receding the $P.C.$ point upon the old tangent. Let the distance between the two parallel tangents be y , and the length of advanced or retreat of the $P.C.$ point be x , the deflection angle being Δ ; then

$$y = x \sin. \Delta.$$

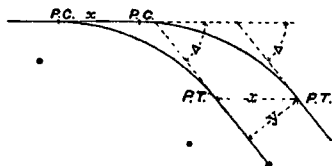


FIG. 845.—To Divert a Curve to a Parallel Tangent.

Case II. Reverse Curves of Equal Radii between Tangents inclined to each other with Unequal Central Angles:—To find the radius, the tangential points and their distance apart being

given. In Fig. 846, let XA and YB be the tangents, A and B the $P.C.$ and $P.T.$; and AB the given distance apart. Draw $Aa=Bb$ respectively $\perp$ to XA and BY , and of any suitable length; through

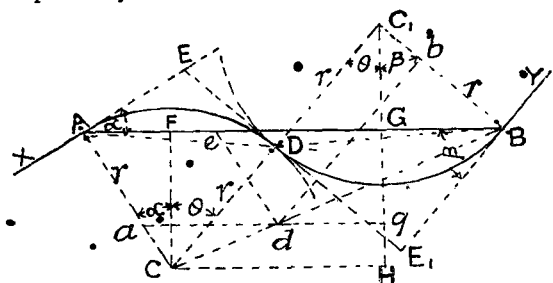


FIG. 846.—Reverse Curve.

draw ag indefinitely parallel to AB ; with the compasses apply $bd=2Bb=2Aa$ cutting ag in d ; through B , draw BdC meeting Aa produced in C ; and through C parallel to db draw CC' meeting Bb prolonged

in C' ; then C, C' are the centres, and AC and BC' are the equal radii of the serpentine curve, ADB : the common normal of the portions AD and DB of the curve being $CDC'=2AC=2BC$.

Proof:—Draw CF and $C'G \perp$ s to AB , and CH parallel to AB . When $AC=BC'=r$; $\angle EAB=\alpha$, $\angle ABE_1=\beta$, and $\angle FCC'=\angle CCH=\theta$, $AF+FG+GB=AB$,

$$r \sin. \alpha + 2r \sin. \theta + r \sin. \beta = AB; \quad \dots \quad (1)$$

$$\text{Again } CC'=2r, \text{ or } CF/\cos \theta + C'G/\cos. \theta =$$

$$r (\cos. \alpha + \cos. \beta) / \cos. \theta = 2r. \quad \therefore \cos. \alpha + \cos. \beta = 2 \cos. \theta. \quad (2)$$

$\therefore r$ and θ are known.

Ex. 12. Given two Non-parallel Tangents XA and BY , and, A and B , the $P.C.$ and $P.T.$ of the S-curve, and AB their distance apart. Find the common radii, if $\alpha=30^\circ$, $\beta=40^\circ$, and $AB=2000'$, $\cos. \theta = \frac{1}{2} (\cos. \alpha + \cos. \beta) = \frac{1}{2} (\cos. 30^\circ + \cos. 40^\circ) = \frac{1}{2} (.86603 + .76604) = .81603$.

$$\therefore \theta = 35^\circ 19' \text{ nearly. } AB = r \sin. \alpha + 2r \sin. \theta + r \sin. \beta;$$

$$\therefore r = AB / (\sin. \alpha + 2 \sin. \theta + \sin. \beta) = AB / (\sin. 30^\circ + 2 \sin. 35^\circ 19' + \sin. 40^\circ) = 2000' / 2.29883 = 874 \text{ ft. nearly.}$$

Case III.—In a Serpentine Curve having Non-parallel Tangents and Unequal Deflection Angles, one radius and its tangential point being given, find the radius and tangential point of the S-curve.

In Fig. 847, from the given tangential point, B , draw $BC \perp$ to tangent BY , and draw the curve BD to some point D where it is found suitable to have its point of contrary flexure; through

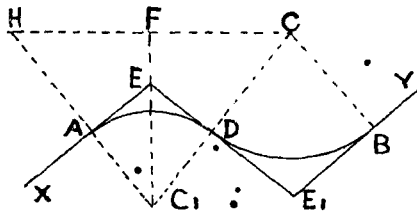


FIG. 847.—Reverse Curve.

C and D draw normal CDC' ; from D draw $DE \perp$ to CDC' to meet tangent XE at E ; make $EA=ED$; and draw $AC' \perp$ to

XE to meet CDC in C^1 , then C^1 is the centre, and $C^1A = C^1D$ is the radius of the curve AD .

Case IV. In an S-curve having non-parallel tangents and unequal central angles, when $P.C.$ and $P.T.$ and one of the radii are given, find the other radius. From the given $T.P.s.$, B and A , draw BC and AH respectively $\perp$ to the tangents BY and AX , and equal to the given radius; join CH , and bisect it in F , draw $FC_1 \perp$ to CH meeting AH prolonged in C^1 , and join CC_1 ; make $C^1D = C^1A$, then C is the centre, and $C^1A = C^1D$ is the radius of the portion AD of the curve as required. (Fig. 847.)

870. The Method of setting out the 2nd Curve in an S or Ogee curve, when an angular instrument is handy, is to set up the instrument (theodolite) at D (Fig. 846), range back on the 1st. $P.C.$, i.e., A , set off an $\angle ADE = \angle EAD$, reverse the instrument (add 180° in the case of Everest theodolite) and lay out the curve from DE , thus obtained. If no theodolite is handy, the first curve is left, and the second taken up thus: When the point D , the point of contrary flexure, is reached when setting off the first curve, by setting only half the versed sine the tangent DE is resumed: if then the half versed sine be set off on the other side of the tangent, the first point in the second curve is obtained. The whole versed sine is used for finding the next point and so on.

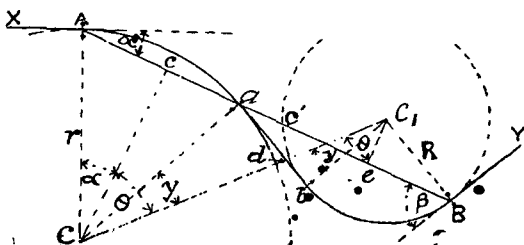


FIG. 848.—Reverse Curves united by a Straight.

Case V. Reverse Curves consisting of two Plain Curves united by a Straight :—Given the radii of the curves, R, r , the distance apart $AB = l$, $\angle XAB = \pi - \alpha$, $\angle ABY = \pi - \beta$ (Fig. 848). Find the lengths of the arcs Aa and Bb , and also that of ab the common tangent of the curves, we have :

$$Cc = r \cos \alpha; C_1e = R \cos \beta; ce = cy + ye = Cc \tan \theta + C_1e \tan \theta.$$

$$\therefore l = Ac + ce + eB = r \sin \alpha + (r \cos \alpha + R \cos \beta) \tan \theta + R \sin \beta.$$

$$\therefore \tan \theta = \frac{l - (r \sin \alpha + R \sin \beta)}{r \cos \alpha + R \cos \beta}. \therefore \theta \text{ is known.}$$

$$\text{Now } CC_1 = (Cc + C_1e) / \cos \theta = (r \cos \alpha + R \cos \beta) / \cos \theta;$$

$$\text{Again } Cd + dG_1 = CC_1 = (Ca + C_1b) / \cos y = (r + R) / \cos y;$$

$$\therefore \cos y = (R + r) \cos \theta / (r \cos \alpha + R \cos \beta).$$

$$\therefore \angle ACa = (\alpha + \theta) - y; \angle BC_1b = (\beta + \theta) - y;$$

and $ab = (R + r) \tan y$, and hence the lengths of arcs Aa and bB and length ab are known.

Ex. 13. Two portions of a railway are to be united with r° and 2° curves having an intervening common tangent. The starting and

closing T.P.s. are 10400' apart, and the straight line joining the T.P.s is found to make angles of 155° and 135° respectively with the tangents. The chainage of the P.C. of the first curve is given as 876+20. Determine the chainage of the P.T. of the first curve and P.C., and P.T. of the 2nd.

$$\begin{array}{l} AC=r \text{ for } 1^\circ \text{ curve} = 5730' \quad | \quad AB = 10,400', \alpha = 180^\circ - 155^\circ = 25^\circ \\ BC_1 = R, \quad 2^\circ \quad \quad \quad = 2865' \quad | \quad \dots \quad \quad \quad \beta = 180^\circ - 135^\circ = 45^\circ \end{array}$$

$$\begin{aligned} \text{Now } \tan \theta &= \frac{10400 - (r \sin. 25^\circ + R \sin. 45^\circ)}{r \cos. 25^\circ + R \cos. 45^\circ} \\ &= \frac{10,400 - (5730 \times .42262 + 2865 \times .70711)}{5730 \times .90631 + 2865 \times .70711} \\ &= \frac{5952.157}{7219.026} = .82450. \quad \therefore \theta = 39^\circ 30' \text{ nearly.} \end{aligned}$$

$$\begin{aligned} \cos. y &= \frac{(R+r) \cos. \theta}{r \cos. 25^\circ + R \cos. 45^\circ} = \frac{8595 \times .77162}{7219.026} = .66320739 \\ &= .91868. \quad \therefore y = 23^\circ 16' \end{aligned}$$

$$\therefore \angle ACA = 25^\circ + 39^\circ 30' - 23^\circ 16' = 41^\circ 14', \text{ and}$$

$$\angle BC_1b = 45^\circ + 39^\circ 30' - 23^\circ 16' = 61^\circ 14'; \text{ and } ab = (r +$$

$R) \tan y = 8595 \tan. y = 8595 \times .42998 = 3695.678$ or $3696'$ say.
 $\therefore$ the chainage of a the P.T. of 1st curve $= (876 + 20) + (41 + 23.3)$
 $= 917 + 43.3$; and the chainage of b the P.C. of the 2nd curve
 $= 917 + 43.3 + (36 + 95.6) = 954 + 38.9$; and that of B ,
P.T. of the 2nd curve $= (954 + 38.9) + (30 + 61.6) = 985 + 0.5$
If XA and BY were parallel, then the $\angle \alpha$ would equal the $\angle \beta$.

Problem IV.—In running the main central line of a Railway it is found that the first tangent, XA , has a bearing of 42° , and A is at ch. 16000 where the direction of the line changes to the right with a bearing of 92° , and continues up to B where the chainage is 19100. Here the line turns with a bearing of 52° to Y . At chainage 17450 a nullah 125' wide is crossed at an angle of 52° . It was decided to erect a bridge at right angles to the stream at a distance of 250' upstream measured along the centre of the stream, and to lay out reverse curves to unite the lines, XA and BY , with a straight 300' on each side of the centre of the bridge. Find the radii and starting and closing points of the curves.

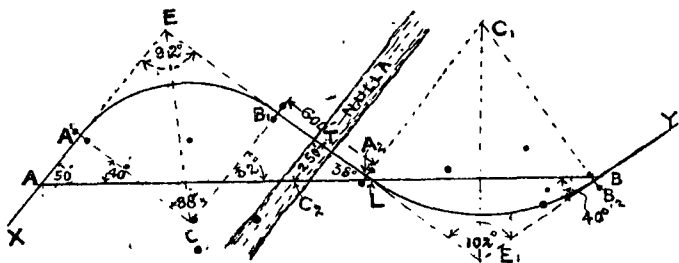


FIG. 849.—Reverse Curves with an Intervening Tangent.

In $\triangle C_2TL$ (Fig. 849), $TL=250' \times \tan. 52^\circ=250' \times 1.27994=319.98'$ $TA_2=300'$; $\therefore A_2L=19.98'$. In the same $\triangle C_2TL$, $C_2L=C_2T \sec. 52^\circ=250 \times \frac{1}{.61566}=406.06'$. $\therefore AL=1856.06'$.

In the $\triangle AEL$, $\frac{EL}{AL} = \frac{\sin. 50^\circ}{\sin. 92^\circ}$; $\therefore EL=1856.06 \times \frac{.76604}{.99939}$
 $=1422.6.'$

Now $B_1L = B_1A_2 + A_2L = 600. + 19.98 = 619.98'$.

$\therefore EB_1 = EL - B_1L = 1422.6 - 619.98 = 802.62'$.

Again in the $\triangle ECB_1$, $CB_1 = \cot. \frac{8.8^\circ}{2} EB_1 = 802.62 \times \cot. 44^\circ$
 $= 831.137 \text{ ft.}$

Now, $AL=1856.06'$, $\therefore LB=3100-1856.06=1243.94'$.

In the $\triangle LE_1B$, LB and all the angles are known. Hence the sides E_1B and E_1L of this $\triangle$ can be found; and $A_2E_1 = A_2L + LE_1$; knowing A_2E_1 , we get A_2C_1 ; and in the $\triangle LBE_1$, BE_1 is found, hence $B B_2$ is obtained. Again in the $\triangle AEL$, which can be solved, we get AE , and we know $A_2E = B_1E$, hence we obtain AA_1 . Then the starting and closing points of the reverse curve, are found and laid down on the ground. It should be noted that XA is produced to A_1 for P.C. of the Curve and P.T. of the reverse curve will be at B_2 on BY .

Problem V.—To unite two curves by a simple curve when the straight joining them falls across an obstacle and to find the points of tangency. Let AB and A_1B_1 are 3° and 5° curves, and BA_1 the straight $=500'$. It is required to set out a 1° curve to unite these two and to find A and B^1 the points of curvature and tangency. Let C and C_1 be the centres of the curves.

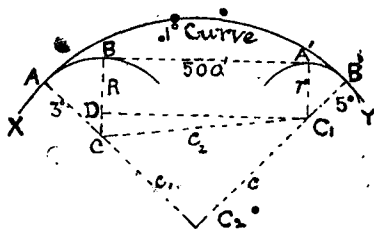


FIG. 850.—Uniting two Curves with a Simple Curve.

Join BC , CC_1 , and A_1C_1 and through C_1 draw C_1D parallel to BA_1 . Then in the $\triangle CDC_1$, $CD=R-r=\frac{5729.7}{3}-\frac{5729.7}{5}$
 $=1909.9-1145.9=764'$; and $\tan DCC_1=500/764=.65445$.

$\therefore \angle DCC_1=33^\circ 12' 10''$ and $CC_1=C_1D/\sin. DCC_1=$
 $500/\sin. 33^\circ 12' 10''=913.07'=c_2$.

$AC_2=\text{radius of } 1^\circ \text{ curve}=5729.7'$; $\therefore CC_2=3819.8=c_1$,
 and $C_1C_2=4583.8=c$.

In the $\triangle CC_1C_2$, we have 3 sides, and $\frac{1}{2}$ of which $s=4658.33 \text{ ft.}$
 $\therefore s-c_2=4658.33-913.07=3745.26$; $s-c_1=4658.33-3819.8=838.53$; $s-c=4658.33-4583.8=74.53$.

$\tan. \frac{CC_1C_2}{2} = \sqrt{\frac{s-c_1)(s-c)}{s(s-c_2)}} = \sqrt{\frac{838.53 \times 74.53}{4658.33 \times 3745.26}}$

$$\therefore \log \tan \frac{CC_1 C_2}{2} = 2.77709 = \log \tan 3^\circ 25' 30'' \text{ nearly.}$$

$\angle CC_2C_1 = 60^\circ 51'$. Similarly $\angle CC_1C_2 = 29^\circ 56'$, $\angle C_1CC_2 = 143^\circ 13'$.

Now $\angle ACC_1 = \angle CC_1C_2 + \angle CC_2C_1 = 29^\circ 56' + 6^\circ 51' = 36^\circ 47'$, and $\angle ACB = \angle ACC_1 - \angle BCC_1 = 36^\circ 47' - 33^\circ 12' = 3^\circ 35'$. Hence AB or length of curve to $P.C.$, i.e., $A = (3^\circ 36' \times 100 / 3^\circ) = 119.44'$; and similarly A_1B_1 the length to $B_1 = (3^\circ 16' \times 100 / 5) = 658.33$. The points of tangency, A and B, are thus found.

The length of curve, $AB_1 = \frac{6^\circ 51' \times 100}{1^\circ} = 685'$.

874. Curve of Deviation or Diversion.—Another way of avoiding a house or any other obstacle situated on or near the straight line of a railway is to make a deviation from it, this is effected by means of 3 curves as follows (Fig. 851):—Let ABCD be a straight

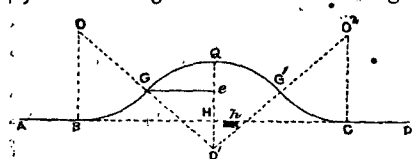


FIG. 851.—Curve of Deviation.

[illegible]

For, from G draw Ge at right angles to QH. Then $Ge = \frac{1}{2}BH$, and $Qe = \frac{1}{2}QH = \frac{1}{2}d$. In the $\triangle GO'e$, $Ge^2 = r^2 - (r - \frac{1}{2}d)^2$. Hence $Ge^2 = \frac{1}{4}d(4r - d)$. $\therefore Ge = \frac{1}{2}\sqrt{d(4r - d)}$, $\therefore BH = \sqrt{d(4r - d)}$.

Again $GQ^2 = Ge^2 + Qe^2 = \frac{1}{4}d(4r-d) + \frac{1}{4}d^2 = dr$, $\therefore GQ = \sqrt{dr}$.

Ex. 14. Make out the necessary computations for setting out a Diversion Curve required during construction of a permanent bridge, the road line being taken over a temporary bridge which is located 1000' up the permanent bridge. The diversion curve is to begin on the river bank at a point 3000' from the centre of the permanent structure.

In Fig. 851, BH=3000,' and QH=1000.' Now BH= $\sqrt{d(4r-d)}$ = $\sqrt{1000(4r-1000)}$; $\therefore r=2500.'$ Now to come to the degree system take a radius, nearer to this value of r from a *Table of Curves*, which will be 2491.29 ft. for a 2° 18' curve; and by substituting this new radius in the above eqn. we get BH=2994 ft. nearly. Now the $\angle GBH = \frac{1}{2} \Delta$ equals the whole tangential angle for each branch of the curve, and $\tan \frac{1}{2} \Delta = \frac{1}{2} \frac{L}{r} = 0.33400$. $\therefore \frac{1}{2} \Delta = 18^\circ 28'$ nearly, and $\Delta = 36^\circ 56'$, $L = (100 \times 36.56) / 2 = 1828$. $\therefore$ 1828 ft. for each branch of the curve and the whole length of the diversion curve=1828 $\times$ 4=7312 ft. Also BG=Long chord= $\sqrt{dr}$ =1578.3 ft. nearly; and BQ=CQ=3156'6".

Problem VI.—Set out a Suitable Diversion with a piece of Straight of suitable length between the two curves with the data given in Ex. 14. In Fig. 852 the

dotted line shows the two curves with $r=2500'$ with G a point half-way between B and Q, which is the point of contrary flexure. From the data in above example, $\tan. y = \frac{1}{2} \Delta = \frac{10000}{80000} = 0.33333$. $\therefore y = 18^\circ 26' 12'' = \angle CQG = \angle CGQ = \angle GBA = \angle BGA$.

Now $BQ = \sqrt{QH^2 + BH^2}$

(Fig. 852), and $BG = \frac{1}{2} BQ = 2' R \sin. y = 2R \sin. 18^\circ 26' 12''$. $\therefore R = 2495' 5$ ft. for 2 curves commencing at B and ending at Q. Also tangents BA, AG, GC, CQ each $= R \tan. y = 2500 \times 0.33333 = 833.333$.

Let the piece of straight required be 80' sufficient for a train running on a diversion curve. Then GG' and GG'' will each equal 40', i e., the former tangents will be lessened by 40'. $\therefore$ the new tangents each will equal $833.333 - 40 = 793.333$. As y is constant, new tangent $= 793.333 = R \tan. y = R \times 0.33333$. $\therefore R = 2380.025$ ft. Also $R = 50 / \sin. \frac{1}{2} D^\circ$; $\therefore D^\circ = 2^\circ 24'$ nearly, and the curves may be laid out with tangential angles of $1^\circ 12'$, or the tangents can be recalculated to get a true value and B, G', G'', Q, are measured to. The points A and C can be pegged down on the ground by measuring from B and Q respectively a distance of 833.333 ft. The $\angle BAC$ may be measured as a check on the work at G' and G''.

875. In laying down a curve in actual practice, it will be sufficient for all practical purposes to fix points at such intervals that the Versed Sine of the intercepted arc does not exceed 0.25', or thereabouts. These points may be obtained by off-sets from the tangents. when the maximum length of such off-sets does not exceed 30' or 35', above this limit the method of off-sets from the chords should be adopted. Compute the number of chords of a constant length (100' or 200') and the length of the remaining chord in the curve, and also the angle contained by 2 successive chords. Proceed to lay them

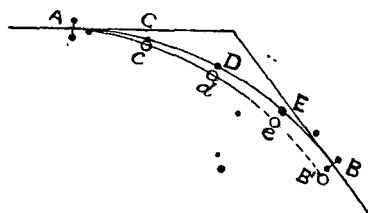


Fig. 853.—Adjustment of Error in Laying out Curve.

with a theodolite and chain, or with a tacheometer, commencing from one end of the tangents; if correctly done, the end of the last chord will fall on the peg marking the end of the other tangent. If there is some discrepancy on closing, and in a long curve this often occurs in direction, seldom in length, correct as under: suppose A, c, d, e, B, be the line laid down on first trial (Fig. 853) and ACDEB the correct one; the distance, BB₁, being measured, the corrections for the other chord ends or distances Bc, Dd, Cc, may be found by taking the space be-

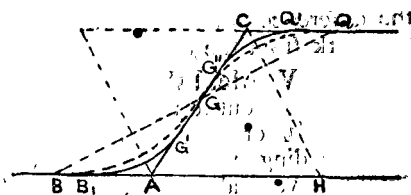


FIG. 852.—Diversion Curve.

tween the 2 curves to be a triangle, and the lines Ee, etc., to be drawn parallel to base BB₁; BB₁ and the lengths of chords being known the corrections are easily obtained (q v., p. 206); then returning over the curve correct the positions of the pegs marking chord ends. If the difference in the length of the curve is much it is better to begin the work from the beginning.

876. Vertical Curves at Changes of Gradients.—Vertical curves are set out to round off ascending and descending grades on a **summit** or **sag**, or they may be inserted between an ascending or descending grade with the level. For uniting steep gradients

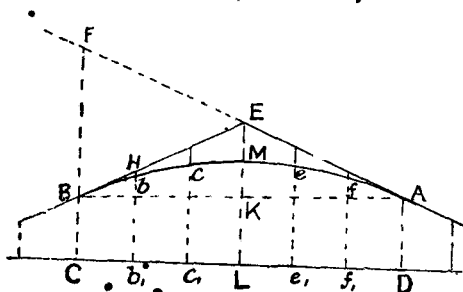


FIG. 854.—Vertical Curve at Change of Gradient.

parabolic vertical curves are usually used. Fig. 854, shows a ready method of computing the levels of a parabolic vertical curve. Let CD be the datum on the section, and AE and BE be the gradients to be united by the parabolic vertical curve AB. Produce one of the gradients, AE, to F, and measure

the off-set FB at the end of the vertical curve. All the other off-sets from A to F as to *b, c, e*, etc., may be calculated, as they are proportional to the square of their distances from A measured along the datum, i.e., to (Db₁)², (Dc₁)², etc. From these offsets the levels of the points, *b, c*, etc., may be calculated and figured on the section as usual.

For most practical purposes, if AE and BE are taken as tangents to a circular curve, the offsets H*b*, Ic, etc., can be found by formula (p. 757):—Offset = (chord)²/2R. The point above *c* on the tangent BE is I, and above *e* and *f* on tangent AE are J and G.

Ex. 15. *The Levels of A, E, and B are found to be 1075, 1095 and 1065 feet respectively and the distance from A to E and E to B each to be 300 feet. The level of F must be 1115, as the gradient is 6.6666 %. ∴ BF = 1115 - 1065 = 50 ft. Then by similar Δs, BF : KE :: AF : AE; ∴ KE = BF × AE/AF = 50 × 300/600 = 25'; ∴ MK = 12.5, assuming AB = AF for all practical purposes. Then as AE is a tangent, ME an off-set = chord²/2R; ME = AE²/2R = 300²/2R. ∴ R = 3600'. And the off-set Hb = 100²/2 × 3600 = 1.39' taking BH = 100'. Offset Ic = 200²/2 × 3600 = 5.55'; Similarly other offsets can be found from the tangents AF and BE. The height of H according to the gradient is 10.00' above B or 20' below E. ∴ The level of H = 1065 + 10 = 1075'. Hence the level of b = 1075 - 1.39 = 1073.61; and the reduced levels of other points on the curve can be similarly found, e.g., of c = 1079.45; of M = 1095 - 12.5 = 1082.5; height of G = 6.66 above A; ∴ the level of f = 1080.27. Height of J = 13.32 + 1075; ∴ Level of e = 1088.32 - 5.55 = 1082.77'. When the gradients are steep offset-intervals should be 50' or even 25'.*

To set out the curve set up the level near peg A with a foot screw in the direction of the line and with the eye-piece just half an inch away from the face of the staff held on peg A; through the object-glass read the instrument height (see Fig. 669, p. 601) to be

4'62'. * Now, hold the staff on the peg E and elevate the telescope by the one footscrew on the level till the reading on the staff at E is 4'62'. The line of sight will now be in the direction of the tangent to the circular curve, and all off-sets to the curve are to be adjusted from this line of sight, e.g., the points in example will for an ascending grade be lower than G, J, E, etc., and the top of the pegs, *f*, *e*, etc. will be correct when the staff placed on those points reads $(4'62 + 1'39) = 6'01$, and $(4'62 + 5'55) = 10'17'$, etc. A theodolite having a vertical arc may also be used.

877. Transition Curves.—The centrifugal force of a running train on a curve tends to make the flanges of the outer wheels press against the outer rail on the curve with a force depending on the speed the curvature of the curve, and the weight of the vehicle. To counteract the effect of the centrifugal force of the train, the outer rail is elevated above the inner rail, and the amount of *super-elevation* or *cant* of the outer rail, which depends on the radius of the curve and the maximum speed at which trains pass while on the curve, is obtained by the formula—

$$E = \frac{WV^2}{1.25 R}; \text{ in which } W = \text{Width of gauge in feet};$$

V = Velocity in miles per hour

E = Super-elevation of outer rail in inches;

R = Radius of curve in feet.

The full amount of super-elevation is required to be given at the commencement and throughout the whole of the curve. In cases like these the rail on the straight, joining the outer rail of the curve, has to be raised above its opposite rail for some distance prior to the point of curvature so that the proper amount of super-elevation may be attained at that point avoiding an abrupt incline on to the outer rail, which would damage the springs and give a lurching movement to the carriages as they leave the straight and run on to the curve. A piece of straight line should be laid between the two curves of a sharp reverse curve on which the super-elevation of one rail may die out and the super-elevation of the other rail may attain its proper amount. The best course would be to ease off one curve to the other or into the straight line by a *curve of adjustment*, i.e., by a change of curvature so graduated that the super-elevation of the rails not only varies gradually, but at the same time is also at each point suitable to the curvature.

The object of a transition curve is to make a gradual change from a tangent into a circular curve. The transition curve also serves to put the cant on the rails gradually, and full cant is reached where the circular crowning curve begins. In addition to this, there is the *widening of the gauge* on curves; this also may be gradually done on the transition curve. Curves of less than 3° do not require a transition curve. It is the practice to put in transition curves on the Continent of Europe and in the United States and South America. It is called the *Uebergang* by the Germans, and constructed as a true parabola. Of the various forms of transition curves, the cubic parabola and the compound transition curve

are in common use. The latter is a combination of short circular curves, forming a close approximation to the parabola, in which, with equal chords, the curvature increases by an equal amount at the end of each chord. Mr. Searles adopts 10 minutes as the basis, and uses chords of from 10 to 100 feet. Whatever its length, the curvature of the first chord is 10, the second 20, the third 30, the fourth 40 minutes, and so on.

The object of a transition parabolic curve is not to dispense with the circular arc, but to attain gradually any suitable radius with which to form a circular connecting or *crowning-curve*, and leave it again by the same gradual transition to join the further tangent. Sharp curves and high speed are the factors that make the demand for the transition-curve. Here will be dealt with a case suitable to a tramway or workshop siding of about 40' radius, because the illustration of a whole curve can be shown. The spiral, even on the sharpest curves, can be made to follow very closely the same ground as a circular arc. Fig. 855 shows clearly how the 2 curves differ and yet the spiral commences with a radius of 573' instead of one of 40'.

American Method.—The spiral adopted in the following pages differs from that of Mr. Searles in having for its base $2''=12$ minutes instead of 10 minutes. For the sharper spirals the radii are given in feet, for the others the degree of curvature per 100 feet chord. Sharp short curves are best put in by offsets (Fig. 856); flatter and longer ones by tangential angles (*q.v.*, p. 762-765).

The following Table I on p. 779 gives the general elements of the decimal spiral which are common to all kinds of spirals. The first Column, *n*, shows the number of the chord from the commencement. The 2nd, *n.c.* gives the curvature in that chord, which is the same whatever its length may be. The length of the short chord, *c*, in feet defines the spiral; thus No. 2 spiral (Table IV, p. 780) is one in which $c=2'$, (L in other Tables when $n=1$). The 3rd Col. *s*, gives the total curvature of the spiral from the commencement—the angle of deflection formed by a tangent at any point *n* with the main tangent, called by Mr. Searles the *spiral angle*. The 4th Col. *k*, is the inclination of any chord to the main tangent = total curvature opposite the middle point of the said chord. It is the basis for the calculations of *x* and *y*. The 5th Col. *i* = tangential angle formed by the long chord, C, at the point of the spiral, S, with the main tangent required for setting out the curve by tangential angles similarly to a circular curve. In other tables the Col. *r*, is the radius of curvature; *d* = the degree or deflection of 100' chord. The col. *x* = ordinate to the main tangent from any point *n*, and *y* = the corresponding abscissa. L gives the length of the spiral. R = radius of the crowning curve does not occur in the tables. C = long chord, c = short chord.

General formulae of the spiral: (1) $s = \frac{1}{10}[n \times (n+1)]$; (2) $k = \frac{1}{10}n^2$; (3) $\tan. i = x/y$; (4) $x = \Sigma \text{ chord} \times \sin. k$; (5) $y = \Sigma \text{ chord} \cos. k$; (6) $C = x/\sin. i = y/\cos. i$; (7) $r = c/2 \sin. \frac{1}{2} n.c.$ (8) sub-tangent along main tangent for any value of $n = y - x \cot. s$; (9) corresponding sub-tangent to $n = x/\sin. s$.

Table IV.

No. of Point.	Elements of No. 2 Spiral for Trunkways.			
	1	2	3	4
<i>n</i>	<i>r</i> , ft.	<i>x</i> , ft.	<i>y</i> , ft.	<i>L</i> , ft.
1	572.96	.003	2.00	2'
2	286.48	.017	4.00	4
3	190.99	.049	6.00	6
4	143.24	.105	8.00	8
5	114.59	.192	10.00	10
6	95.49	.318	11.99	12
7	81.85	.488	13.98	14
8	71.62	.711	15.97	16
9	63.66	.993	17.95	18
10	57.30	1.34	19.92	20
11	52.09	1.76	21.88	22
12	47.75	2.26	23.82	24
13	44.68	2.84	25.73	26
14	40.93	3.51	27.61	28
15	38.20	4.27	29.46	30
16	35.81	5.14	31.26	32
17	33.71	6.10	33.02	34
18	31.84	7.18	34.70	36
19	30.16	8.35	36.32	38
20	28.65	9.64	37.85	40
21	27.29	11.03	39.29	42

Table V.

Elements of No. 15 Spiral for Light Railways.				
2	3	4	5	6
<i>ds</i> , °	<i>r</i> , ft.	<i>x</i> , ft.	<i>y</i> , ft.	<i>L</i> , ft.
1.33	4297.1	.02'	15.00	15
2.67	2148.7	.13	30.00	30
4.00	1432.4	.37	45.00	45
5.33	1074.3	.78	59.99	60
6.67	859.6	1.44	74.98	75
8.01	716.2	2.38	89.95	90
9.34	613.9	3.66	104.89	105
10.68	537.2	5.33	119.80	120
12.02	477.5	7.45	134.65	135
13.36	429.7	10.05	149.42	150
14.71	390.7	13.20	164.09	165
16.05	358.1	16.93	178.62	180
17.40	330.6	21.29	192.97	195
18.75	307.0	26.32	207.10	210
20.10	286.5	32.06	220.96	225
21.45	268.6	38.54	234.49	240
22.81	252.8	45.79	247.62	255
24.18	238.8	53.83	260.29	270
25.54	226.2	62.66	272.41	285
26.91	214.9	72.31	283.90	300
28.28	204.7	82.74	294.67	315

Table VI.

Elements of No. 100 Spiral for Trunk Lines.				
2	3	4	5	6
<i>ds</i> , °	<i>C</i> , ft.	<i>x</i> , ft.	<i>y</i> , ft.	<i>L</i> , ft.
0.2	100.00	6.17	100.00	100
0.4	199.99	0.87	199.99	200
0.6	299.98	2.44	299.98	300
0.8	399.97	5.23	399.94	400
1.0	499.94	9.60	499.85	500
1.2	599.86	15.88	599.65	600
1.4	699.70	24.42	699.29	700
1.6	799.45	35.56	799.66	800
1.8	899.04	49.65	897.66	900
2.0	998.39	67.02	996.14	1000
2.2	1097.44	87.98	1093.92	1100
2.4	1196.11	112.85	1190.78	1200
2.6	1295.71	141.92	1286.46	1300
2.8	1391.77	175.47	1380.67	1400
3.0	1488.52	213.73	1473.06	1500
3.2	1584.15	256.94	1563.24	1600
3.4	1678.85	305.27	1650.79	1700
3.6	1772.36	358.85	1735.22	1800
3.8	1863.42	417.77	1816.02	1900
4.0	1948.59	482.05	1892.63	2000
4.2	2040.48	551.64	1964.44	2100

The angles λ and μ in Fig. 855 are formed by the semi-chord of the crowning curve with a normal to the main tangent and central radius of the curve respectively. $\theta = \frac{1}{2}\Delta$; $\lambda = 90^\circ - \frac{1}{2}(\frac{1}{2}\Delta + s)$; $\mu = 90^\circ - \frac{1}{2}(\frac{1}{2}\Delta - s)$ and as a check $\lambda + \mu + \frac{1}{2}\Delta = 180^\circ$.

The point to be aimed at with spirals, as with ordinary curve-ranging, is to obtain the easiest curve possible within the limits prescribed by the situation. With tramway curves, it is often the apex which fixes its other elements. The formulas for putting in a spiral to conform to a pre-determined apex-distance require the finding of angles λ and μ . A piece of spiral is fixed upon, and the distance of the point of spiral S from the point of intersection, I, and the radius, R, of the crowning curve are found by the formulas:—

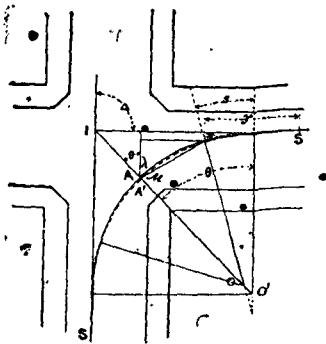


FIG. 855.—Curve at a Square Street Turning.

$$R = (AI \cos. \theta - x) / 2 \cos. \lambda \cos. \mu \quad \dots \quad (1)$$

$$IS = y + \tan. \lambda [(AI \cos. \theta) - x] + AI \sin. \theta \quad \dots \quad (2)$$

These formulas are very useful for putting in a spiral upon a tramway; but it is not always practicable to fix the apex-distance in this way. The data for a crowning curve of any required radius can be found by the following equations for a spiral of any chord-length and any curvature. Let R = Radius of crowning curve, $\theta = \frac{1}{2}$ total curvature; s = total curvature of one spiral; x = ordinate to end of spiral; y = abscissa to end of spiral; IS = distance of point of spiral from the intersection point; AI = apex-distance. Then

$$IS = y + \tan. \theta (R \cos. s + x) - R \sin. s. \quad \dots \quad (3)$$

$$AI = (R \cos. s + x) / \cos. \theta - R \quad \dots \quad (4)$$

The radius of an approximately corresponding circular curve should be first ascertained for the whole curvature, remembering that the spiral will somewhat sharpen the rate of curvature at the apex.

The next operation is to select from one of the tables a portion of the spiral which will lead in a convenient manner into a crowning curve of not much less radius than the trial curve just found. The third step is to work out the data, either from a fixed apex-distance by (1) and (2), or from the fixed radius of crowning curve by (3) and (4). If not limited by a fixed apex-distance, a radius of crowning curve will be chosen which forms a harmonious transition from the spiral, but any radius may be chosen to join any spiral, provided s is less than θ . When $s = \theta$, the 2 spirals meet to form a continuous parabola or nearly so.

Ex. 16. Tramway Spiral (Table II).—Fig. 855 represents the very common case of a square street-turning, the streets being only 30 feet wide, with 5 feet footpaths, and the centre-line is placed

somewhat to one side in order to give equal clearance at the corner, and on the further side of the street. This, of course, is only a secondary matter, and has nothing to do with the principle of the spiral; but the figure will serve to show how, when the limit of apex-distance has been fixed by a trial curve, a spiral can be put in which will differ very slightly from it, and have more clearance at the tight corner. The data are as follows:—

$\Delta = 90^\circ$; $\theta = 45^\circ$; from which a 40 feet radius is selected for a trial curve, and the sub-tangent ID, on IS and apex-distance A'I found by rules on p. 756. It is presumed that the limit of curve-radius is 35 feet. A'I is found to be equal to 16.57 feet. In order that the spiral may approach co-incidence with the trial curve, the crowning curve must lie outside, and the spiral inside of it. It is found that an approximately constant relation exists between the ordinate x of the end of the spiral, and the distance AA' between the trial curve and the crowning curve at the crown, and for curves of this character it will do to diminish A'I by one-tenth of x .

To select a spiral, on examination of Table II it will be seen that any of the points 11 to 14 are of a curvature greater than 40 feet, so that in using them there will be no danger of the crowning curve flattening the spiral. Whichever point we choose, we first reduce the apex-distance by one-tenth of x . Supposing we use point 11.

Our data are, $r = 52.09$ feet; $s = 13^\circ.2$; $x = 1.76$ feet; $y = 21.88$ feet. Deducting from A'I one-tenth of x we have AI = 16.40 feet.

$$\lambda = 90^\circ - \frac{1}{2}(\theta + s) = 60^\circ.9; \text{ and } \mu = 90^\circ - \frac{1}{2}(\theta - s) = 74^\circ.1;$$

$$\text{and by (1), } R = \frac{16.40 \times \cos. 45^\circ - 1.76}{2 \cos. 60^\circ.9 \times \cos. 74^\circ.1} = 36.93 \text{ feet};$$

$$\text{and by (2), } IS = 21.88 \text{ feet} + 16.40 \times \sin. 45^\circ + \tan. 60^\circ.9 \times (16.40 \cos. 45^\circ - 1.76) = 51.16 \text{ feet.}$$

The transition at end of spiral will be from $r = 52.09$, to $R = 36.93$, or a change of 15.16 feet.

If we were to end the spiral at point 13, we should have AI = 16.57 - .28 = 16.29, and adapting λ and μ to the altered data, R would become 35.74 feet, and IS = 51.36 feet. The transition would then be from $r = 44.08$ to $R = 35.74 = 8.34$ feet. The change would be more gradual, but the crowning curve would be .19 feet sharper. In either case the travelling public would be none the wiser, the rolling stock would not be affected, and it would be difficult to say which of the points of 11, 12, and 13, would be preferable.

If it were not necessary to fix the apex-distance absolutely and it was desired to have a crowning curve with a perfectly harmonious we should choose for a value of R that of r next in order to the point we selected as the end of spiral. Thus with $n = 13$ we should take $R = 40.93$ feet: which is the value of r when $n = 14$. We do not then need λ or μ ; and by (3)—

$IS = 25.73 + \tan. 45^\circ (40.93 \times \cos. 18^\circ.2 + 2.84) - 40.93 \sin. 18^\circ.2$;
and by (4)— $AI = (40.93 \cos. 18^\circ.2 + 2.84) / \cos. 45^\circ - 40.93$;

or $IS = 54.67$ feet; and $AI = 18.07$ feet.

The point of spiral would not be at an inconvenient length, but the apex would be 1.67 feet nearer to the footpath.

When finally selected and calculated, the curve should be tabulated for reference, and a working drawing made to a large scale in the form of Fig. 856. *Any practical tracklayer can then put it in or replace it with nothing more than a chalk-line, a set-square, and a steel tape, graduated to feet and hundredths.*

When the survey has been made previous to construction, as it always should be, if possible, the data of each curve should be worked out and the rails bent at the rolling mills to the required spirals and crowning curves, painted and stamped so as to identify them.

When there is no survey, and where no facilities for accurate bending are available at the job, a number of spiral rails should be included in the shipment, from 12 to 30 feet long, having holes for fish-bolts at every two feet in all lengths exceeding 12 feet, so that they will only need cutting at the most suitable value of n for each particular curve. A template will be made for the centre line of the spiral track, and each of the rails adjusted to it; they will need a little humouring with the 'Jim Crow' to bring them to gauge, not being perfectly concentric circular arcs.

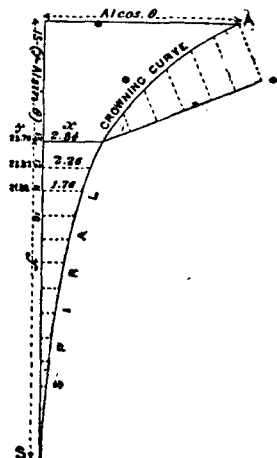


FIG. 856.—Tramway Spiral.

Another and more exact method is to calculate the spiral for the outer rail, and make a second working drawing from it by laying off ordinates at every two feet equal to the gauge.

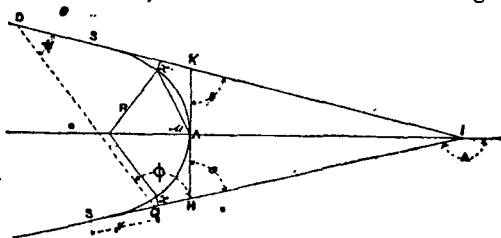


FIG. 857.—Horse Shoe Spiral.

The ordinates for the crowning curve can be put in from the tangent by formula on p. 759 (Kröhnkes tangential system), or from the chord as shown on Fig 856:

by formula in Trautwine's 'Pocket Book.'

The Horse-Shoe Spiral (Tables II, III, V).—When the total curvature approaches a semicircle (Fig. 857) it is impossible to run

the tangents to an intersection. Long before so great a curvature is reached, as shown in the example, it becomes very inconvenient to do so, and recourse should be had to the apex-tangent, HAK. The deflection Δ of the main tangents can be measured by running an auxiliary base-line, DG, from one tangent to the other. The sum of the interior angles IDG, IGD is equal to Δ , but if the instrument is kept to the astronomical bearing the difference of bearing between the two tangents will give their deflection, even though two or three auxiliary base-lines have been run between them.

The point A is generally fixed by inspection of the ground and measured; HAK being set out with angle $IHK = \theta$.

In exceptional cases, such as when A falls in the middle of a torrent, H and K being on the sides of a precipitous ravine, it is inconvenient to measure HK with the chain, and telemetric measurement may be also difficult on account of heavy brushwood; we can then measure the distance HG from the end of the auxiliary base to the point opposite the intended apex, and calculate AH by the following formula:

$$AH = \frac{DG \sin. \phi}{2 \sin. \theta} - HG \cos. \theta - DG \sin (\theta - \psi) \cot. \theta. \quad \dots \quad (5)$$

Although lengthy, the only angles involved are θ and the two interior angles IDG, IGD, so that the formula is less tedious than it looks. When the distance, AH, has been obtained, the radius of a circular curve is calculated which will join the tangents and pass through A by the formula

$$R' = AH. \cot. \frac{1}{2}\theta. \quad \dots \quad \dots \quad \dots \quad (6)$$

R' should be from 5 to 10 per cent greater than the curve-limit of the survey, in order to allow for the diminution due to the spiral. If R' is too small, a fresh point A' must be chosen somewhat further from the intersection point. It is generally possible, even in very rough country, to obtain a sufficient approximation to AH by stepping, or even guessing in order to be within the limit of radius. For instance, supposing θ to be 76.25° , and the curve-limit 250 feet. AH must not be less than $250 \times \tan. \frac{1}{2}\theta = 198.8$ feet, therefore before selecting A, HK is roughly measured to be sure that it is over 400 feet. If the object be the staking out of the curve for actual construction, it will not do to put in A with the tachometer, it must be fixed to a tenth of a foot by the chain, but for the first approximation to HK it can be measured by the tachometer, by pacing, or by the two foot rule, explained on p. 127. Flags are fixed at two points on the first main tangent, such as D and S, and the assistant, keeping himself in line with them, moves forward along the tangent until he is in the line of HK, the direction of which is given by the surveyor at H with either theodolite or prismatic compass.

When a point A has been found which appears to suit the

ground best, and also to be within the curve-limit, it is finally determined either by the exact measurement of AH (=HK) or by calculation with formula 5. To select a spiral, examine any of the tables suitable to the class of curve intended, and find one which attains a radius r or somewhat $> R'$ within a suitable length of spiral. For instance, although it is possible to apply No. 2 spiral to a 1° curve on a trunk railroad, it would be no use whatever, because it attains a radius of that degree at a distance of eight feet.

No. 50 would attain that radius in a length of 200 feet, or No. 100 in 400 feet, so that one of the three spirals (Nos. 50, 75 and 100) would be chosen. The Elements of No. 50 spiral: ds is double that of No. 100 and x, y, L are half that of No. 100 spiral.

Similarly the Elements of No. 75 spiral: ds is $\frac{4}{3}$ and x, y, L are $\frac{2}{3}$ that of No. 100 spiral.

At the transition point x or end of spiral, s should be somewhere between one-fourth and one-half of θ ; one-third is best.

When the spiral is chosen, angles λ and μ are calculated (see page 781) and the data of point of spiral KS and radius of crowning curve R are given by the following formulas:

$$R = \frac{AH \sin. \theta - x}{2 \cos. \lambda' \cos \mu} \quad \dots$$

$$KS = y - x \cdot \cot. \theta + \frac{2R}{\sin. \theta} \cos. \mu \quad \dots \quad (8)$$

Ex. 17. (see Fig. 857) Let the curve-limit be 250 feet.
 $\theta = 76^\circ 25'$. AH is measured = 217 ft.

Selecting No. 15 spiral at point 15, we have further data,
 $x = 32' 06''$; $y = 220' 96''$; $s = 24' 00''$. From these we obtain $\lambda = 39^\circ 8' 75''$,
 $\mu = 63^\circ 8' 75''$, $R = 217 \times \sin 76^\circ 25' - 32' 06'' \div 2 \cos. 39^\circ 8' 75'' \cos. 63^\circ 8' 75'' = 264' 43''$;

$$\bullet KS = 220' 96'' - 32' 06'' \cdot \cot. 76^\circ 25' + \frac{2 \times 264' 43''}{\sin. 76^\circ 25'} \cos. \mu = 318' 69''.$$

This will give a perfect transition; r being 285.5 at the transition point. The length of spiral, 225 feet, will be ample for a curve of this character, being as long as many narrow-gauge trains. If we had chosen No. 10 spiral at point 10, we should have had a crowning curve of 274' 32'' radius, the last value of r being 236' 52''. The transition would be equally good, but the length of spiral, 100', would not be sufficient to be thoroughly effective. Spirals up to No. 15 inclusive are more suitable for setting out by the ordinates of x and y , although values of i , the tangential angle, are given from point 10 onwards in No. 15 spiral. Supposing a horse-shoe of total curvature 170 degrees were required with a curve-limit of 200 feet radius, it would be advisable to adopt No. 15, spiral at point 21, rather than No. 10 spiral at point 13, and the ordinate x of 82' 74'' would be inconveniently large for tape measurement.

The Mountain Spiral (Table III).—This term has been chosen

to distinguish spirals Nos. 25 and 50 as suitable for sharp curves in a standard-gauge line or ordinary curves in a narrow-gauge line.

The ordinates of x and y will not be used in the field, as they become inconveniently large. The curve will be ranged by the tangential angle, z , similarly to the ranging of circular curves.

Ex. 18.—Having a total curve deflection of 30° , it is desired to put in a 5° crowning curve with a uniform transition from a spiral. Choosing a No. 25 spiral we find the transition point is at $n=6$ and the data are:

$$\theta = 15.0; s = 4^\circ.2; x = 3.97; y = 149.91. \quad r = 1,193.6; R = 1,146 \text{ feet.}$$

$$\text{By (3) } IS = 149.91 + \tan. 15^\circ.0 (1,146 \cos. 4^\circ.2 + 3.97) - 1,146 \sin. 4^\circ.2 \\ = 373.28 \text{ feet.}$$

$$\text{By (4) } AI = \frac{1,146 \cos. 4^\circ.2 + 3.97}{\cos. 15^\circ.0} - 1,146 = 41.33 \text{ feet.}$$

The apex-distance will here only differ by 0.9 foot from that of a 5° curve throughout.

The Trunk Line Spiral (Table V.—VI.)—This term has been chosen to distinguish spirals 75 and 100, but No. 50 is also applicable in many cases.

Ex. 19.—Having a total curve deflection of 45° between the two main tangents, it is desired to put in a 1° crowning curve, so as to form a uniform transition from the spiral.

Choosing a No. 100 spiral, we find the transition point for a 1° curve to be where $n=4$, and the data are:

$$\theta = 22^\circ.5; s = 2^\circ; x = 5.23 \text{ feet; } y = 399.94; R = 5,730 \text{ feet.}$$

$$\text{By (3) } IS = 399.94 + \tan. 22^\circ.5(5,730 \cos. 2^\circ + 5.23) - 5,730 \sin. 2^\circ \\ = 2,574.19 \text{ feet.}$$

$$\text{By (4) } AI = \frac{5,730 \cos. 2^\circ + 5.23}{\cos. 22^\circ.5} - 5,730 = 474.00 \text{ feet.}$$

The apex-distance will here only differ by two feet from that of a 1° curve throughout. For practice the reader might take the apex-distance, 474 feet, as the fixed quantity, and find the radius and distance IS by formula (1) and (2). The results will agree with the foregoing assumptions within a small fraction of a foot, but inasmuch as AI is small as compared with R , in order to get R correct to two places of decimals, AI should be given to 3 or 4 places, which is not necessary for practical curve-ranging.

Ranging the Spiral from an Intermediate Point.—

Hitherto the spiral has been supposed to be visible from end to end. Such is generally the case, because when the first trial lines are run, the bush is cleared, and it is only when unusual obstructions, such as sharp rock-points, interfere with the view that the spiral cannot be ranged continuously. When however, from any cause, a break or turning-point has to be made, the operation is analogous to that described for circular curves on p. 764, except that the tangential angle i is not half the deflection or spiral angle s .

Supposing a No. 25 spiral is being ranged from a main tangent whose bearing is $33^\circ.34$, and it is desired to make a turning-point, where $n=16$, the curve being to the right. Point 16 will be

ranged from s , with a tangential angle $i = 9^{\circ}33'$; consequently on a bearing of $33^{\circ}34' + 9^{\circ}33' = 42^{\circ}67'$, on shifting the instrument to u , we require for the tangent at that point a bearing of $33^{\circ}34' + s$, that is $60^{\circ}54'$, we therefore clamp the vernier at $42^{\circ}67'$ for the back-sight, clamp the external axis, reverse the telescope to the forward position, and set the vernier to $60^{\circ}54'$, which will bring the line of sight on to the tangent.

In ranging the remainder of the curve either by ordinates or tangential angles, a separate set of values of k' , x' , y' , and i' have to be calculated and used and entered in a Table. If the curve is ranged by ordinates, the values of x' and y' have to be calculated by simple proportion from those of No. 100 spiral given in the table, for instance a No. 25 spiral will have values of x and $y = 25$ per cent. of those in the table. They are found by the same equation as those for the primary tables, *viz.* $x' = z \cdot \text{Chord. sin. } k'$, $y' = z \cdot \text{Chord cos. } k'$. If the curve is ranged by tangential angles i' , no calculation is required, as i is the same for all spirals.

By keeping the instrument on the astronomical or any continuous bearing, the transition point or end of spiral can be very conveniently checked, however many turning-points may have been taken. For instance, supposing a curve to the right, and calling the bearing of the main tangent B, the bearing of the transition point when viewed from S will be $B + i$. With one turning-point it will be $B + i + i'$; and so on. When the transition point is reached, the direction of the tangent to the crowning curve is found by taking a back-sight to the last turning-point with the bearing $B + i + i'$, *etc.*, and reversing the telescope, with the external axis clamped, and the vernier released, the line of sight is then set to the bearing $B + s$. It can easily be seen on the ground whether this is a tangent to the last chord of the spiral, and so check the calculations.

It is also useful as an independent check to lay off and measure the sub-tangent from the transition point, or if necessary from any turning-point, to the main tangent which is done by the formula on pp. 778, 782.

878. Froude's Curve of Adjustment.*—To make the change of curvature gradual, Froude's curve of adjustment is one of the best practical forms of transition curves. This approaches a cubic parabola and forms a transition curve between a tangent and a circular curve of uniform radius. In Fig. 858, let AB and AC be the two tangents to be joined by a circular curve with curves of adjustment, and let the angle of deflection $\angle DAC =$ central angle of curve, be measured as usual. Let BC be the ordinary circular curve with P.C. and P.T. at B and C. The operation of the adoption of transition curves consists in *shifting* the curve BC to EF, BC and EF being concentric. Let r be the radius of curve EF, then $r + S =$ radius of curve BC, where $S =$ shift. The length of the straights AB and AC is found by $AB = AC = (r + S) \tan. \frac{1}{2} \Delta$. Let l be the length of the transition curve $= 300 \times$ super-elevation (the question of the amount of

transition curve in connection with a circular curve is described as follows :—

Ex. 20. In Fig. 858, let AB and AC be the straights to be joined by a circular curve with transition curves. They are produced on the ground and the angle of deflection $\angle BAC$ is measured as usual with a 5" theodolite to be $80^{\circ} 20'$ which also equals the central angle included between the radii BO and CO. A 5° circular curve with a radius of 1146'.3 is adopted in this case as that would fit the locality conveniently. The highest speed of the train allowed for is 40 miles per hour. Width of gauge = 5'6".

$$\therefore \text{Super-elevation in inches} = 5\frac{1}{2} \times \frac{40 \times 40}{1.25 \times 1146} = 4.5".$$

The length of the transition curve is then to be fixed.

A length of 240' = l for the transition curve is fixed in this case, as it may be obtained even in difficult situations. (Froude recommended a gradient of 1 in 300 to allow for the super-elevation being gradually applied, then the length of transition curve would have been $= 300 \times \text{super-elevation} = 112\frac{1}{2}'$. The length of a transition curve is often made greater for curves of larger radius. Curves of over 2500 or 3000 feet radius do not require transition curves as the small amount of super-elevation and widening of gauge is gradually applied on the straight. When sharp curves follow each other closely there is sometimes no space to set out transition curves, but in practice transition curves from 180' to 400' long usually cover all cases). The length of the transition curve having been decided on; the shift to the circular arc is obtained by the equation — $\text{shift} = (\text{length of curve of adjustment})^2 \div 24 \text{ radius} = 240^2 \div 24 \times 1146 = 2'09 = BE = CF$. In this method the transition curve is extended 120' in each side of the shift and its bisecting point is 1'.045 from the straights at B and C (Fig. 858). Having laid out the offset 1'.045 (BN) from the straight at a distance 120' (GN) on the curve or tangent from G any other offset is easily calculated as these offsets from the tangent to any point in the transition curve are proportional to the cubes of the distances along the curves from G or H, each being measured square off the tangents. The following calculations should be made before setting out the curve :—

The Sub-tangent AG or $AH = (r + S) \times \text{nat. tan. } \frac{1}{2} \Delta + \frac{1}{2} l = 1148.39 \times .844068 + 120 = 969.32$ (= AB or AC) + 120 (= BG or CH) = 1089.32 feet.

The Apex-distance $AK = \text{Secant } AP + \text{shift} \cdot PK = (r + S) \times (\text{Nat. sec. } \frac{1}{2} \Delta - 1) + S = 1148.39 \times .308606 + 2'09 = 354.40$ (AP) + 2'09 (PK) = 356.49 feet.

The Apex-distance due to shift $AM = AK - KM$.

$KM = r (\text{nat. sec. } \frac{1}{2} \Delta - 1) = 1146.3 \times .308606 = 353.75$ feet

$\therefore AM = 356.49 - 353.75 = 2.74$ feet.

The Length of Curve $EF = \frac{1}{2}(100 \times 80^{\circ}20') = 1606.66$ feet or more accurately $= (60 \times 40^{\circ}10' \times 1146.3) / 1718.87 = 1607'.2$

Having made the necessary computations for setting out the curve, the apex-distance AK is ranged, bisecting $\angle BAC (=99^{\circ}40')$ and its distance $356.49'$ is measured. A flag or ranging rod is then set up at K , the apex of the curve, with a view to check the tangential angles for the pegs on arriving at one-fourth the total angle of deflection, *viz.* $20^{\circ}5'$; also to check the chainage at apex, K , $1210+3.3$ feet. The distances $AB=969.32'$ and $BG=120'$ were then chained from A along the straight and pegged; G with 3 pegs. At B the offset for shift BE , $2.09'$, is then measured square off the straight to obtain the point where to plant the theodolite when ranging the circular curve, $L L_1$, afterwards. On the other straight equally similar distances AC and CH are measured and pegged, also the offset for shift CF . The point G was found to correspond with peg $1200+80'$ in the chainage of the centre line of the proposed railway and is marked with 3 pegs. The offsets from the tangent are then calculated for the pegs of the curve of adjustment and are given in the following table. The last offset, $8.36'$ marks the junction point L of the transition and circular curves, and always equals $4S = 4 \times 2.09 = 8.36'$. It also equals the square of the chord divided by twice radius + shift $= 120^2 \div 2 \times 1146.3 + 2.09 = 8.37'$. The offset pegs of the transition curve are driven at distances from the tangent as given in the following table (Fig. 858):—

Half Transition Curve.	Distance along Curve from G.	Offset at middle of Transition Curve = Half Shift.	Offset from Tangent GA in feet.	Chainage Peg.
	0		0.000	1200+80'
120°	20°	1.045	0.004	1201
120°	70°	1.045	0.207	1201+50'
...	120°	Half transition curve	$1.045 = \frac{1}{2}$ shift	1202
120°	170°	1.045	2.971	1202+50'
120°	220°	1.045	6.439	1203
120°	240°	1.045	8.360	1203+20'

The junction point L of the transition and circular curves is marked also with 3 pegs. In order to range the circular curve, the theodolite is set up at E , the offset point for shift, and a flag or ranging rod is planted $2.09'$ ($=$ shift) from the straight GA , some distance away from the instrument giving the direction of the shifted tangent from which to lay off the single tangential angles which are each equal to half the degree of the curve $= 2^{\circ}30'$. One half of this was used for 50-foot chords and proportionally for shorter lengths.

The ranging is then to be started by setting out the total

tangential angle 3° for L, at 120' from G and continued in the ordinary manner for circular curves by 50-foot chords. The point fixed by the angle 3° and chord 120' must coincide with the end offset point L, at 8'36" previously measured square off the tangent. The chainmen should be instructed as to the chord offset to be used for 50-foot lengths. The following is a copy of the field-book for the tangential angles:

Table of Tangential angles for circular Curves.

Chainage Pegs.	Tangential Angles.	Total tangential Angles.	REMARKS. Instrument at E.
1203+20 ft.	3°	3°	Angle for 120 ft. (half transition curve, giving point coinciding with end offset 8'36 feet.
1203+50 "	$0^{\circ} 45'$	$3^{\circ} 45'$	Angle for 150 feet.
1204 "	$1^{\circ} 15'$	5°	" " 200 "
1204+50 "	$1^{\circ} 15'$	$6^{\circ} 15'$	" " 250 "
1205 "	$1^{\circ} 15'$	$7^{\circ} 30'$	" " 300 "
1205+50 "	$1^{\circ} 15'$	$8^{\circ} 45'$	" " 350 "
1206 "	$1^{\circ} 15'$	10°	" " 400 "
1206+50 "	$1^{\circ} 15'$	$11^{\circ} 15'$	" " 450 "
1207 "	$1^{\circ} 15'$	$12^{\circ} 30'$	" " 500 "
1207+50 "	$1^{\circ} 15'$	$13^{\circ} 45'$	" " 550 "
1208 "	$1^{\circ} 15'$	15°	" " 600 "
1208+50 "	$1^{\circ} 15'$	$16^{\circ} 15'$	" " 650 "
1209 "	$1^{\circ} 15'$	$17^{\circ} 30'$	" " 700 "
1209+50 "	$1^{\circ} 15'$	$18^{\circ} 45'$	" " 750 "
1210 "	$1^{\circ} 15'$	20°	" " 800 "
1210+36 "	$5'$	$20^{\circ} 5'$	" " 803'6 Apex of curve.

The transition curve at the other tangent AH is to be pegged out in the same way but working backwards. The chainage at H is obtained by adding up the lengths of the transition and circular curves and adding them to the chainage at G $= (120 \times 1607 \cdot 2 + 120) + (\text{peg. } 1200 + 80) = \text{peg. } 1219 + 27 \cdot 2$ feet. Field-books are given below.

Table of Offsets for Transition Curve.

Half Transition curve.	Distance along curve from H.	Offsets at middle of Transition curve = Half shift.	Offsets square off the tangent HA in ft.	Chainage Peg.
	0		0.000	1219 + 27.2
120 ^s	27.2 ^s	1.045	0.012	1219
120 ^s	77.2 ^s	1.045	0.278	1218+50
120 ^s	127.2 ^s	Half transition curve	1.045	1218+17.2
120 ^s	177.2 ^s	1.045	1.245	1218
120 ^s	227.2 ^s	1.045	3.365	1217+50
120 ^s	277.2 ^s	1.045	7.635	1217
120 ^s	327.2 ^s	1.045	8.360	1216+87.2

Table of Tangential Angle for Circular Curve.

Chainage Peg.	Tangential Angle.	Total Tangential Angle.	REMARKS. Instrument at F. . .
1216+87.2	3°	3°	Angle for 1.0'.
1216+50	0° 56'	3° 56'	" " 157'·2'
1216	1° 15'	5° 11'	" " 207'·2'
1215+50	1° 15'	6° 26'	" " 257'·2'
1215	1° 15'	7° 41'	" " 307'·2'
1214+50	1° 15'	8° 56'	" " 357'·2'
1214	1° 15'	10° 11'	" " 407'·2'
1213+50	1° 15'	11° 26'	" " 457'·2'
1213	1° 15'	12° 41'	" " 507'·2'
1212+50	1° 15'	13° 56'	" " 557'·2'
1212	1° 15'	15° 11'	" " 607'·2'
1211+50	1° 16'	16° 26'	" " 657'
1211	1° 15'	17° 41'	" " 707'·2'
1210+50	1° 15'	18° 56'	" " 757'·2'
1210+3.6	1° 9'	20° 5'	" " 803'·6' Apex of Curve.

Ex. 21. Transition Curve.—In the Fig. 859 from *a* to *b* is the transition curve, which is a cubic parabola ; from *b* to *d* and onwards the curve is circular.

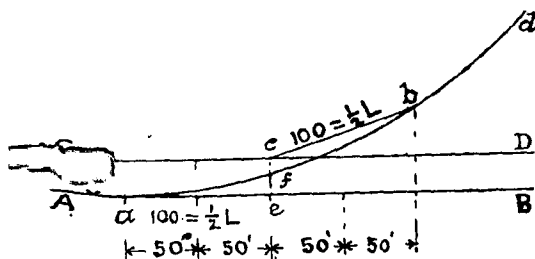


FIG. 859.—Transition Curve.

AB is the tangent to the main line, CD the tangent to the circular curve. From the apex on the main line the tangent for the circular curve is measured and set out to the point *e* ; from *e* the "shift" (*S*) *ec* is set out at right angles. Through *c* and parallel to AB is set the tangent CD to the shifted circular part of the curve. It is necessary to assume either the length (*L*) or the shift (*S*) in order to work out the transition curve ; and for choice the length is assumed and made in feet equal to the number of degrees of deflection multiplied by 40.

This is the ratio selected by Mr. T. A. Ross, A.M.I.C.E., suited for speeds up to 30 miles an hour. From the point *e* half the length is set out along the tangent AB : *a* is the springing point of the transition curve. From the point *c* again half the length *cb* is set out to the circular curve ; *b* is the point where the transition curve ends and the circular curve begins.

The following are data for the transition curve :—

- length of transition curve (assumed) = $40 \times$ number of degrees of deflection of circular curve in feet. $R =$ radius of circular curve in feet.
- shift, or distance between tangents of transition curve and of circular

curve in feet. y = offset from tangent to transition curve at any point at distance x from a in feet.

$$S = \frac{L^3}{24 R}; \quad \text{and} \quad y = \frac{4 S}{L^3} x^3.$$

As an example we take a curve of 5 degrees deflection the radius of which will be 1146 feet. Hence—

$$L = 5 \times 40 = 200 \text{ feet and } \frac{1}{2} L = 100 \text{ feet. } S = \frac{200 \times 200}{24 \times 1146} = 1.454 \text{ feet.}$$

$$y \text{ at } 50 \text{ ft.} = \frac{4 \times 1.454}{8,000,000} \times 50 \times 50 \times 50 = 1.09 \text{ inches.}$$

$$y \text{ at } 100 \text{ ft.} = \frac{4 \times 1.454}{8,000,000} \times 100 \times 100 \times 100 = 0.727 \text{ feet (This is half } S).$$

$$y \text{ at } 150 \text{ ft.} = \frac{4 \times 1.454}{8,000,000} \times 150 \times 150 \times 150 = 2.454 \text{ feet.}$$

$$y \text{ at } 200 \text{ ft.} = \frac{4 \times 1.454}{8,000,000} \times 200 \times 200 \times 200 = 5.816 \text{ feet. (This offset would}$$

fall beyond the point b where the circular curve begins.)

With a transition curve of this length the super-elevation of the outer rail if given in accordance with the usual formula could be run out in its own length. Thus super-elevation in inches = $V^2 G / 1.25 R$ = $3\frac{1}{2}$ inches nearly. If taken out at one inch in every 60 feet we have $200/60 = 3\frac{1}{3}$ inches. This is for the 5' 6" gauge and a speed of 30 miles per hour which would ordinarily not be exceeded on a 5° curve. If it were permissible to maintain a higher speed then the transition curve could be lengthened by adopting a higher multiplier than 40 for the number of degrees of deflection.

879. Transition Curves between Two Reverse Curves.—

When there is no room to put in a piece of straight between two reverse curves, the change of superelevation may be easily effected by joining the two reverse curves by a transition curve by shifting both curves towards their respective centres and the transition curve would then pass through what would have been the point of contrary flexure. In Fig. 860, let ABC be two reverse curves meeting at D, shift each

curve the proper amount to GH and FE. Compute the transition curve, which will in this case = $300 \times$
sum of cants.

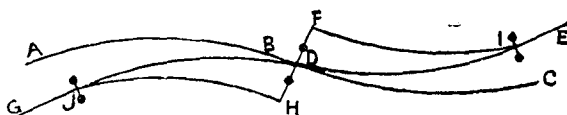


FIG. 860.—Transition Curve between two Reverse Curves.

The middle point of the transition curve will bisect the total shift FH at D, and the ends of the transition curve will be at I and J, DI and DJ being each equal to half the length of the transition curve. Set off the curve DJ by offsets from HJ making each offset proportional to the cube of its distance from J, and the curve DI set off similarly. The final centre line is then GJDIE.

880. Transition Curves in Tramway Tracks.—As tramways

are generally laid in existing streets, and have often to be carried round corners formed by streets which intersect at angles of 90° , or even less, without the tracks going at any place beyond the limits of the street widths, they necessarily contain a great many curves of very small radius. The curves are often set out as arcs of circles, but it will be found that, in order to ensure smooth running, a transition curve of some sort must be introduced between the circular arc and the straight track. In order to find the best form of transition curve, the reasons which render a transition curve necessary have first to be considered. The reason usually given for the introduction of a transition curve in railway work, *viz.*, that it allows of the necessary super-elevation of the outer rail being gradually introduced, does not apply here, as the car-speeds are not sufficiently great to require super-elevation, and in any case the rails have generally to be laid to suit street-surface levels. The development of centrifugal force fails likewise to account for the necessity for a transition curve, because the force developed in a body, passing from a tangent on to a curve, is no greater than that developed in the body, passing from point to point round a curve of the same radius.

The necessity for a transition curve will be found in the fact that, when a car passes round a curve, in addition to being carried bodily round the curve, it is twisted about its own axis by the force of a couple acting through the leading and trailing wheels, and at right angles to the longitudinal centre line of the car, and it is the inertia of the mass of the car, resisting twisting, which cause the shock experienced, if a car of short wheel-base be run direct from a straight track on to a circular arc of small radius. This will be seen, if the centre line of a car be considered as always tangent to the curve, on which the car rests, because in such a case we have the car, when entering a curve, first moving in a straight line without any twisting, and, immediately it passes the tangent point of the curve, twisting about its vertical axis at a velocity depending on its speed and the sharpness of the curve. The car is forced to twist by the pressures of the rails in the wheel-flanges, and, theoretically, in this case, these pressures would be infinite.

In actual practice, however, the centre line of the car is not tangent to the curve, but is a chord, equal to the length of wheel-base, so that, when the car is passing the tangent point, the leading wheels will be on the curve, while the trailing wheels are on the straight, and it will be found that, in effect, this is equivalent to the introduction of a transition curve, such as would give a uniform twisting acceleration to the car. This transitional condition lasts, of course, only from the time the leading wheels pass the tangent point till the trailing wheels pass the same point, so that the time between no twisting of the car and maximum twisting is very short, consequently the twisting forces are very high, and, in short-wheel-base cars, are higher than the centrifugal forces developed.

To show this, let us take a typical short-wheel-base car, with wheel-base 6ft. long, and body 30ft. over all. Let half the weight of

the car, $\frac{1}{2} W$ be in the truck, with radius of gyration of 3ft. so that weight per axle = $\frac{1}{4} W$. Let other half of the weight be in the body, with radius of gyration of 8ft., so that the equivalent weight at 3ft. radius, as regards twisting, will be

$$\frac{W}{2} \times \frac{8^2}{3^2} = \frac{32}{9} W, \text{ or } \frac{16}{9} W \text{ per axle.}$$

When passing from the straight track on to the curve of any radius R , so that the leading wheels of the car are on the curve, while the trailing wheels are on the straight, the acceleration of the leading wheels, normal to the centre line of the car, will be V^2/R (where V = velocity of car), and as the acceleration of the trailing wheels is zero, the acceleration of the leading wheels about the trailing wheels is V^2/R , or of the leading and trailing wheels about the vertical axis of the car is $V^2/2R$. The forces of the couple required to give this acceleration to the car are therefore =

$$= \frac{V^2}{2R} \left(\frac{W}{4g} + \frac{16W}{9g} \right) = \frac{73}{72} \frac{V^2 W}{Rg},$$

while the centrifugal force at the leading axle is only = $V^2 W/2Rg$ and the centrifugal force at the trailing axle is still less, owing to its being on a curve of large radius. The pressure of the trailing wheels, while twisting of car is being accelerated, will thus be towards the inside of the curve, and this explains why, at the beginning and end of a curve, over which short wheel-base cars are run, there are two distinct tracks worn in the rail groove, one being worn by the leading wheels, and the other by trailing wheels.

As the function of a transition curve is therefore to allow of the rate of twisting of the car being gradually increased from zero to the maximum necessary for the particular radius of the curve being dealt with, probably the best form of curve will be that which will give the car a uniform angular acceleration about its axis when travelling at a uniform velocity. The curve should be as long as possible, as the longer the curve is the longer will be the time between no twisting of the car and maximum twisting and the less will be the amounts of twisting acceleration and the forces of the twisting couple. **The following example** shows the form of such a curve, and the method of obtaining it.

The radius of the principal curve being given, and any convenient speed taken for the car, the method followed is to fix the length of the transition curve as a multiple of the wheel-base length, to find the rate at which the car will be twisting about its own axis when travelling round the principal curve, to calculate the uniform twisting acceleration required to correspond with these, find angular position of car at end of consecutive periods occupied by car in travelling the length of its wheel-base, and to set off the points in curve obtained. Thus in Fig. 861, let $ae fg$ be a transition curve joining the straight ua to the curve ghs . If, for convenience, we assume that the car travel a distance equal to its wheel-base length

per second, ae , ef , fg , and gh will represent the position of the car in successive seconds, and if ghs is the principal curve, sh

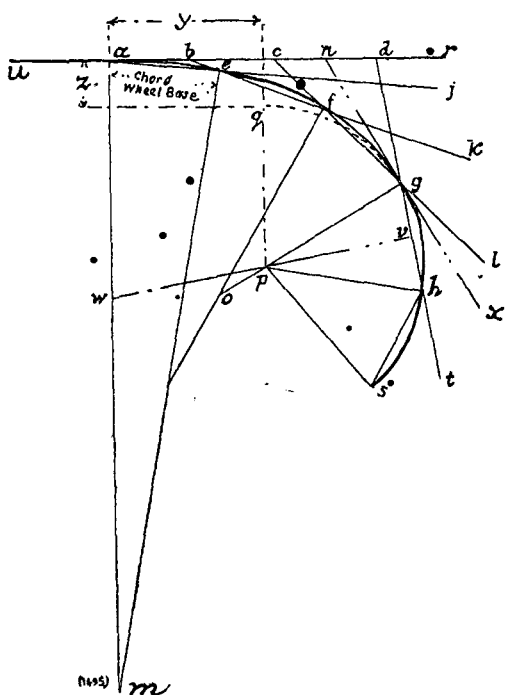


FIG. 861.—Tramway Transition Curve.

is the angle through which the car will twist in one second when on the curves, that is to say, it is the rate of twisting to which the car must attain. Representing this rate by r , $r = AT$, where A is the angular acceleration of the car about its axis, and T is time in seconds, so that, $A = r/T$.

The angles jar , kbr , lcr , and tdr represent angles twisted through $= S = \frac{1}{2} AT^2$ at any time T , and, if these angles are calculated, successive chords may be set off, and points in the curve $ae fg$ and h fixed. When the car reaches the position gh it will lie on the principal curve and

the position on the principal curve of the point s , where the transition curve terminates, is found as follows:— vp bisects the angle gph , so that angle vpg is equal to an angle whose sine is $= (\frac{1}{2} \text{ wheel-base})/R$ and the angle vva is equal to the angle hdr , which has already been found, therefore the angle $gpq = \text{angle } hdr - \text{angle } gpv$, and the new tangent to the principal curve makes with the original tangent, the angle $xnr = \text{angle } hdr - \text{angle } gpv$.

As regards the curve joining the points fixed, they will be found to be arcs of circles, each pair of consecutive arcs having a common tangent at the point between them, as will be seen from the following considerations. While the leading wheels of the car move from a to e , the trailing wheels will move along the straight ua , and as velocities along ae and ua are practically constant, and the acceleration of the trailing wheels is zero, the acceleration of the leading wheels normal to the centre line of the car must be constant if the twisting acceleration of the car is to be uniform, and the curve must, therefore, be an arc of a circle giving the acceleration $= V^2/R$.

Similarly, when the leading wheels are moving from e to f , the

trailing wheels are on the arc ae , and moving with a uniform acceleration V^2/R normal to the centre line of the car, so that, in order to have a constant acceleration of the leading wheels about the trailing wheels, the acceleration of the leading wheels must again be constant while on ef , and therefore ef must be an arc of circle.

The radii of these arcs are obtained thus :—

If the principal curve started direct from the tangent, the acceleration of the leading wheels about the trailing wheels would be $=V^2/R - V^2/\alpha$, but, as the acceleration extends over n stages, the successive radii will be obtained from the equations

$$\frac{V^2}{R_1} - \frac{V^2}{\alpha} = \frac{\frac{V^2}{R} - \frac{V^2}{\alpha}}{n}, \text{ which gives } R_1, \quad \dots \quad (1)$$

$$\frac{V^2}{R_2} - \frac{V^2}{R_1} = \frac{\frac{V^2}{R} - \frac{V^2}{\alpha}}{n}, \text{ which gives } R_2, \quad \dots \quad (2)$$

and so on to $\frac{V^2}{R_n} - \frac{V^2}{R_{n-1}} = \frac{\frac{V^2}{R} - \frac{V^2}{\alpha}}{n}$, which gives $R_n = R$.

The curve can be set out from the tangents by setting off the ordinates and abscissae of the various points efg with a as origin: Thus the ordinate of, say, f is $= \text{chord} \times \sin. jar + \text{chord} \times \sin. kbr$, and abscissa of f is $= \text{chord} \times \cos. jar + \text{chord} \times \cos. kbr$.

The shift z is given by the difference between the ordinate of g , with a as origin, and the ordinate with g as origin, and the advance y of the tangent point is given by the difference between abscissae of g with a as origin, and with g as origin.

Ex. 22. The following are the calculations for a transition curve at the beginning of a curve of 50ft. radius :—Radius of principal curve = 50ft. Length of wheel-base of car = 6ft. Assume speed of car = 6ft. per second = V . Rate r at which the car twists, when on the curve, is equal to twice the angle, whose sine is $= \frac{1}{2} V/R = 6/2 \times 50 = 0.06$.

$$\therefore r = 2 \times 3^\circ 26' = 6^\circ 52' = A. T.$$

Time T required to traverse the transition curve 18ft. long is $= \frac{1}{6} \times 18 = 3$ seconds. $\therefore$ Acceleration $A \frac{1}{3} \times 6^\circ 52' = 2^\circ 17\frac{1}{3}'$.

Angular position of the car at ends of the successive seconds are :

$$\text{(First second)} \quad \frac{AT^2}{2} = \frac{2^\circ 17\frac{1}{3}' \times 1}{2} = 1^\circ 8\frac{1}{3}'.$$

$$\text{(Second second)} \quad \frac{AT^2}{2} = \frac{2^\circ 17\frac{1}{3}' \times 4}{2} = 4^\circ 35'.$$

$$\text{(Third second)} \quad \frac{AT^2}{2} = \frac{2^\circ 17\frac{1}{3}' \times 9}{2} = 10^\circ 18'.$$

The car has therefore turned through an angle of $10^{\circ}18'$ when passing off the transition curve, so that the angular distance of the first radius of the principal curve from the tangent point is $10^{\circ}18' - \frac{1}{2} \times 6^{\circ}52' = 6^{\circ}52'$. **Tabulating, we have**

Angles	...	$1^{\circ}8\frac{1}{2}'$	...	$4^{\circ}35'$	...	$10^{\circ}18'$	
Sines	...	0.0199	...	0.0799	...	0.1788	
Cosines	...	0.9998	...	0.9968	...	0.9839	
Ordinates =	...						
sin. $\times$ chord	...	0.1194	0.1194	0.4794	0.5988	1.0728	1.6716 ft.
Abscissae =	...						
cos. $\times$ chord	...	5.9988	5.9988	5.9908	11.9796	5.9034	17.8830 ft.
Points in curve	...		e				g

Ordinate of point *g* on principal curve with the original tangent point *g* on the diagram as origin = $R - R \cos. \angle gpg$

$$= 50 - 50 \times \cos. 6^{\circ}52' = 50 - 50 \times 0.9929 = 0.355 \text{ ft.}$$

$$\text{Abcissa} = R \sin. \angle gpg = 50 \times \sin 6^{\circ}52'$$

$$= 50 \times 0.1195 = 5.975 \text{ ft.}$$

$$\therefore \text{Shift} = 1.6716 - 0.355 = 1.3166 \text{ ft.}$$

$$\text{and advance of tangent point} = 17.883 - 5.975 = 11.908 \text{ ft.}$$

A form of transition curve to give other than a uniform twisting acceleration to the car might be preferred, but the foregoing remarks are intended chiefly to show how the necessity for a transition curve arises, and the bearing the type of rolling stock has on the form of the curve.

APPENDIX I.—Stanley's, Cooke's and Watt's Zeiss Pattern Levels.

The following New English levels of the Zeiss Pattern are most likely to come into the hands of the Indian surveyors and hence their descriptions will be found useful.

Stanley's Engineer's and Surveyor's Precision Level.—

Fig. 862 shows the instrument in its normal position with the long bubble on the left. Fig. 863 shows the level in its reversed position with the telescope turned over, bringing the long bubble on the right, while Fig. 864 is a longitudinal sectional view of the instrument.

The telescope is $10''$ solar focus, with an objective of $1\frac{1}{8}''$ clear aperture. Instead of this being rigidly fixed to a vertical center as usual, it is mounted in a collar or sleeve, *K*, in which it can revolve half a turn. This collar carries 2 pivots, one on each side, fitted into 2 uprights supporting the telescope. From below the collar is a projecting bar which engages between the end of the micrometer screw, *E*, and spring piston. By turning this screw

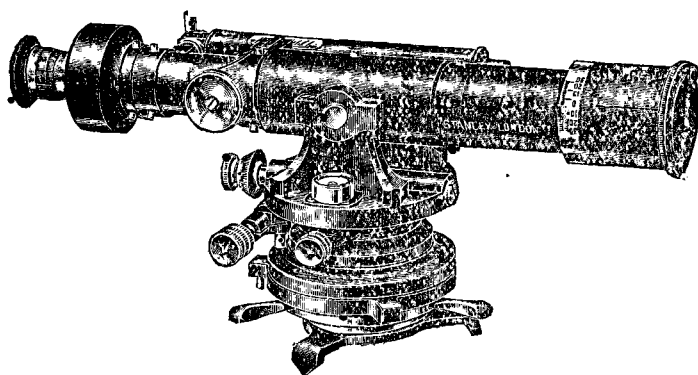


FIG. 862.—Stanley's Precision Level in its Normal position with the Long Bubble on the Left.

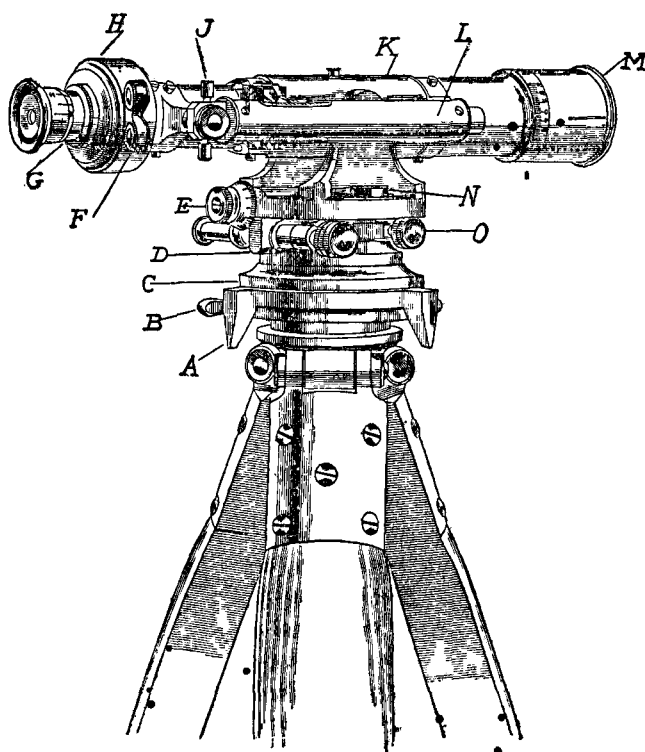


FIG. 863.—Stanley's Precision Level in its Reversed Position with Telescope Turned over, bringing the Long Bubble on the Right.

the telescope will be inclined up or down in a vertical direction. The telescope and the collar are turned from solid castings, thus ensuring a degree of accuracy difficult to obtain from drawn tubes owing to their liability to distortion.

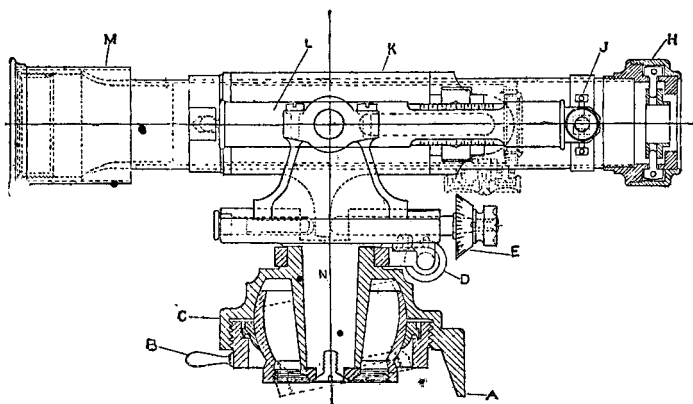


FIG. 864.—Stanley's Precision Level, Longitudinal Sectional view.

The Spirit-Levels.—Two spirit levels are fitted, namely, (1) a *small circular* one on the horizontal plate, which enables the body of the instrument to be made approximately level; (2) a *long sensitive* one fixed to the side of the telescope, and used for finally setting it truly level, by means of the screw E. The method of using this level is new, the bubble being set central by bringing one end of it opposite the pointer that projects across the top of the glass-tube. Behind the pointer is a reflecting prism which enables the reader to observe the bubble when standing at the eye-end of the level, and to set it from this position. The screws O and D are the clamp and tangent adjustment for moving the telescope horizontally; an advantage with a swaying levelling staff in windy weather, or when using a diaphragm with points.

The Levelling Arrangement consists of the quick-setting spherical joint at C, one of the clamping levers of which is shown at B. Only the preliminary levelling operation of setting the body of the instrument approximately level, is effected by this means. The final levelling is carried out by using the screw, E.

The Base is fitted with 3 feet to enable the level to be used without its stand when necessary when working underground in shallow tunnels or when it is desired to place it on a wall or girder.

The Optical Arrangement is on the internal focussing system which makes the telescope damp and dust-proof.

The Diaphragm or Graticule is fitted with, both for sighting and stadia, platinum-iridium points, although any other form may be adopted. The diaphragm is instantly removable by simply withdrawing from its socket. The adjusting screws of the diaphragm

are protected by a cap, H, which screws on to the end of the telescope.

The Eye-piece, usually supplied, gives a magnification of 20 diameters, and has a screw focussing adjustment, and is graduated in diopters, so that once the user has ascertained the focus that suits his eye-sight, he can always adjust the eye-piece immediately to the correct position.

The Ray Shade is similar to that described on p. 580.

The Tripod Stand is similar to that shown in Fig. 641*d*, p. 579.

Accuracy.—It is now admitted that levels having an independent vertical movement to the telescope that is adjustable at each sight, after the instrument has been set up in the usual manner are capable of greater precision than levels of the older form in which the telescope is rigidly attached to the vertical centre, the reason being that with latter instruments the sensitiveness of the spirit-level can only be in conformity with the accuracy of workmanship in the fitting of the vertical centre and levelling arrangements, as if it be too sensitive, great difficulty is found in getting the instrument to reverse. In this instrument these limitations do not apply, as the long bubble is only used after the telescope has been directed on to the staff, and it is not shifted at all between the operation of setting it level and taking the staff reading.

To Use the Level.—Having set up the stand with the legs well-spread, screw the instrument firmly upon it, and release the spherical joint by slightly unscrewing the clamping ring by means of the levers, B (there is another lever on opposite side not shown in illustration). Then take hold of the instrument near the ends of the telescope and set it approximately level by the air-bubble of the circular level, N, being in the centre of the glass cover. Then clamp it in this position by use of the levers.

Next, focus the eye-piece to give the clearest possible view of the points of the graticule, release the clamp screw, O, and direct the telescope on to the levelling staff. When the staff is seen in the field of view, focus it carefully and tighten clamp, O, making the final adjustment of bringing the points of the graticule up to the graduations of the staff by means of the tangent-screw, D.

Then set the telescope truly level by turning the screw, E, until the end of the air space in the long bubble is coincident with the pointer. The position of the bubble end and pointer are observed through the reflecting prism placed between it and the body of the telescope which is then truly level and the staff reading may be taken. The final adjustment of setting the long bubble to the pointer is made previous to every sight, and it is as well to glance at the bubble again after taking the reading to make sure it has not been displaced.

When using the level for the first time in the day, observe whether, when the bubble end is set to the pointer, the bubble is central in

the tube. If not, bring the bubble central by the screw E, and then push the pointer along its slide until it is in exact coincidence with the end of it. This adjustment becomes necessary when there is such a considerable change in the temperature as will alter the length of the bubble. If the temperature changes greatly during the day, or the level is used sometimes in direct sunshine and at others in the shade, the test should be made at each change.

To Measure Distance of the Levelling Staff.—This is effected by means of the two stadia points on the left of the diaphragm as described on p. 582 subject to the only provision that $5'' = 0.44'$ must be added to all measurements taken in this way, as this is the *constant* for all distances.

Long distances are measured by means of the screw E, which, in addition to providing the vertical adjustment to the telescope is also a *gradienter screw*. The measurements are made by use of a stadia rod fitted with sighting targets at any distance apart, usually 10' from centre to centre. The rod is held vertical at the spot the distance of which is required. The central horizontal line or point of the diaphragm is then set to one of the targets and the reading of the gradienter is noted. This screw is then turned until the same line or point of the diaphragm is brought on to the second target, when the gradienter reading is again noted. The formula for finding the distance is—Length of stadia rod $\times 10,000 \div$ Difference between the 2 gradienter readings.

To set out a gradient is simple, as each complete revolution of the gradienter represents an inclination of 1 in 100, and each division of the head 1 in 10,000. Therefore, having set the telescope level, turn the gradienter screw through as many hundredths of a foot as the gradient represents in 100 feet. The screw reads up to 1000, so it is applicable for all gradients up to at least 1 in 20 in either elevation or depression.

The Adjustment.—(1) The adjustment of the bubble index has already been described.

(2) **The Adjustment of the Bubble tube and the Diaphragm.**—As these are securely held, it is unlikely that either will be disturbed unless through accident or very rough handling.

(a) **To Test the Adjustment of the Diaphragm, i.e.,** to find whether it is duly central on the collimation line, sight the telescope on any distinct spot at a long distance and focus it, having previously focussed the eye-piece to give the clearest possible view of the point in the diaphragm. Then revolve the telescope a half-turn in its tubular bearing, and observe if the central point of the diaphragm has shifted off the observed spot; if not, the graticule is in adjustment. If, however, it no longer coincides with the spot sighted, unscrew the protecting cover, G, and shift the position of the graticule, by means of the adjusting screws, one-half of the distance that it was off the mark at the second observation. A pin for turning the adjusting screws is in the instrument case. Before tightening any

APPENDIX I.] IMPROVED COOKE'S LEVEL.

of the adjusting screws, the one on the opposite side must be correspondingly slackened. When the adjustment is completed all the screws must be left tight. Now adjust the screws, D and E, to bring the central point of the graticule again on to the spot originally sighted, and revolve the telescope back again the half-turn to make sure the adjustment is corrected.

(b) **The Adjustment of the Long-Bubble-Tube.**—Having set the instrument approximately level by the aid of the spherical joint and the **Circular** level, adjust the screw E to bring the end of the bubble to the centre of its run, revolve the telescope a half-turn as before, and observe if the bubble maintains its position; if so, it is in perfect adjustment, but if not, the adjustment at J must be at fault, so slacken the clamping nut and adjust the bubble-tube by means of the **two small screws** above and below it, shifting the bubble **one-quarter** the distance of the apparent error, and shifting the telescope the same amount by means of the screw, E, always revolve the telescope after the operation to make sure that the adjustment is perfect. Then reset the pointer to the end of the bubble.

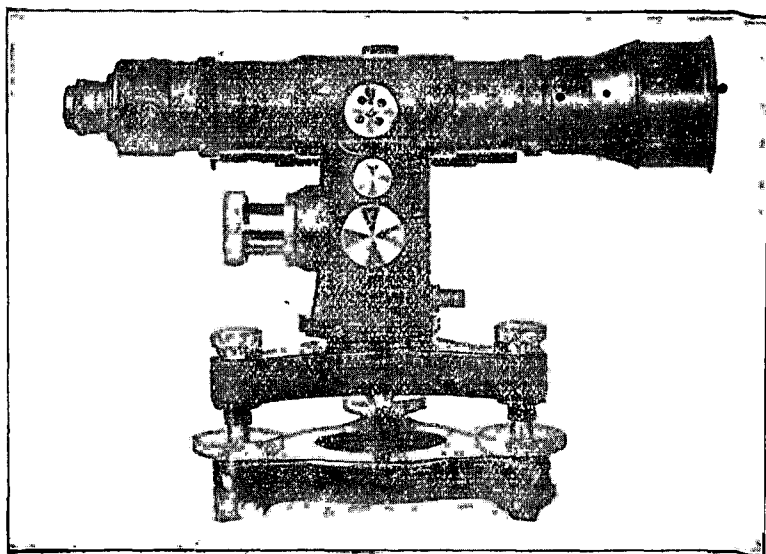


FIG. 865.—Cooke's Level.

Engineering.

Improved Cooke's Engineer's and Surveyor's Level*—
Figs. 865, 866 show its general appearance from two sides, and Fig. 867 shows its view from the eye-piece end with the detachable magnetic compass mounted above the telescope. The compass is so arranged that the bearing can be read on the graduated circle

* Reprinted by permission of the Editor, *Engineering* (June 3, 1921).

from the eye-piece-end of the telescope. In some respects it resembles the Zeiss level, but it is not intended to be taken to pieces

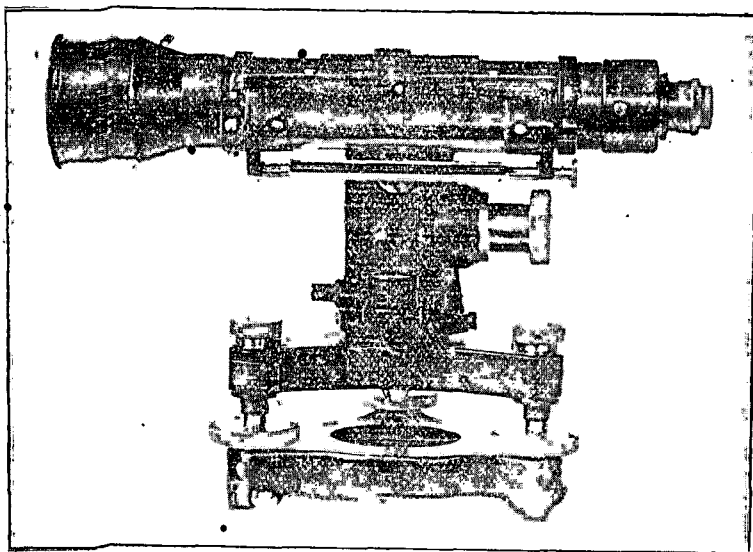


FIG. 866.—Cooke's Level.

Engineering.

for transport. It can be packed in a box, $11\frac{1}{2}'' \times 9'' \times 6\frac{3}{4}''$, weighing $13\frac{1}{4}$ lb complete with the instrument. The adjustments to be made by the user have been reduced to one only, *viz.*, for bringing the tangent to the bubble-tube parallel with the collimation line of the telescope. This level is of the internal-focussing type, in which the distance between the object-glass and diaphragm is kept constant, and focussing is effected by altering the position of an internal negative lens mounted in a short tube and moved by means of a rack and pinion. The optical principles involved in this method of focussing are shown by Fig. 868 in which R R represent the approximate parallel rays from a distant object, and, but for the negative lens, N, these rays would be brought to a focus at F by the object-glass O. Similarly the divergent rays, R^1R^1 , from a near object would be brought to a focus at F^1 . The introduction of the negative lens, N, however increases the focal-length of the combination, and brings the Rays, R, R, to a focus at D on the Diaphragm, and by moving the negative lens into the position N^1 , the divergent rays, R^1R^1 , can be made to focus in the same plane. The usual draw tube for focussing is dispensed with the advantages, of which are that (1) the over-all length of the telescope is reduced, (2) the ends can be hermetically sealed to exclude dirt, dust, and water. The eye-piece is of the screw focussing type fitted with a dust

cover. The instrument has an apparent field-of-view of 40° the whole of which is exceptionally flat and free from distortion.

A common source of Error in levels and other surveying instruments arises from a slight bending of the telescope-tube, as

these tubes when made

of drawn brass or

German Silver are very

difficult to make and

to keep straight. The

effect is shown in Fig.

869. Assuming the

telescope to be correctly

collimated at infinity

focus, with mechanical

and optical axes coincident,

the line of sight,

as indicated by the

graticule, will be shifted

by racking out the

draw-tube, to focus on

a near object, unless

the axis, of the draw-

tube remains exactly in

the line of collimation.

If this is not the case,

a serious error will be

introduced in sighting

near objects as is

shown by the diagram.

In the Cooke level the

telescope body, made

of Duralumin, is turned

accurately out of the

solid ensuring perfect

straightness. This to-

gether with the use of

internal focussing device

practically eliminates this source of error

above referred to. Any slight lateral movement of the negative lens

due to slackness of the internal tube would produce a collimation

error only one-sixth as great as would be caused by an equal

movement in the draw-tube of a telescope of ordinary design.

Another common source of collimation error, often over-

looked, is also minimised by the internal focussing arrangement.

For perfect collimation there must be perfect coincidence of the

mechanical and optical axes in all focussing positions, but even if

ordinary focussing tube has been made to rack in and out without

the least deviation from the optical axis, the error shown by Fig.

870 will still exist. In this diagram, the position of the eye-piece and

and diaphragm, when the telescope is at infinity focus, is shown

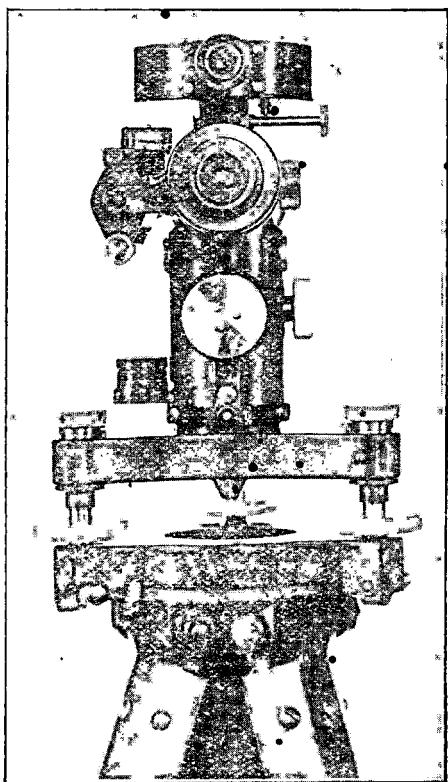


FIG. 867.—Cooke's Level. *Engineering.*

by full lines while the position of these parts when a near object is focussed is indicated by dotted lines. In the first position the line of sight through a horizontal cross-line b , off the optical axis ad , will be in the direction of the line ba . The operation of focussing however will bring the line b to the position b' , so that the line of sight will shift from the direction ba to the direction $b'a$, and the angle between these two lines is an indication of the error produced by the movement. Unless the horizontal line on the diaphragm lies on the optical axis, the line of sight will be altered by focussing by the ordinary draw tube, but with internal focussing, since the eye-piece and diaphragm are not moved, the error due to this cause will be greatly reduced. The internal focussing telescope is not absolutely anallatic, hence it is necessary to use a constant when using this level for stadia work.

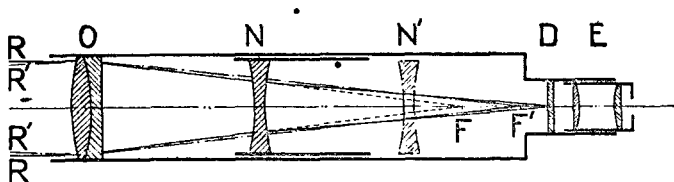


FIG. 868.

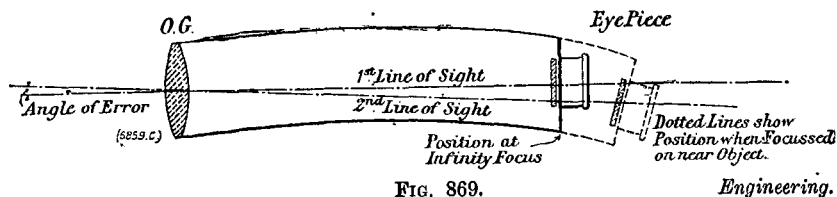
Engineering.

FIG. 869.

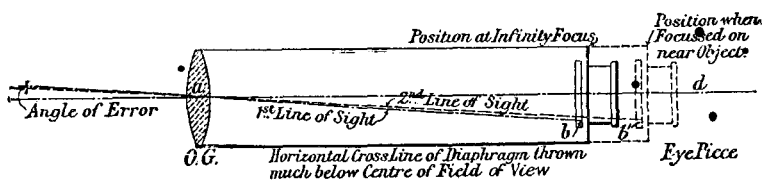
Engineering.

FIG. 870.

Engineering.

Again this level differs from the ordinary surveyor's level in that accurate results are not dependent on the perfect vertical axis. It is now considered to be better practice for accuracy and rapidity in working to **adjust the telescope to the horizontal position for each individual sight**, and this Cooke and Zeiss levels are worked in this way: The instrument is first levelled roughly by means of the 3 foot-screws, a circular bubble (see Figs. 863 and 864) being provided to assist in this operation. The fine adjustment, which is made before each sight is taken, is effected by means of the horizontal gradienter-screw seen below the eye-end of the

telescope. By giving this screw a fraction of a turn the telescope and the main bubble are tilted slightly in a vertical plane to bring them exactly horizontal, this position being indicated by the main bubble. The bubble-tube is enclosed in an outer glass tube to insulate it as far as possible from the effect of temperature changes. The bubble is illuminated by a tilting mirror (Fig. 863) which can be adjusted to reflect light upwards through the tubes, and by means of a system of prisms, the operator can see the two ends of the bubble without moving his head from the eye-piece end. The optical arrangements are such that when the telescope is not horizontal two separate images of the two ends of the bubble are seen through the prisms, and to bring the telescope horizontal it is only necessary to turn the gradienter screw until the two images coincide. During this operation movement in a azimuth is, of course, prevented by a clamp, the screw operating which is shown by Fig. 864 on the right hand side of the instrument. When required a tangent-slow-motion screw can be fitted. This arrangement expedite field-work.

Watt's Surveyor's Level (Fig. 871).—The parts of this instrument are named as under :—

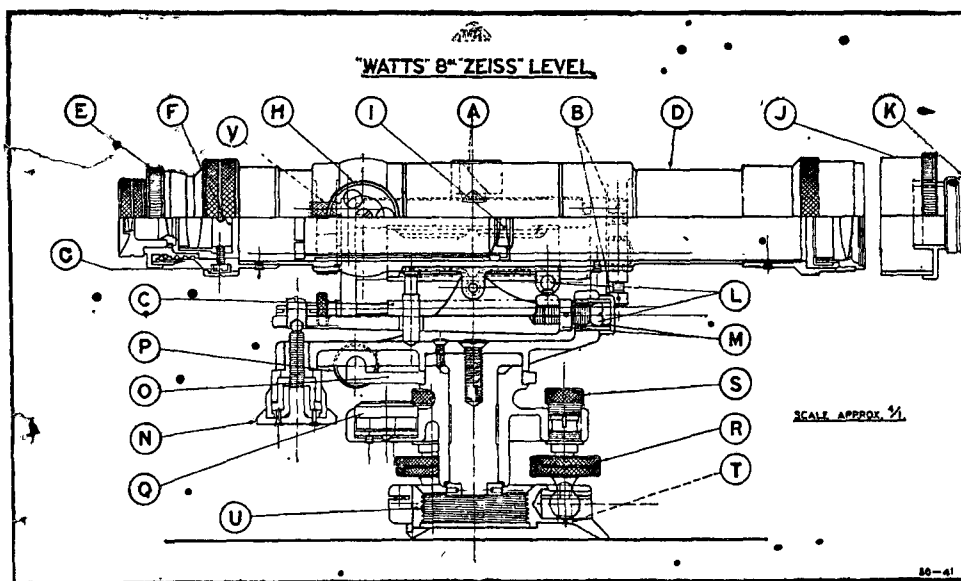


Fig. 871.—Watt's Level.

A.—Main Bubble Reading Prism. B.—Bubble Adjusting Screws, (Seldom used, see V). C.—Bubble Reflector. D.—Internally Focussing Telescope. E.—Screw Focussing Eye-piece. F.—Diaphragm. G.—Diaphragm Adjusting Screws. H.—Focussing Millhead. I.—Focussing Lens. J.—Object End Cap. K.—Position of Eye-piece, when reversing Telescope. L.—Patent Lever Movement. M.—Lever Adjustment for Drum Values. N.—Drum for

Elevating and Depressing Collimation Line. O.—Clamp for Azimuth Motion. P.—Millhead for Azimuth Motion. Q.—Circular Bubble. R.—Adjustable and Dust-Tight footscrews. S.—Foot Screw Adjusting Cap. T.—Foot Screw Ball Adjustment for Wear. U.—Standardised Stand Thread. V.—Bubble Adjusting Millhead.

The Chief advantages of the instrument may be enumerated as follows :—

(1) Its extreme portability combined with a very high degree of accuracy. The complete instrument only weighs 4 lbs. 6 ozs., and is packed in a box measuring $9\frac{1}{2}$ inch by $5\frac{1}{2}$ inch by 4 inch. overall, while the magnification of the telescope is 20 diameters enabling the observer to read .01 of a foot on the staff at 400 ft. distance which is equivalent to a 14 inch. level.

(2) The ease with which the collimation of the instrument can be adjusted from one station. To accomplish this with the older types of instruments, it was necessary to make the telescope removable from the standards in order to reverse it. The great disadvantage of this was that there was great danger of disturbing the levelling of the instrument in removing the telescope and on account of dirt and grit collecting in the telescope bearings the accuracy of the reading was liable to be affected.

With the new Zeiss pattern level these disadvantages are entirely obviated, the adjustment being accomplished as follows :—

- (1) A first reading is taken with the telescope in its normal working position as shown in the illustration.
- (2) A second reading is taken with the telescope turned over 180° (that is upside down)
- (3) A third reading is taken with the eye-piece K placed in the recess "O" provided for it in the telescope cap put on the object-glass end.
- (4) Finally a fourth reading is taken with telescope turned back through 180° but with the eye-piece as in 3.

The Mean of the above four readings should be absolutely free from instrumental errors, and it only remains to place the telescope in its normal position and by means of the elevating screw M cause it to read the above ascertained normal mean, and bring the bubble to the centre of its run by means of the screw and lock-nut S.

(3) The telescope being hermetically sealed at both ends, is impervious to dust and damp; an obvious advantage in the tropics and is provided with a graticule (Stadia lines) by which distance can be accurately estimated without the use of a constant. This entirely does away with chaining, a very great convenience when surveying in a rough or uncleared country.

(4) A special feature of the instrument is the ingenious arrangement of prisms whereby the bubble is viewed from the eye-piece end. The bubble is so reflected that two half images appear in the prism levelling being accomplished by causing the two images to coincide by turning the elevating screw M. In this manner a very accurate setting of the bubble is obtained.

APPENDIX II.—Solar Attachment to a Theodolite.

Burt's Solar Attachment to a Theodolite.—This appliance, (Figs. 872, 873) by Messrs. W. & L. E. Gurley, of Troy, N. Y., is an adaptation to the theodolite of the solar compass of W. A. Burt, of Michigan, which was made to replace the magnetic compass in determining a true meridian, or north and south line, by observation of the sun only. It was brought into general use in the surveys of the United States public lands. The solar compass consists mainly

of three arcs of circles by which the latitude of a place, the declination of the sun, and the hour of the day can be set off

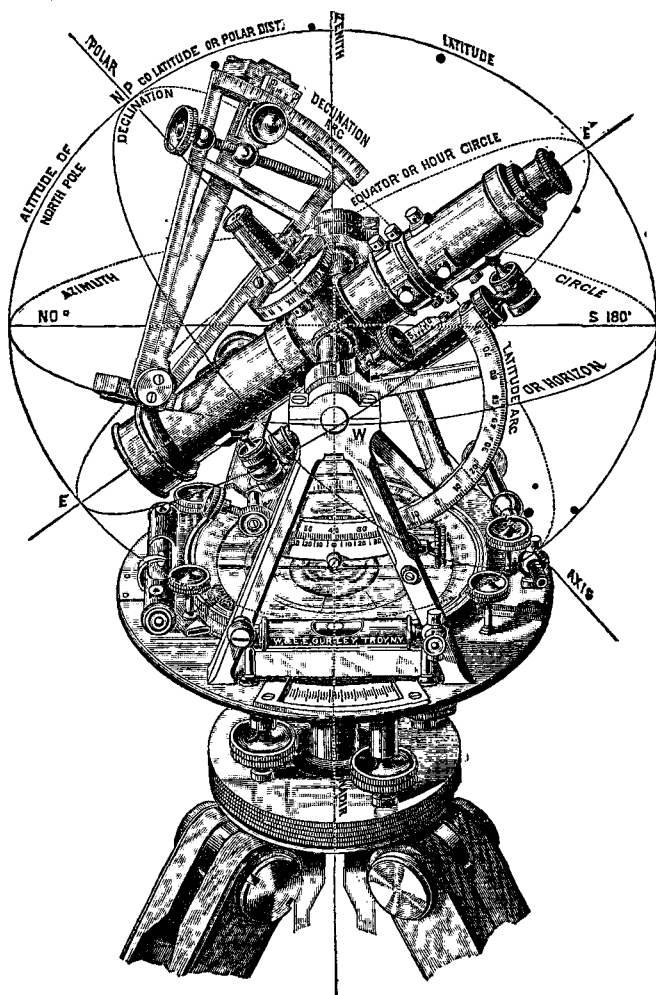


FIG. 872.—Gurley's Solar Attachment.

Polar axis—The solar attachment is placed on the cross bar of the transit (Fig. 873). A disc $1\frac{1}{2}$ " in diameter, having a pivot projecting above its upper surface, is screwed to the telescope axis. Upon this pivot rests the enlarged base of the polar axis, MS, which is firmly connected with the disc by 4 capstan head screws passing from the under side of the disc into the base. These screws serve

to adjust the polar axis. In the solar attachment to the theodolite the latitude arc is found unnecessary, as this is formed by the vertical arc of the theodolite.

The Hour-Circle, H, (Fig. 873,) surrounding the base of the polar axis, **MS**, and movable about it, is fixed upon the centre of

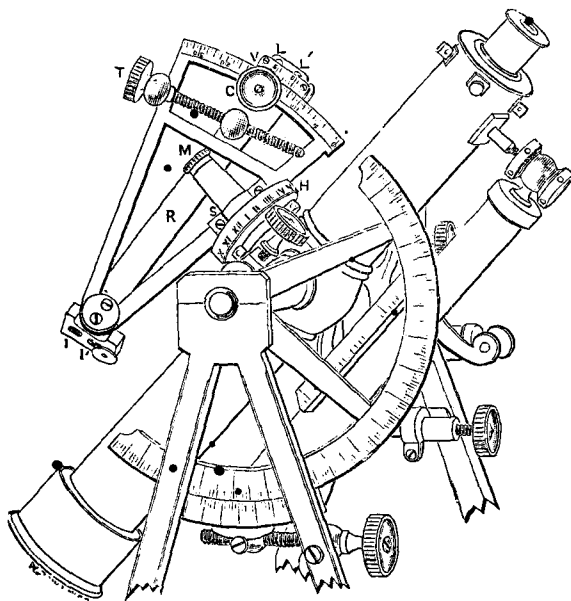


FIG. 873.—Burt's Solar Attachment to a Theodolite.

the telescope upon a socket axis, **S**, which is placed perpendicularly to the optical axis and to the transverse axes or pivots of the theodolite. This circle is divided to read five minutes of time, and is figured I to XII., twice or I to XXIV, the index being a fine line on a plate from the lower arm of the **declination arc**, which is fixed to the socket **S**. The hour-circle when set to any reading may be clamped to this position by means of the milled head placed over the socket, **M**.

The Declination Arc is of 5 inches radius, divided to quarter degrees and read on the same plane with a vernier, **V**, to single or $\frac{1}{2}$ minutes of arc. The Vernier arm is fixed by a clamp at **C**, which carries tangent adjustment, **T**. At the back of the vernier arm two spur-pieces are carried out directly from it, **L** and **I**. These are blocks of metal about $1\frac{1}{2}$ by $1\frac{1}{4}$ by $\frac{1}{4}$ inches, which carry each a lens of a focus **L** to **I**, and a silver plate to be presently described. upon which the sun's image is received in one direction or the other.

The Image Plate, Fig. 874, is marked with two intersecting each other at right angles. The lines **b b** **hour Lines**, the lines **c c**, "equatorial lines," these lines having reference respectively to the hour of the day and the position of the sun in relation to the equator. The intervals between the lines **b b** and **c c** are just sufficient to include the circular image of the sun formed by the solar lens on the opposite end of the revolving vernier arm. The axes of the solar lenses and corresponding

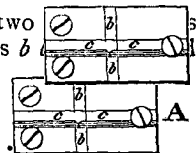


FIG. 874.—Image plate of Solar attachment.

image plates are placed parallel with each other, and with the direction of the vernier arm. Below the lower line *c* three other lines are cut at 5 minutes apart. These are useful for making allowance for refraction.

When the instrument is made perfectly horizontal, the equatorial lines and the opposite lenses being accurately adjusted to each other by a previous operation, and the sun's image brought within the equatorial lines, his position in the heavens with reference to the horizon will be defined with precision.

Suppose the observation to be made at the time of one of the equinoxes ; the arm, *R*, set at zero on the declination arc, *V* ; and the polar axis, represented by the axis of the telescope, is placed exactly parallel to the axis of the earth. Then the motion of the arm, *R*, if revolved on the spindle of the declination arc around the hour-circle, *H*, will exactly correspond with the motion of the sun in the heavens on the given day and at the place of observation ; so that if the sun's image is brought between the lines *cc* on the image plate in the morning, it will continue in the same position, passing neither above nor below the lines as the arm is made to revolve in imitation of the motion of the sun about the earth.

In the morning as the sun rises from the horizon, the arm, *R*, will be in a position nearly at right angles to that shown in the cut, the lens being turned towards the sun, and the silver plate, on which his image is thrown, directly opposite. As the sun ascends, the arm must be moved around, until when he has reached the meridian, the graduated side of the declination arc will indicate XII on the hour circle, and the arm *R*, the declination arc, *V*, and the latitude arc, that is the vertical arc of the theodolite, will be in the same plane. As the sun declines from the meridian the arm, *R*, must be moved in the same direction, until at sunset its position will be the exact reverse of that it occupied in the morning.

***Allowance for Declination.**—Let us now suppose the observation made when the sun has passed equinoctial point, and when his position is affected by declination. Then by referring to the **Nautical Almanac** and setting off on the arc his declination for the given day and hour, we are still able to determine his position with the same certainty as if he remained on the equator.

When the sun's declination is south, *i. e.*, from the 22nd of September to the 20th of March in each year, the arc, *R*, is turned towards the plates of the transit in the opposite position to that shown in the engraving, using the solar lens at *L*, with the silver plate opposite at *L*, while during the remainder of the year the arc is turned from the plates, and the lens at *L* and plates at *I* are employed in the position shown in the figure.

When the solar attachment is accurately adjusted and transit plates made perfectly horizontal, the latitude of the place and the declination of the sun for the given day and hour being also set off on

their respective arcs, the image of the sun cannot be brought between the equatorial lines until the polar axis is placed in the plane of the meridian of the place, or in a position parallel to the axis of the earth. The slightest deviation from this position will cause the image to pass above or below the lines and thus discover the error.

We thus, from the position of the sun in the solar system, obtain a direction absolutely unchangeable, from which to run lines and measure horizontal angles. The transit theodolite will without the solar compass, perform the same functions; but by means of this instrument the calculation of position is much more simple.

Explanation of the Solar Attachment: When the telescope is set horizontal by the spirit level, the hour circle will be in the plane of the horizon, the polar axis will point to the zenith, and the zeros of the vertical arc and its vernier will coincide. If we incline the telescope, directed north, the polar axis will descend from the direction of the zenith. The angle through which it moves, being laid off on the vertical arc, will be the co-latitude of the place where the instrument is used, the latitude itself being found by subtracting this number from 90° . When the sun passes above or below the equator, its declination, or angular distance from it, as given in the Nautical Almanac, can be set off on the declination arc, and its image brought into position as before. In order to do this, however, it is necessary not only that the latitude and declination be correctly set off upon their respective arcs, but also that the instrument be moved in azimuth until the polar axis points to the pole of the heavens, or in other words, is placed in the plane of the meridian. Thus the position of the sun's image will indicate not only the latitude of the place, the declination of the sun for the given hour, and the apparent time, but it will also determine the meridian, passing through the place where the observation is made.

Definitions.—Right and Left swing.—A swing is a continuous rotation of the telescope horizontally beginning and ending with a zero. Several angles may be observed during a swing. It is clock-wise or anti-clockwise, ordinarily called right or left swing.

A setting is the exact division on the limb or arc to which a vernier is clamped at the beginning of a round of observations. The zero includes a pair of settings on different faces, right and left, and may be 0° and 180° , 60° and 240° , 90° and 270° , etc. If the first zero 0° and 4 zeros are to be observed on, the next zero is found as follows: $360^\circ/\text{No. of Verniers} \times \text{No. of zeros}$. Thus with 2 verniers and 4 zeros the next zero will be 45° and the 4 zeros will be 0° , 45° , 90° , 135° ; with 3 verniers and 4 zeros the next zero will be 15° , etc., with 2 verniers and 2 zeros the zeros will be 0° and 90° .

APPENDIX III.—Examples and Examination Papers.

QUESTIONS ON SCALES.

1. Draw a scale of 3" to a mile, to read multiples of 10 yards. State its R. F.
2. Construct without the aid of a protractor a pentagon of 2" side, and make by simple construction a square equal in area to $\frac{3}{4}$ of the above pentagon. And construct a simple scale for the above figures assuming the side of a pentagon 1 mile 2 furlongs in length.
3. Construct a comparative diagonal scale to one of 1 mile = 1", of knots to show decimals of a knot to two places of decimals. 1 knot = 6080'.
4. Construct a scale of 16" to a mile to read to 10'. What is its R. F.?
5. Make a scale of knots comparative to a scale of 10 miles = 1". Knot = 1.15 miles.
6. Construct a scale of 32" to a mile to read to 10 links. What area does one square inch represent?
7. Construct a vernier for a Barometer to read to thousandths of an inch.
8. Explain the principle of (a) Diagonal scales, (b) Vernier Scales; and say in what cases each kind is peculiarly applicable. Construct a scale, suitable for a map drawn to a scale of 32" to a mile, the scale to show 100 metres and metres. The metre is equivalent to 39.3709.
9. Why are scales used by engineers and architects? What do you understand by the terms "Vertical Scale," "Diagonal Scale," "Vernier Scale," and "Isometric Scale"? By what expressions are the scales $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, and $\frac{1}{128}$ ordinarily alluded to?
10. A survey of a tract of country 14 miles by 10 miles in extent is required to be plotted so as to fit on to an "imperial" sized sheet of paper. Note what scale you would adopt.
11. Draw a diagonal scale of $\frac{1}{80}$ to read feet and inches; for what purposes the following scales are used? (a) $\frac{1}{2}$ " to the foot; (b) $\frac{1}{16}$ " to the foot; (c) 50' to the inch; (d) 8 miles to an inch. Give their R. Fs.
12. Draw a rectangular figure with an area of 45 square inches, and state to what scale it should be measured to give an area of 20 square feet.
13. On a map a distance, known to be 19 miles, measures $7\frac{1}{2}$ "; construct a diagonal scale to read miles and furlongs, and long enough to measure off 15 miles.
14. Draw comparative scales of $\frac{1}{3600}$ to represent English feet, French metres, Greek cubits, 1 cubit = 45 metre.
15. Construct a diagonal scale 6" long for an R. F. of $\frac{1}{80}$ so as to measure feet and inches.
16. Sketch the relative positions of the Vernier and the fixed scale of a theodolite, the latter being graduated to half-degrees, when the reading is found to be $31^{\circ} 39'$.
17. On a map it is found that 100 acres are represented by 359.48 sq. inches, but the scale not being attached to it, it is required to calculate what it is. In the answer give the representative fraction, and also express the scale as so many feet or decimals of a feet per mile.
18. In a diagonal scale, show how tenths are measured, and prove that hundredths are measured by moving the compasses along the diagonals.
19. Construct a scale of 2' to the mile to read to 10 yards, and what is its representative fraction?
20. Construct a scale of 5' to the inch. Show sufficient of it to represent

40, and divide it so that you may read tens and units. Neatly ink it in in the usual way in which scales are shown on drawings, and above it write its representative fraction.

21. You have a scale of inches divided into tenths;—what will be the length of a vernier scale which will enable you to read to the eightieth part of an inch?

22. What is the object of a vernier scale? And how is this object attained? The limb of a theodolite shows 20', and it is desired to read to 1'. Sketch the vernier required.

23. What is the representative fraction of a scale? Draw a scale of 16" to the mile reading to yards, and shew 137 yards on it. What is its R. F.?

24. The scale of a plan is $\frac{1}{2500}$. How many inches to a mile does that correspond? Make a $\frac{1}{2500}$ scale showing yards.

25. One arc being graduated to half degrees, how do you make a vernier to read minute? Prove your answer.

26. Construct a scale of $\frac{1}{10000}$ showing yards.

27. Draw a scale of which the representative fraction is $\frac{1}{10000}$ showing miles and chains of 100'. Draw on this scale a vernier placed to read $1\frac{1}{2}$ miles.

28. A metre is about 39.37079 English inches, a Russian verst is about 1066.78 metres; make a scale to show $\frac{1}{1000}$ of a verst upon the Indian G. T. S. maps whose scale is an inch to the mile.

29. Your proportional compass falls, and a little bit is found to be broken off one of the points (say $\frac{1}{32}$ of an inch). How would you set the compasses right so as again to work accurately?

30. What is the R. F. of the Indian Atlas scale of 4 miles to one inch?

31. In making a rough reconnaissance you observe that your steady average walking pace is a 12 mile march in $3\frac{1}{2}$ hours: in noting objects met with on the march you record the time only as read from your watch: make a scale to show, on the atlas sheet, distances marched every five minutes.

32. Construct a scale of 16 inches to the mile, showing chains (100'). Distances up to 2000' are required to be taken off. Give the R. F.

33. Draw a scale whose R. F. is $\frac{1}{32}$ so that any measurement from an inch to 12' may be read off it.

34. Construct a scale of 10' to the mile showing feet, tens of feet, and hundreds of feet. Give R. F.

35. Construct a scale to show yards and feet, the R. F. being $\frac{1}{72}$.

36. Construct a diagonal scale of $1\frac{1}{2}$ " to the mile to show miles, furlongs and chains; and mark on it a length of 2 miles, 8 furlongs, and 2 chains. What is the R. F. of this scale?

37. Construct a scale of 32' to the mile to read feet. What is its R. F.?

38. Construct scale of 1 mile to an inch showing miles and furlongs.

39. Construct a scale of 6" to the mile showing chains (100'). Distances up to 5000' (=50 chains) are required to be taken off. Give its R. F.

40. Construct a scale of 2 miles to an inch, showing miles and furlongs. Give its R. F.

41. Construct a scale of 2' to the inch, showing feet and inches: Give its R. F.

42. Construct a scale of 12" to the mile, showing chains (100'). Distances up to 2000' (20 chains) are required to be taken off. Give R. F.

43. Construct a scale of 9 miles to the inch, showing furlongs diagonally.

44. Construct a scale of 4' to the inch, showing feet and inches.

45. Construct a scale of $\frac{1}{2500}$ to show miles and furlongs.

46. Draw a scale of 150' to the inch to show tens and hundreds of feet: Give its R. F.

47. Construct a scale of 5' = 1", showing feet and inches. Give its R. F.

48. Construct a scale, whose R. F. is $\frac{1}{27}$ showing feet and inches. Distances up to 10' are required to be taken off.

49. If 40 acres are represented on a map by 225 square inches, what is the scale of the map in inches to a mile?

50. Construct a diagonal scale of $\frac{1}{270000}$ to show furlongs.

51. Design a vernier: (1) to read to hundredths of a foot, the scale

APPENDIX III.] QUESTIONS ON SURVEYING INSTRUMENTS. 815

divisions being each $\frac{6}{10}$ of an inch; (2) to read to thousandths of a foot, the scale divisions being each $\frac{6}{10}$ of a foot.

52. Draw a scale of $\frac{1}{100}$, and attach a vernier to it so as to read to the tenth of a foot. On this scale mark off a length of 186'.

53. Draw a diagonal scale of $\frac{1}{10000}$ to read to $\frac{1}{100}$ th part of a furlong, and lay off 5.76 furlongs.

54. Construct a scale of chords, and by its aid lay off a bearing of $225^{\circ}30'$.

55. A Russian verst = 1166.6 yds. Construct a scale whose R.F. is $\frac{1}{11666.6}$ to read to single versts.

56. Draw a diagonal scale of $\frac{1}{30000}$ which will read to 5' on the diagonals.

57. Attach a vernier to a scale of $\frac{1}{50}$ th to read to decimals of a foot.

58. Construct a scale of 8 miles to the inch to show 50 miles and diagonally 10th of a mile.

59. The limb of a theodolite reads to 20 minutes, and the vernier to 30 seconds. Draw a part of the arc with the vernier when the reading is $68^{\circ}17'30''$.

60. Construct a scale of 2 miles to an inch, showing 15 miles reading to furlongs.

QUESTIONS ON SURVEYING INSTRUMENTS AND THEIR USES.

1. Name the surveying instruments required for: (i) a chain survey; (ii) chain and compass survey; (iii) a Plain table survey; (iv) Theodolite Survey.

2. Describe the different kinds of chain in common use, stating the peculiar advantage of each. How many Gunter's chain are there in $\frac{1}{8}$ of a mile? How do you convert measurements taken with the Gunter's chain into feet and yards respectively, and also into acres and decimal parts?

3. Describe the construction and use of the measuring chains.

4. What points should be kept in view in chaining?

5. What errors is the chain liable to? And state the precautions you would take to guard against them.

6. How would you test a chain? How will you allow for its elongation in the distances measured during the day?

7. Explain briefly what would be the effect on your field-book—(a) if the chain were too long; (b) if it were too short.

8. What is the correct length of a chain? And show by sketches how the arrows should be placed with reference to the handles of a chain. What would be the result of posting them wrongly?

9. How is the surveyor to know if his chain is in the correct line?

10. What errors is the surveyor liable to in using the chain? And state what precautions you will take to avoid them.

11. What will be the result if the chain gets out of the line?

12. How are stations marked, so that they may be recognised in the field during work or afterwards if the work is to be tested?

13. Describe the different kinds of surveyor's cross. Describe their construction and use. How would you test a cross-staff?

14. For what purposes are tapes used? And state the precautions you will take in using them. What is an offset rod? Describe its construction and use. Describe the Steel-band noticing its superior features over the metallic tape.

15. Describe the construction of optical square and how you test it.

16. Supposing a line to be correctly ranged, explain clearly the duties of the surveyor and chainmen in its measurement.

17. What form of arrow would you use in hard ground? Show by sketch how you will mark your station points on such ground.

18. How do you find out the true length of a line which has been measured by an incorrect chain?

19. Why is it necessary that a chainage should be recorded in the field-book each time a change of arrow occurs? Supposing an off-set falls at 20 links on the chain, and the follower has 5 arrows in his hand, and 3 changes of arrow entered in the field-book, what is the chainage done? Ans. 3520 or 35·2 chains.

20. Describe the prismatic compass and its uses: what is the use of the mirror attached to it and that of the dark glasses? Why are graduations reversed and written upside down?

21. What is the difference between bearings and angles? What is understood by saying that a line bears 50° from the magnetic north?

22. Define the terms:—"bearing," "magnetic variation," "azimuth," "relative azimuth," "magnetic azimuth," "meridian," "magnetic meridian," "magnetic bearing."

23. What are the properties of a magnet and how is it utilised in surveying? Explain the use of the magnetic needle in the rectangular box with the planetable.

24. Describe an ordinary surveying compass. What are the points of difference between the surveying compass and the prismatic compass?

25. Describe the compass usually attached to a level, and show by diagram how its ring is graduated and how bearings are taken with it.

26. To what errors is the surveying or the prismatic compass liable? For what purposes are they used? How do you test their correctness?

27. What is the bearing of the south pole when the magnetic variation is 14° west? What would be the bearing of the sun at noon observed with a compass having a magnetic variation of 17° E.? If the bearing of the true north is observed to be 355° , what is the variation of the compass with which the bearing was taken?

28. From a point b bears 270° , and the point c bears 10° . What angle does bc subtend at a ?

29. Describe the plane table and its uses. Discuss its advantages over the prismatic compass for land surveying. What are its demerits?

30. If a particular accuracy is required, explain how bearings should be taken. Explain why it is that the east and west points of the compass card are marked directly opposite the cardinal points they represent.

31. Describe two easy practical methods of finding the variation of a compass:—(1) by the sun's shadow; (2) by the Pole star. How do you obtain the true meridional bearing of a line?

32. Show by diagram that the variation of the compass must be added to or deducted from the observed magnetic bearing according as the variation is east and west to obtain the true bearing.

33. Show by diagram that if a line is to be relaid according to field-books giving true bearings, the magnetic variation of the compass has to be deducted from or added to the true bearing according as the variation is east or west.

34. What are the causes which affect the variation of the needle? Compare the pocket sextant and compass as angle-measuring instruments.

35. What kinds of angles are measured by them?

36. Describe the theodolite, noticing its principal parts and their use.

37. What are the points of difference between the Transit Theodolite and the Everest Theodolite, and that between the latter and the Y theodolite? Describe how would you obtain the reading on the horizontal limb. Describe how you will set off a given angle on the horizontal limb.

38. What is the advantage of having 2 or 3 verniers on the horizontal limb of a theodolite instead of one? What is meant by the eccentricity of a graduated circle? Prove that in measuring angles the error due to eccentricity is removed by taking the mean reading of the verniers.

39. What is meant by "changing face" of a theodolite, and what by changing pivots? Why are these manipulations performed, and what is the difference between them?

40. What are the temporary and permanent adjustments of the transit theodolite? When and how are they performed?

41. How would you obtain a correct altitude and a correct horizontal angle without making the adjustments of collimation in a theodolite?
42. How would you replace the cross-wires in the diaphragm and a bubble-glass of a theodolite or level when they are broken?
43. Explain the terms 'diaphragm', 'focusing', 'face', 'transit axis', 'reversing the telescope on its Y's', 'inverting the Telescope', 'the zero of vernier', 'face left', 'face right', 'zero of the limb', "Graticule," "Reticule."
44. Describe the difference between the Pocket Sextant and the Nautical Sextant and between the latter and the Hughes's Double Sextant.
45. How will you reduce an oblique angle to its horizontal projection?
46. Show by sketch how you will observe small angles with the artificial horizon.
47. Describe the Line-ranger, the optical cross, the apomecometer, and their uses.
48. What is the use of the pocket compass and the pocket prismatic compass? When are they used?
49. How will you find the North and South line approximately by means of a watch? And by observing Delta Cassiopeæ over the vertical of *polaris*.
50. Describe the use of the omni-telemeter, and range-finder.
51. What additional parts the 8-inch theodolites have? Describe them with sketches.
52. What are the special features of the Hoskold mining and Stanley's telemetric theodolites?
53. What is the difference between the ordinary planetable and Stanley's planetable? Describe the special features of Gurley's and Kern's Plane tables.
54. Describe Casella's Altazimuth and its use.
55. Describe the construction and use of the semi-circumferentor, Miners' dial, Leach's dial, Hedley dial, and Grubb Miner's dial.
56. After measuring 15 apparent miles of a road a man discovers that his chain is 2" too long. Find the error of measurement. What would be the error in measuring a square field which apparently contains 12 acres 2 roods?
57. Your chain instead of being 66' measures 66' 2", and you measure 2000 chains. What is the correct distance chained over?
58. Explain the term "bearing". And show how a station may be found by observation of two points already known. Show also how an angle may be measured with the prismatic compass. Give an example.
59. In using a chain mention the points to be attended to in order to obtain an accurate measurement. What is the length in feet of Gunter's chain and why is it of this particular length?
60. After measuring a distance of 3147' with the chain of 100', you find by measurement of the chain that it is two inches too long, what is the true length of the line measured?
61. What is the length of the Gunter's chain and why is it used for land surveying? As a measuring chain is not always of correct length, what tests would you adopt to ensure accuracy of measurement?
62. Show how to determine accurately the true meridian; also state how the same may be approximately ascertained.
63. In chain surveying why is it advantageous to use a chain that is a little too long and why is an old chain preferable to a new one?
The chainages and chain errors with a 100' chain are as follows:—First day—at starting chain 2' short, chainage 117 chains 47 links, chain error at finishing 1' too long; 2nd day—chainage 65 chains 29 links, chain error at the end of the day 3' too long; 3rd day—chainage 213 chains 19 links, error at end of day 3½' too long. What is the total actual distance chained?
64. What is meant by the variation of the compass? Is it the same in all places? Does the variation increase or decrease annually? Give two simple methods of finding it.
65. What is the usual amount and direction of the variation of the compass in Bengal, Madras and Bombay Presidencies, and the Punjab?
66. Sketch the graduations of the (a) prismatic compass, (b) Surveying compass, and explain the methods of figuring the cards.

67. What is the index-error of a sextant and how is it rectified? Explain how you would obtain the sun's altitude by means of a sextant.

68. How do you take the altitude of an object on land with the sextant? What is an artificial horizon? Describe the use of the artificial horizon.

69. Give a description of the construction and method of use of the plane-table. For what purpose is it best suited?

70. Show how to find heights and distances by the aid of the pocket sextant. How would you find your position in a Survey by its means?

71. Explain the permanent adjustments of the transit theodolite. Describe also the temporary adjustments to be attended to every time the instrument is used. Note the difference in permanent adjustments between this and the Everest Double Arc Theodolite.

72. Explain (1) how you would repeat an angle with a theodolite; and (2) how you would obtain accurately the magnetic bearing of an object with the same instrument.

73. Describe a box sextant and make a neat sketch of the instrument. What is the objection to the use of the sextant as a surveying instrument?

74. Explain the principle of the sextant and show by an example how you would find the height of an inaccessible object by the aid of this instrument. Describe the way you would check the divisions on its horizontal plate. What corrections have to be made in all angles of altitude taken to objects at a considerable distance off on the Earth's surface?

75. Select 5 points, 3 of which are free from local attraction, while 2 consecutive ones are known to be subject to such attraction. Make the survey, finding length and forward and reverse bearings of every side. Find what the true bearing of each line is, and plot to obtain the error of closure.

76. A reconnoitering party of two men are provided with a pocket sextant and have procured a few straight sticks, but have no means of measuring length accurately. A and B are two points clearly visible by the party, but separated from them by a river which cannot be crossed. Show how the party can mark out, on their side of the river, a line equal in length and parallel in direction to AB, and how the accuracy of the result can be checked.

77. What is the object of repeating an angle with the theodolite? Describe the process. What is the advantage of repeating an angle?

78. Explain what is meant by the expressions:—(1) Optical axis of telescope; (2) Line of collimation; (3) Virtual line of sight, and also describe the process of 'focussing' and removing 'parallax'.

79. (a) Sketch the relative positions of the vernier and fixed scale of a theodolite (the latter being graduated to $\frac{1}{2}^\circ$) when the reading is found to be $31^\circ 39'$. (b) Also sketch a part of the horizontal limb with vernier A so as to read $315^\circ 19'$.

80. A base-line is measured with a 66' chain. During the first day 13459 links were measured, average length of chain $66' 3"$, on the 2nd day 12600 links were measured before the chain broke, its average length having been $66' 45"$. The remaining length 17538 links was measured with the mended chain only measured $65' 8"$. What is the true length of the base-line?

81. Show the form of field-book you would keep when making a traverse survey with the prismatic compass.

82. If a line has been measured with an incorrect chain, show how the true length of the line may be obtained. Suppose the chain to have stretched $4'$, and that the measured distance is 875 links; what is the true distance?

83. A chain is found to be half an inch too short, and the measured distance 3,423 links: What is the true distance? How is the chain tested?

84. How would you ascertain whether a prismatic compass given to you to work with was a reliable instrument to use?

85. Why is an ordinary offset tape unreliable as a means of checking the length of a chain? How would you check the length of the chain you were using, the only appliances at hand being the ordinary ones supplied to a leveller?

86. If a hundred feet chain at the beginning of a day's work was $2'$ too short, and at the end $4'$ too long, what length would you plot a line which measures 1072 feet by the chain?

17. Find the area of a Δ r field whose sides are 100, 1200 and 1500 links. Plot the above field to a scale whose R. F. is $\frac{1}{1000}$ and divide it into three parts by fencing which starts from the angle opposite the side 1500 so that A's share is one-half and B's and C's one quarter each.

18. A chain which measures 99.2 feet makes the area of a plot of ground to be 1464.6 acres. What is the true area?

19. Give the methods for determining the breadth of a river without the use of an angular instrument.

20. An obstacle, such as a tank, comes in the way of the chain line. Show how you would avoid it and pick up your chainage again correctly.

21. In chaining a line between two stations *A* and *B*, the follower loses sight of the station *B*. How would you proceed with the chaining of the line, *A* being still in view?

22. The area of a plot of land, surveyed with a 66' chain which was 3" longer than its supposed length, was found to be 655 acres. Find the true area.

23. Explain by a diagram, how you would reduce an irregular offset-piece to a right-angled Δ .

24. How would you find the diameter of a $\odot$ r tank, too large to chain across? Show how to find the distance of an inaccessible object by means of a rhombus and equal figures.

25. *a* and *b* are two points on the opposite banks of a river; show how you would find the distance *ab* by the method of equal figures on the bank at *a*.

26. How many cases of obstacles occur in practice? Show by diagram how you will avoid them.

27. Explain how can you fix position on a Survey by means of the plane table and compass, and also without the compass, three known points having been already plotted on the table. Describe the 2-point and 3-point problems.

28. In the traverse of a road with the plane table what check should be adopted by the surveyor? What is meant by *interpolation*?

29. How can you fix a number of points when two points are known and are fixed by the method of *Intersection*? Explain by diagrams the methods of surveying by *Radiation*, *Resection*, and *Traverse*.

30. Describe 2 methods of finding your position in a Survey by means of the plane table. Of the two methods selected which do you prefer and why?

31. How will you test by calculation the length of a tie-line *CD* of any ΔABC , measured from the angle *C* to the side *AB* so as to divide *A B* into two parts *AD* and *BD*, of which *AD* is adjacent to *AC*, and *BD* adjacent to *CB*?

$$\text{Ans.} - CD = \sqrt{\left(\frac{AD \cdot CB^2 + BD \cdot AC^2}{AD + BD} - AD \cdot BD \right)} \quad \left(\begin{array}{l} \text{See Art 241, Prob. } \times V. \\ \text{Case I (c). page 140} \end{array} \right).$$

And if the tie-line *CD* is at right angles to the side *AB* then

$$CD = \sqrt{CB^2 - DB^2} = \sqrt{AC^2 - AD^2}.$$

32. What are the four principal methods of procedure in Surveying by which the position of a number of points, requisite to obtain a sufficiently close and adequate representation of the features of the country, and the natural and artificial objects there existing, may be determined for record, and finally reduction to plan? Ans:—They are:

1. By distances, or distances and offsets (See Chap. IV.)

2. By triangulation or network of Δ s (See Chap. IV & V).

3. By traversing, or a continuous combination of distances with angles or bearings (See Chap. V).

4. By determining the latitudes, longitudes, and azimuths of detached points (See Chapter on Practical Astronomy).

These modes of procedure may be adopted either severally or in conjunction with each other, whether measurement or calculation or both happen to be the means of carrying out these modes.

33. In Fig. 839, p. 761, the chord *AB* is measured to be 130', the angle of deflection α is observed with a compass to be 18° . Find the radius of the

curve and the ordinates every 13' apart, Ans:—The radius suitable to the case is found by the formula $C=2R \sin. a=2\sqrt{R(V-V^2)}$; where C =chord, R =radius, a =angle of deflection, and V =versed sine.

∴ $R=130 \div 2 \times 0.309=210$ feet. And $V=C^2 \div 8R=130^2 \div 8 \times 210=10$ feet. (See Method 5, Page 761). To set out the curve, four other ordinates are determined by the formula $O=\sqrt{R^2-X^2}-(R-V)$, thus:

The 1st or 9th ordinate = 3'4" feet.	The 4th or 6th ordinate = 9'6 feet.
„ 2nd or 8th „ = 6'3 „	The 5th or middle ordinate being 10 feet.
„ 3rd or 7th „ = 8'4 „	

34. Find the horizontal length and bearing of a line joining two inaccessible points both of which can not be seen from any one position.

Ans:—Let A and B be the inaccessible points; measure a base CD such that A is seen from C, and B from D. Auxiliary bases and Δ s may be used to find lengths of AC and BD. Knowing AC and CD and the included angle, compute AD in distance and bearing. The $\angle$ ADB may now be found which, with the adjacent sides AD and BD known, enables the side AB to be found in bearing and distance.

QUESTIONS ON CHAIN SURVEYING.

34. What is meant by the terms?—*Stepping, Station points, Station or chain lines, Rectangular or right offsets, oblique offsets, Base-line, Tie-line, Main and Subsidiary station lines.*

35. Explain the principle on which the chain surveying is based. What preliminary operations should be made before commencing the actual survey in the field?

36. How is a chain survey executed in an open country?

37. In a chain survey why should the ground be divided into Δ s and why should they be made as large as practicable? Does not any other rectilineal figure fulfil the same conditions as the Δ s?

38. What is the object of tie or check lines and in accordance with what maxim are tie-lines measured?

39. What are offsets? And show how are they taken and recorded. Why is it desirable that offsets should be as near as possible to the main line? Show by diagram that there is a greater error for a point more distant from the chain line if it be a little deviated. What would be the length of the longest offset you would take?

40. What are the two great principles to be borne in mind in executing a survey, and how far does the method of chain survey accord with them?

41. If a lake, wood or river or a compound with a building can not be divided into Δ s and chained across for surveying, how would you go on to execute their survey?

42. What is meant by surveying by “chain angles” and what objection is there in this method of surveying?

43. What is meant by planning a survey? What points should be borne in mind when making the triangulation?

44. How would you proceed to lay out in the field your stations and Δ s for a survey? Describe the successive processes. How many men will you require to assist you? Explain the duties of each.

45. Describe the field-book and show how it is kept for a chain survey.

46. Why is it desirable in a chain survey to have one base-line at least about the centre of the survey measured twice or thrice for the sake of accuracy?

47. When would you recommend a chain survey? When does it become inconvenient?

48. Show by diagram how you will plot a chain survey.

49. In all surveying operations the work should be from “whole to part and not from part to whole.” Explain this maxim.

“This means that the survey should be so carried out that the utmost

accuracy is secured in the triangulation and measurement of the main Δ s which should be as large as possible into which the whole land is divided, and that the accuracy of the filling in of the internal work either by offsets or by subsidiary Δ s and by offsets on the latter should be made to depend on that of the external work.

50. Plot to a scale of $100' = 1''$ the fields in Fig. 166, and find out the area of the field $C F_1 G D B_1$.

51. How are stations marked in the field expressed in field-books? Why have survey field-books to be written from the bottom of the page upwards? How are stations left and found again when actually surveying? Explain this with the aid of a diagram.

52. How will you fix the position of a distant point by means of oblique offsets and what check will you adopt? How will you pick up a gate or a corner of a fence or a building when chaining? Explain by sketches.

53. Explain the methods of finding out approximately true horizontal distances of sloping sides of hills. And show by diagram how you will find out the horizontal distance of a line along undulating ground. What is this method called? How is the necessary correction calculated after chaining on a slope?

54. Why is it necessary to know the scale of a survey before commencing actual measurements in the field? How would you range a long line across elevations and depressions? Explain by diagram. Why is it necessary to post a flag in front of every hedge crossed by a chain line?

55. (a) In a chain surveying the ground is divided into a series of imaginary triangles. Why is this? (b) What is the most acute angle the sides of a triangle should be allowed to meet at, in practice and why? (c) What is a tie-line? If you wanted to drop a tie from any point A on an unobstructed line BC , what point in BC would be best for your purpose and why? (d) How would you take an offset? If a point were a little too far off for the offset to be very accurate, explain how you would fix its position with the chain only.

56. A piece of ground in the form of a trapezium has to be surveyed with the chain only; No diagonal can be measured. How would you survey it so as to be able to lay down on paper?

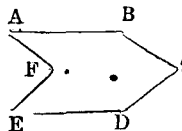
57. From A to B on a base-line the mean angle of inclination is $2^\circ 55'$ and the measured distance 442.7 links. What are the horizontal and vertical distances between the two points?

58. Give a concise description of the method of surveying with chain only, a piece of ground of limited area, aiding your description by a diagram.

59. Supposing you have selected the site and marked out the axis line of a proposed structure, you are provided with a land plan on which the positions of two adjacent structures are shown; and that you are equipped with a note-book, a pencil and a 50' tape; indicate how you would lay down the site of the proposed structure on the land plan.

60. Describe the process of laying out the lines of a masonry road bridge preliminary to its construction. It has three spans of 15', piers 3' thick, Roadway 16' wide, height above bed of stream 12 feet.

61. (a) Show how you would survey by chain only the piece of country ABCDEF represented in the margin below supposing it has a dense jungle inside. (See Arts. 301, 324, 325).



(b) If you make a prismatic compass traverse on the lines AB, BC, CD, DE, EF and FA, you find the traverse closes on the starting-point. In plotting the field observations you find there is a closing error. How would you eliminate it? The bearings have been corrected for local attraction before plotting.

62. What are the chief points to attend to in laying out a chain survey? What are the duties of the leader and follower in chaining?

63. What errors in chain survey chiefly liable to, and to what points must the surveyor's attention be directed to avoid such errors?

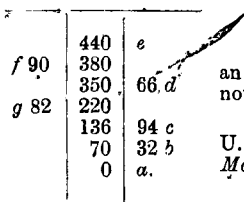
64. Describe, how you would make a survey, with the chain only of a few fields of which, it is required to plot the boundaries and ascertain the

areas. Give a neat specimen of the field-book you would keep showing clearly what offsets would be taken to enable you to plot the boundaries. Explain what measures you would take to check the accuracy of the field-work.

65. In a chain survey a, b, c, d, e are five stations, no three of which are in the same straight line; what lines must be measured?

(i) To give the positions of these points. (ii) To check the work.

66. In making a chain survey of a plot of ground, the following measurements were made:— a to $b = 530'$, b lying due north of a ; b to c (going westwards) $= 425'$; c to $a = 622'$; c to d (going southwards) $= 380'$; and d to $a = 555'$. Plot to a scale of 200' to an inch, and find the length of the check line db . Also by the use of a protractor determine the bearings of the lines bc, ca, cd and da .



67. Find the area in acres and decimals of an acre of the field $abcdefg$ from the marginal notes, the measurements being in feet.

Find what the *areas* would be in Bengal, U. P., Bombay and the Punjab *Bighas*, Orissa *Mans*, Madras *Cawnys*, and Burmese *Pegadis*.

QUESTIONS ON COMPASS AND THEODOLITE SURVEYS.

68. How will you execute a compass survey of a field? In how many ways can you survey a field with a chain and compass? Which is the more correct method of surveying—survey by compass and chain or survey by chain only?

69. How would a surveyor find his position in the field on a map of the same with a compass? Show by an illustration.

70. How will you fix the position of a spire or any other object far away from your chain line and to which it is not convenient to chain? Explain how points can be fixed by the plane table or compass without actually measuring to them. What precautions would you take to ensure absolute accuracy.

71. Write out a field-book generally kept for a compass survey. Point out the difference between this field-book and the one kept for a chain survey.

72. What is a "closed traverse"? In a closed traverse done with a compass, what field and office checks are there on the accuracy (1) of the bearings taken, (2) of the measured lengths?

73. How will you plot your compass survey? In how many ways can you plot it? Show them by diagrams.

74. For what class of work a compass survey is resorted to?

75. What is meant by the terms "forward" and "back-bearings"? What are the uses of the back-bearings?

76. The Latitude of B from A is $106^{\circ}42' N.$, and its departure is $273^{\circ}6'2 W.$ What is the bearing of the line AB?

77. How would you fill in the details with the aid of a compass? Show by sketches. How would you interpolate a point in a compass survey and verify its position?

78. Show how you will determine the breadth of a river by the compass.

79. Explain by diagram how you would prepare your paper with meridian lines for plotting direct in the field. Describe an accurate method of plotting an angle and a bearing.

80. Plot the following: Scale 6" to one mile, one of the bearings is wrong correct it giving reasons— a to b bearing 45° , distance 2 furlongs; b to c bearing 315° , distance 2 furlongs; c to d bearing 225° , distance $\frac{1}{2}$ mile; d to a bearing 137° , distance 20 chains (Gunter's).

81. Plot the following open traverse lines from the notes of an actual road survey given below to a scale of $400' = 1''$:—

From station	To station	Bearings	Distances.
A	B	149°00'	100'
B	C	141°30'	550'
C	D	151°00'	120'
D	E	130°30'	230'
E	F	142°15'	550'
F	G	134°00'	250'

82. Explain a method of finding the bearing of a station point by a pocket compass.

83. Under what circumstances would you prefer to conduct a prismatic compass survey instead of a chain survey? Which is generally considered to be the more accurate. Give reasons for your answers.

84. After completing a traverse $abcdef$ with a compass you find, on making the interior angle check an error (of about 3° say). What may this error be due to? and how may you scrutinise your field-book to locate the station at which the faulty reading or readings were taken?

85. Supposing that you have map handed to you showing the boundary between two villages, with instructions that you are to lay this boundary down upon the ground. Describe in detail how you would set to work and what method of check you would adopt.

86. How can the position of a solitary object, which is too far off to admit of an offset being taken to it, be determined?

87. Having plotted a prismatic compass traverse of the boundary of a certain area, explain how you would set up the plane table inside the boundary to commence filling in the interior details.

88. In what respects is the method of surveying with the compass more convenient than surveying with chain only? What are the disadvantages of the former method?

89. What are the different conditions to be fulfilled, when a traverse makes a complete circuit?

90. How is the bearing of a line obtained from the observed inward angle of two lines? What are latitudes and departures?

91. Show how you would find the distance of an inaccessible point with the aid of an instrument for measuring angles.

92. If the following survey were made with a compass having 23°W

- | | |
|--------------------------|---|
| 1. S 30° W ... 4 chains. | } variation and plotted on a plan by the true meridian without being reduced to it. What would be the magnitude of the error in position of the last point? |
| 2. N 50° W ... 8 " | |
| 3. N 50° E ... 9 " | |
| 4. N 53° W ... 8 " | |

93. A traverse survey is run round an irregular field $abcdefga$. ab bears 63° and the $\angle s abc, bed, cde, def, efg, fga$ are 216°, 287°, 230°, 256°, 258°, 62° respectively. What are the bearings of the different sides of the field?

94. Describe the prismatic compass, and how you would make a small traverse survey with it. Give a specimen of the field-book you would keep and show how the survey would be plotted.

95. Two points bear 45° 39' and 113° 15' respectively. What is their angular distance—that is to say, the angle between, the lines joining them with the centre of the compass?

96. Describe how you would make a traverse survey of a road with the prismatic compass. Give a neat specimen of the field-book you would keep to enable you to make a plan of the road and adjacent objects, such as boundary pillars, houses, large trees, and tanks.

97. Explain how bearings can be plotted by means of a scale of chords.

98. What do you mean by a Gale's traverse? How are measurements made and how would you plot your results.

99. A line is run out with the magnetic bearing N. 13° 15' E. At the same time the declination is noted as being 0° 55' W. What will be the magnetic bearing of the line at a later date when the declination is 1° 05' E?

100. Explain the method of reducing and plotting a survey by rectan-

gular co-ordinates. What are the advantages of this method as compared with the ordinary mode of plotting with the scale and protractor? Explain the traverse tables. State the Universal Theorem for finding area.

101. In a closed traverse, how could you prove that the interior angles were correct?

102. Sketch and explain the action of the tangent-screw and clamp as applied to any parts of a theodolite.

103. Plot to a scale of 2" to the chain the following traverse survey, and determine the length and bearing of the line joining *A* and *E* :—

Lines.	Meridian Angle.	Length.
AB	... 226°	2 Chains 78 links.
BC	... 123°	2 " 40 "
CD	... 242°	1 " 82 "
DE	... 172°	2 " 95 "

104. A theodolite is set up in line with two telegraph poles, 150' from the nearer pole and 450' from the further pole. The top of the further pole subtends an angle of 18°, the line of sight passing through a hole exactly half way up the nearer pole. Required the heights of the two poles, the theodolite standing 5' above the ground.

105. Calculate the co-ordinates, of the following traverse survey, calculate the length and bearing of the line *GA*, and plot by co-ordinates to a scale of 1 chain to the inch :—

Traverse Survey of polygonal area made by double fore-sight method with a right-handed theodolite reading to 30 seconds; the theodolite was originally set in the true meridian, true north reading 360°00'00"

Line.	Observed Angle.	Length in Links.	Line.	Observed Angle.	Length in Links.
<i>AB</i>	14°48'00"	245	<i>EF</i>	271°33'30"	480
<i>BC</i>	198°06'30"	310	<i>FG</i>	268°01'30"	607
<i>CD</i>	284°01'30"	480	<i>GA</i>		
<i>DE</i>	200°12'30"	709			

106. Calculate the area of the above polygon in acres, etc., by the method of co-ordinates.

107. What are latitudes and departures? Determine a formula for calculating areas by means of them. What is "double meridian distance."

108. Explain the method of surveying with a theodolite by adopting a fixed meridian. What checks are applicable when a circuit is completed and how is the traverse usually plotted?

109. Give a specimen of the field-book you would keep in running a traverse with theodolite and chain.

110. The main divisions of the circular arc of a theodolite show degrees which are subdivided into 4 parts representing 15 minutes each. The main divisions on the vernier show minutes and the subdivision 15 seconds. Make a rough sketch of the scale to read 23°17'45".

111. Prepare a traverse table from the following data and plot the survey to a scale of 200' = 1".

Station.	Observed inward Angles.	Bearing.	Distances.
1	.	1 to 2 = 114°10'	.
2	225°1'	.	238
3	112°30'	.	370
4	100°0'	.	362
5	123°15'	.	595
6	112°50'	.	268
7	143°15'	.	356
1	83°10'	.	351.5.

112. What is the amount of error allowed in the Revenue Survey for the correction of latitudes and departures? How is the bearing of a line obtained from the observed inward angles of the two lines?

113. Explain the mode of surveying by theodolite traverse and the method of plotting according to Gale's system.

114. Show how you would find the distance of the inaccessible points with the aid of an instrument for measuring angles.

115. If you are entrusted with a town survey (of such a place, for instance, Howrah) state precisely how you would conduct it from the outset up to the production of a complete survey map.

116. Find the true length of the base-line from the following copy of a page from a field-book.

	Measurements.	Mean of Measurements.	Angles of Inclination.			Mean Angles of Inclination.			REMARKS.
			Degrees.	A	B	Degrees.	Min.	Sec.	
1		569.33				—1	29	37	Length of chain before measurements of base = 100.00. Do after = 100.27.
2		258.19				0	0	0	
3		396.73				+0	59	13	

117. What is a base-line, how would you measure it? What is a Satellite station? Explain the method of calculating corrections necessary for one. What is an *Eccentric Station*?

118. The field-work of a trigonometrical survey having been completed, what calculations have to be made?

119. A, B and C are the points of a Δ ; A is the weather-cock on a steeple; B is a basket on a pole; C is a point on the ground indicated by a flag. The horizontal distance A B is known and the angles are measured with a pocket sextant. What further data are required before the Δ can be plotted. By what steps is the determination of the Δ arrived at?

120. Describe the method of surveying with a theodolite by triangulation. How would you select and measure the base-line?

121. Show how the angles subtended at an inaccessible station may be obtained by means of angles taken at a Satellite station.

122. Describe the measurement of a base-line for a survey with chain and theodolite: also describe the method of measuring a round of angles. What are bases of verification?

123. By what considerations would you be guided in choosing the trigonometrical stations for a large survey. Why is it desirable to avoid ill-conditioned Δ s.

124. What is meant by *reduction to the sea level*, *reduction to the horizon*, *reduction to the centre*, and how is the last effected?

125. Describe the system of Trigonometrical Survey from a base-line and the various checks on your work which you would think necessary.

126. Plot the figure A B C D E A to a scale of 100' = 1'', and determine the last distance and the bearing, giving a geometrical check of the bearing after Enc. I-32 Cor. The data are A B ($33^{\circ} 15'$) 111', B C ($118^{\circ} 38'$) 143', C D ($105^{\circ} 45'$) 203', and D E ($260^{\circ} 0'$) 99'; the figures in bracket are forward bearings, and the others are distances in feet.

127. Draw an angle-book of triangulation and make entry of angles for three stations A, B, and C at corners of a Δ ABC so as to have their mean

values $43^{\circ} 21' 30''$, $77^{\circ} 19' 45''$ and $59^{\circ} 18' 45''$ respectively. If in the above A B is the base-line $421' 37''$ long, calculate the other sides and plot the figure to a scale of $200' = 1''$

The following are copied from log. tables :—

Angles.	L. Sine.	No.	Log.
$43^{\circ} 21' 30''$	9.8366778	42137	6246636
$59^{\circ} 18' 45''$	9.9344800	33641	6268614
$77^{\circ} 19' 45''$	9.9892924	47805	6794760

128. In taking angles by the theodolite, which station is observed first, the left-hand one or the right-hand one? What will the difference be if done otherwise? Would there be any variation in the reading if the telescope is turned clockwise or anti-clockwise?

129. Are any of the above rules to be observed in surveying with a magnetic compass? Give your reasons.

130. What is ordinary traversing by a plane table, what important advantage Gale's system has over it, and is the trough compass supplied with plane table a needful adjunct in traversing.

131. Describe Coradi's Co-ordinatograph. To what purpose is it applied?

132. What will you do when the circuit is too large to admit of the stations being plotted from the first meridian.

133. What is meant by balancing the traverses? How do you apply corrections to the traverses? What do you mean by *closing* a circuit?

134. What is meant by *Putting up* a circuit? How do you deduce and reduce bearings? How is the angular work proved to be correct?

135. State the rule for finding the area of a polygon obtained by the Gale's system. Draw up a *Formula* for the area also.

136. In Fig. 244 (Art. 484, p. 274) the angles on the lower side of the river from 1 to 7 are: $62^{\circ} 15'$, $48^{\circ} 25'$, $49^{\circ} 30'$, $55^{\circ} 14'$, $59^{\circ} 00'$, $46^{\circ} 24'$, $68^{\circ} 17'$, $46^{\circ} 05'$, $65^{\circ} 18'$. If the bearing of 1-3 is $S 18^{\circ} 30' E$, find the bearing of 7-8.

137. Determine a general method of obtaining the length of an inaccessible line.

138. State the method to be used in the above problem when there is only one point from which both ends of the line are visible.

139. State the method when there is no point from which both ends of the line are visible.

140. Find the correct bearing of the line AB in the following traverse:

	Fore-sight	Back-sight	
AB ...	N. $15^{\circ} 00' E$.	... S. $15^{\circ} 00' E$.	Is this problem possible? If so, under what circumstances?
BC ...	S. $14^{\circ} 35' W$.	... N. $14^{\circ} 30' E$.	
CD ...	N. $25^{\circ} 10' E$.	... S. $23^{\circ} 55' W$.	
DE ...	S. $85^{\circ} 05' W$.	... N. $85^{\circ} 05' E$.	

141. Compute the error of closure, its bearing, and the ratio of closure in the following closed traverse:

Line.	Bearing.	Length.	Line.	Bearing.	Length.
AB ...	N. $84^{\circ} 50' E$.	6.88 ch.	CD ...	S. $84^{\circ} 50' W$.	8.75 ch.
BC ...	S. $29^{\circ} 28' E$.	4.90 ,,	DA ...	N. $6^{\circ} 55' W$.	4.70 ,,

142. Supply the missing data in the following closed traverses:

(a) Line.	Bearing.	Length	(b) Line.	Bearing.	Length.
AB ...	N $13^{\circ} 05' W$.	... 417.3 ft.	AB ...	N. $25^{\circ} 13' E$.	... 915.3 ft.
BC ...	...	... 503.0 ,,	BC ...	...	... 450.0 ,,
CD ...	S. $45^{\circ} 00' E$.	... 400.0 ,,	CD ...	S. $37^{\circ} 00' W$.	... 375.0 ,,
DA ...	...	... 317.5 ,,	DA ...	N. $43^{\circ} 10' E$.	...

143. The points on a village boundary are fixed by means of a theodolite traverse. How would you complete the detailed survey of the village and how could the accuracy of your work be checked?

144. Describe the method of Bar—subtense survey. Where is it specially suitable?

145. State the method of surveying with the tachometer and the advantages it has over the prismatic compass.

EXAMINATION PAPERS ON LEVELLING AND CONTOURING, CURVE-RANGING, ETC.

1. Obtain an Expression for the error in levelling due to the curvature of the Earth. How is this modified by the error due to refraction? Why are these corrections very rarely applied in practice?

2. What is meant by Contouring? How would you contour a hilly country? Describe how you would fix and plot the contour lines on a survey? Show by diagram how to find the cubic contents of a reservoir from contours.

3. What are the errors for which a Y-level may need adjustment and how would you test it for each?

4. What precautions must be observed in using the aneroid barometer for levelling in a reconnaissance survey? What pocket instruments would be useful to check and supplement aneroid observations?

5. Check and complete the following portion of a Level-book.

No. of Station.	B.S.	I.S.	F.S.	Distance.	Bearing.	Rise.	Fall.	Reduced Level.	REMARKS.
1	4.70			5 miles Och.				77.06	B. M. on milestone.
				5 1				77.37	
				... 2				79.92	
				... 3				79.89	
				4				80.28	
				5				80.38	
				6				80.89	
				7				80.45	
2	11.85			8				80.68	
				9				80.62	
				10				80.01	
				11				86.19	
				12				89.68	

6. Plot the Section to a Scale of $200' = 1''$ horl., $20' = 1''$ vertl. Draw on this Formation line for a road at 5 miles 1 chain, R. L. 88.00; thence to 5 miles 6 chains there is a falling gradient of 1 in 350; then a rising gradient of 1 in 220 in the remaining portion of the Section.

7. In the above section, the slope of the side long ground is 1 in 5 from left to right; width at formation 30'; slopes 2 to 1. Draw a plan showing thereon the lines of the top width, and the outer edges of the side slopes in cutting and embankment.

8. Describe some of the methods employed for checking the field-work of a line of levels, as the work proceeds. Do you know of any instruments, besides a level by which differences of altitudes of stations can be ascertained?

9. Mention briefly the permanent adjustments required for (a) the Y-level; (b) the Dumpy level. Explain clearly in what respects and why the adjustments of the two instruments differ.

10. Explain the terms: Datum line; Reduced level; Line of Collimation; Collimation error; Parallax; Check levels; Contouring; Clinometer.

11. How do curvature and refraction affect levelling observations, and why are their effects not usually allowed for in ordinary operations. What is the usual way of eliminating the effect of all errors other than faults in observation? If prevented from employing this method at one observation, how can the error be compensated at the next?

12. What are the permanent and temporary adjustments of a Dumpy level? Account for the difference in the arrangement of the Cross-wires in a level as compared with a theodolite. Fill up an imaginary page of a level-book giving the headings to each column and inserting 12 or 15 entries.

13. What are "Bench-marks"? How and why are they chosen and how marked? What kinds of objects are best suited for bench-marks, and how should the position of bench-marks be recorded in the field-book?

14. In what way is the Barometer utilized in Survey operations?

15. Explain fully the permanent adjustments of Cooke's "Reversible level." How would you correct the position of the cross-wires?

16. A and B are two pegs 450' apart. A level is set up between the two pegs at a distance of 50' from B, and the readings on the staves held at A and B are 5.39' and 6.81' respectively. The level is then shifted along the line AB so that its distance from A is 150', and the readings recorded in this position are 4.32' and 5.14' at A and B. Determine the correct difference in height between A and B.

17. To select a suitable alignment for a line of railway through difficult hilly country, it is necessary to prepare a contoured map of the country (scale 200' = 1") the contours being 10' apart in height and extending to 150' vertically above and below the rough grade line traced between obligatory points. Explain how you would set to work and what instruments you will require. Detail the points to be observed in the selection of Bench marks.

18. What are the uses of an Abney Level or Clinometer?

19. Complete the entries in the following field-book, check the results, and plot the plan and section. Scales : horl. 100' = 1", vertl. 10' = 1".

Station.	B. S.	Inter.	F. S.	Dist.	Bg.	R. T.	F.	R. L.
1	3.56			0	112°			100.00
		5.60		155				
		2.15		160				
2	1.60		7.14	163	86°			
		6.71		186				
		8.31		200				
		5.60		260				
	2.24		13.52	300				
3		8.91		350	42°			
4	6.00		6.70	400	310°			
5		2.14		500				

20. Describe how you would carry out the permanent adjustments of Zeiss Pattern levels. Describe the instruments with sketches.

21. In running a long line of levels what precautions would you adopt to avoid errors?

22. Explain the term 'line of collimation'. How would you correct it in Dumpy and Y levels?

23. What are the different ways of levelling with a theodolite.

24. Define a "horizontal line" and "a level line."

25. With a level in perfect adjustment, a distinct point known to be 3 miles 5 furlongs distant, appears to intersect the cross-wires. Taking refraction and curvature into account what is the true relative position of the point with respect to the eye. Prove the formula you apply.

26. Describe a water level.

27. Describe contour surveying with a spirit level and with a theodolite. At what vertical distance apart may contour lines be generally most usefully applied in Bengal and for what purposes are they mostly required.

28. Explain in what case it would practically be necessary to make

corrections in levelling on account of curvature and refraction and what corrections would have to be made.

29. • Work and plot the following extract both in plan and section and check the results.

Station.	Back sight.	Inter.	Fore-sight.	Bg.	Distance. Ft. •	Rise.	Fall.	R. L.
A	3·01	5·62		9°	0			100·00
B		7·80		203°	25			
		9·63			240			
	1·06	13·46			290			
		3·72	14·01		300			
		5·91			330			
C		2·16		186°	350			
	12·16				400			
		7·30	0·32		450			
D		4·26		135°	490			
		2·06			550			
					700			

The lines BC and CD in above are to be connected by a curve of radius 500 ft. Find the angle between them and calculate the tangential distance and offsets for setting out the curve.

30. If you were instructed to make a preliminary report on a projected line of road in a country of which no detailed surveys existed, describe what information you would collect and what rough survey work would be needed to make your report of value.

31. The following is an extract from a level-book. Explain the meaning of each term used, show how the figures were obtained, and give the usual checks to the accuracy of the numbers—

Back.	Intermediate.	Fore.	Rise.	Fall.	Reduced.
1	2	3	4	5	6
10·71	10·68		0·03		745·60
	7·91		2·77		745·63
	5·44		2·47		748·40
12·72		2·08	3·36		750·87
	6·54		3·18		754·23
	2·96		3·58		760·41
	2·25		0·71		763·99
		2·56		0·31	764·70
					764·39

32. Describe with sketches a dumpy level. Explain the adjustments necessary and the method of using it. The bubble-tube of a dumpy level is out of adjustment. How would you detect this error and adjust it?

33. Draw up an imaginary level-book: show two back, eight intermediate and two fore-sights. Work out the reduced levels, that of the starting point being 23·76 and check your numerical work.

34. How would you ascertain whether a dumpy level was in permanent adjustment ; if found to be incorrect, how would you adjust ?

35. What is the approximate error in 700' due to earth's curvature in levelling operations ? Would this error as read on the staff, raise or lower the true height of the point ?

36. The bubble-tube of a Dumpy level is found to be in adjustment. How would you proceed to ascertain whether the line of sight was horizontal ?

37. You are given a 6' Ordnance map with contours of 100' vertical interval. How would you interpolate contours at 25' interval, using a clinometer ?

38. Two lines meeting at an angle of 80° are to be joined by a 4° curve. A, the starting-point of the curve is at chainage 10,560'. You are pegging out the curve at 100' intervals. Setting up the theodolite at A, you peg out the curve to chainage 10,800', and then find that an obstacle prevents you from seeing any further. Explain how you would peg out the remainder of the curve.

39. A look-out on a ship is 20' above the water and the light of a light-house 120' high is just visible on the horizon. What is the distance from the ship to the light house.

40. From two points on a level area and in the same vertical plane with a hill top, the angles of elevation to the top are determined as $39^\circ 37'$ and $27^\circ 40'$. The distance between the two points is 3810'4". Find the elevation of the top above the points.

41. The line connecting two points A and B is 500' long, and at an inclination of 30° to the horizontal. The angles of elevation to a point in the same vertical plane with A and B are 60° and $49^\circ 19'$. Find the elevation of the point above A

42. Three points x, y, z are in the same horizontal line, the distances xy and yz being respectively equal to m and n . The angles of elevation to the top of a tower are X, Y, Z . Find the elevations of the top of the tower above the points.

43. You have to make a longitudinal section along a centre line which has been pegged out in the field. What instruments, equipment and labour would you take into the field with you ?

44. How would you determine the instrument constant and focal correction of a telescope fitted with stadia lines ?

45. What are the practical uses of a Contour survey. Describe the different methods of carrying it out

46. Describe the permanent adjustments of a Dumpy level. Explain particularly the effect of moving the various screws used in the adjustments.

47. What are the corrections to be introduced in measuring the height of a distant peak by Trigonometrical survey ? Explain fully the object of introducing these corrections.

48. What is the function of a telescope in surveying and levelling ? Explain fully the object of having a bubble-tube attached to it.

49. The R. Ls. of the centre line of an embankment taken 100' apart are ; 108.87, 102.52, 103.81, 105.97, 107.02, 109.13, 110.28, 112.00, 113.21, 115.83, 116.48, 117.83, 118.29, 120.81. Make out the field book to correspond to the above readings, and plot the centre line to a scale in use in the Irrigation Department.

50. Calculate the tangent length, secant length and the length of curve whose radius is 4000' and angle of intersection 145° . If an obstacle renders the position of the 3rd and 4th pegs impossible. How will you lay out the 5th peg.

51. Describe the operation of tracing contour lines at 5 ft. difference interval along a hillside of moderate slope

52. What surveys are necessary for a complete Railway project.

53. How would you tell whether the horizontal wire of a Dumpy Level is in adjustment. If not, how would you correct the error.

Back sight.	Inter sight.	Fore sight.	Height of Instrument.	Reduced Level.	Distance.	REMARKS.
3.84				84.91	9	B.M. Mile stone
4.71	4.60	3.98			100	
	4.15				200	
	4.50				280	
		6.70			320	
					350	Cross road

54. Why ordinarily two different scales are used in Longitudinal sections of roads and canals, one for horizontal and the other for the vertical distance?

55. Fill up the following Level-book from the figure given, and draw the section to a suitable scale :

56. Make the necessary calculations and explain how a 4° curve is laid out ; angle of deflection = $28^{\circ} 36'$ (tan. $14^{\circ} 18' = 0.254908$). How would you find points at every 50' on the curve ?

57. Sketch and describe clinometer. How would you use it in laying out the formation level of a hill road.

58. What are the special points of gradient telemeter Level. Rule out a page of a field-book which you would use with it. Enter readings for 2 settings of the instrument and reduce.

59. A line is to be deflected to the right through an angle of 16° by a 2° curve. Points 400' apart are to be fixed by a theodolite and the intermediate points 100' apart by offsets from the chord. Explain how you set out the curve and give necessary calculations.

60. How would you make a Contour of a large tract of tolerably flat country for the purpose of preparing a drainage project.

61. Where are Indian Trigonometrical survey level datums on the coasts of India ?

62. The Chainage at the point of intersection of the tangents of a Railway curve is 3549 links and the angle between them is 124° . Find the chainage at the starting point of the curve, if it is 40 chains radius, and compute the angles required to set out the curve with a theodolite.

Personal Error in Levelling, etc.—Different persons do not see things exactly alike. Each person may have an inaccuracy peculiar to himself. One may take an observation different from another equal in skill. Also one's right and left eye may differ. This difference in persons is called their **personal equation**. To try the accuracy of the eye, turn the head sideways, so that the eyes are in the same vertical line, and sight to the staff held horizontally. Read where the vertical hair cuts the staff. Then turn the head the other way so as to reverse the position of the eyes, and sight again. The average of the two readings is correct.

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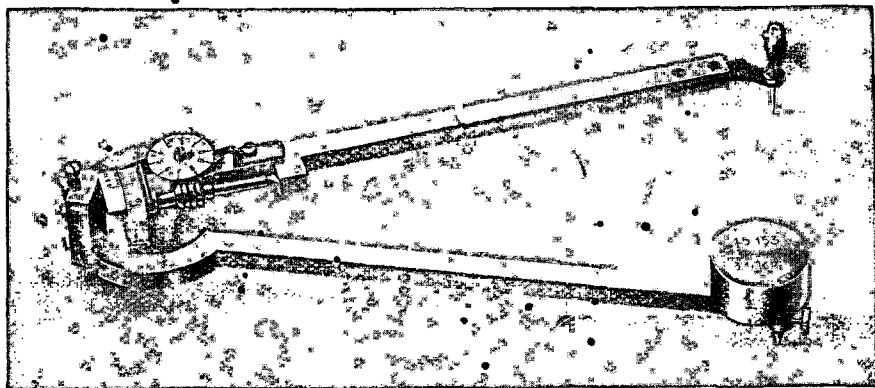
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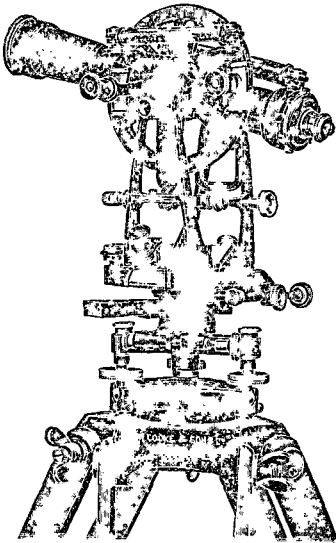
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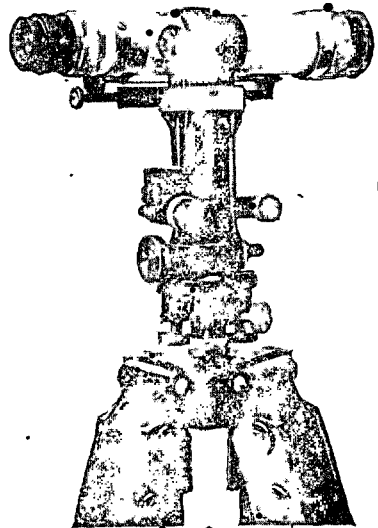
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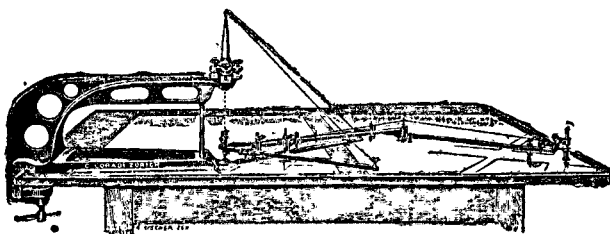
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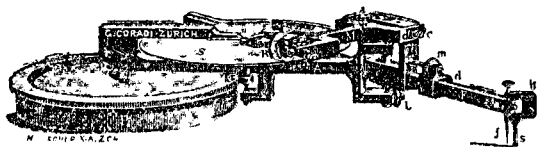
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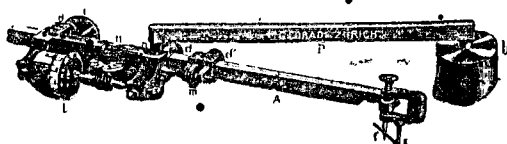
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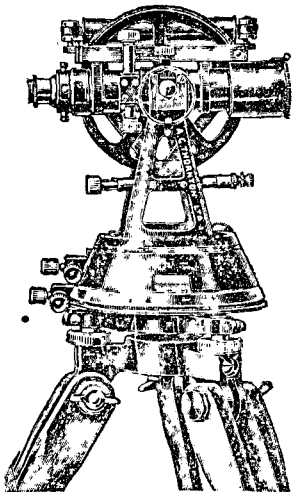


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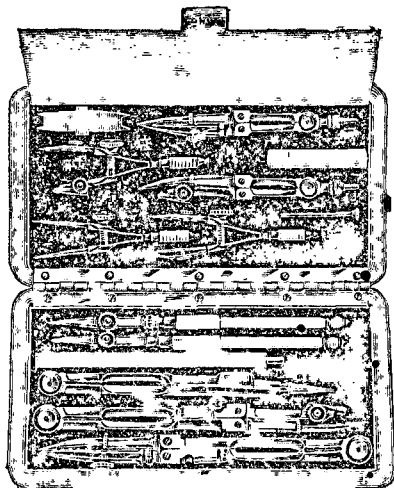
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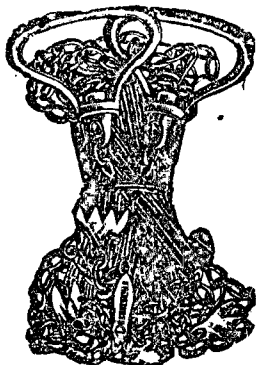
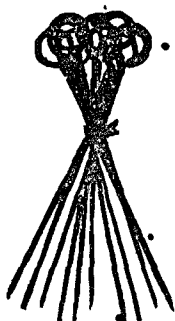
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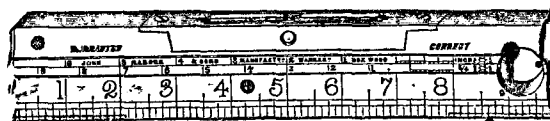
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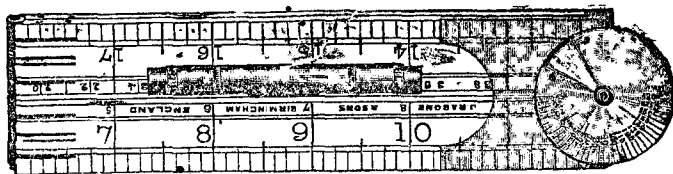
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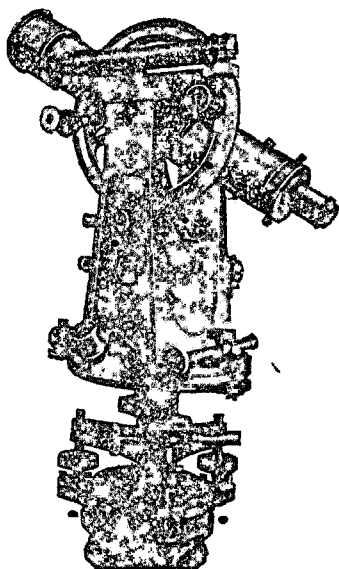
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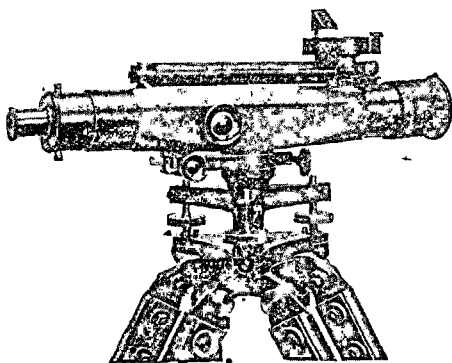
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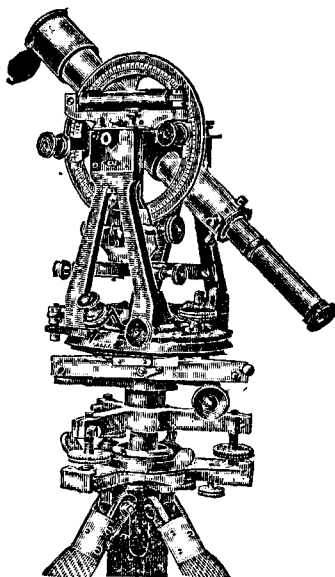
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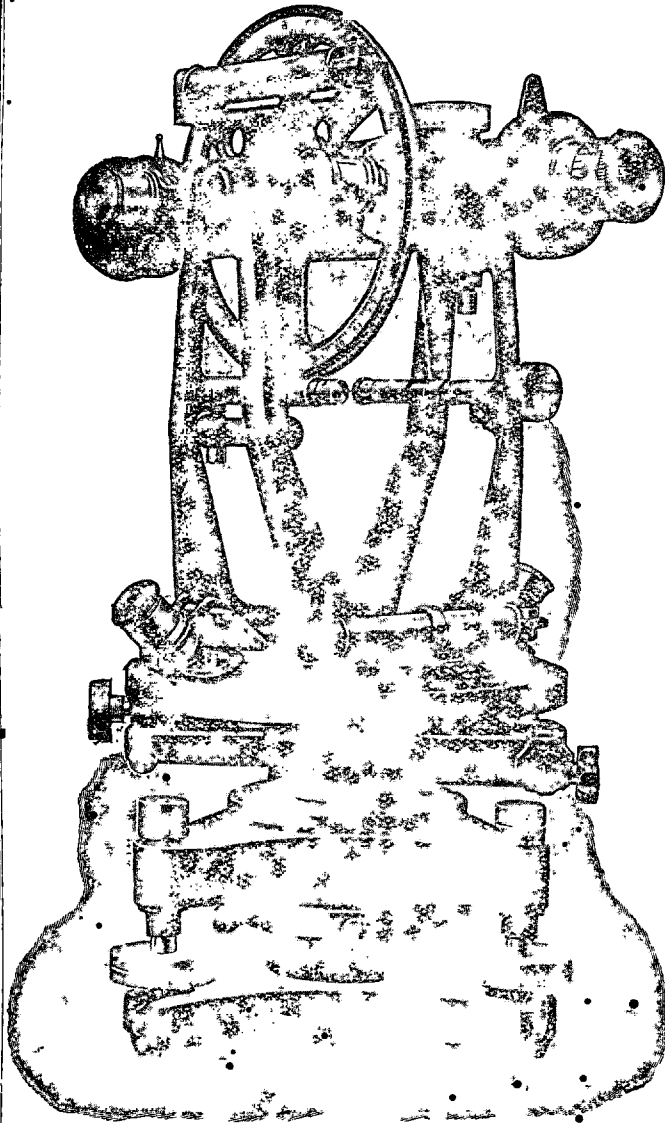
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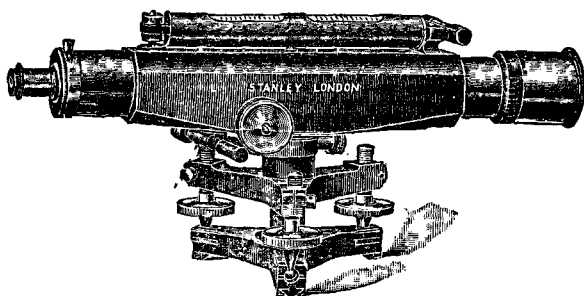
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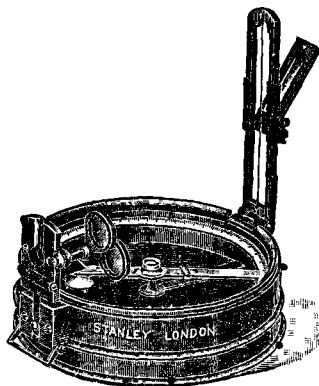
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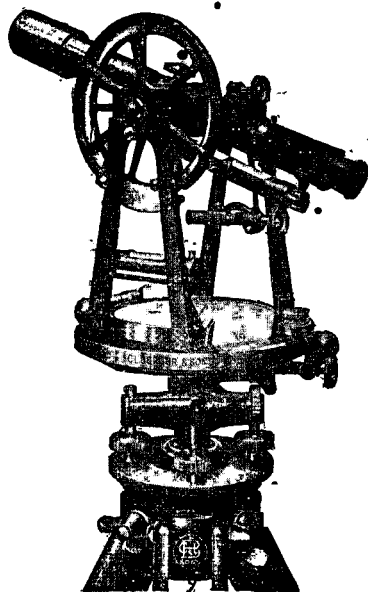


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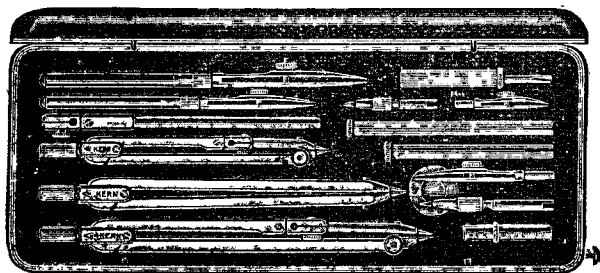
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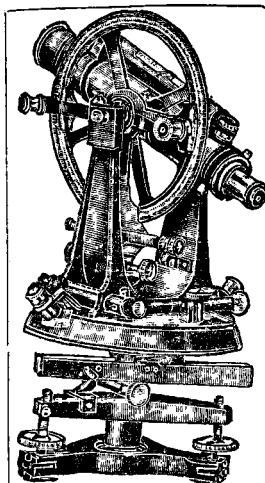
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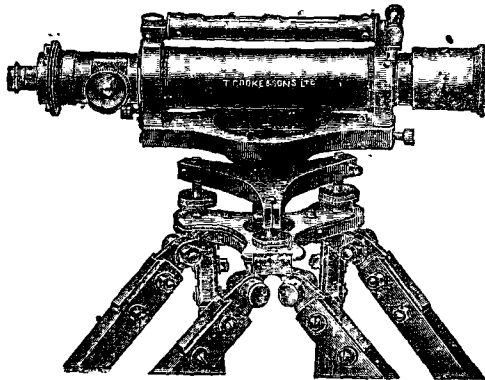
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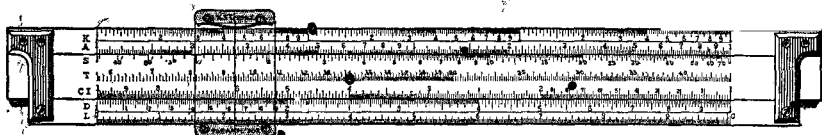
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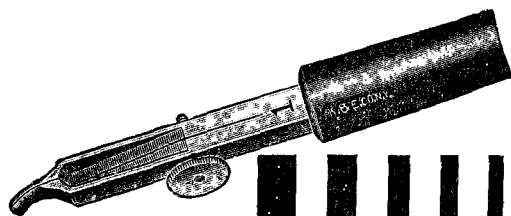
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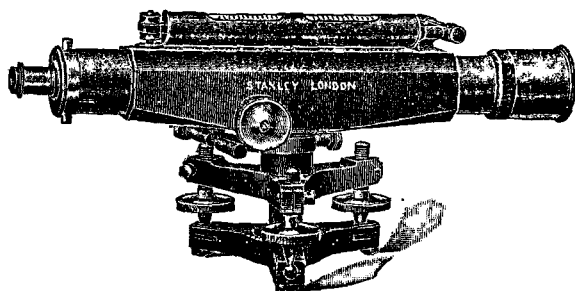
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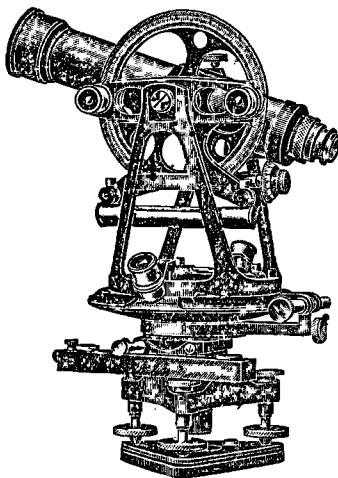
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